Emergent Insights into Mathematical Intelligence from Cognitive Science[1]

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Over the past 20 years, complexity scientists have been labouring to identify the sorts of conditions that enable the emergence of complex systems —how, for example, ants interact to form anthills, species couple together within ecosystems, cells knit themselves into organs, and organs into individuals, and individuals into societies and so on. Among the necessary conditions for these happenings, the following six seem to have a particular relevance to the work of the mathematics teacher:

- Internal Diversity—Internal diversity refers to the pool of possibilities that a system has to choose from when it's faced with a novel circumstance. It is the basis of the collective's intelligence. A system in which all of the components are expected to do the same thing at the same time will not be an intelligent one.
- Internal Redundancy—That being said, it's important that the agents in a system have enough in common to be able to work together, whether talking cells, birds, people or social systems. Redundancy is also necessary for a robust system. If one agent fails, another can step in.

  Some redundancies among participants in a collective have a lot to do with actions and competencies that are automatized. This is where traditional mathematics teaching has focused. The only way that a system's diversity can be a source of intelligence is if its agents are sufficiently alike for the bit of diversity to be embraced and elaborated.
- Neighbour Interactions—This condition might seem ridiculously obvious. Of course the agents in a system need to interact if that system is to become a system.

  But in the context of the classroom, the agents that need to interact aren't necessarily people. They can also be ideas or interpretations. As already mentioned, mathematics knowledge emerges as new ideas are blended with old ones. These blendings open up spaces for more powerful notions. So the phrase *neighbour interactions* doesn't refer to pod seating or group work, but to ensuring there is a sufficient density of diverse thought represented for the possibility of new ideas, as in the example of the varied interpretations of multiplication.
- Liberating Constraints—Consider these three tasks:
  1) Write down all that you know about three fourths.
  2) Write down two fractions equal to three fourths.
  3) Write down three things that you know about three fourths.

  In most classroom contexts, the first of these is much too broad to generate much that is interesting. The second suffers from being much too narrow, but has the same result—it likely won't generate much that's interesting either. But the third, like Baby Bear's porridge and bed, might be just right. It's open enough to allow for diverse possibilities, but sufficiently constrained to ensure that ideas won't be too diverse to prevent them from working together. (Of course, whether it is suitable depends on the collective.)

  Complex systems have to maintain this delicate balance between so much structure that they lock into place and so little structure that they decay into chaos. And the tasks that you set will determine whether or not intelligent—once again, appropriate and innovative—action can emerge.
- Organized Randomness—With a complex system, there's always a bit of randomness. Some of that randomness is ignored by the system—which is to say, it doesn't really affect what the system does. Other bits of randomness come to be really important—the unexpected observation, the sudden insight, the fact that this student's father is a painter and he knows the world doesn't work the way the question about ratios says it should work. Really intelligent systems, it seems, take advantage of more of these random events, and they're able to do so because they have strategies to organize such events.
- Decentralized Control—One of the big changes at Microsoft, Apple, IBM, Hewlett Packard and other locations of cutting-edge knowledge production has been an abandonment of the top-down model of centralized management in favour of a more distributed sort of control. Intelligent collective action can't be orchestrated into existence, at either the individual or the collective level. Space to negotiate the parameters and possibilities is needed.