

## Grade 11 U/C Summative Assessment Unit

**Activity:** Review Stations

**Day:** 4 and 5

**Purpose of Activity:** To review important concepts and make connections between various parts of the course.

### **Overall Expectations Addressed:**

#### **Station 1 – Graphing Trigonometric Relations**

##### **TFV.03**

- determine, through investigation, the relationships between the graphs and the equations of sinusoidal functions;

##### **TFV.04**

- solve problems involving models of sinusoidal functions drawn from a variety of applications.

#### **Station 2 – Solving Equations Using Graphs and Algebra**

##### **TFV.02**

- demonstrate an understanding of the meaning and application of radian measure;

##### **OCV.01**

- demonstrate facility in manipulating polynomials, rational expressions, and exponential expressions;

#### **Station 3 – Match My Graph**

##### **OCV.02**

- demonstrate an understanding of inverses and transformations of functions and facility in the use of function notation;

#### **Station 4 – Connecting Functions and Sequences**

##### **FAV.01**

- solve problems involving arithmetic and geometric sequences and series;

##### **FAV.02**

- solve problems involving compound interest and annuities;

##### **OCV.02**

- demonstrate an understanding of inverses and transformations of functions and **facility in the use of function notation;**

#### **Station 5 – Bacteria Counting**

##### **OCV.01**

- demonstrate facility in manipulating polynomials, rational expressions, and exponential expressions;

##### **FAV.02**

- solve problems involving compound interest and annuities;

#### **All stations:**

##### **OCV.03**

- communicate mathematical reasoning with precision and clarity throughout the course.

**Activity Description:**

**Station 1 – Graphing Trigonometric Relations** – Students use Geometer’s Sketchpad to generate data of a trigonometric relation. They are then to determine both graphical and algebraic models for the data.

**Station 2 – Solving Equations using Graphs and Algebra** Students graph both sides of an equation and then find the intersection points. This is followed by an algebraic solution to verify their graphical solution.

**Station 3 – Match my Graph** -This activity requires students to use their knowledge of basic graphs and transformations to match graphing calculator screen images to an equation. This activity is to be done without the aid of a graphing calculator.

**Station 4 – Connecting Functions and Sequences** – Students will be presented with scenarios of both discrete and continuous data leading to a discussion comparing sequences and functions. The scenarios presented will reinforce connections between simple interest, arithmetic sequences, and linear functions as well as connections between compound interest, geometric sequences, and exponential functions.

**Station 5 – Bacteria Counting** – Students will be presented with a scenario involving exponential growth and they have to create an equation that models the situation. They also have to use this model to make predictions.

**Management Suggestions:**

These stations should be done in groups. Each station probably takes about 20 minutes. Therefore students should work through 3 stations on the first day and 2 on the next. This will allow time on the second day for a whole class discussion of various solutions.

The class could be divided into 6 groups and 2 copies of Stations 1 – 3 could be set up in the class. Each of the 6 group then moves through the 3 stations. The following day, 3 sets of Stations 5 and 6 could be set up for the 6 groups.

Geometer’s Sketchpad is required for Station 1. If computers are scarce, then 1 computer per group could be set up (2 computers in total). Students will need access to graphing technology for a variety of the stations.

**Assessment:** No formal assessment is suggested. Students may be using informal self-assessment to diagnose areas where they need to do more work to prepare for further summative assessment activities.

### ***Station 1 – Graphing Trigonometric Relations***

Using Geometer's Sketchpad, create  $\triangle ABC$  with a right angle at B. Fix the length of BC and make sure that it doesn't change as you do the following constructions:

- ◆ Record the value of  $\angle A$  and the length of AB. Move point A so that  $\angle B$  remains at  $90^\circ$  and the length of BC stays at fixed
- ◆ Record the value of  $\angle A$  and the length of AB.
- ◆ Continue to move point A in the same way, taking and recording measurements.
- ◆ Collect at least 10 different sets of measurements

Graph your data using  $\angle A$  as the independent variable and AB as your dependent variable. Describe your graph.

Using what you know about trig and the method of constructing and recording your data, can you determine an equation for your graph?

Describe all of the features of this graph.

## ***Station 2 – Mixture of equations***

Solve each of the following equations. However, start with a graph to help you visualize the possible solutions. Using graphing technology, graph the left side of the equation, and the right side of the equation as 2 separate functions. See if you can determine the intersection points and then use an algebraic method to verify your graphical result.

1.  $6x^2 = 7x - 2$

2.  $3x^2 + 4x + 1 = 0$

3.  $2\cos^2\theta - 1 = -\cos\theta$

4.  $2^{2x} = 12 + 2^x$

5.  $\cos\theta + 13 = 15\sin^2\theta$  , find  $\theta$  to the nearest hundredth of a radian,  $0 \leq \theta \leq 2\pi$  .

### Station 3 - Match my Graph

Several graphs and equations are shown below. The scales are the same on all graphs.

a. Match the graphs to the equations and explain your reasoning in each case.

b. State the domain and range of each relation.

This question is to be done without the aid of graphing technology. Once you have completed the activity, you may want to check your solution with the graphing technology.

1.  $y = 2(x + 2)^2 - 3$

2.  $y = 2^x - 2$

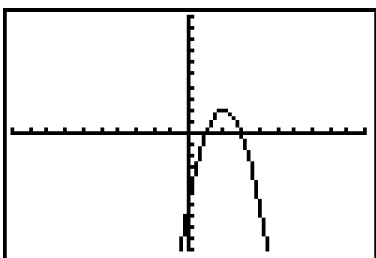
3.  $y = \frac{2}{x}$

4.  $y = -2\sqrt{x+1} - 2$

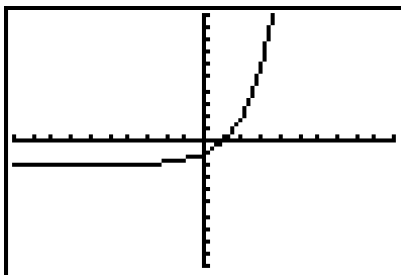
5.  $y = -2(x - 2)^2 + 2$

6.  $y = 3\sqrt{x-2}$

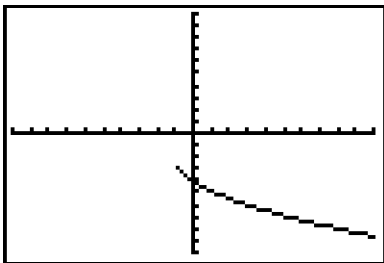
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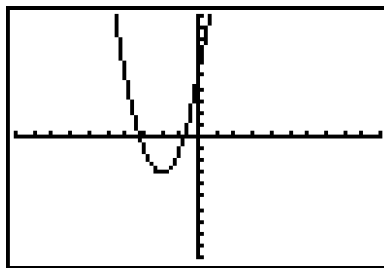
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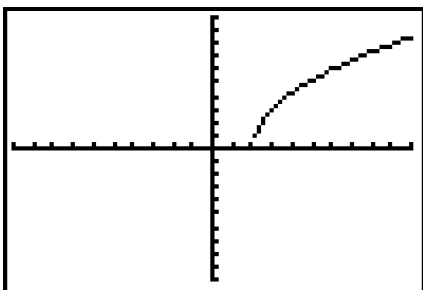
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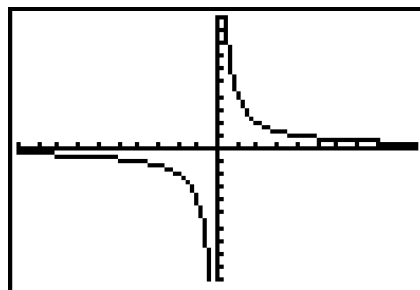
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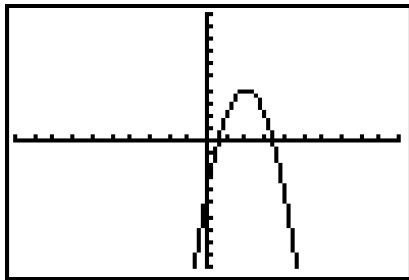
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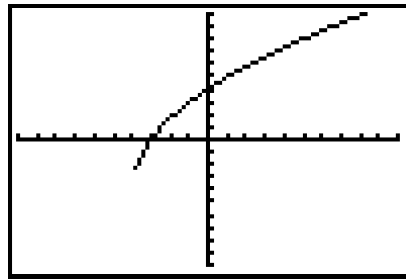
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- c. Create equations for each of the following graphs and explain how you determined the equations.



G



H

### Station 4 – Connecting Functions and Sequences

Christopher's grandmother gives him \$1 000 for his 16th birthday to invest in an account of his choice. He decides to invest it in an account that pays 8%/a compounded annually:

- (i) Determine the amount in the account at the end of each year for 6 years.
- (ii) Express these amounts as the terms of a sequence.
- (iii) What type of sequence is this? Explain how you know.
- (iv) Determine an expression for  $t_n$ .
- (v) Enter this data into a table and plot the graph. Remember that this is discrete data. Why?
- (vi) If we connected the discrete points with a curve of best fit, describe the information that the new points represent.
- (vii) How much money will Christopher have after 2.5 years? How do you know?
- (viii) We can now think of our curve as a continuous function,  $f(x)$ . Determine an equation for this function.

You have used  $t_n$  notation to indicate the term of a sequence. Sequence formulas generate **discrete points**.

Sequences and their terms are identified with subscripts, such as,  $t_n = 2n+1$ .

This can also be written using function notation as  $f(x) = 2x+1$ . as long as the relation can be continuous

Complete the following :

$t_1 =$	$\Rightarrow$	$f(1) =$
$t_2 =$	$\Rightarrow$	$f(2) =$
$t_{10} =$	$\Rightarrow$	$f(10) =$

If you want to show what would happen for real-number values of the independent variable, such as 2.5, then the graph will need to show that the points are connected.

Write an equation in the form  $y = \text{some expression in } x$  for each situation described below. Use your graphing calculator to **plot** the points for the relation and then enter the equation into the  $y=$  position to draw the graph that contains the points. The graph will be a smooth, unbroken (*continuous*) curve indicating that  $x$  can be any real number. Comment on the appropriateness of connecting the points. (i.e. in the given context should the graphs be discrete or continuous?)

- A. In a small lecture hall there are 10 rows of seats. For the first few rows the number of seats are 10, 16, 22, 28, ....

Set up a table where  $x$  represents the row and  $y$  represents the number of seats.

$x$	1	2	3	4	...	10
$y$	10	16	22	28	...	

- B. Dan drops a rubber ball from a height of 100m. After each bounce, the height of the ball's bounce is  $\frac{3}{4}$  the height of the ball's previous bounce.

Set up a table where  $x$  represents the bounce and  $y$  represents the height.

$x$	1	2	3	4	...	10
$y$	100				...	

### **Comparing Compound & Simple Interest :**

Now, back to Christopher:

Recall that Christopher decided to invest at 8%/a compounded annually. He did have another choice. Another account was available that pays 8%/a, simple interest.

The formula for simple interest is :  $A=P(1 + rt)$ , A = Accumulated amount, P = Principal invested, r = annual interest rate, t= time(years).

- (i) If he had used the simple interest account, what amount would be in the account at the end of each year for 6 years. What type of sequence is this?
- (ii) Enter this data into a table. Sketch the graph of this relation. Can you consider the relation to be discrete or continuous? Why?
- (iii) Determine an equation for the relation. What type of relation is this?
- (iv) Compare the two investments at the end of 6 years, 25 years.

Chris wants to purchase a computer so he decides to remove his money from the account that pays 8%/a, compounded annually exactly three and one-half years after opening the account.

- (i) How much is in the account?
- (ii) Since this is between compound periods, the bank pays compound interest for the first 3 years but only pays simple interest for the amount after the third year. How much will Chris have available to buy his computer?
- (iii) What is the difference between this and the amount he would have had if the bank used compound interest for the full  $3\frac{1}{2}$  years?
- (iv) Show what the graphs would look like for compound and simple interest between the third and fourth year.

Summarize the connections that you can make between sequences and relations.



### ***Station 5 – Bacteria Counting***

The cells of a certain strand of bacteria divide every hour producing two new cells. Two cells from this strand are placed in a large Petri dish with an unlimited food supply and allowed to reproduce. One of the cells is treated with a steroid that allows it to reproduce in half of the time. The other cell is left untreated. The two cells are placed in the dish at the same time.

A researcher is interested in knowing how many hours it will take for the number of bacteria cells in the Petri dish to reach 4160.

1. Determine an equation to model the combined growth of the bacteria and justify your equation.
2. Use your equation to determine the number of hours it takes to reach 4160 bacteria.
3. Approximately, how long will it take for the number of bacteria cells to reach one million?
4. How would your answers have been different if both of the original cells were treated with the same steroid?

