Part I – Instructional/Assessment Tasks
Grade 11 U/C Summative Assessment Unit

Activity: Extreme Ballooning **Day: 1 & 2**

Purpose of Activity: Instructional

Overall Expectations Addressed:

OCV.01
· demonstrate facility in manipulating polynomials, rational expressions, and exponential expressions;
OCV.02
· demonstrate an understanding of inverses and transformations of functions and facility in the use of function notation;
OCV.03
· communicate mathematical reasoning with precision and clarity throughout the course.

Activity Description:

- Students are presented with a variety of data within the context of Hot Air Ballooning.
- They are required to choose from the mathematical functions presented in this course to model this data in order to solve problems.

Management Suggestions*:*

- Students work in groups of 3 or 4
- Suggested structure for the two days:
  whole class discussion – introduce activity and brainstorm strategies
  groups work together
  whole class consolidation at various teachable moments mixed with continued group work for example ask leading questions such as "What methods did your group try?", "What type of model did you find fits best?", "How do you know if your model is a good fit?",
  For closure, have groups present different parts of the activities to other groups (whole class or groups with groups in the case of large classes – jigsaw may be a consideration)

Assessment:
Although this activity is instructional, this is an opportunity to assess students' ability to communicate mathematical reasoning with precision and clarity throughout the course. (Overall Expectation)
Specific Expectations:

OC3.01 – explain mathematical processes, methods of solution, and concepts clearly to others;
OC3.02 **–** present problems and their solutions to a group, and answer questions about the problems and the solutions;
OC3.03 – communicate solutions to problems and to findings of investigations clearly and concisely, orally and in writing, using an effective integration of essay and mathematical forms;
OC3.04 – demonstrate the correct use of mathematical language, symbols, visuals (e.g., diagrams, graphs), and conventions;
OC3.05 – use graphing technology effectively (e.g., use appropriate menus and algorithms; set the graph window to display the appropriate section of a curve).

"Check-bric" to Assess Communication included

Eric's Extreme Ballooning (Student Activity Sheet)

You have been offered a summer job with a recreational hot air ballooning company. The company is a seasonal business that takes groups of tourists on scenic balloon rides over the city. Usually the tours go up 1000 m, then cross the city and descend. To promote themselves, the owners of the company are planning a record 10 km climb at the end of the month, and have hired you to analyze some data and prepare a report for them about what conditions they should expect. Up to now the highest they have climbed is 6 km.

Part A: **What Should We Wear?**

Last week the owners took the balloon up to 6000 m and recorded the temperature every 500 m. The results are given below.

Altitude (in m)	Temperature (in ^{o}C)
50	14.9
500	11.8
1000	8.5
1500	5.3
2000	2.0
2500	-1.2
3000	-4.5
3500	-7.8
4000	-11.0
4500	-14.2
5000	-17.5
5500	-20.8
6000	-24.0

a) What possible mathematical models could fit this data, relating temperature and altitude?
b) Determine an equation of the curve of best fit. Justify your choice of curve of best fit.
c) Use your curve of best fit to predict the temperature at 10 km.
d) Advise the owners of how they should dress for their trip.

Part B: **How Far Will We be able to See?**

Assuming their eyes (and/or photographic equipment) are strong enough, the owners want to know how much of the region will they be able to see from their new height. The table below gives their viewing distances from various heights. (Viewing Distance is defined to be the distance to the furthest point that can be seen,)

Altitude (in m)	Viewing Distance (in km)
50	25
500	78
1000	111
1500	136
2000	157
2500	175
3000	192
3500	207
4000	221
4500	235
5000	247
5500	260
6000	271

a) What possible mathematical models could fit this data, relating *Viewing Distance* and *Altitude?*
b) Determine an equation of the curve of best fit. Justify your choice of curve of best fit.
c) Use your curve of best fit to determine how far the owners are able to see when they are 10 km above the ground.

Part C: **How Will We see our Home Base?**

"The Extreme Ballooning Company" offer tours at night. When ballooning in the dark, the operators shine a spotlight from the ground. This spotlight acts as a beacon to find their way back. The bulb used in the spotlight depends on the distance the balloon operator plans to travel from the spotlight.

The table below gives the owners suggestions on which bulbs to use for various trips.

Maximum Distance (in m)	Recommended Bulb (in Watts)
1800	60
2000	75
2300	100
2800	150
3300	200
5100	500
7300	1000

a) What possible mathematical models could fit this data, relating the wattage of the bulb to the distance from the spotlight?

b) Determine an equation of the curve of best fit. Justify your choice of curve of best fit.

c) Use your equation of the curve of best fit to determine the wattage of a spotlight the balloonists need if they were planning to return from their record climb of 10 km after dark. Assume the trip is straight up and then straight down.

d) If the spotlight uses a 2000 W bulb on the record 10 km climb, how far off course (horizontal distance) can the balloon drift and still see the light?

Part D: **How Thin is the Air Up There?**

Another factor the team must take into consideration is that air gets thinner as the balloon gets higher. They want to know exactly what the air pressure will be at 10 km.

The following table gives the air pressure at various altitudes.

Altitude (in m)	Air Pressure (in kilopascal)
50	101.0
500	95.4
1000	89.9
1500	84.6
2000	79.5
2500	74.6
3000	70.1
3500	65.8
4000	61.6
4500	57.7
5000	54.0
5500	50.5
6000	47.1

a) What possible mathematical models could fit this data, relating altitude to air pressure?
b) Find an equation of the curve of best fit for this relationship? Justify your choice of curve of best fit.
c) Use your equation of best fit to determine the air pressure at 10 km.

You find the following clipping from an old textbook. You have more data for this relationship.
When you graph the new data, along with the original data, the graph's shape reminds you of graphs of financial relationships dealing with compound interest, present value and depreciation. The exponential function modelled these situations.

Altitude (in m)	Pressure (in kPa)
25 000	2.5
30 000	1.2
35 000	0.56

d) Find a curve of best fit that incorporates the new data. Justify your choice of curve of best fit.
e) Using your new curve of best fit, determine the air pressure be at an altitude of 10 km.
f) Air pressure less than 55 kPa requires the use of oxygen masks in order that humans remain conscious. Use your curve of best fit to determine at what altitude the balloonists would need oxygen masks.

Part E: **What about our mascot "E*xtreme Eric"?***

The balloonists have a small inflatable Viking doll mascot (called *Extreme Eric*) made out of very lightweight and stretchable rubber. The owners hang the doll from their basket on each flight. The tourists enjoy the gimmick because, as the balloon rises, the air inside the doll expands and they can see *Eric* grow.

The table below gives the size of *Extreme Eric* at various different air pressures.

Air Pressure (in kilopascal)	Eric's Volume (in L)
101.0	40.8
95.4	42.8
89.9	45.0
84.6	47.3
79.5	49.7
74.7	52.3
70.1	55.0
65.8	57.9
61.6	61.0
57.7	64.4
54.0	67.9
50.5	71.7
47.1	75.8

a) How does air pressure affect the volume of the doll?
b) In previous mathematics courses, you may have described this relationship as having a negative correlation and modelled it using the equation of a line with a negative slope. Determine a **line** of best fit that would have a close fit to this data.
c) Since then, you have seen other relationships where one variable increases as the other decreases. Explore models to determine a **curve** of best fit. Justify your choice of **curve** of best fit.
d) Use your curve of best fit to determine the volume of the doll at 10 km. (See Part D for air pressure at 10 km.)

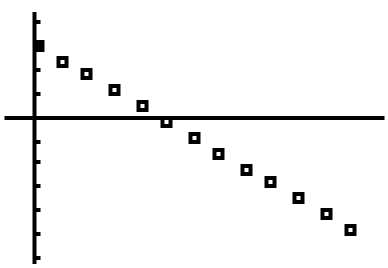
Eric's Extreme Ballooning – Solution

Data Table

Altitude (m)	Temperature (°)	Viewing Distance (km)	Air Pressure (kPa)	Inverse Pressure 1/(kPa)	Volume (L)	Maximum Height (m)	Watts
50	14.9	25	101	0.009900990099	40.8	1800	60
500	11.8	78	95.4	0.01048218029	42.8	2000	75
1000	8.5	111	89.9	0.01112347052	45	2300	100
1500	5.3	136	84.6	0.01182033097	47.3	2800	150
2000	2	157	79.5	0.01257861635	49.7	3300	200
2500	-1.2	175	74.6	0.01340482574	52.3	5100	500
3000	-4.5	192	70.1	0.01426533524	55	7300	1000
3500	-7.8	207	65.8	0.01519756839	57.9		
4000	-11	221	61.6	0.01623376623	61		
4500	-14.2	235	57.7	0.01733102253	64.4		
5000	-17.5	247	54	0.01851851852	67.9		
5500	-20.8	260	50.5	0.0198019802	71.7		
6000	-24	271	47.1	0.02123142251	75.8		

Part A: What Should We Wear? - Temp vs. Altitude (Linear Relationship)

This is an easy one to start with. Potentially it can be done as a whole class activity, in order to outline the steps.



Linear Regression (ax+b)
$\text{regEQ}(x) = -.006521 \cdot x + 15.0891$
r_ = -.99999
r2_ = .999981

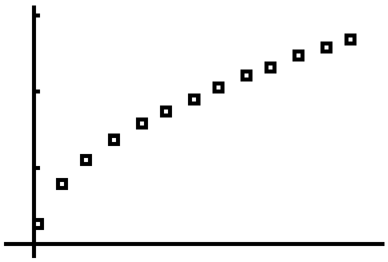
a) The students should recognize that there is a relationship and that it is linear.

b) The regression equation above gives the equation. $T = -0.0065h + 15$

c) According to this formula the temperature at 10 km is -50ºC.

d) Answers will vary, but the students should recognize that -50 is colder that any winter day!

Part B: How Far Will We Be Able to See?- Viewing Distance vs. Altitude (Square Root Variance)



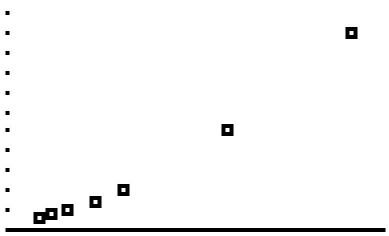
a) The relationship is a square root function.

b) The relationship appears to go through the origin. By trial and error, students can determine the vertical stretch factor. One possible equation is $V = 3.5\sqrt{h}$.

c) At an altitude of 10 km, the potential (or theoretical) viewing distance would be 350 km.
--Limitations of vision, weather, fog, may come into play here.

Part C: How Will We See Our Home Base? - Wattage vs. Altitude (Quadratic Variance)

This one involves many approximations, but because of the inverse square law of light, the power (wattage) of the bulb will have a quadratic dependance on the distance desired.



Quadratic Regression

$\mathrm{regEQ}(x) = .000018 \cdot x^2 + .008132 \cdot x + -14.1717$

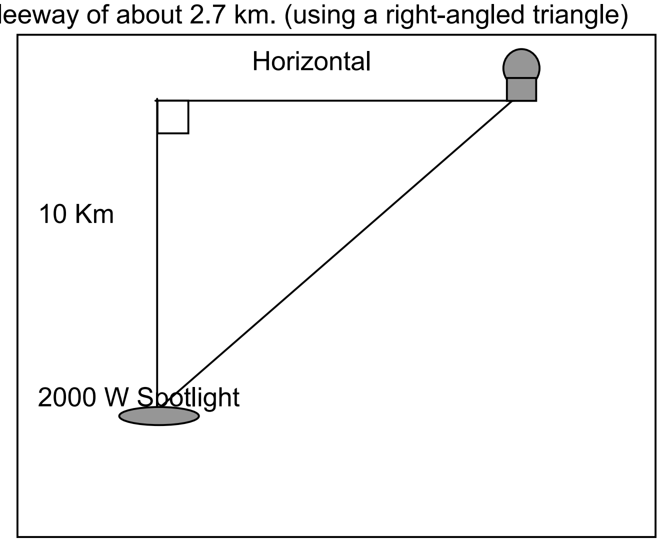
$\mathrm{bigr2\_} = .99985$

a) The higher the flight, the more powerful bulb would be needed. The relationship has a quadratic shape.

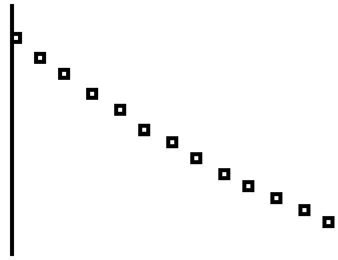
b) The regression equation is $W = 0.00001.8h^2 + 0.0081h - 14$.

c) From a distance of 10000 m, the wattage required is 1867 W, but the next available wattage is most likely 2000 W.

d) To answer this question the students examine how far off course the 2000 W bulb will let them wander. Using the curve of best fit, a 2000 W bulb will provide enough light for the balloonists to see the spotlight from a distance of about 10 350 m.
This will give them a horizontal leeway of about 2.7 km. (using a right-angled triangle)

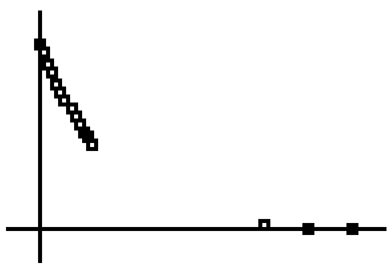


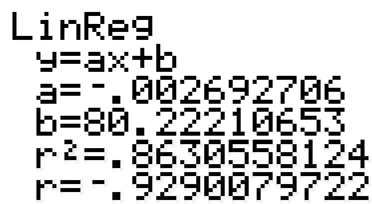
Part D: How Thin's the Air Up There? - Pressure vs. Altitude (Exponential Variance)
This one is interesting because the exponential curve is not examined until grade twelve, but they use exponential growth during the financial unit. It appears linear, but when extra data points are added to the graph, it is clearly not a linear relationship. The regressions also, make it obvious that the exponential fit is better.



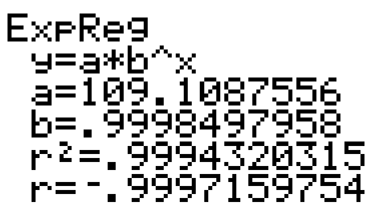
Linear Regression (ax+b)
$\text{regEQ}(x) = -.009003 \cdot x + 98.7191$
r_ = -.996346
r2_ = .992705

a) At first it may appear to be linear.
b) The equation would be $P = -0.009h + 98.7$.
c) Using this equation the air pressure would be 8.7 kPa at an altitude of 10 km.
d) It can be verified by using the regression equation of the new data or by adding the three new points to the plot.





e) We can see how the linear regression is no longer sufficient.
We must try others:



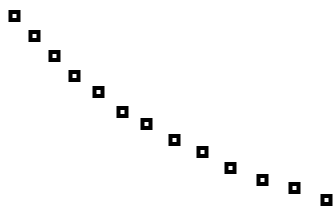
Note: Instead of using the calculator's regression capabilities, students can find the curve of best fit by the method of guess, check and refine. (trial and error)

f) The new equation of the curve of best fit is $P = 109.1(0.9998498)^h$
This gives the air pressure to be 24 kPa at 10 km. (much closer to the true value)

Part E: What About Our Mascot "Extreme Eric"? - Pressure vs. Volume

Note: If Part D was not completed, the students will need some additional information. They will need to know that the air pressure at 10 km is around 24 kPa.

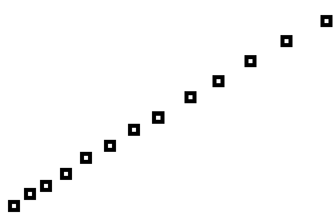
Below is the scatterplot of Volume (*y*) vs. Pressure (*x*)

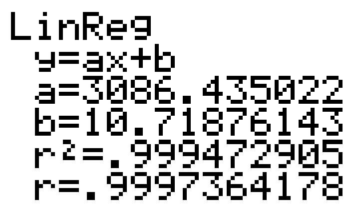


a) As the air pressure drops, the doll expands.

b) After trying various regressions, without success, the students should recognize that this is a reciprocal relationship.
One method of getting a curve of best fit would be to take the reciprocal of the independent variable, pressure.

Below is a graph of Volume (*y*) vs. Pressure^{-1} (*x*)





This is clearly a linear relationship

c) We replace the variable *x* with its value, which is the reciprocal of the pressure.
This gives the equation of the curve of best fit:: $V = \dfrac{3086}{P} + 10.7 \quad or \quad V = 3086P^{-1} + 10.7$

d) At 10 km (or 24 kPa) of pressure, the doll would have a volume of approximately 139.3 L. This is over three times its original size.

Eric's Extreme Ballooning
Group Presentations
"Check-bric" for Assessment of Communication

	Level 1	Level 2	Level 3	Level 4
	Slightly	**Somewhat**	**Adequately**	**Extensively**
➢ Explains mathematical processes, methods of solution, and concepts clearly to others				
➢ Connects graphs and equations to support the model				
➢ Demonstrates the correct use of mathematical language, symbols, visuals, and conventions				
➢ Makes effective use of graphing technology				

Comments:

Student name:____________________________