From Arithmetic to Algebra

Understanding Teaching and Learning from a Complexivist Perspective

OAME Board Meeting

February 9, 2008

presented by

Mary Lou Kestell and Kathy Kubota-Zarivnij

Sessions Goals

During this session, participants will become familiar with complexity theory notions in relation to:

- the structure of mathematics knowledge (*recursive elaboration*)
- the nature of mathematics knowledge production within problem solving contexts using bansho (*teacher as consciousness of the collective*)
- the design principles of mathematics teaching and learning (*conditions of a complex learning system*)

Teacher as Consciousness of the Collective

Second, further developing the notion of communal cognition, the classroom community can and should be understood as a learner—not a *collection of learners*, but a *collective learner*—with a coherence and evolving identity all its own (see Davis & Simmt, 2003). Third, drawing on recent con-

The metaphor of "teacher as the consciousness of the collective," then, is a suggestion that the teacher is responsible for prompting differential attention, selecting among the options for action and interpretation that arise in the collective.

consciousness is more a commentator than a controller. It doesn't direct, it orients as it contributes to the reworking of the parts the constitute our perceptual wholes. When something bubbles to the surface of consciousness, it frames what we might notice or look for, without dictating what will be

possible. Rather, teaching seems to be more about expanding the space of the possible and creating conditions for the emergence of the as-yet unimagined. So complexivist teaching is not about prompting a convergence onto pre-existent truths, but about divergence into new interpretive possibilities. The emphasis is not only on what is, but also what might be brought forth. It comes to be a participation in a recursively elaborative process of opening up new spaces of possibility by exploring current spaces.

Let's Do Math - Butterfly Problem

3 butterflies landed on a bush.

4 more butterflies came.

Then 8 more butterflies came.

- How many butterflies are on the bush?
- Show at least two different solutions.



Different solutions to a problem as a *recursive elaboration*

Butterfly Problem – Range of Responses 3+4+8 (Addition of 3 Addends)

1. Counting by 1s (counting "literally" by representations of butterflies in problem)
2. Counting on from the first number (3)
3. Adding numbers two at a time
4. Counting on from the first sum (3+4=7)
5. Counting on from the larger number (8)
6. Decomposing to make 5s and 10s (e.g., (7 + 3) + 5)

- Regrouping to same number (3+4+4+4)
- Regrouping to 3s (3+3+3+3+3+3) – groups of, skip counting on NL
- Regrouping to 5s (5+5+5) – groups of, skip counting on NL
- Doubles plus one (7+7+1)

11. Doubles subtract one (8+8-1)

Let's Do Math as a Teacher!



- **Pick a number.** → 13
- **Double it.** → 26
- **Add 6.** → 32
- **Double again.** →
- **Subtract 4.** →
- **Divide by 4.** →
- **Subtract 2.** →
- **What's your final number?** → ?

Why did you end up with that number?

How does it work?

Problem design as a *recursive elaboration*

What Does It Look Like Algebraically?

$$\left(\frac{2\,(2x + 6) - 4}{4} \right) - 2 = x$$

Pick a number, double it, add 6, double again, subtract 4, divide by 4, and subtract 2.

Problem design as a *recursive elaboration*

Let's Do Math - Legs Problem

There are 44 legs in the room. People are sitting on stools around one table with no empty stools. One stool has 3 legs. One table has 4 legs.

a) How many people are around the table?

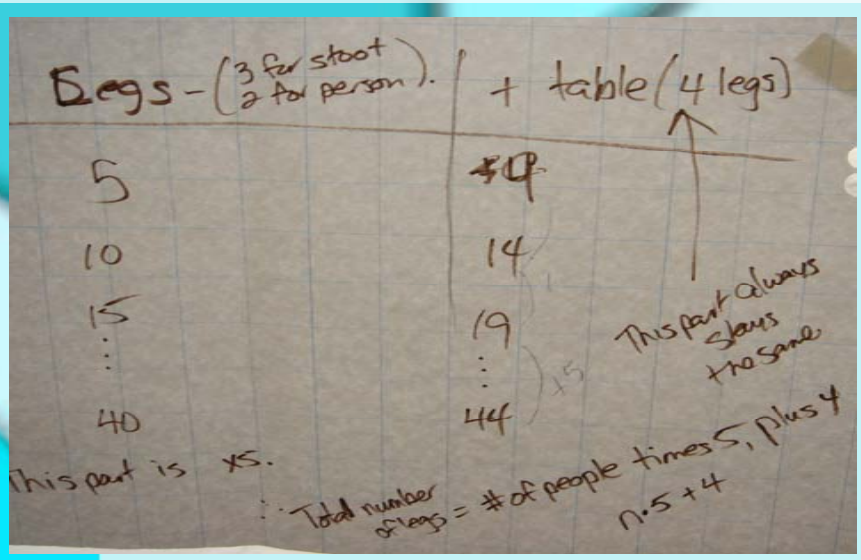
b) Show 2 different solutions.

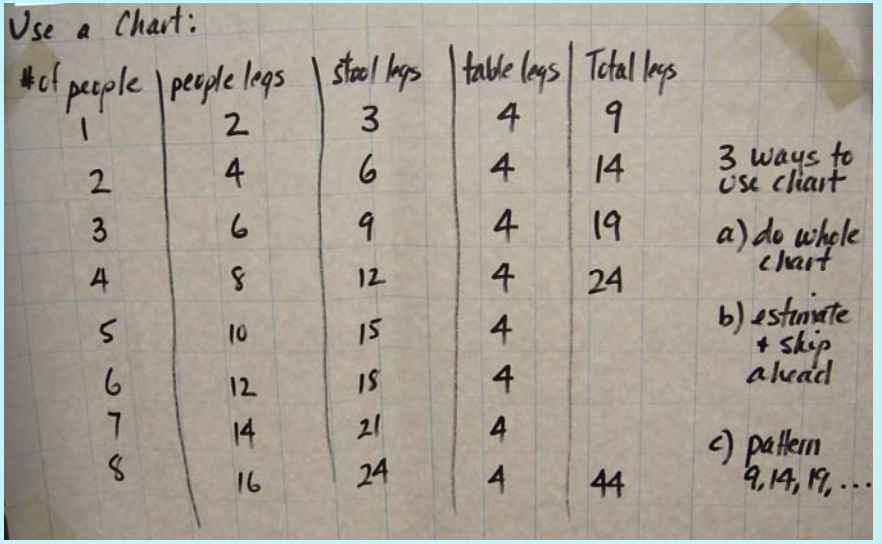
c) How does the range of solutions show the relationship between arithmetic and algebra?

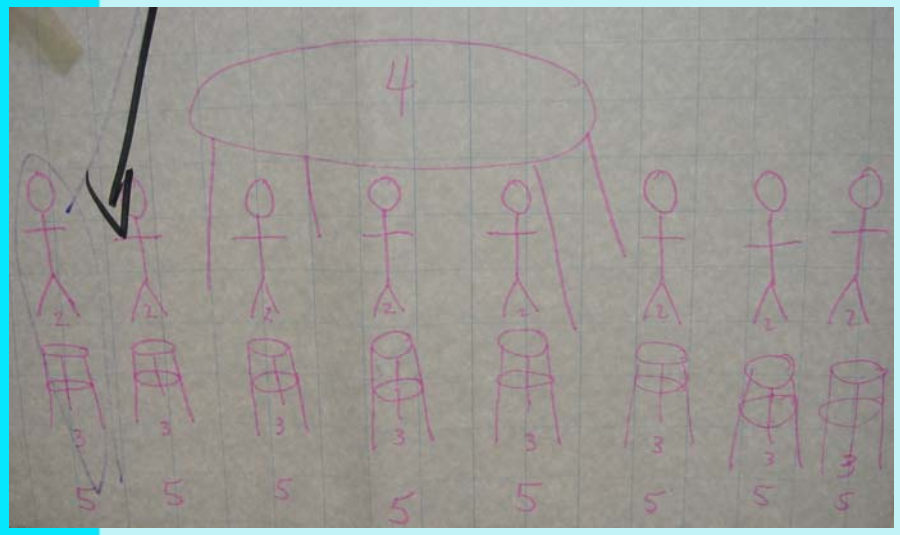


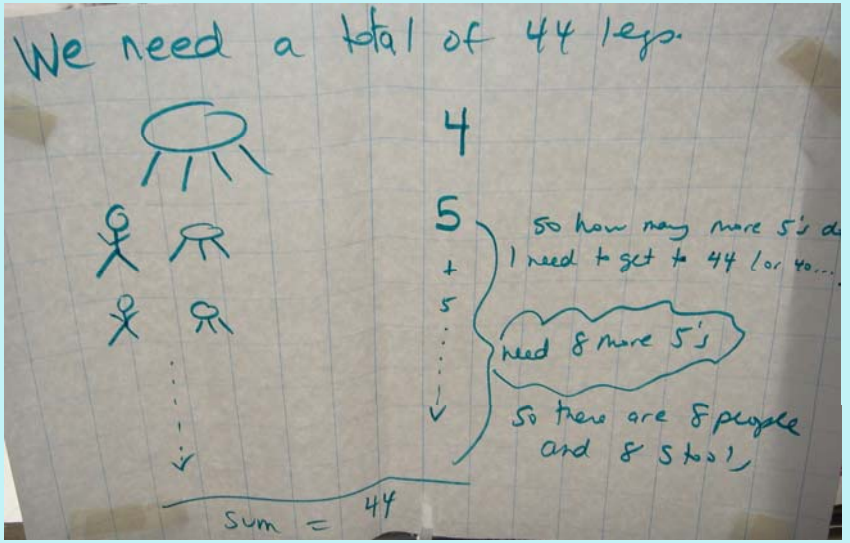
Arithmetic to algebra as a *recursive elaboration*

Leg Problem - A Range of Solutions



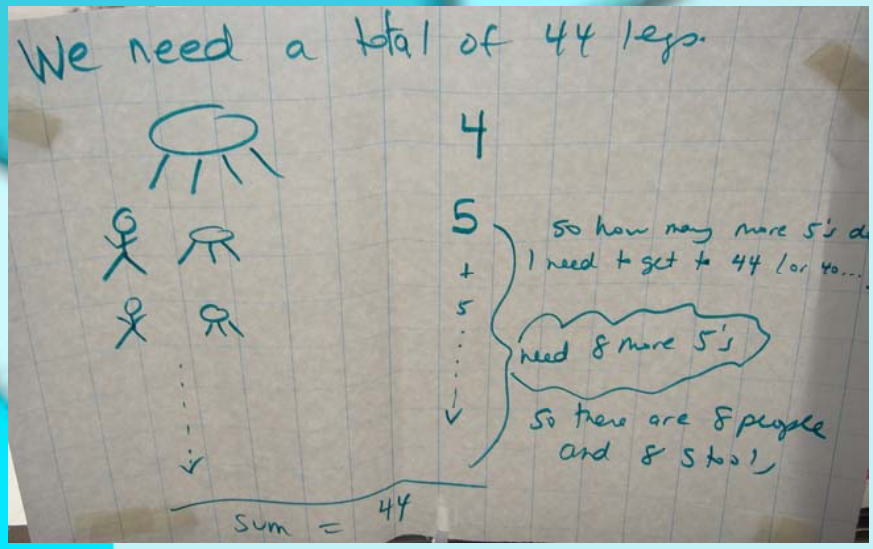


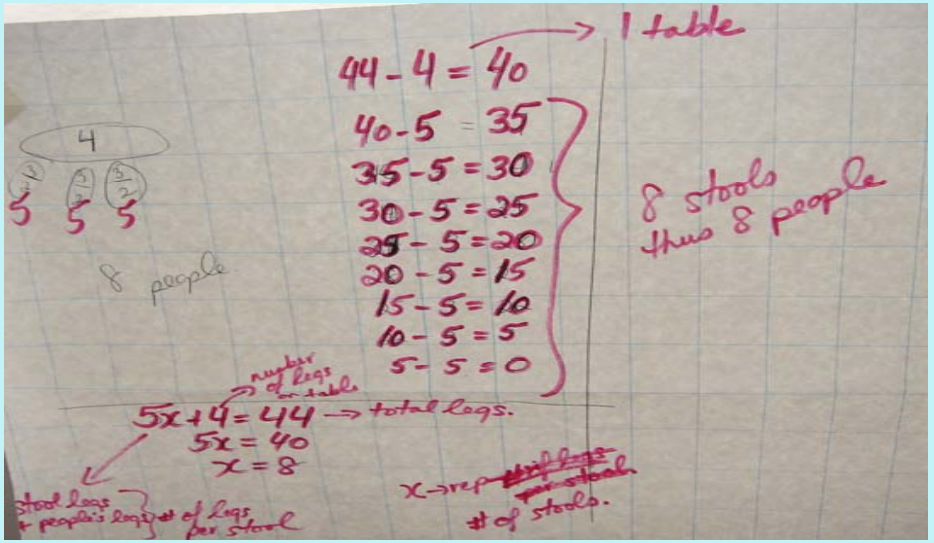


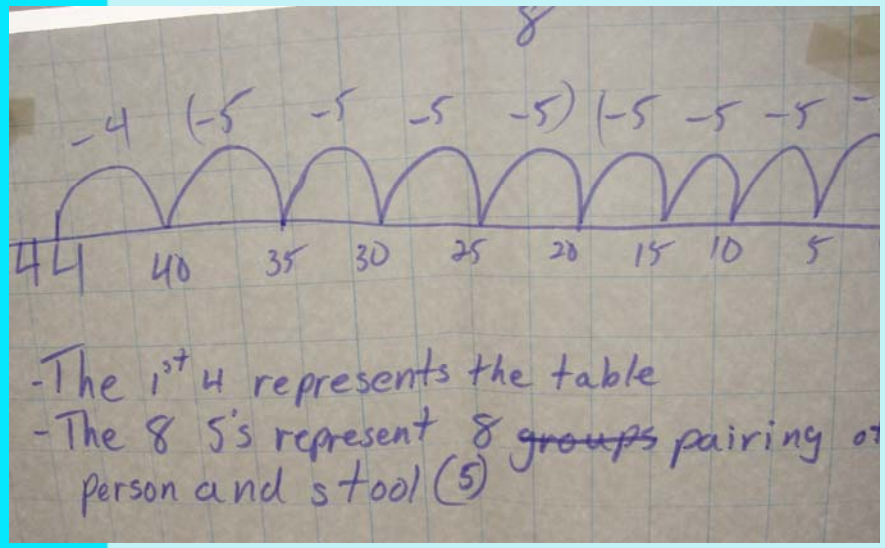


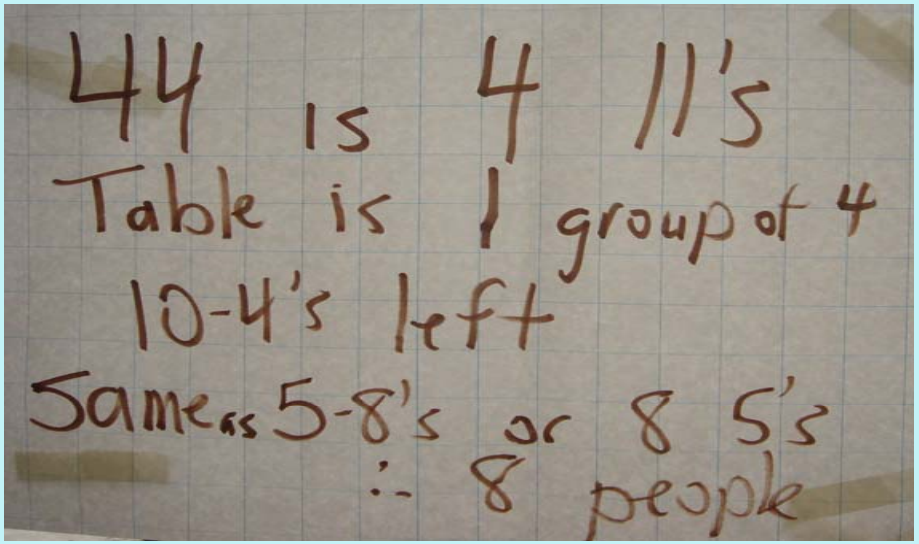
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Legs Problem- A Range of Solutions



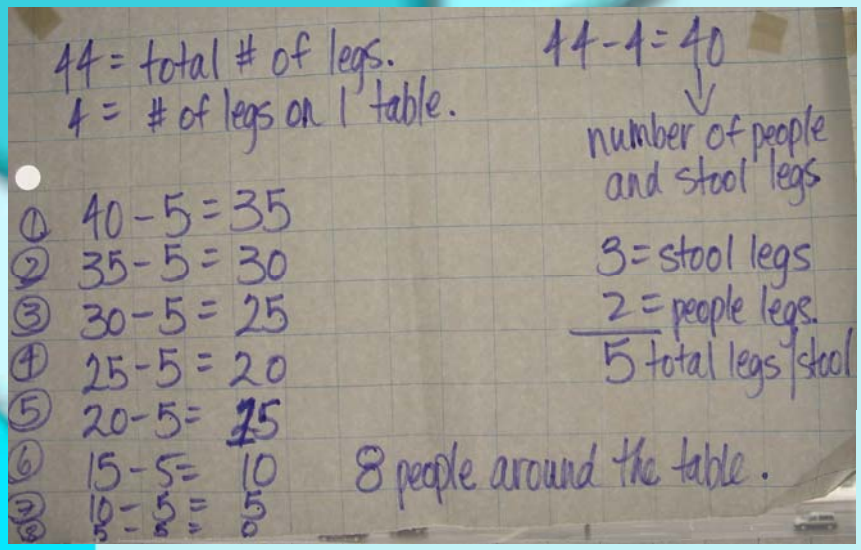


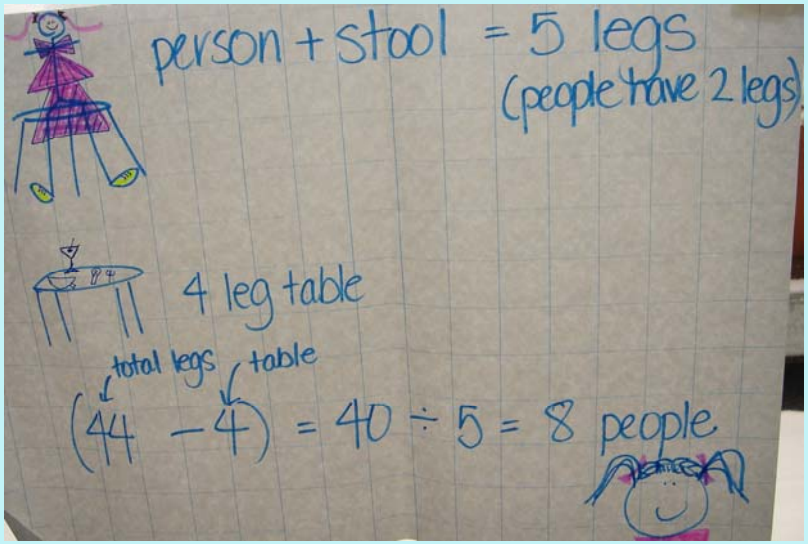


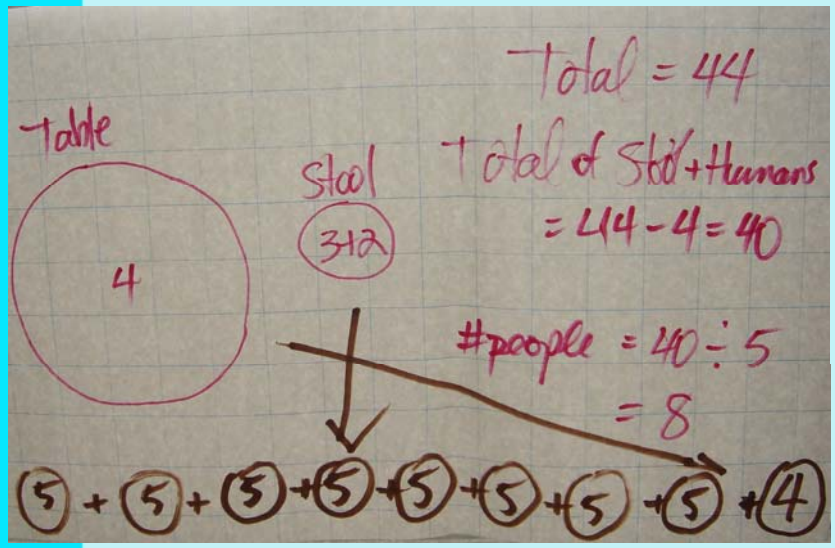


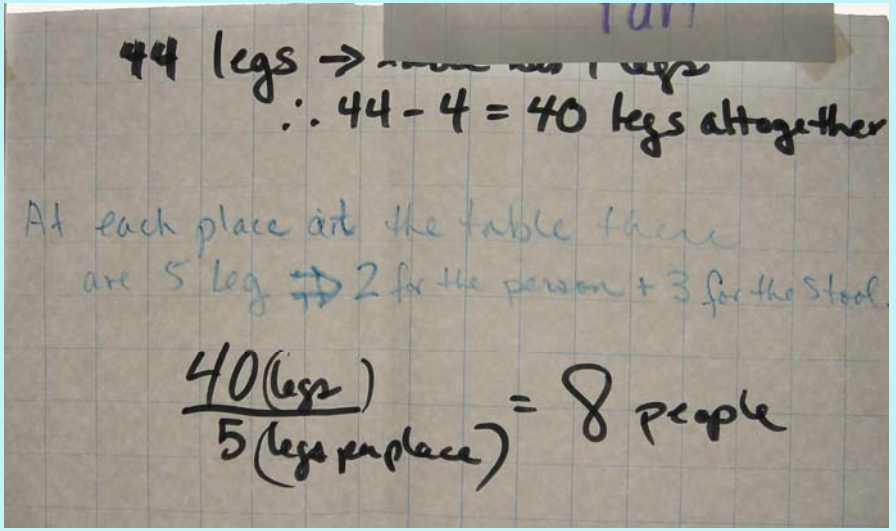
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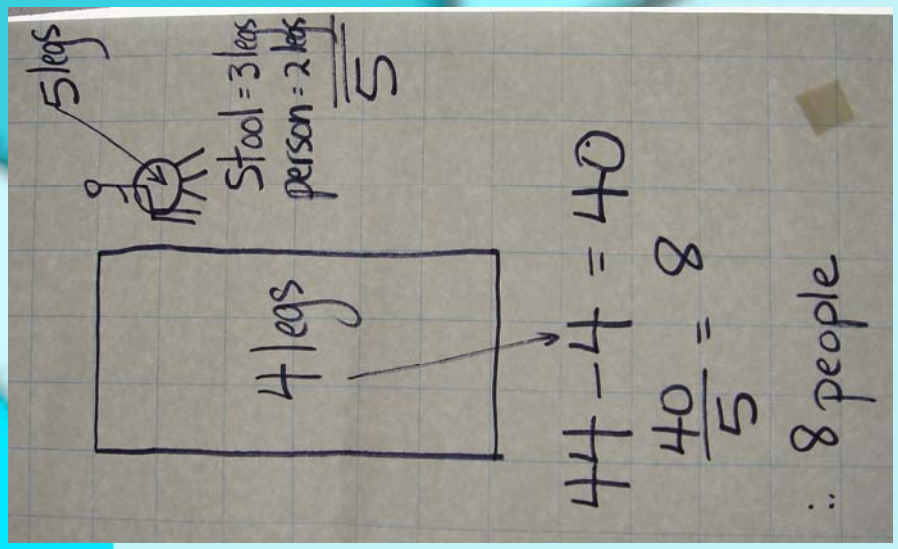


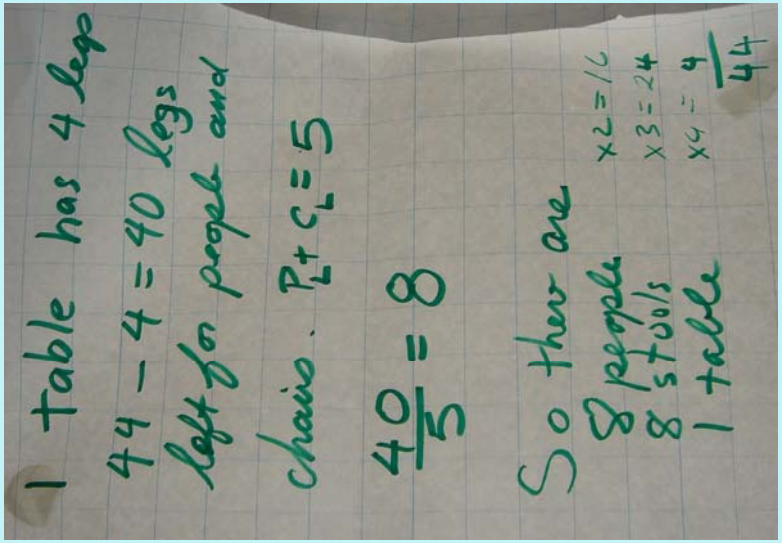


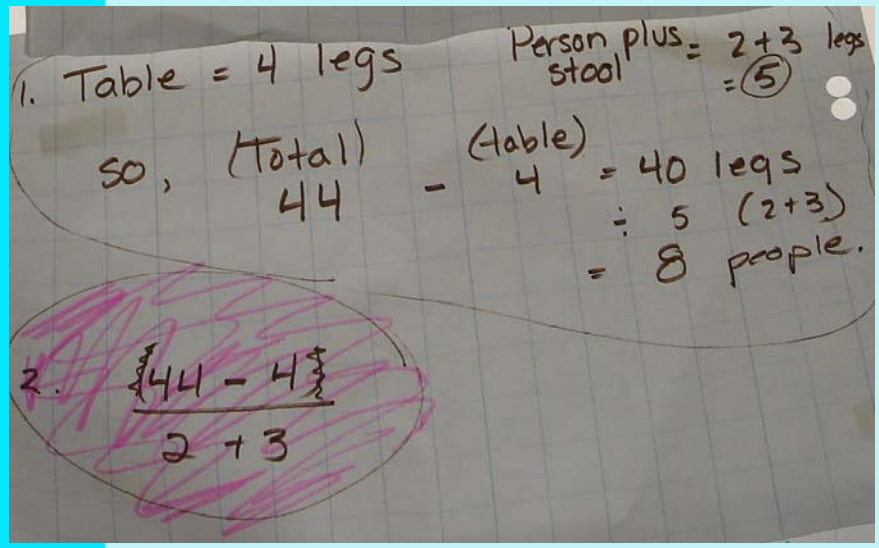


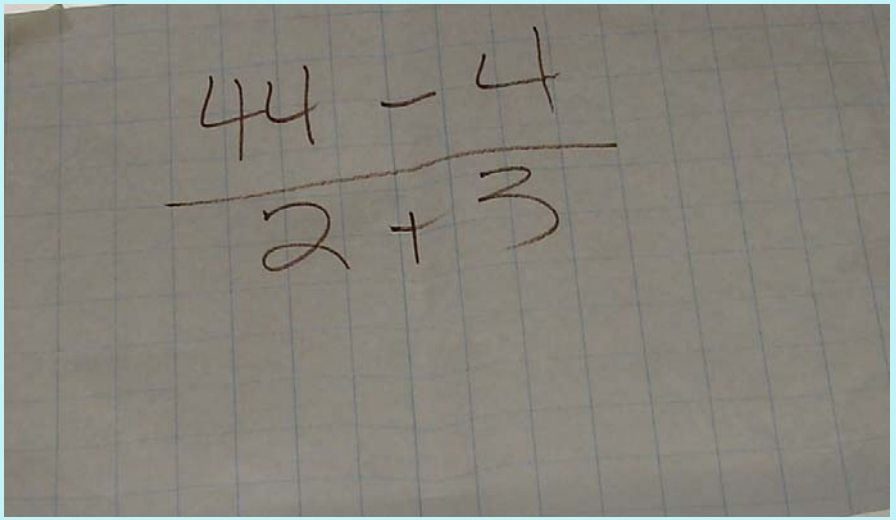
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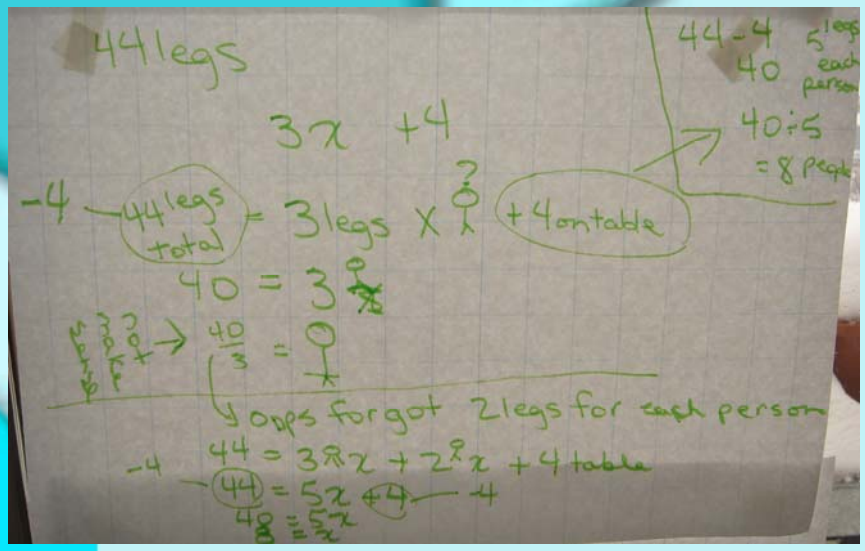


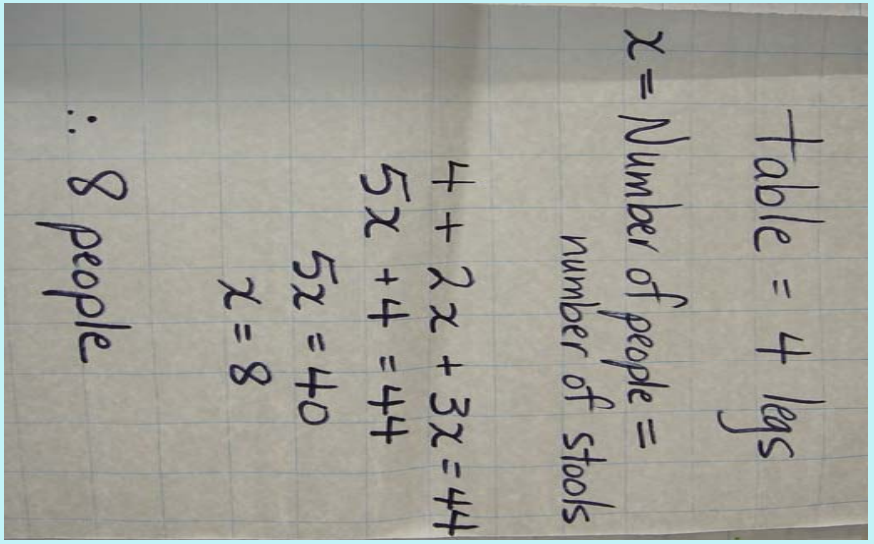


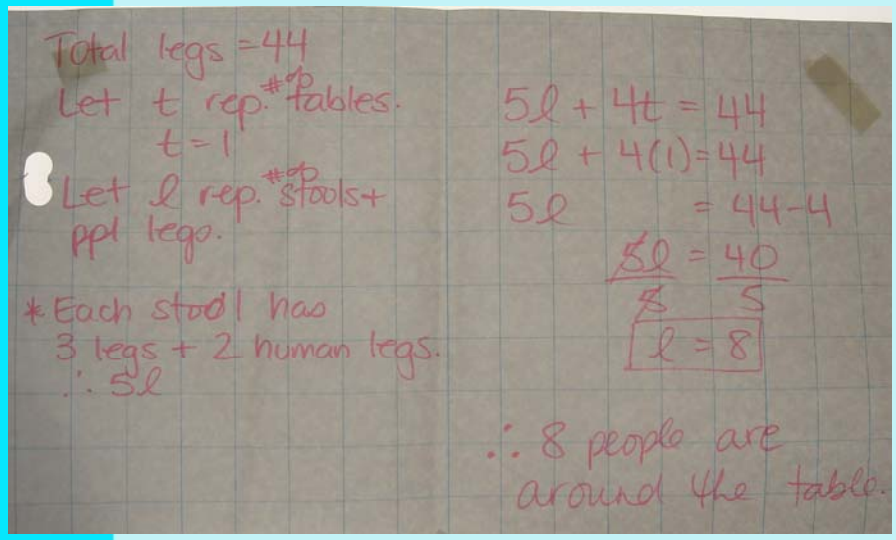


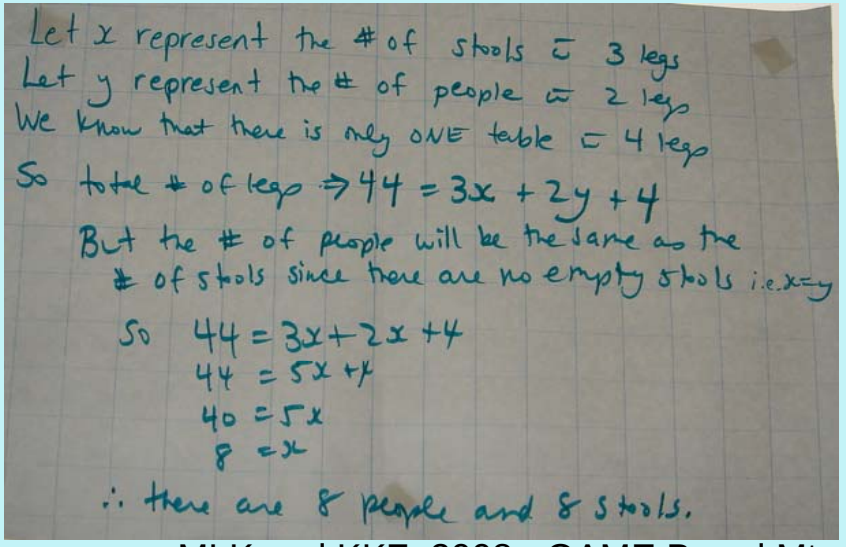
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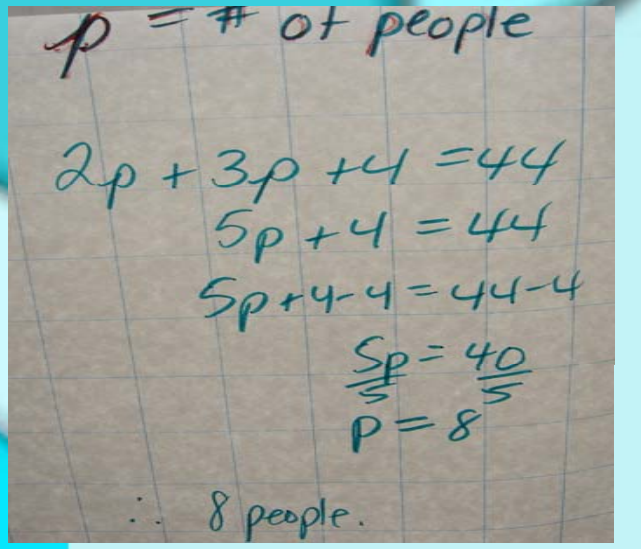


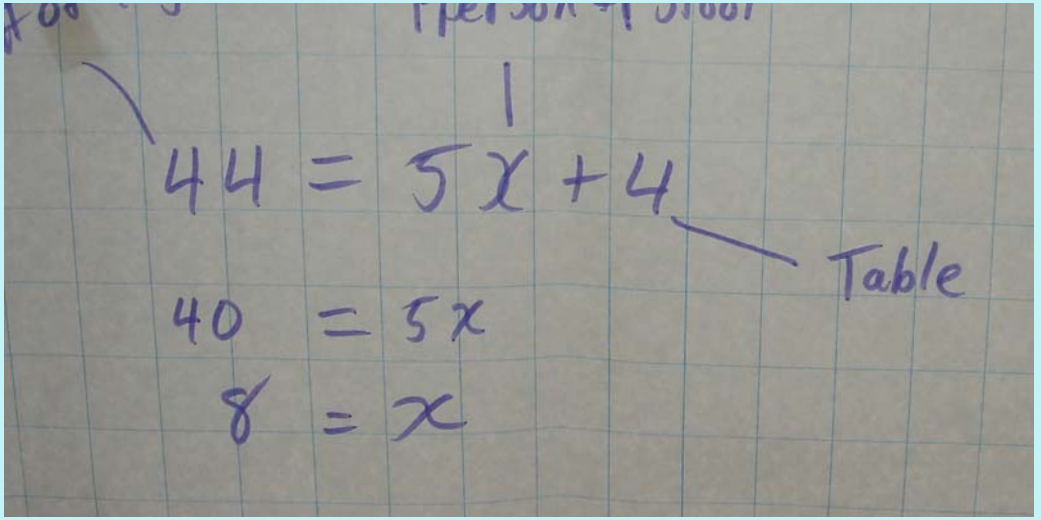


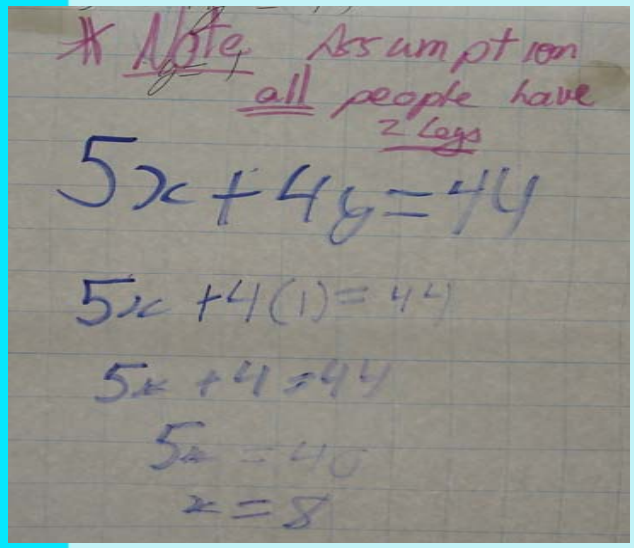


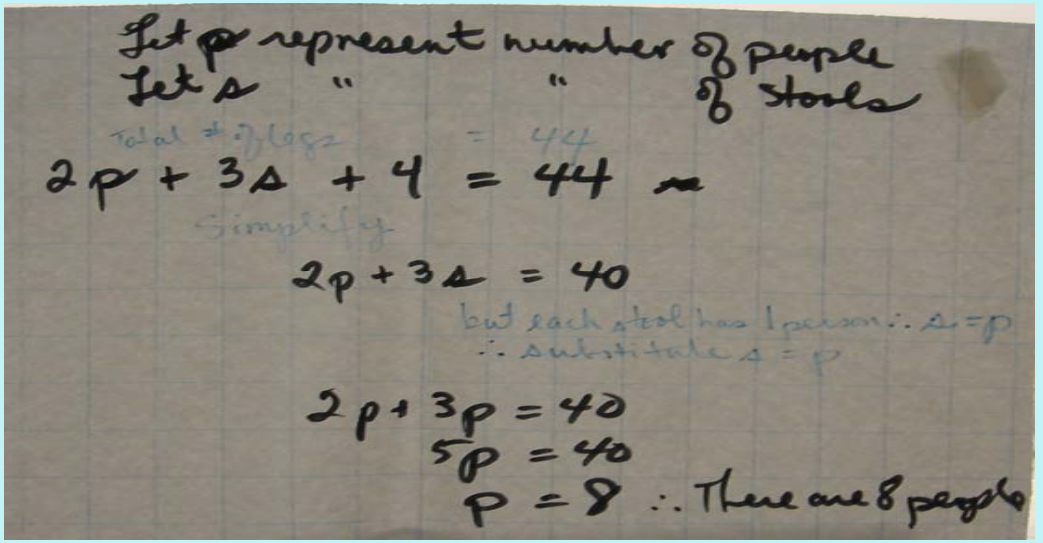
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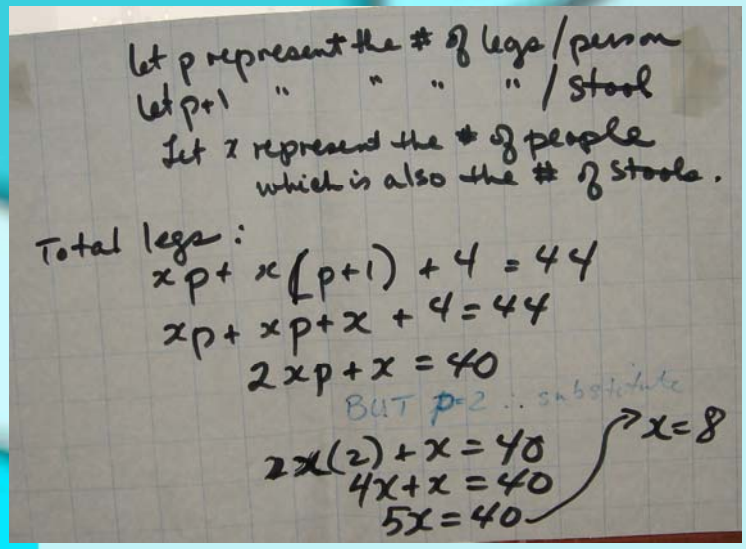


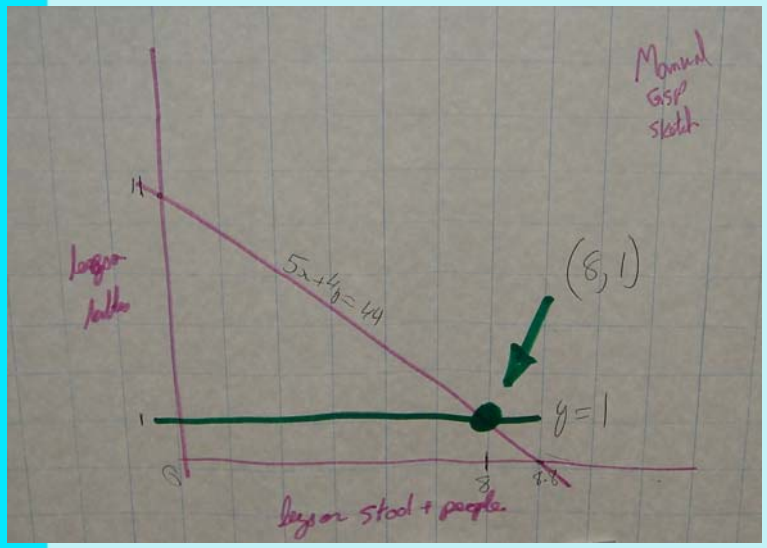






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Legs Problems
- Bansho Reinvented for “Ontario” classrooms

- a **mathematics instructional strategy** that provokes students’ mathematical thinking to be explicit when solving problems through the organization and annotation of student work samples and classroom discourse
- a **classroom artefact** produced collectively by students and teacher that publicly displays the mathematical relationship among students’ solutions; could be used as a mathematics landscape for learning or as a mathematics anchor chart
- a **professional learning strategy** that develops teacher’s knowledge of mathematics *for teaching*



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Conditions of a Complex Learning System

Internal Diversity—Internal diversity refers to the pool of possibilities that a system has to choose from when it's faced with a novel circumstance.

Internal Redundancy—That being said, it's important that the agents in a system have enough in common to be able to work together, whether talking cells, birds, people or social systems. Redun-

Neighbour Interactions—This condition might seem ridiculously obvious. Of course the agents in a system need to interact if that system is to become a system.

Organized Randomness—With a complex system, there's always a bit of randomness. Some of that randomness is ignored by the system—which is to say, it doesn't really affect what the system does. Other bits of randomness come to be really impor-

Decentralized Control—One of the big changes at Microsoft, Apple, IBM, Hewlett Packard and other locations of cutting-edge knowledge production has been an abandonment of the top-down

Liberating Constraints—Consider these three tasks:

Multiplication as a *Recursive Elaboration*

Furthermore, Davis, Simmt, and Sumara (2006) describe multiplication as a complex conceptual blend or a recursive elaboration of numerous interpretations consisting of images and analogies. Such interpretations are knitted together as a web of interconnections that constitute the concept of multiplication. Some interpretations include:

- repeated addition (e.g., 2 x 4 = 2 + 2 + 2 + 2 or 4 x 2 = 4 + 4)
- equal grouping (e.g., 2 x 4 means 4 groups of 2)
- number-line hopping (e.g., 2 x 4 means make 4 jumps of length 2)
- sequential folding (e.g., 2 x 4 refers to the action of folding a page into two parts, then folding again to have a result of 4 parts, and again to have a total of 8 parts)
- many-layered (e.g., 2 x 4 means 4 layers, each of which contains 2 layers)
- ratios and rates (e.g., 4 L at $2/L costs $8 or $2/L x 4L = $8)
- array-generating (e.g., 2 x 4 array has 2 rows of 4 columns or 2 x 4 = 8 objects)
- area-producing (e.g., a 2 linear units x 4 linear units rectangle has an area of 8 square units or 2 linear units x 4 linear units = 8 square units)
- number-line stretching or compressing (e.g., 2 x 4 = 8 can mean that 4 corresponds to 8 when a number line is stretched by a factor of 2)

Knowing Multiplication

A Recursive Elaboration …

Multiply:

$$\begin{array}{r} 49 \\ \times\ 25 \\ \hline \end{array}$$

What Different Ways Could 49 x 25 = ? Be Re-presented? *Recursive Elaboration*

Knowing Multiplication

Multiply:
$$\begin{array}{r} 49 \\ \times\ 25 \\ \hline \end{array}$$

Compensate

49 -> 50

50 x 25 = 5 x 25 x 10

So, 5 x 25 = 125

and 125 x 10 = 1250

1250 - 25 = 1225

Decompose

49 x 25 => 40 x 20 = 800

=> 9 x 20 = 180

=> 40 x 5 = 200

=> 9 x 5 = 45

1225

Using Coins - Quarters

4 x 25¢ = 100¢

10 x 100¢ = 1000¢

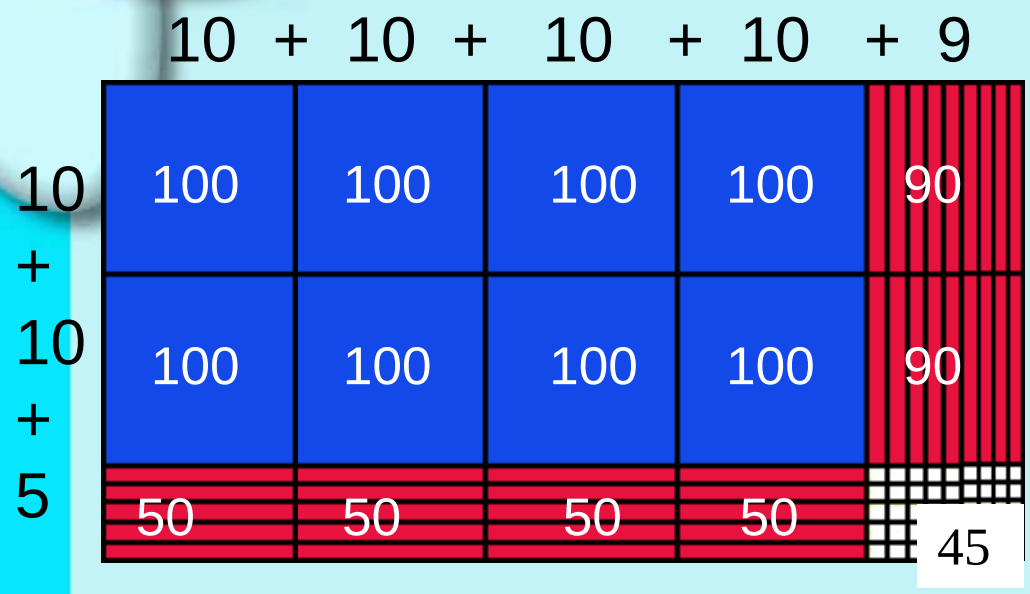
So, 40 x 25¢= 1000¢

9 x 25¢ = 8 x 25¢ + 1 x 25¢

= (4x25¢x2)+25¢ = 225¢

1000¢ + 225¢ = 1225¢

What Does It Look Like?
Whole Numbers … 49 x 25 = ?



	40	+	9
20 +	800		180
5	200		45

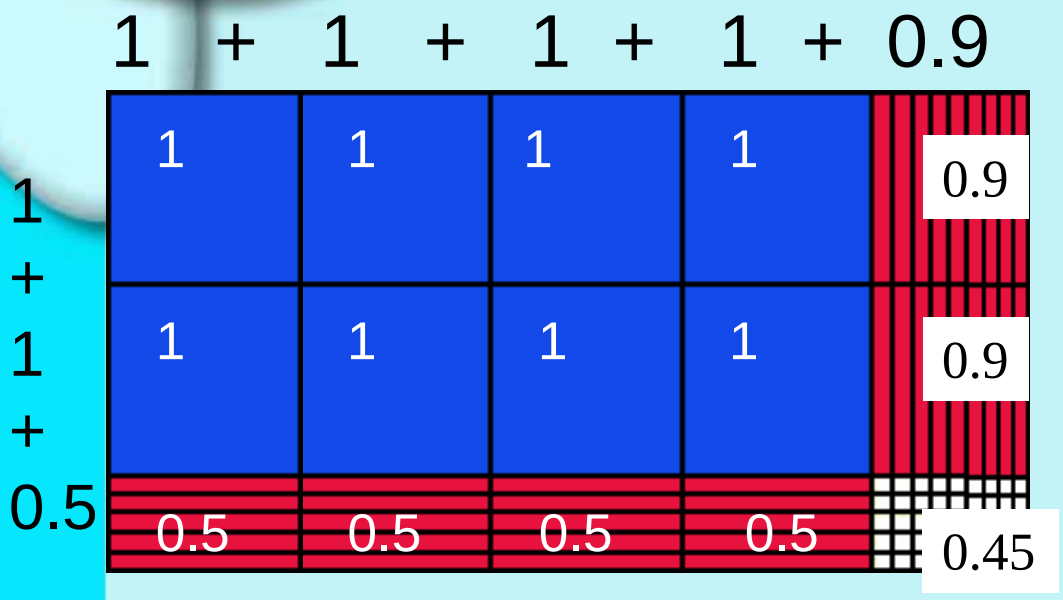
800 + 180 + 200 + 45

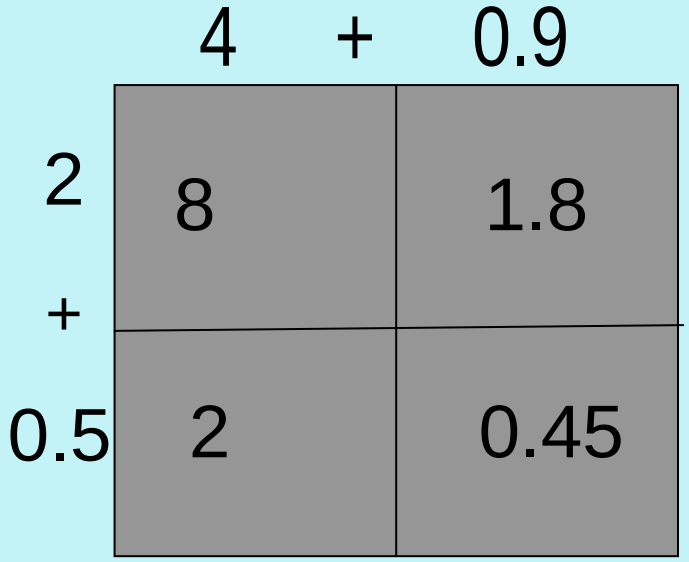
= 1100 + 120 + 5 = 1225

Multiplicaton as a *recursive elaboration*

What Does It Look Like?

Decimals … 4.9 x 2.5 = ?





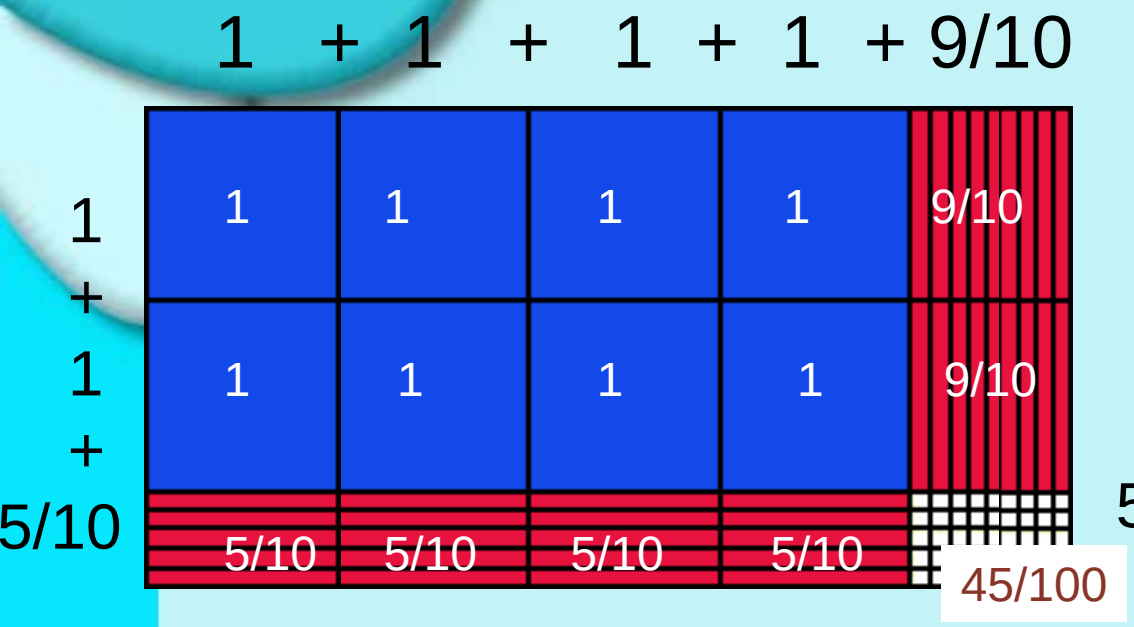
8 + 1.8 + 2.0 + 0.45

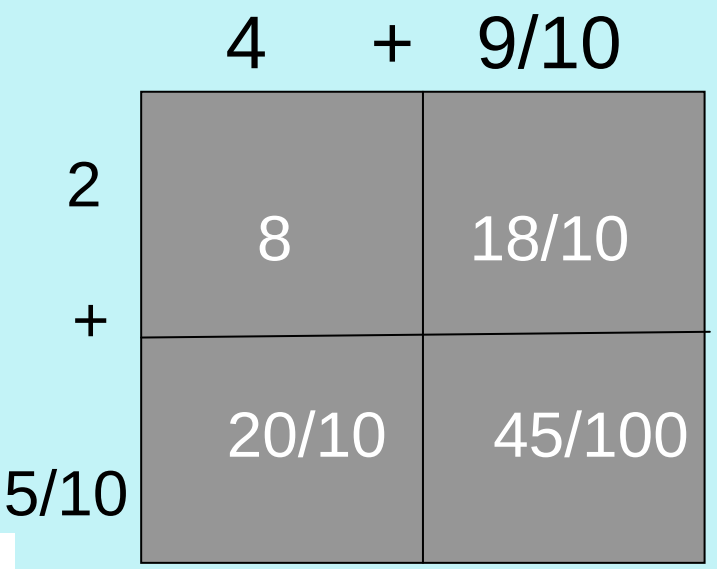
= 11 + 1.2 + 0.05 = 12.25

Multiplicaton as a *recursive elaboration*

What Does It Look Like?

Fractions … 4 9/10 x 2 5/10 = ?



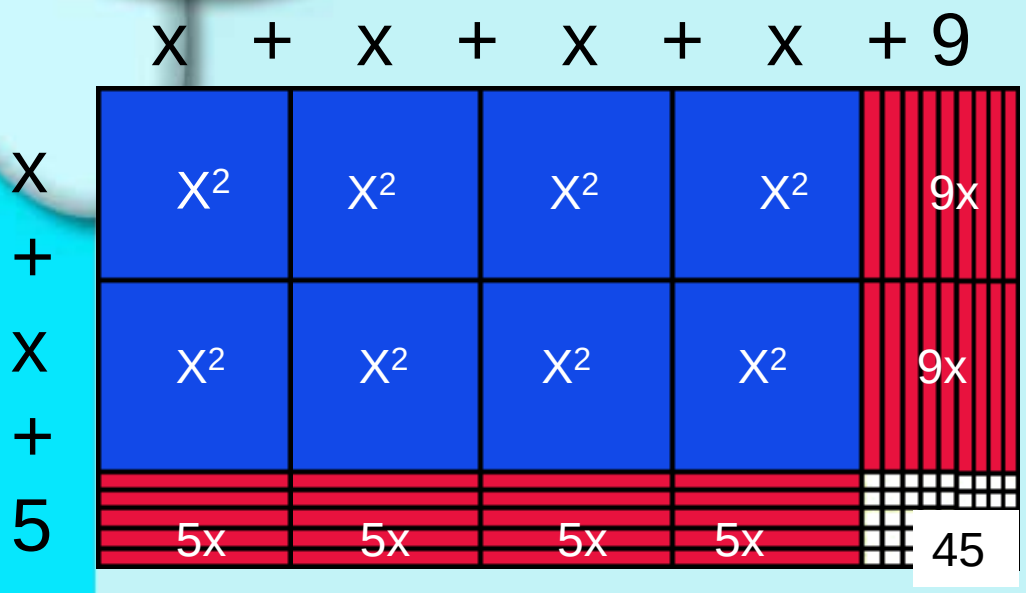


$$8 + 2(9/10) + 4(5/10) + 45/100$$
$$= 8 + 18/10 + 20/10 + 45/100$$
$$= 8 + 3 + 8/10 + 45/100 = 12 + 25/100$$

Multiplication as a *recursive elaboration*

What Does It Look Like?

Algebraic (4x + 9) (2x + 5) = ?



	4x	+	9
2x	$8x^2$		18x
+			
5	20x		45

$$(4x + 9)(2x + 5)$$
$$= 8x^2 + 38x + 45$$

Multiplicaton as a *recursive elaboration*

Key Terms - Complex Learning Systems

- Recursive elaboration
- Internal diversity
- Internal redundancy
- Neighbour interactions
- Decentralized control
- Organized randomness
- Liberating constraints