

## Investigating Hyperbolic Navigation-LORAN

After the second world war, the need for a Long Range Navigation system was addressed with the development of LORAN. A set of radio beacons was established throughout the world. A ship trying to determine its position could determine the difference in signal time between two fixed beacons. As you have seen, the locus of points with a constant difference in distance between two fixed points, is a hyperbola. Hence LORAN is referred to as a hyperbolic navigation system. Maps were produced with families of hyperbolas from different beacons. Seafarers located their position as the intersection of two specific hyperbolae.

Open the Geometer's Sketchpad file Map.gsp

The grey area represents land and the blue areas represent shallow and dangerous water.



Loran beacons are located at A B and C.

- ✓ Construct point  $S$  on the map (you will have to use the label tools to relabel the point)
- ✓ Measure the distances from  $S$  to  $A$  and  $S$  to  $B$ .
- ✓ Use the calculator to calculate the difference in the distances  $SA-SB$
- ✓ Select point  $S$  and choose Trace from the display menu.
- ✓ Drag point  $S$  so that the value of  $S$  remains as constant as possible (this may take some practice)



Reopen the map or undo all your work.

On this sketch, do the following

- ✓ Draw a line segment  $DE$  to represent the path of a ship
- ✓ Construct a point  $S$  on line  $DE$  to represent the location of a ship.
- ✓ Measure the distances  $SA$  and  $SB$
- ✓ Calculate the difference  $SA-SB$
- ✓ Can you adjust the location of the line  $DE$  so that as you drag the location of the ship the difference is always the same (hint: the difference will be 0)

Unfortunately, this path takes you through dangerous water.

Open the script 2fp\_hyp.gss which will construct a hyperbola given two foci and a point on the hyperbola

- ✓ *Place a point  $S$  on the sketch to represent the location of a ship*
- ✓ *Use the script tool `2fp_hpy.gss` to construct the family of hyperbolae with foci  $A$  and  $B$  and passing through  $S$*
- ✓ *Use the measure and calculate tools to calculate the difference  $SA-SB$*
- ✓ *Use the script to construct the family of hyperbolae with focus  $A$  and  $C$  and passing through  $S$*
- ✓ *Use the measure and calculate tools to calculate the difference  $SA-SC$*

Our loran has a device which will sound an alarm if specified conditions on the differences  $SA-SB$  and  $SB-SC$  are met.

**Describe a set of conditions which will set off the alarm when the ship passes through dangerous water (blue regions).**

**The conditions should be of the form**

**IF  $SA-SB < p$  AND  $SA-SC < q$  THEN ALARM.**

**You may use multiple statements to define the dangerous region.**

