Part II - Sample Examination Questions

The following questions serve as a "test bank" of samples of possible examination questions. In designing an examination, teachers will want to be sure that they have a balance between the strands of the curriculum as well as a balance between the types of questions and the Achievement Chart categories.

For each of the following questions, the curriculum expectations and suggested Achievement Chart category are given. Although communication is given as a category for some questions, some teachers find it more useful to mark the entire exam, in general, for communication using a rubric.

As well as listing expectations and Achievement Chart categories, some questions also include a guide to the solution of the question and possibly comments for marking the question.

Short Answer Questions

1. A student writes, "The inverse of y = $\sqrt{x-4}$ is y = $x^2 + 4$". Is this statement true? Explain why or why not.

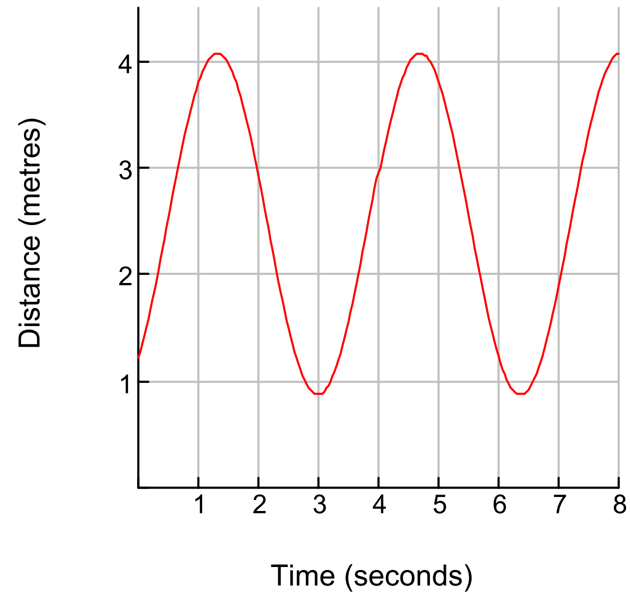
> **Curriculum Expectations:**
> **OCV.02**
> · demonstrate an understanding of inverses and transformations of functions and facility in the use of function notation;
> **Achievement Chart Category:** Knowledge, Communication

2. Use graphing technology to determine the values that make the following statement true. Verify your answer.

   $\cos^2 x - \sin^2 x = 2\cos^2 x - 1$

> **Curriculum Expectations:**
> **TF2.05**
> – prove simple identities, using the Pythagorean identity, $\sin^2 x + \cos^2 x = 1$, and the quotient relation, $\tan x = \frac{\sin x}{\cos x}$;
> **TF2.06**
> – solve linear and quadratic trigonometric equations (e.g., $6\cos^2 x - \sin x - 4 = 0$) on the interval
> $0 \le x \le 2\pi$;
> **TF2.07**
> – demonstrate facility in the use of radian measure in solving equations and in graphing.
>
> **Achievement Chart Category:** Thinking, Inquiry and Problem Solving

3. Tom used a CBR to collect motion data of his son Nick having a swing ride. Tom calculated an equation based on the data he collected. Below is a graph of the equation.



An equation of Nick's motion is y = 1.60061sin(1.87246x -0.904861)+2.47325.

Compare this equation to the general equation y = a sin(bx - c)+d.

Explain the significance of the numbers representing a, b, c, d in Nick's equation to this situation.

Curriculum Expectations:
TFV.02
· demonstrate an understanding of the meaning and application of radian measure;
TFV.03
· determine, through investigation, the relationships between the graphs and the equations of sinusoidal functions;
TFV.04
· solve problems involving models of sinusoidal functions drawn from a variety of applications.
Achievement Chart Categories: This question could be assessed for Application since students are being asked to go in and out of a context and to apply their knowledge of the parameters of a sinusoidal function.

Solution:

a = 1.60061 represents the amplitude of the graph or half the distance between the highest and lowest points on the graph.
This means that half the width of Nick's total swing (front to back) was 1.60061 m.
The total width of his swing was 3.20122 m.

b = 1.87246 represents 2B/(the period) of the graph. Therefore the period of the graph is about 3.35558 seconds. This means that it took Nick 3.35558 seconds to complete one front to back to front motion (one complete swing).
c = 0.904861 represents the phase shift times 1.87246. Therefore the phase shift is about 0.48525 seconds. This means that the CBR did not start tracking Nick at the very front of his swing. The CBR began tracking 0.48525 seconds into Nick's backward motion.
d = 2.47325 represents the vertical shift of the graph. This means that the closest that Nick got to the CBR was 2.47325 – 1.60061 = 0.87264 m. That is, the amount a standard sine graph was shifted up less the amplitude. The furthest that Nick got from the CBR was 2.47325 + 1.60061 = 4.07386 m.
Category:
I think this question could be assessed for Application since students are being asked to go in and out of a context.

4. Paul invests his money with his wealthy grandmother. He excitedly tells you that since his grandmother is going to double the interest rate, the value of his investment will double.
   a) Explain to Paul why his statement is wrong.

   b) If the interest rate doubles, what is one statement that Paul could truthfully make about the effect on the investment? Explain how you know this.

Curriculum Expectations:
Specific:
FA3.01
– analyse the effects of changing the conditions in long-term savings plans
OC3.01
– explain mathematical processes, methods of solution, and concepts clearly to others;

Achievement Chart Category:
Thinking, Inquiry, and Problem Solving, Communication
The question is open-ended and students have to develop a plan, select and sequence a variety of tools, and re-evaluate their strategies as they go along, Therefore the question is well suited to the TIPS category. It could also be assessed for communication.

5. Kepler's 3rd Law states that the squares of the periods of the planets are proportional to the cubes of their radius of revolution. (The period is the time it takes to complete one revolution of the Sun).
   The equation that states this relationship would be:
   $\frac{T_a^2}{T_b^2} = \frac{R_a^3}{R_b^3}$, where a and b are two different planets, T is the length of the period, and R is the radius of revolution.
   a. Rearrange this equation and give an equation for R_b without using fractions.
   b. The planet Venus rotates in orbit with a radius of approximately 1.08 x 10^8 km. It takes Venus 227 days to complete one revolution of the sun. It is

known that Mars takes 555 days to revolve around the sun. How far is Mars from the Sun?

Curriculum Expectations:
OC1.07
– simplify and evaluate expressions containing integer and rational exponents, using the laws of exponents;
Achievement Chart Categories: Application

6. Describe the transformations that would be applied to the graph of $y = 2x^2$ to obtain the graph of $y = 4x^2 + 3$.

Curriculum Expectations:
OC2.06
– represent transformations (e.g., translations, reflections, stretches) of the functions defined by
$f(x) = x$, $f(x) = x^2$, $f(x) = \sqrt{x}$, $f(x) = \sin x$, and $f(x) = \cos x$, using function notation;

Achievement Chart Category: Application

7. Is y=0.01x a good approximation of the trigonometric function $y = 0.1\sin(0.1x)$ for $-10\langle x\langle 10$? Justify your reasoning.

Curriculum Expectations:
OC3.05
– use graphing technology effectively (e.g., use appropriate menus and algorithms; set the graph window to display the appropriate section of a curve).
TF3.04
– sketch the graphs of simple sinusoidal functions
[e.g., $y = a \sin x$, $y = \cos kx$, $y = \sin(x + d)$, $y = a \cos kx + c$];
Achievement Chart Category: Thinking, Inquiry, and Problem Solving

8. Determine the roots of the equations and verify your solution graphically:

   a. $x^2 – 6x + 13 = 0$
   b. $6x^2 – x – 2 = 0$

Curriculum Expectations:
Specific Expectations
OC1.05
– determine the real or complex roots of quadratic equations, using an appropriate method (e.g., factoring, the quadratic formula, completing the square), and relate the roots to the *x*-intercepts of the graph of the corresponding function;
OC3.05
– use graphing technology effectively (e.g., use appropriate menus and algorithms; set the graph window to display the appropriate section of a curve).

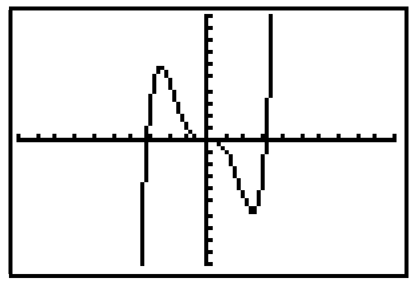
Achievement Chart Category: Knowledge and Understanding

9. Explain how to find both the graphical and algebraic representations of the inverse of: $y = -2x + 4$.

Curriculum Expectations: **OC2.04** – explain the relationship between a function and its inverse **OC3.01** – explain mathematical processes, methods of solution, and concepts clearly to others; **Achievement Chart Category:** Knowledge and Understanding, Communication

10. The following homework question was given to a grade 11 math class.

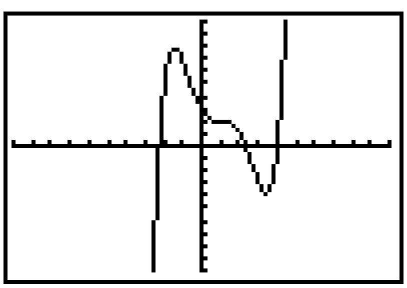
Given the graph of F(x)



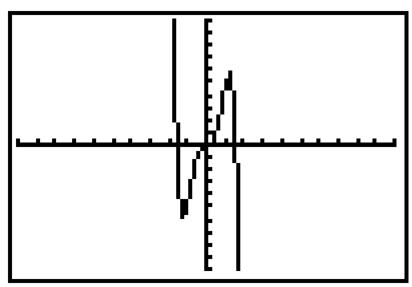
Sketch the graphs of:
a) F(-x) b) F(x - 1) + 2 c) -F(2x)

Jessica completed her homework at the library, but forgot to label her graphs. When she arrived at home she realized she had lost one of the graphs. The two remaining graphs are shown below.

i)

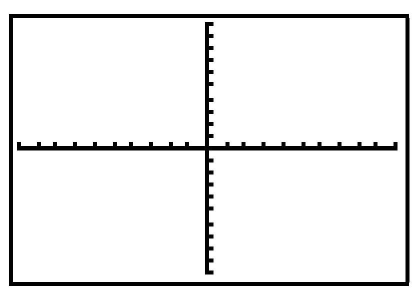


ii)



a) Which graphs are they?
b) Use the terminolgy of functions and transformations to explain how you are able to determine the equation of each graph.

c) Draw a sketch of the inverse of F(x).



d) Give the domain and range of the inverse of F(x).

Curriculum Expectations:
OC2.06
– represent transformations (e.g., translations, reflections, stretches) of the functions defined by
$f(x) = x$, $f(x) = x^2$, $f(x) = \sqrt{x}$, $f(x) = \sin x$, and $f(x) = \cos x$, using function notation;

OC2.07
– describe, by interpreting function notation, the relationship between the graph of a function and its image under one or more transformations;
OC2.04
– explain the relationship between a function and its inverse
Achievement Chart Category: Knowledge/Understanding and Communication

Extended Response Questions

7. Two companies opening for business in 2002 agree with their employees that they will contribute to local charities using formulas that will be renegotiated over time. Apex Industries agrees to use the formula $t_n = 10n + 500$ while Boomerang Inc. agrees to use $t_n = 10^n + 500$. In each sequence t_n refers to the amount of money, in dollars, contributed annually to local charities and n refers to number of years the company has been in business.

a) Explain the significance of the 500 in each formula.

b) Describe what happens to the charitable contribution for each additional year that Apex Industries is in business.

c) In order that Boomerang Inc is able to make the charitable donations that it has promised by the given formula, describe what the company's profits will need to be, year after year.

d) In what year(s) will Apex Industries and Boomerang Inc. report the same charitable contributions?

e) If maximum charitable donations for tax write-offs is set at $1 000 000, when will each company reach its maximum contribution using these formulas?

Curriculum Expectations:
Overall:
FAV.02
· solve problems involving compound interest and annuities;

OCV.01
· demonstrate facility in manipulating polynomials, rational expressions, and exponential

Specific:
FA2.04
– demonstrate an understanding of the relationships between simple interest, arithmetic sequences, and linear growth;
FA2.05
– demonstrate an understanding of the relationships between compound interest, geometric sequences, and exponential growth.
OC1.01
– solve first-degree inequalities and represent the solutions on number lines;

Achievement Chart Category:
Application. This problem is highly scaffolded, therefore it is not really problem solving because students are not expected to select a variety of tools and sequence them appropriately. Students are asked to select and fit a particular tool to each section of the problem.

7. Sandy receives an offer from her investment company to reinvest the funds she has in a program that is being phased out. Unfortunately she spills some calligraphy ink on the offer as shown to the right.

   Sandy assumes that One-In-A-Million will provide more interest, but must be a bit riskier, since it has a higher interest rate. Is Sandy correct in her assumption? Explain your reasoning.

Two Great Offers!

Limited time to make up your mind how you would like to invest

Golden Opportunity 10[redacted] pacsa

One-In-A-Million 10¼% pacq

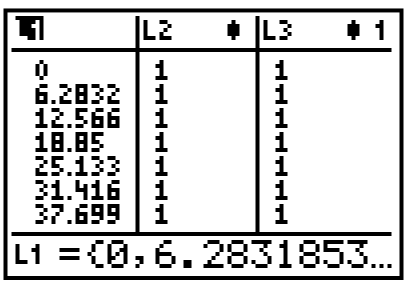
Contact your investment cou[redacted]ely to
[redacted]

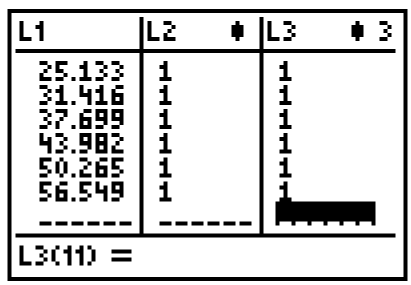
Curriculum Expectations:
FA3.01
– analyse the effects of changing the conditions in long-term savings plans (e.g., altering the frequency of deposits, the amount of deposit, the interest rate, the compounding period, or a combination of these) (Sample problem: Compare the results of making an annual deposit of $1000 to an RRSP, beginning at age 20, with the results of making an annual deposit of $3000, beginning at age 50);
FA3.05
– communicate the solutions to problems and the findings of investigations with clarity and justification.
FA2.02
– solve problems involving compound interest and present value;
Achievement Chart Category: Thinking, Inquiry, and Problem Solving.

4. Mrs. Ingalls' test contained a question that asked students to decide whether or not $\cos\beta = \sqrt{1-\sin^2\beta}$ is an identity. She starts grading the assignments and sees very different solutions in the first 4 papers she marks.

a) Which student(s) is/are correct?
b) Write notes to each student who has an error, to help them see their error in logic and improve their understanding of the situation. Provide comments that are appropriate to the type of model the student used. You may write on the students' solutions, much like Mrs. Ingalls would.

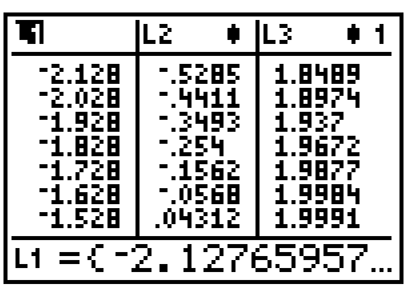
Student A's Solution
I created numerical models for the Left Side and Right Side of the equation using a TI83+ calculator.
I entered seq(x,x,0,20π , 2π) into L1.
I entered "cos(L1)" into L2
I entered "$\sqrt{1-(\sin(L1))^2}$ " into L3.
These are the top and bottom of the list screens that I saw:

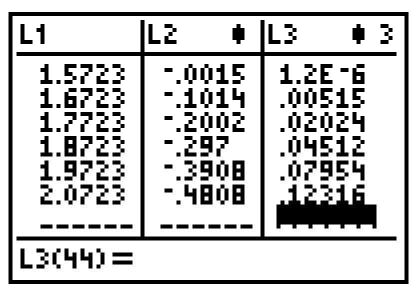




It looks like an identity, but I know that the cosine function is not constant, so I will change L1 to get a more detailed set of values for $\cos\beta$ and $\sqrt{1-\sin^2\beta}$.

I changed L1 to seq(x,x, $-\frac{\pi}{2}$, $\frac{\pi}{2}$,0.1)
These are the top and bottom of the list screens that I saw:





Since all the values in L2 and L3 are identical, no matter whether I take large increments of 2π or small increments of 0.1, I conclude that the trig functions they model are identical for all β values. Therefore $\cos\beta = \sqrt{1-\sin^2\beta}$ is an identity.

Student B's Solution:

$If \ \cos\beta = \sqrt{1-\sin^2\beta}\,, then$

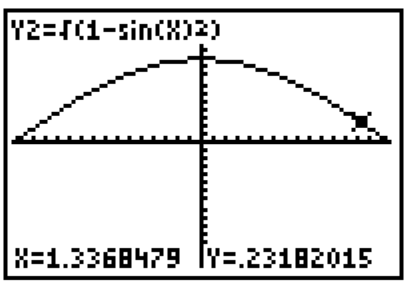
$\cos^2\beta = 1-\sin^2\beta$ by squaring both sides

$\therefore \cos^2\beta = \cos^2\beta$ using the Pythagorean Identity $\sin^2\beta + \cos^2\beta = 1$

$LS = RS$

$\therefore \cos^2\beta = \sqrt{1-\sin^2}\beta \ is\ an\ identity$

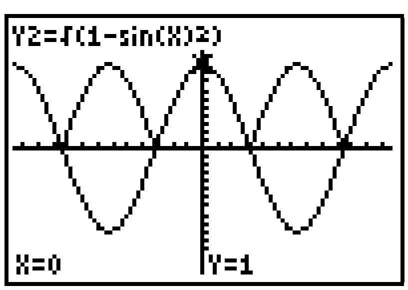
Student C's Solution:
I made the left side of the equation into a function $y_1 = \cos x$. I made the right side of the equation into a function $y_2 = \sqrt{1-(\sin x)^2}$. I compared the graphs of these functions with my TI83+ calculator. I set the window to graph from $-\frac{\pi}{2}$ to $\frac{\pi}{2}$ for x and from –1.5 to 1.5 for y.
As I watched the graphs appear on screen, I saw that $y_2 = \sqrt{1-(\sin x)^2}$ fell exactly on top of the graph of $y_1 = \cos x$. My screen looked like:



WINDOW
Xmin=-1.570796…
Xmax=1.5707963…
Xscl=.5
Ymin=-1.5
Ymax=1.5
Yscl=.1
Xres=1

Since the graphs of the LS and the RS are identical, I conclude that $\cos\beta = \sqrt{1-\sin^2\beta}$ is an identity.

Student D's Solution:
I made the left side of the equation into a function $y_1 = \cos x$. I made the right side of the equation into a function $y_2 = \sqrt{1-(\sin x)^2}$. I compared the graphs of these functions with my TI83+ calculator. I set the window to graph from $-2\pi \ to\ 2\pi$ for x and –1.5 to 1.5 for y. My screen looked like:



WINDOW
Xmin=-6.283185…
Xmax=6.2831853…
Xscl=.5
Ymin=-1.5
Ymax=1.5
Yscl=.1
Xres=1

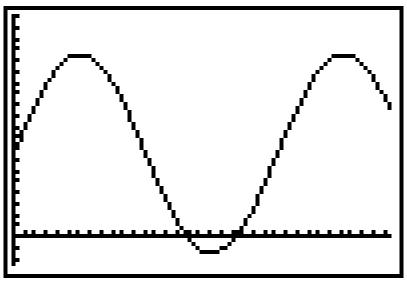
Even though the graphs of $y_1 = \cos x$ and $y_2 = \sqrt{1-(\sin x)^2}$ are identical in some parts of the domain, they are not identical for the entire domain. Therefore I conclude that $\cos\beta = \sqrt{1-\sin^2\beta}$ is not an identity, rather a trigonometric equation that has solution $-\frac{\pi}{2} + 2k\pi \le \beta \le \frac{\pi}{2} + 2k\pi$, $k \in I$.

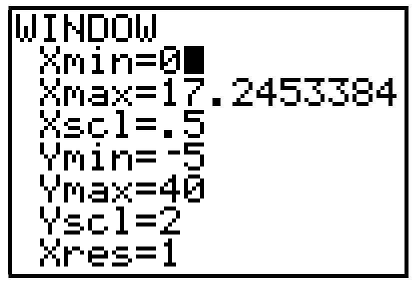
Expectations Addressed:
OC3.01
– explain mathematical processes, methods of solution, and concepts clearly to others;
TF4.03
– predict the effects on the mathematical model of an application involving sinusoidal functions when the conditions in the application are varied;
TF4.04
– pose and solve problems related to models of sinusoidal functions drawn from a variety of applications, and communicate the solutions with clarity and justification, using appropriate mathematical forms.
OC3.05
– use graphing technology effectively (e.g., use appropriate menus and algorithms; set the graph window to display the appropriate section of a curve).
Achievement Chart Categories: Knowledge/Understanding, Thinking, Inquiry and Problem Solving, and Communication
Solution Guidelines:
a) Students should be able to identify that student D's answer is the only correct one.
b) The solution could be scored with a marking scheme or this rubric could be used for assessing the solution. K/U: Level ____; TIPS: Level ____; C: Level ____
Note: Teachers may choose to only include 2 or 3 student solutions, rather than all 4.

Criteria	Level 1	Level 2	Level 3	Level 4
Knowledge/Understanding				
Understanding of the concept of a trigonometric equation as opposed to a trigonometric identity	Shows limited understanding of the required concept	Shows some understanding of the required concept	Shows considerable understanding of the required concept	Shows thorough understanding of the required concept, along with a broader view of the application of the concept
Thinking, Inquiry, and Problem Solving				
Reasoning [i.e., following an argument, judging the validity of an argument, making an argument]	Demonstrates reasoning that is limited	Demonstrates reasoning that is somewhat limited	Demonstrates reasoning that is logical	Demonstrates reasoning that is logical and adds to the argument given or places provisos on the argument
Making inferences, conclusions and justifications that connect to the problem solving process and models presented	Presents justification of the answer that has a limited connection to the problem solving process and models presented	Presents justification of the answer that has some connection to the problem solving process and models presented	Presents justification of the answer that has a direct connection to the problem solving process and models presented	Presents justification of the answer that has a direct connection to the problem solving process and models presented, with evidence of reflection/insight
Communication				
Using Appropriate mathematical vocabulary	Sometimes uses mathematical vocabulary correctly when expected	Usually uses mathematical vocabulary correctly when expected	Consistently uses mathematical vocabulary correctly when expected	Consistently uses mathematical vocabulary correctly, recognizing novel opportunities for its

				use
Integrating narrative and mathematical forms of communication	Provides either mathematical or narrative form, but not both	Provides both mathematical and narrative, but the forms are not integrated	Provides both mathematical and narrative forms integrates them	Provides a variety of mathematical forms and narrative, integrated and well chosen
Clarity in explanations and justifications in reporting	Provides explanations and justifications that have limited clarity	Provides explanations and justifications that have some clarity	Provides explanations and justifications that are clear for a range of audiences	Provides explanations and justifications that are particularly clear and detailed

5. The following graph has equation $y_1 = 18\sin(0.52x) + 15$, where x is in months, and represents, fairly accurately, the average high temperature in degree Celsius for Snowbell, Ontario, over 3 years in the 1700’s.



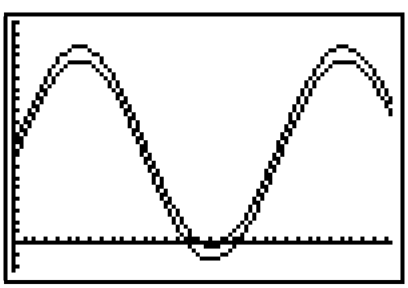


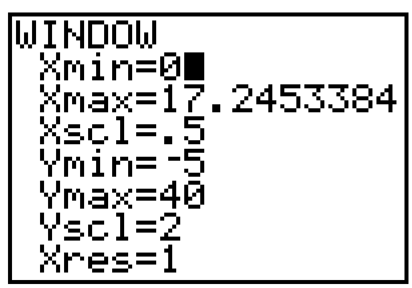
a) What would be the effect on the graph of changing the 18 to 15?

b) What would be the effect on the description of the average high temperatures of changing the 18 to 15?

c) What is the significance of 0.52 in this context?

d) The average high temperature in degree Celsius for Snowbell is measured again in 1997-2000 and displayed along with the graph from the 1700’s, as shown below.





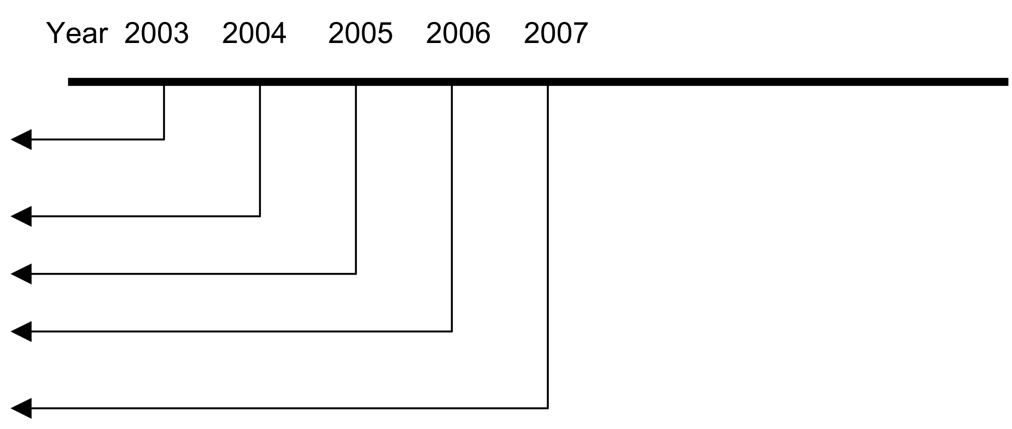
What is the equation of the new graph? Explain what has happened to the average high temperature in degree Celsius for Snowbell between the 1700’s and the end of the 1900’s.

e) Snowbell has a twin city, the same distance from the equator but in the southern hemisphere, where the seasons are the reverse of ours in Ontario –winter there is summer here and vice versa. What hypothesis could you make about the graph of the average high temperature in degree Celsius for Snowbell's twin city? Explain your reasoning.

Curriculum Expectations:
TF3.02
– determine, through investigation, using graphing calculators or graphing software, the effect of simple transformations (e.g., translations, reflections, stretches) on the graphs and equations
of $y = \sin x$ and $y = \cos x$;
TF3.05
– write the equation of a sinusoidal function, given its graph and given its properties;
TF4.03
– predict the effects on the mathematical model of an application involving sinusoidal functions when the conditions in the application are varied;
TF4.04
– pose and solve problems related to models of sinusoidal functions drawn from a variety of applications, and communicate the solutions with clarity and justification, using appropriate mathematical forms.
TF2.03
– represent, in applications, radian measure in exact form as an expression involving □ (e.g., $\frac{\pi}{3}$, 2□) and in approximate form as a real number (e.g., 1.05);

Achievement Chart Category: Application, Communication

6. Karen's mother is planning an investment for her. The timeline she begins to develop looks like the following.



a) Describe the investment plan Karen's mother is developing. Include as many of the financial terms from this unit of study as is appropriate to giving a full description.

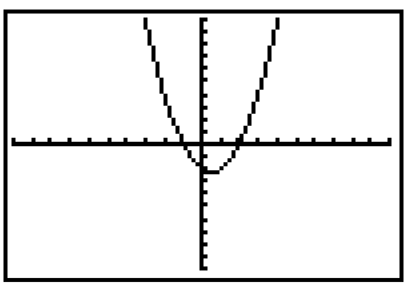
b) What would be the 10^{th} term of the sequence of terms shown on the left?

c) What formula would you develop for finding the sum of the first 10 terms from this timeline?

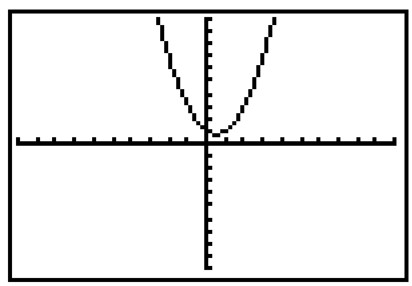
Curriculum Expectations:
FA1.02
– determine a formula for the nth term of a given sequence (e.g., the *n*th term of the sequence
$\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \ldots$ is $\frac{n}{n+1}$);
FA1.05
– determine the sum of the terms of an arithmetic or a geometric series, using appropriate formulas and techniques.
Achievement Chart Categories: Application

7. The equations of the following functions have integral coefficients between –5 and 5.

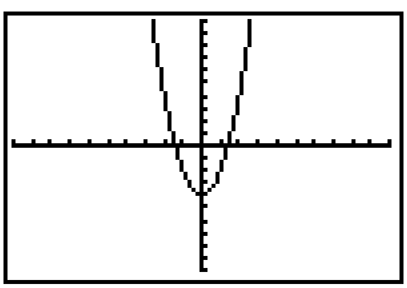
   a) Hypothesize the equations that correspond to each graph, explaining your reasoning.
   b) How would you test your hypotheses?



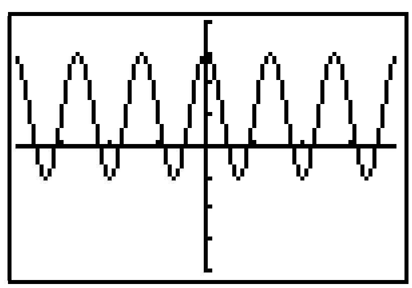
A



B



C



D

Curriculum Expectations:
OC1.05
– determine the real or complex roots of quadratic equations, using an appropriate method (e.g., factoring, the quadratic formula, completing the square), and relate the roots to the *x*-intercepts of the graph of the corresponding function;

OC3.05
– use graphing technology effectively (e.g., use appropriate menus and algorithms; set the graph window to display the appropriate section of a curve).
TF3.02
– determine, through investigation, using graphing calculators or graphing software, the effect of simple transformations (e.g., translations, reflections, stretches) on the graphs and equations
of $y = \sin x$ and $y = \cos x$;
TF3.04
– sketch the graphs of simple sinusoidal functions [e.g., $y = a \sin x$, $y = \cos kx$, $y = \sin(x + d)$, $y = a \cos kx + c$];

Achievement Chart Categories: Application, Communication
Solution Guidelines: The actual equations of the given graphs are:
A y=(x-2)(x+1), B y=(x-2)(x+1)+3, C y=2(x^2-2) , D y=2cos3x+1