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ABACUS

Ontario Mathematics Gazette

OAME – ONTARIO ASSOCIATION FOR MATHEMATICS EDUCATION

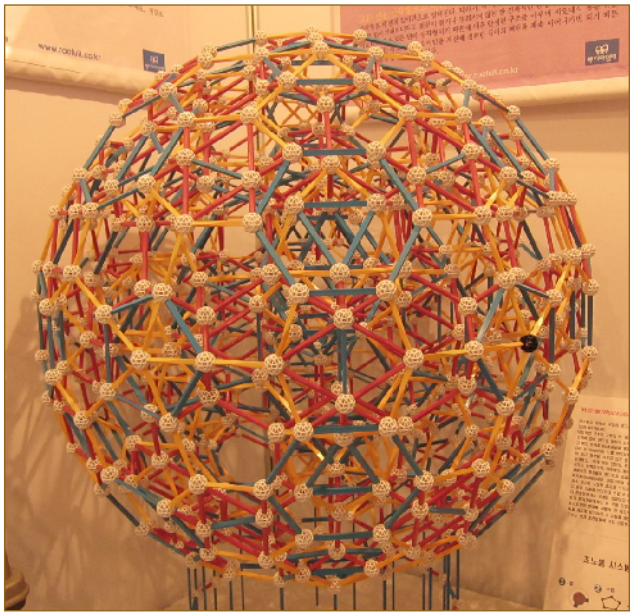
AOEM – ASSOCIATION ONTARIENNE POUR L'ENSEIGNEMENT DES MATHÉMATIQUES

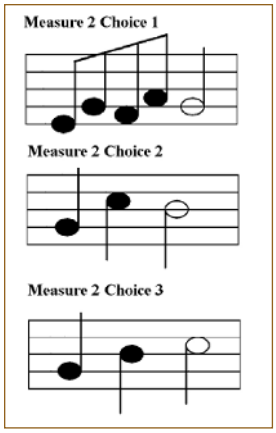
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September 2012

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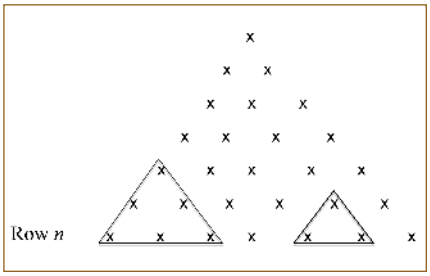
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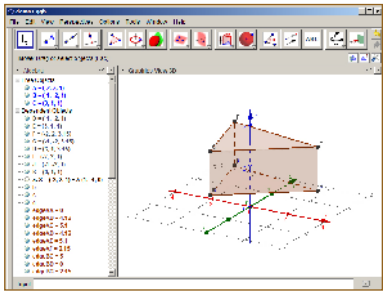




See In The Key Of Mozart: Composing Probabilistic



See Visual Reasoning K–12



See Technology Corner – Graphing Technology

See TEAMS PRESENTS OAME 2013 – THINK BIG: Reflecting on the Big Ideas in Mathematics Education



Submission of Articles

The *Ontario Mathematics Gazette* (*OMG*) is looking for news items, articles, and good ideas that are useful to mathematics teachers and mathematics teacher education. We are seeking submissions, preferably from mathematics teachers K–12 and other mathematics education professionals, that describe innovative and creative approaches to mathematics teaching.

Please keep in mind the following criteria when making submissions to the *OMG*:

- The ideas/activities must be of interest to the readership.
- The ideas/activities must be fresh and innovative.
- The mathematics content must be appropriate for the readership.
- The mathematics content must be accurate.
- The article must be well written and easily understood.
- The article and its ideas must be free of sexual, ethnic, racial, or other bias.
- The article must not have been previously published, nor should it be out for review by other publications.
- The article must be original.

Articles must be word-processed in MS Word, double-spaced with wide margins, not exceeding 10 numbered pages of text, and prepared according to the *Publication Manual of the American Psychological Association, Sixth Edition*. Figures and diagrams should be drawn by computer, if possible, or drawn in black ink in camera-ready form. Embedded images must also be submitted separately in jpeg or tif format. Proof of the photographer's permission is required, and for **photos of students** under the age of 18, the written permission of a **parent or guardian is required**.

You must submit **one complete copy** of your article, embedded with any tables, figures, and captioned photographs or graphics, to the Editor, Marian Small, along with **separate files for each of the text, graphics, and/or photographs**. Please e-mail all files to Immaculate K. Namukasa at **immaculate.namukasa@oame.on.ca**.

Your name should not appear anywhere in your article, including websites, so that your article can be sent out for blind review. Your name, full mailing address, and e-mail address must be included on a separate sheet. Upon review, you will be notified as to whether your article has been accepted for publication (as is, or pending minor or major revisions) or rejected.

The Editor reserves the right to edit manuscripts prior to publication. Once an article is published, it becomes the property of OAME.

PERMISSION TO REPRINT: Permission to reprint any part of this publication for instructional use or for inclusion in an affiliate or other publication must be obtained through the Editor, Immaculate K. Namukasa. Full credit must be given to the author and to the *Ontario Mathematics Gazette*.

The views expressed or implied in this publication, unless otherwise noted, should not be interpreted as official positions of OAME.

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About the Ontario Mathematics Gazette

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Submission of Advertisements

Advertisements for publication in the *Ontario Mathematics Gazette* should be sent to **Robert Sherk** at the above address. Courier is recommended to avoid possible delays. Deadlines for advertisements are January 23 for the March issue, April 1 for the June issue, July 1 for the September issue, and October 1 for the December issue.

Full-page advertisements are to be on 8.5" by 11" paper with a minimum of 0.5" margins and single sided. Each advertisement should be camera ready and colour advertisements should have no bleeds.

Advertising Rates

Advertising rates are available by telephoning, e-mailing, or writing to the Advertising Manager.

Editor's Message

IMMACULATE KIZITO NAMUKASA
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Immaculate Kizito Namukasa is an associate professor at the University of Western Ontario. Her current research interests in mathematics education include mathematical thinking and activity, mathematics teacher education, non-routine problem solving and learning environments, complexity research framework, and critical mathematics education. Namukasa is also interested in globalization and internationalization of schools and schooling in general.

A warm hello to all readers of the *Ontario Mathematics Gazette*.

My editorship experience for these first two months has already been a wonderful learning experience. I very much look forward to my term of service.

I would like to thank members of the OAME executive committee who trusted me with this responsibility; many of them, whom I have contacted in the past few months, have made me feel very welcome and supported.

Dr. Tim Sibbald, who encouraged me to apply for this job, noted that four Western professors— Barry Onslow, Eric Wood, Bob Smith, and Arnold Harris—were previous editors of the *Gazette*. To this my Dean, Dr. Vicki Shwean, encouraged that "Immaculate is in great company." The shoes of the past *Gazette* editors are much bigger to step into. Information about past editors is now avaialble at the OAME Archives-Gazette Editors **http://www.oame.on.ca/main/index1.php?lang=en&code=archives&type=12**

Congratulations and many thanks to Dr. Marian Small for her successful term of service. It is amazing how she managed to squeeze editorship work into her schedule.

In this first issue of the fifty-first volume of the *Gazette*, readers will find several regular columns, a new column on *Mathematics by and 4 Teachers*, some news from EQAO, Fields and OAME, and a write-up about the OAME awards. You will also find some not-so-good news from Statistics Canada about the ending of the Statistics Canada's Education Outreach program. In this write-up, in the Data Rocks column and in a special

feature on CANSIM, the authors suggest alternative learning resources and web sites. Two articles are published in this volume: one on visual reasoning and the second part of #anyqs (see pp 24-27), presenting stories of a mathematics teacher.

September is an exciting time of the year for parents, teachers, educators, and all those whose work centres on or is related to students' learning. I would like to take this opportunity to wish all of the readers, contributors, authors, reviewers, and others who support the work of this magazine a wonderful school year.

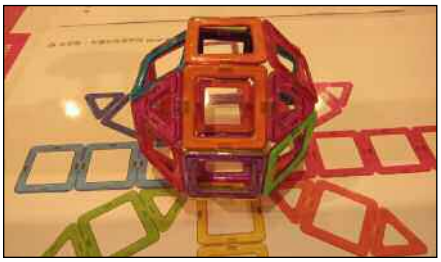
I personally bring to my new academic year energy and inspiration from two conferences. In July, together with several Canadian mathematics educators, I had the opportunity to attend The 12th International Congress on Mathematical Education, ICME-2012 in Seoul, Korea. What a huge gathering of over 3600 participants including practising teachers, mathematicians, educators, researchers, publishers, subject specialists, and graduate students, all from 84 countries of the world. Notably unique at this conference was the Mathematics Carnival and Plaza, a program directed toward students, teachers, and families. I share selected pictures here. Besides publisher and country exhibitions, participants engaged in mathematics activities; observed live televised mathematics classrooms; and visited fairs of mathematics manipulatives, art, puzzles, games, and magic at the carnival. I am sure several of you have been inspired by various events over the summer.

Have a very pleasant and productive academic year. ▲









▲ President's Message

CATHY HALL, PRESIDENT
E-MAIL: cathy.hall@oame.on.ca



Welcome to another super-charged year in the world of mathematics! As your new OAME President, I'm excited about the initiatives and challenges before us—and I hope you are too. As members of one of the most focused and visionary professional organizations in any subject discipline, we have the unique opportunity to make a real difference in both the direction and delivery of math education for teachers and students alike.

As we head into a new school year, it might be a good idea to remind ourselves of where we're headed and how we hope to get there….

The mission of OAME/AOEM is to promote excellence in mathematics education throughout the Province of Ontario:

- excellence for students by striving for a curriculum that will prepare them to meet the mathematical challenges in their lives, by fostering familiarity with the methods of mathematics and an admiration for its principles, and by giving recognition and respect to their own mathematical achievements
- excellence for educators by promoting their role as facilitators and resource persons, by supporting a variety of teaching methods to meet the varied learning styles and needs of students, by encouraging their students to gain facility in learning both co-operation and independence, and by celebrating the mathematical excellence of both individuals and groups

One of the main strengths of our organization is our grassroots nature. Our membership is comprised of educators from Kindergarten to Grade 12 and beyond—and our members represent all aspects of the math profession, from those in the trenches to those in supervisory and administrative roles. Individually and at the regional chapter level, our members are achieving amazing results, while at the same time, keeping an eye on the big picture so that, collectively, OAME is a force to be reckoned with!

Of course, all of this does not magically occur without clear-sighted leadership, in this case by your Executive

and Board of Directors. If you attended this year's Annual General Meeting during OAME 2012 in Kingston, you already know that we are in the midst of a restructuring of our governing body. Over the next three years, we will be phasing in a pared-down Executive and Board of Directors. Our goal in doing this is geared toward increased efficiency, fiscal responsibility, and hands-on involvement and input at the chapter level. No longer are we tied to three or four face-to-face meetings per year; we have begun to employ interactive Google docs, Adobe Connects, and virtual meeting rooms as effective and cost-saving add-ons.

Another of our strengths as an organization is the publication of a high-calibre professional magazine for our members. It's hard to believe that The OAME Gazette has been in circulation for 40 years now—and every year, it gets better and better! Within the past few months, most of our members have opted for the new online version of The Gazette. From both cost-saving and environmental perspectives, I applaud this move on OAME's part! I would like to take this opportunity to thank our retiring Gazette Editor, Dr. Marian Small, for a job well done… and welcome our new Editor, Immaculate Namukasa!

On behalf of all our members, I would like to offer a vote of thanks to Connie Quadrini for her innovative thinking and tireless efforts on our behalf during her past term as President. She has set the bar especially high for those of us following in her shoes. On a personal note, I would like to say how much I have appreciated all her help and mentorship, especially when it came to such things as her invaluable pointers on how to run a successful annual conference. But perhaps the most important lesson I learned from Connie was that we should always try to find a way to "pay it forward" for our membership. It just takes a bit of creativity and outside-the-box thinking to find ways to offer rich learning experiences for as many members as possible, whether it be with a gift of a professional resource for everyone attending a professional development event or through a "pay it forward" contest, where winners are sent to an OAME conference with most expenses paid. Our ultimate goal is capacity building within the Ontario math community, so the more educators who can benefit from this professional development, the better!

That said, my term as President is aptly sandwiched between OAME 2012 "CON.NETWORKS" and OAME 2013 "Think Big" —perfect positioning for my personal platform of OUTREACH. While I firmly believe in the powerhouse that is the present-day OAME, I can also see areas where we need to improve, and I would like to enlist your help in helping to raise our visibility and building an even bigger presence on the Ontario math scene. It never ceases to amaze me when some folks in the education field have no knowledge whatsoever about OAME's existence, let alone the accomplishments of our members. Why is it that some school boards, Directors of Education, Supervisory Officers, math coaches/consultants, and, especially, math teachers have never heard of our professional organization? Clearly we need to kick our communication skills up a notch if we hope to let people know about the good things we do and how we've been working to make OAME's vision come alive. I need your help in this regard. It's as simple as networking with your colleagues and taking every possible opportunity to share OAME news within your chapter. If we are more vocal about how we are helping to "make math happen," I believe it will open up opportunities for more individuals to take part in the excellent professional development opportunities afforded by OAME mini-conferences, the Fall Leadership Conference, and the Annual Conference in May of each year. It will also help us garner support and involvement in such events as Ontario Math Olympics, both within our chapters and provincially. That said, when you receive information concerning chapter events or conferences, please take the time to think seriously about attending—and inviting a non-member to join you. OAME events offer top-quality, cutting-edge professional development, and are open to everyone!

Lastly, as you are no doubt aware, in the near future, we will be faced with the prospect of a provincial curriculum review in math. Our mandate is to be as prepared as possible to offer informed feedback from an OAME perspective. To this end, our Curriculum Committee has already developed and been testing a curriculum-analysis tool that can be used to gather valuable insights and information. As the school year progresses, watch for opportunities to get involved in this initiative, either online at **oame.on.ca** or within your chapter. We will be approaching our membership for pertinent feedback. I would urge you, therefore, to be doing some directed and serious thinking about the curriculum presently in place, so that you can pinpoint possible gaps or conceptual inconsistencies within your grade level or division, as well as taking a look at "the big picture" that represents the math continuum from K–12.

Before I sign off, I just want to wish you a wonderful start to the new school year, no matter where you are or what you're doing. Remember to have fun and take time to marvel at its magic as you "make math happen" in your world! ▲

▲ Technology Corner – Graphing Technology

MARY BOURASSA
E-MAIL: mary.bourassa@oame.on.ca

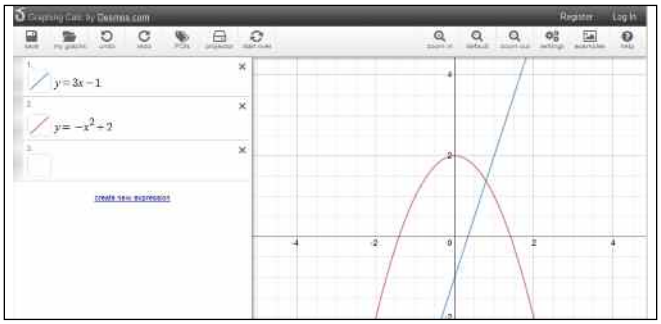


Mary teaches mathematics at West Carleton Secondary School in Ottawa. She is a strong advocate for the appropriate use of technology in the classroom. She has presented workshops internationally, authored mathematics resources, is a past VP of OAME and a Past President of COMA. An award-winning teacher, Mary continually strives to learn new and better ways of helping students learn and love mathematics.

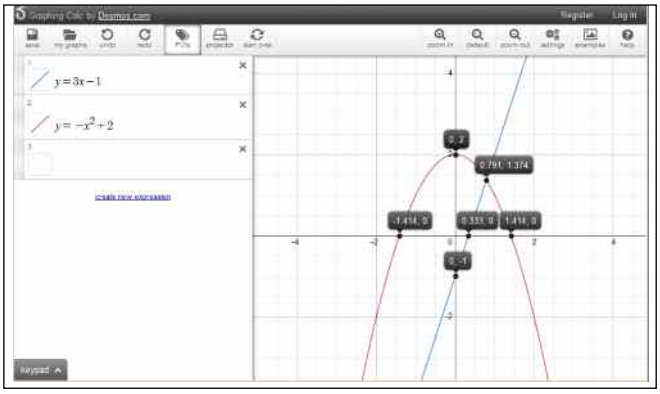
Many teachers and students use graphing technology every day to help develop connections between numerical, algebraic, and graphical representations. For some, this means using a graphing calculator such as a TI-84 or TI-Nspire™. For others, it may mean using software like SMART™ math tools or an app on a tablet or smartphone. Of the many options available, three are showcased here. They are all free and each has great features, yet I use each for different purposes.

As more teachers and students move to using online applications, programs like the graphing calculator by Desmos.com become great tools to allow exploration and experimentation. You and your students can use this if you have Internet access and a computer, smartphone, tablet (such as an iPad), or iPod Touch. Launch the calculator from **www.desmos.com**.

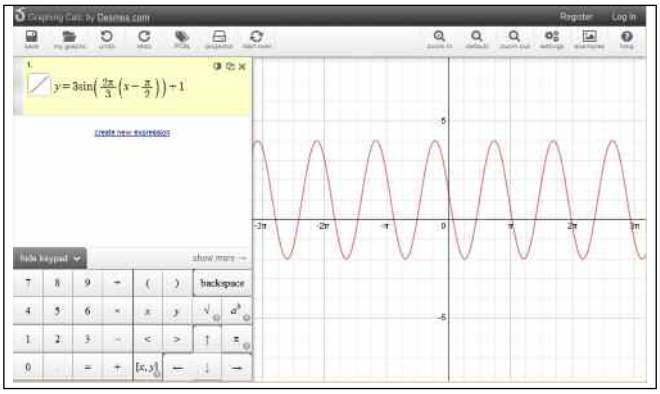
Getting started is a breeze – simply type in the equation or inequality you want to graph.



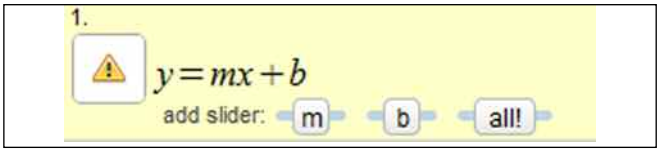
You can adjust the window settings through the "settings" panel or by grabbing the page and moving it. You can trace along a graph and display a point's coordinates by clicking anywhere on it, then dragging. It is also simple to edit the colour of a graph. Points of interest can be shown by clicking on the POIs tab. Clicking on each point will display its coordinates, as shown.



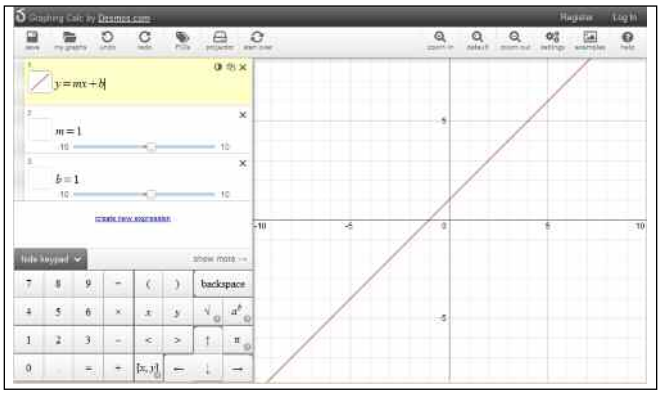
Trig graphs will also provide explicit and important information, especially when the *x*-axis is scaled in terms of π.



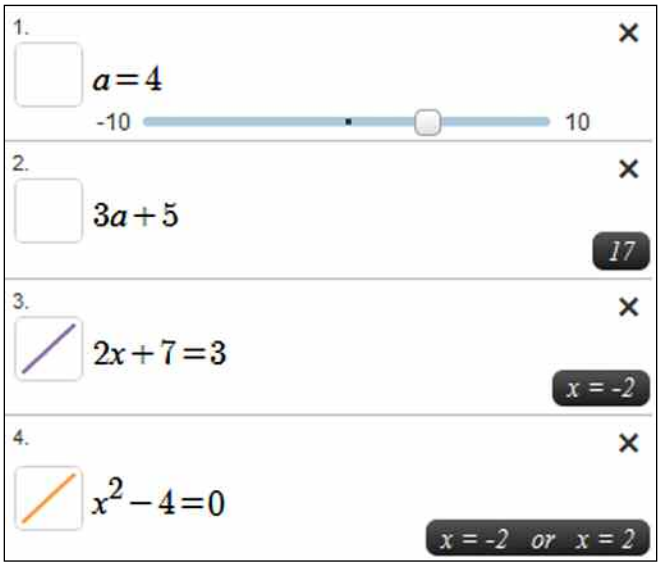
Adding sliders is fast and easy. Create an equation with parameters and you will be given the option of adding a slider for each:



Here is the result:



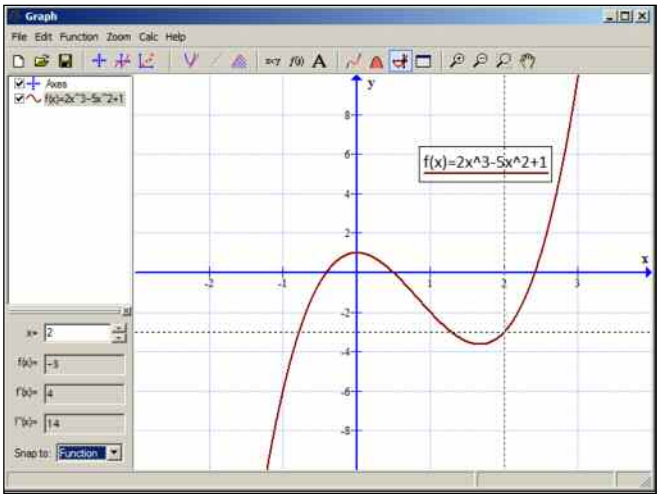
It can also perform calculations and solve equations.



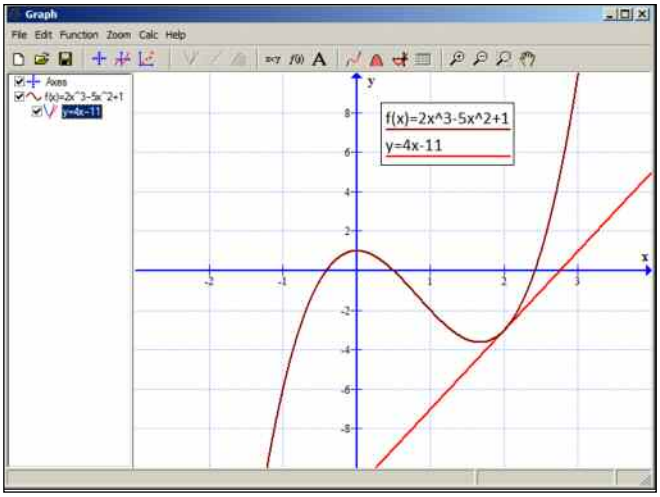
There are many cool examples featured on the Desmos.com website. I suggest you check them out!

A colleague told me about Graph, which is an open-source graphing program. It is fantastic for pasting graphs into documents, as it is easy to use and allows you to copy the graph window alone (go to Edit > Copy Image). Graph 4.0 can be downloaded from **www.padowan.dk**.

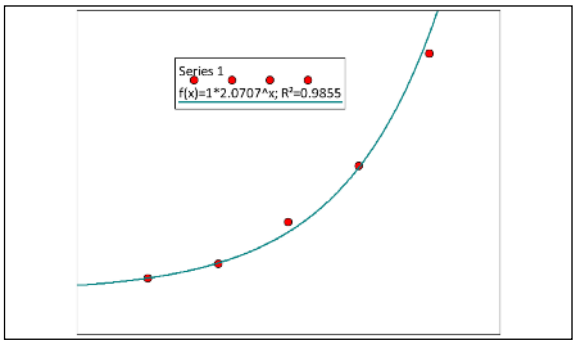
Graph allows you to plot functions, relations, and shade. It can plot the tangent and normal to a function and can symbolically calculate the first derivative. A cubic function is shown below (unfortunately the label isn't in the preferred form), along with the value of the function at $x = 2$ (note the dashed horizontal and vertical lines) and the value of the first and second derivatives at that point.



This shows the tangent at $x = 2$ along with its equation – I did not calculate anything! You can display the normal and its equation in the same way.

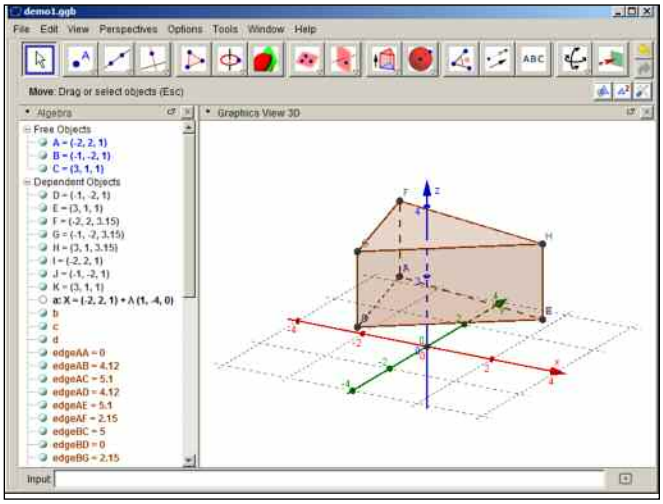


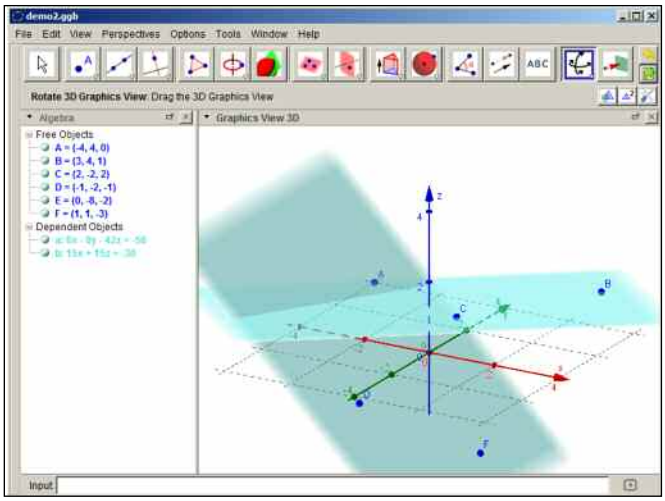
You can also plot a data series along with a trend line, as shown. I used the "Copy Image" option to demonstrate what a graph would look like when pasted in another document.



Graph is very easy to learn to use and is great for teachers who may not be comfortable using a lot of technology.

The third graphing software I want to mention is GeoGebra. I wrote a column about GeoGebra in 2009, but I thought it was worth revisiting, as version 5.0 is currently in beta and does 3D graphing. And it's still free from **www.geogebra.org**. This software unites geometry, algebra, and calculus, and is very powerful. The 3D capabilities are useful for looking at the intersection of lines and planes, but are also useful when developing formulae for the volume of some shapes (using the extrude tool). You can also do such things as measure the angle formed by points in 3-space, construct parallel or perpendicular planes, and rotate the 3D graphics view.





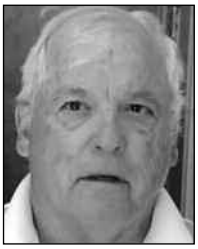
GeoGebra is an all-around great graphing package. You can download it, download a portable version, or run it as an applet. Fantastic software that keeps getting better!

One more mention – Sketchometry (**www.sketchometry.com**) is a free Euclidean geometry web app that is also worth investigating.

Do you have a favourite graphing tool or app? I would love to hear what you are using! Please tweet me @MaryBourassa. ▲

▲ OAME/NCTM Report September 2012

TODD ROMIENS
E-MAIL: todd.romiens@oame.on.ca



Todd recently retired from teaching K–12 methodology in the Faculty of Education at the University of Windsor. He is a past president of OAME and a life member, as well as a previous editor of the Gazette.

As you return to school to begin another year, remember that NCTM has a great number of resources that you can use to help with your teaching. There is now a page on the NCTM website dedicated to "Resources for Canadian Math Teachers." It is found under the lessons and resources tab at the top of the main page. Information about Canadian provincial affiliates is listed, as well as opportunities to find out about the following:

- Teacher training and certification in Canada
- Information for students' families
- Math for the classroom
- Writing for NCTM
- Canadian representation on NCTM boards

When the NCTM Standards document came out, it was the document that set forth position statements on how math should be taught in schools. At that time, Ontario teachers were already on board with many of the initiatives spelled out in the document. With its additions and revisions, it remains today as the document which spearheads mathematics teaching in the United States and Canada.

During 2011, NCTM issued six new or revised position statements. The basic position statements and links to more information are provided below:

What should schools teach about the metric and the customary systems of measure?

Because the metric system is an effective, efficient base-ten measurement system used throughout the world, students need to develop an understanding of its units and their relationships, as well as fluency in

applying it to real-world situations. At the same time, since some non-metric units of measure are still widely used in day-to-day life in the United States, American students also need to develop familiarity with these units of measure.

For more information on this statement, go to **www.nctm.org/about/content.aspx?id=29000**.

What are the roles of intervention in mathematics teaching to support individual students in meeting grade-level goals?

Without identifying specific interventions, we endorse the use of increasingly intensive and effective instructional interventions for students who struggle in mathematics. Teachers must use a variety of formative assessments to target strategic instructional techniques that are tailored to meet individual students' needs. When implementing appropriate interventions for all mathematics learners, teachers must possess strong backgrounds in mathematical content knowledge for teaching, pedagogical content knowledge, and a wide range of instructional strategies.

For more information on this statement, go to **www.nctm.org/about/content.aspx?id=30506**.

What is the role of calculators in the elementary grades?

Calculators have an important role in supporting and advancing elementary mathematics learning. The benefits of their selective and strategic use are twofold. Calculators can promote the higher-order thinking and reasoning needed for problem solving in our information- and technology-based society, and they can also increase students' understanding of and fluency with arithmetic operations, algorithms, and numerical relationships.

For more information on this statement, go to **www.nctm.org/about/content.aspx?id=30558**.

What is the role of technology in the teaching and learning of mathematics?

It is essential that teachers and students have regular access to technologies that support and advance mathematical sense making, reasoning, problem solving, and communication. Effective teachers optimize the potential of technology to develop students' understanding, stimulate their interest, and increase their proficiency in mathematics. When teachers use technology strategically, they can provide greater access to mathematics for all students.

For more information on this statement, go to **www.nctm.org/about/content.aspx?id=31734**.

What does it mean to hold high expectations for students in mathematics education?

To hold high expectations means to engage all students in cognitively challenging tasks that are simultaneously within reach and rich enough to stretch students as far as they can go. Holding high expectations does not necessarily mean accelerating coursework or presenting material that is more difficult or should be done faster. Teaching with high expectations for all students ensures greater understanding for every student.

For more information on this statement, go to **www.nctm.org/about/content.aspx?id=31777**.

What factors should be considered in evaluating the effectiveness of teachers of mathematics?

Teacher effectiveness is one of the most important controllable in-school factors in student learning. Both teachers and students stand to benefit from a comprehensive system of teacher evaluation that considers data from multiple domains of professional practice over time. Current evaluation trends that include students' test scores and value-added measures of teacher effectiveness should not obscure the fact that these aspects of effectiveness should constitute at most one factor in an array of factors considered in evaluating teachers of mathematics.

For more information on this statement, go to **http://www.nctm.org/about/content.aspx?id=31267** ▲

▲ Call for Manuscripts

The *Ontario Mathematics Gazette* is inviting manuscripts for all grade levels.

Instructions for submission of manuscripts are found on page 1 of any *OMG*.

Contact the Editor for further details.

▲ What's the Problem? Making it Right

SHAWN GODIN
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Shawn is the head of Mathematics, Business and Library at Cairine Wilson S.S. in Orleans, Ontario. He is currently Editor-in-Chief of Crux Mathematicorum with Mathematical Mayhem.

Welcome back to a new school year, problem solvers. Last time, I left you with the following problem:

> Two cans, X and Y, both contain some water. From X, Tim pours as much water into Y as Y already contains. Then, from Y, he pours as much water into X as X already contains. Finally, he pours from X into Y as much water as Y already contains. Each can now contains 24 units of water. Determine the number of units of water in each can at the beginning.

This problem is question #1b from the *British Columbia Secondary School Mathematics Contest 2010, Junior Final Round, Part B*. It appeared in the column *Skoliad Corner* from the October 2010 issue of *Crux Mathematicorum with Mathematical Mayhem*, and the solution appeared in the May 2011 issue. You can check out more problems at the Crux Mathematicorum with Mathematical Mayhem website, **journals.cms.math.ca/CRUX/**.

As with many problems where the solution doesn't reveal itself immediately, we can gain some insight by doing trial and error. Take a few minutes for yourself and try out some values.

We could solve the problem by just trying every possibility from 0 in X and 48 in Y, all the way to 48 in X and 0 in Y, or 49 different possible starting configurations. Almost immediately, you should realize that container X must have at least as much as container Y, or else you can't even start. This cuts our possibilities significantly. On the other hand, if the amount of water in Y is too small, then even after it is doubled after the first step, there won't be enough to complete the second step. For example, if we start with 37 units in X and 11 in Y, then the first step leaves 26 in X and 22 in Y, and we don't have enough in Y to complete the second step. These revelations and some patterns in the values should lead to a solution fairly quickly.

Of course, if the numbers were larger, trial and error might take too much time. As such, the problem lends itself nicely to algebra. If we let x represent the number of units of water in X at the start, and y represent the number of units of water in Y, we can model the problem algebraically. To help, the table below uses the example of starting with 35 units in X and 13 in Y beside the algebraic model to see how they work:

Numeric

X	Y
35	13
35 – 13 = 22	2 x 13 = 26
2 x 22 = 44	26 – 22 = 4
44 – 4 = 40	2 x 4 = 8

Algebraic

	X	Y
Start	x	y
Step 1	$x - y$	$2y$
Step 2	$2(x - y) = 2x - 2y$	$2y - (x - y) = 3y - x$
Step 3	$2x - 2y - (3y - x = 3x - 5y$	$2(3y - x = 6y - 2x$

Notice that at each stage of the algebraic model, the sum of the amount in the X can and the Y can is $x + y$. So the solution to our problem is the solution to the system of equations:

$$3x - 5y = 24$$

$$6y - 2x = 24$$

which has solution $x = 33$ and $y = 15$. So Tim starts with 33 units in can X and 15 units in can Y.

The nice thing about the algebraic solution is that we can use it to solve any variation of the problem. For example, if instead of them both ending up with 24 units we wanted 37 units in X at the end and 106 units in Y at the end, we would just solve the system

$$3x - 5y = 37$$

$$6y - 2x = 106$$

to get $x = 94$ and $y = 49$. You have to be careful because the algebra doesn't take into account steps that are not allowed, like having negative values for x or y at some intermediate step.

Of course, the algebraic solution isn't always the "best." I will leave you with my personal favourite solution for this problem. A powerful problem-solving technique is to work backwards. In this problem, one of the cans gets its water doubled, the water coming from the other can. If we play this in reverse, one of the cans gets its water cut in half and the extra water goes to the other can. If we play backwards from the end (the water in Y was doubled at the end), we get:

X	Y	
24	24	Step 3
24 + 12 = 36	24 ÷ 2 = 12	Step 2
36 ÷ 2 = 18	12 + 18 = 30	Step 1
18 + 15 = 33	30 ÷ 2 = 15	Start

which is our solution. Now for your homework:

Determine the maximum possible value of the function

$$f(x) = \frac{2x^2 - 8x + 17}{x^2 - 4x + 7}$$

Until next time, happy problem solving. ▲

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▲ Not-quite-so-elementary Mathematics: Rolling Dice In The Key Of Mozart: Composing Probabilistic Piano Pieces

LYNDA COLGAN
E-MAIL: colganl@educ.queensu.ca



Lynda Colgan's career has included roles as a classroom teacher, a university professor, and newspaper columnist. Her contributions to mathematics and its teaching have been recognized through awards such as the Marshall McLuhan Foundation Distinguished Teacher Award. Lynda always exhibits a passion for mathematics and views her professional mission as dispelling the myth that math is the bad guy.

> "Music is a secret arithmetical exercise and the person who indulges in it does not realize that he is manipulating numbers."
>
> **Gottfried Liebniz** 1646–1716
> an inventor of the calculus and a contemporary of J.S. Bach

This summer, I had the privilege and pleasure to be a workshop leader at SIET 2012: The Summer Institute for Elementary Teachers that was held at the Canada Museum of Science and Technology, in Ottawa. There I met over 30 teachers who had travelled from as far away as The Yukon, Newfoundland and Egypt, and Thunder Bay and Quebec City to participate in this three-day professional development opportunity. The mix of teachers in the room was eclectic: Anglophone and Francophone, novice and veteran, and, sciencephile and sciencephobe. The workshops were a mix of hands on, field trip, and lecture; were led by museum educators, university faculty, and field experts; and focused on not only "how to" but "why to" engage students in particular science, technology, engineering, and mathematics (STEM) learning activities.
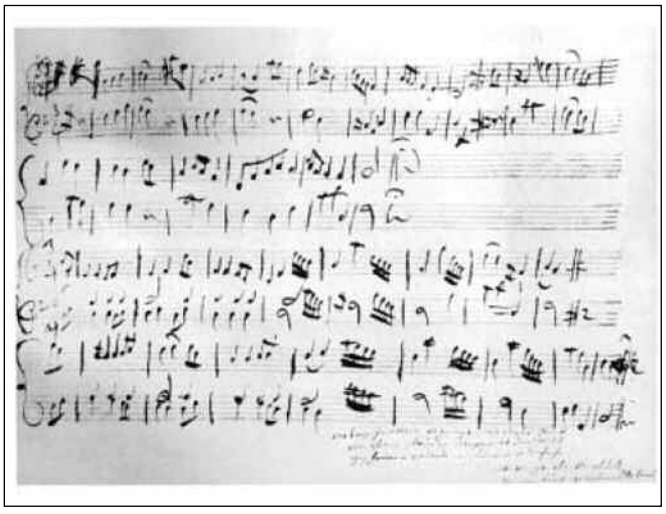
I chose to focus on curriculum integration: specifically, embedding mathematics in The Arts. I designed tasks to illustrate anthropometric ratios and proportions based on da Vinci's Vetruvian Man and Jonathan Swift's Gulliver. Together, the teachers and I investigated built-in "body

rules" and their application to life drawing; used Escher's famous metamorphosing designs to create our own original mutating tessellations; and, generated a geometric quilt square based on The Drunkard's Path code from the Underground Railroad.

It was such a rich learning opportunity for me to prepare the resources for the SIET teachers, that I decided to extend the experience and generate a curriculum-integration activity for the first edition of the new volume of The Gazette. I hope you and your students enjoy the background information and activity instructions for investigating how probability and music can be integrated in enjoyable and meaningful ways… thanks to Mozart.



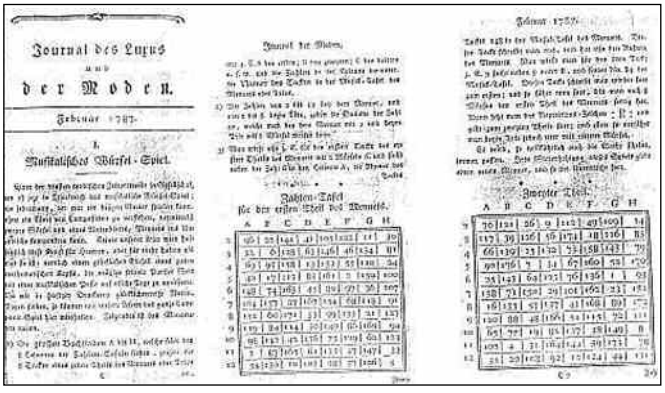
Wolfgang Amadeus Mozart (1756–1791), an Austrian musician and composer, was a child prodigy—he started playing the keyboard at the age of three; composed his first piano pieces at age five (his *Minuet in G and Trio in C for Piano (K.1)*, composed at the age of five in 1761, is shown below); symphonies at nine and complete operas at twelve. During his lifetime, Mozart composed 626 original works. If you were to sit down and listen to all of his compositions, you would need more than 200 hours, and in that time, you would have heard symphonies, operas, concertos, sonatas, serenades, masses, and minuets.



In addition to music, we are told that Mozart loved games and puzzles of all sorts, and enjoyed performing card tricks and charades. His hobbies included billiards, fencing, and horseback riding, and tending to his considerable collection of pets—which included dogs and cats and a starling that could sing the main tune from the last movement of his own *Piano Concerto No. 17.*

The most interesting biographical details about Mozart, however, are in those excerpts that describe his passion for mathematics. One biography says that as a young child learning arithmetic, he once covered the walls of all the rooms in his house with numbers and calculations. According to his sister, Nannerl, Mozart "talked of nothing, thought of nothing but figures" during his school days and often wrote to her, asking her to send him more mathematical tables and exercises in mathematics. We are told that he even jotted mathematical equations in the margins of some of his compositions, including *Fantasia and Fugue in C Major*, where he calculated his odds of winning a lottery.

Mozart used mathematics in a variety of ways to guide how he composed some of his musical pieces. One of the most interesting ways in which Mozart composed music was by using a dice game, *Musikalisches Würfelspiel*. The idea of the musical dice game is to cut and paste pre-written fragments of music randomly together to create a new composition. It is said that Mozart often played this as a party game, challenging his friends to contests in composing minuets, using dice.


Journal des Luxus
und
der Moden.
Februar 1787.
Musikalisches Würfel-Spiel.
Zahlen-Tafel
für den ersten Theil des Menuets.
Zweyter Theil.

The basis of the musical dice game consists of 176 pre-written musical measures (bars) and a table of rules used to select specific measures given a certain dice roll. The musical composition that results has 32 measures—16 randomly selected minuet bars and 16 randomly selected trio bars. What is amazing is that no matter which combination is chosen, the musical snippets fit together to make a pleasant composition.

To select the first measure, a player would roll two dice and sum the total, then refer to the table. The numbers in the far left column represent the possible sums from rolling the two dice and the numbers in the top row represent the number of the measure. For example, in composing measure 1, if you rolled a 9, you

would play measure 119 of the 176 possible minuet measures. In composing measure 2, if you rolled a 7, you would play measure 157. This is done 16 times to complete the first part of the composition.

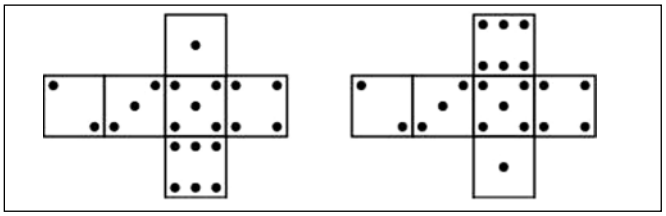
•	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
2	96	22	141	41	105	122	11	30	70	121	26	9	112	49	109	14
3	32	6	128	63	146	46	134	81	117	39	126	56	174	18	116	83
4	69	95	158	13	153	55	110	24	66	139	15	132	73	58	145	79
5	40	17	113	85	161	2	159	100	90	176	7	34	67	160	52	170
6	148	74	163	45	80	97	36	107	25	143	64	125	76	136	1	93
7	104	157	27	167	154	68	118	91	138	71	150	29	101	162	23	151
8	152	60	171	53	99	133	21	127	16	155	57	175	43	168	89	172
9	119	84	114	50	140	86	169	94	120	88	48	166	51	115	72	111
10	98	142	42	156	75	129	62	123	65	77	19	82	137	38	140	8
11	3	87	165	61	135	47	147	33	102	4	31	164	144	59	173	78
12	54	130	10	103	28	37	106	5	35	20	108	92	12	124	44	131

To compose the second part of the composition, only one die is used. The composer needs to roll the die 16 times in total and use the table below. The numbers in the far left column represent the possible numbers on the die. The numbers in the top row represent the number of the bar. The numbers in the body of the table represent the pre-written musical measure to be played.

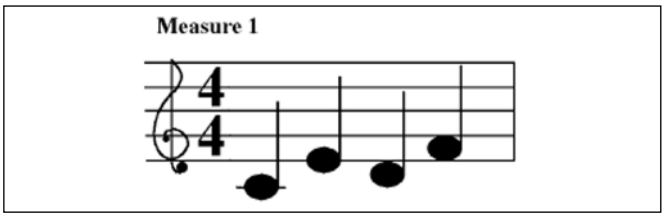
•	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
1	72	6	59	25	81	41	89	13	36	5	46	79	30	95	19	66
2	56	82	42	74	14	7	26	71	76	20	64	84	8	35	47	88
3	75	39	54	1	65	43	15	80	9	34	93	48	69	58	90	21
4	40	73	16	68	29	55	2	61	22	67	49	77	57	87	33	10
5	83	3	28	53	37	17	44	70	63	85	32	96	12	23	50	91
6	18	45	62	38	4	27	52	94	11	92	24	86	51	60	78	31

In principle, the system produces some 45 949 729 863 572 161 possible waltzes. “The number is so large that any waltz you generate with the dice and actually play is almost certainly a waltz never heard before,” mathematical games columnist Martin Gardner once remarked in an article about automated music composition. “If you fail to preserve it, it will be a waltz that will probably never be heard again.”

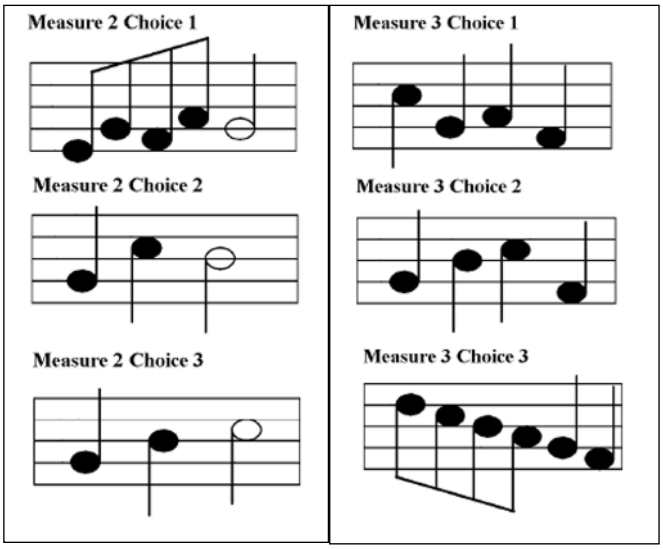
If you want to try your own simple version of a musical dice game, here are some musical measures and the rules. You will create a four-measure melody that you can hear by playing it on a xylophone or recorder. If you need to make your own die, here is a template.

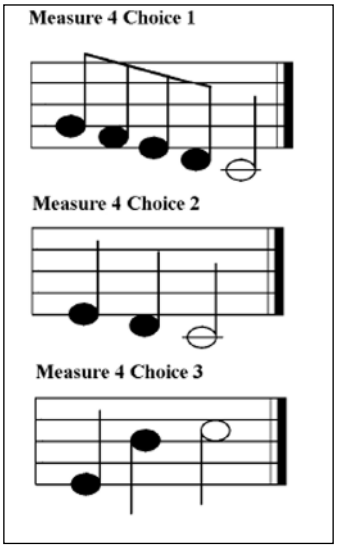


There is only one choice for the first measure.



To choose each of the other three measures, roll the die. If you roll 1 or 4, choose choice 1 for that measure. If you roll 2 or 5, choose choice 2; and if you roll 3 or 6, choose choice 3.





How many possible different four-bar melodies can you make with this set of rules? What is the probability of choosing choice 1 for measure 2? What is the probability of choosing choice 1 for measure 2, choice 2 for measure 3, and choice 3 for measure 4?

Now that you’ve mastered a simple cut-and-paste composition, try your hand at composing a 32-bar musical creation, using dice and Mozart’s tables, by going to this award-winning website: **sunsite.univie.ac.at/Mozart/dice/**. You’ll even be able to listen to each of Mozart’s 176 musical measures and play your original work of art as a Midi file.

Who knew that mathematics could be ear opening? It’s mathemagic. ▲

▲ Data Rocks – Teachers Are Encouraged To Switch From E-stat To Cansim

JOEL YAN
E-MAIL: joelyan@rogers.com



Joel Yan, a volunteer education consultant for Statistics Canada, is a mathematician who is keen to help educators use Canadian data in mathematics teaching and learning.

E-STAT is a popular online learning tool, specially designed by Statistics Canada for use by the educational community. With E-STAT, you have been able to generate dynamic tables and customize graphs and maps, using Statistics Canada's Census of Population and socio-economic data. E-STAT and the Statistics Canada website (**www.statcan.gc.ca**) are referenced as recommended resources in the Ontario mathematics curriculum for several courses between Grades 6 and 12.

However, as noted elsewhere in this magazine, a major change is coming. Please note that E-STAT will be archived on June 30, 2012. E-STAT will remain accessible as is until June 30, 2013, but its content will no longer be updated. It will be discontinued July 1, 2013. For this reason, E-STAT users are encouraged to start using the free CANSIM database and to visit the census portal for timely information.

CANSIM is Statistics Canada's key socio-economic database. Updated daily, CANSIM provides fast and easy access to a large range of the latest statistics available in Canada. Since E-STAT's inception, first as a CD-ROM and then as a website, CANSIM has been the source of the time-series data on E-STAT. However, until February 2012, access to the complete updated CANSIM master database had been restricted to government and other stakeholders, as well as paying clients. And once a year, the CANSIM time-series data on E-STAT were updated from the master CANSIM database. This was usually done over the July 1 weekend so as to not interfere with the school year.

Given the free and open access to the complete updated CANSIM database, teachers and students are encouraged to make the transition from E-STAT to CANSIM. There are several advantages to using CANSIM over E-STAT, including ease of use and timeliness of the data. The one disadvantage is that there are not (yet) any built-in data-graphing capabilities in CANSIM as there are in E-STAT. However, CANSIM data can be easily downloaded and then graphed in a spreadsheet program, Fathom, or other graphing software.

Learning to Use CANSIM

E-STAT users should find it very easy to use CANSIM as it is very similar to E-STAT. Also, the CANSIM table numbers are the same as those in E-STAT. And CANSIM provides many online aids and help features.

To get started using CANSIM, we encourage you to:

- watch the informative 5-minute video Tutorial: *Introduction to the New CANSIM Interface* at **www.statcan.gc.ca/about-apercu/video/cansim-eng.html**
- click the "Help using this page" tab on the left side of many pages
- once you have opened a CANSIM table, click on the "Help" tab above the table
- work through the activity below, developed by Angela McCanny

CANSIM provides several options to help users find specific statistical data of interest:

- Search by text, table number, or series number
- Browse by Subject or by Survey (the specific survey that generated the data)

Do not forget to use the "Add/remove data" tab, if necessary, to customize the data you wish to extract from the selected table.

Other Resources to Help You Use CANSIM

With over 43 million time series on CANSIM from over 300 different surveys and administrative data sources, perhaps the biggest challenge is finding the specific data you want. Below are resources to help you use and find data on CANSIM.

1) **YouTube**

There are several useful videos available on YouTube that provide tips on searching and downloading CANSIM data, developed by Professor Frances Woolley from the Department of Economics at Carleton University.

2) ***The Daily***

One technique I have found very useful is to first search *The Daily* on **www.statcan.gc.ca**. Find a news item in The Daily that relates to a keyword that interests you (e.g., bees). Read the article to gain a summary of the news about this topic and the mathematical trends in the data. Usually the detailed data will be contained within the huge CANSIM database. Click on the link in the actual *Daily* article to the specific CANSIM table(s) containing the data of interest. *The Daily* article will also provide links to the specific surveys that captured the data, publications containing the results (free on the Internet), and experts you can contact for further information.

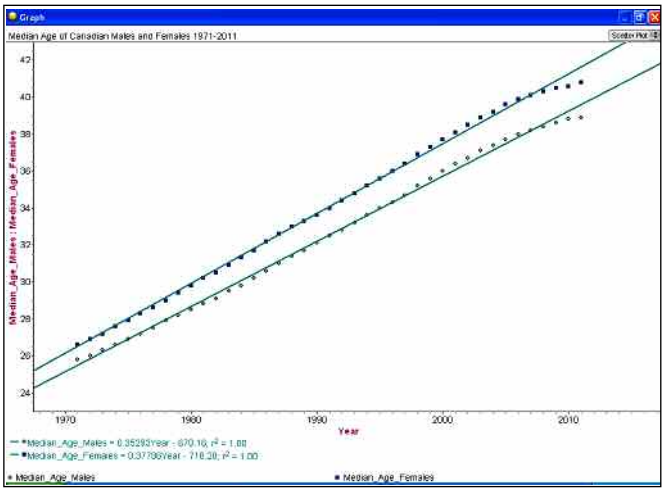
3) **Browse by Subject**

If you have a broad topic that interests you, scroll down from the home page at www.statcan.gc.ca and select Browse by Subject. Select a subject that interests you from the list provided (e.g., Children and youth). Then select a Subtopic of interest (e.g., Health and well-being). This lists and links to several resources, including Detailed tables from CANSIM. This way, you are immediately directed to CANSIM tables related to your selected topic. You also are given links for the same subtopic to the latest news releases, summary tables, publications, and more.

Activity for Teaching Yourself to Use CANSIM and Download Data into Excel/OpenOffice Calc, or Fathom

Angela McCanny has developed a complete activity (included in this issue) to help teachers become familiar with use of CANSIM, while answering the interesting question, "How has the median age of Canada's population changed over the last 40 years?" The median age data follows a very interesting increasing linear trend as Canada's population ages, and can lead to a discussion of the societal impact of an aging population. The resulting graph and suggested questions are given below.

Note: This activity is also available on **www.teacherweb.com/ON/Statistics/Math/** in the "CANSIM, E-STAT, and Function Modelling" folder.



Discussion Questions Based on This Graph of Change in Median Age of Canadians

Note: Suggested answers are shown in italics in brackets.

1) The linear model seems to be a very good predictor for median age between 1971 and 2011. Could we use this equation to predict the median age of Canadians in the year 1900? Explain why the predicted value would be reliable or not. *(If we substitute 1900 into the years, the calculated median age is very close to zero. This is not physically possible for Canada's population.)*
2) What change in the fit of the data to the linear model do you notice in the most recent years? *(The slope of the actual median age trend has definitely decreased, as recent data points all lie below the calculated least squares line.)*
3) For how many years into the future would the linear model with the equations shown be a good predictor *for the median age of Canadians? (Since the shape of the graph is changing, we would be better to recalculate a model based on data for the last few years.)*
4) What would be happening in Canada if the graph of median age continued to be linear with the same equation for the line of best fit until the year 2100? *(The median age of Canadians would then be over 60. It could be very difficult to maintain our economy given a much reduced labour force, and much increased health care expenses compared to today.)*

Website to Support Mathematics Teaching with Canadian Data

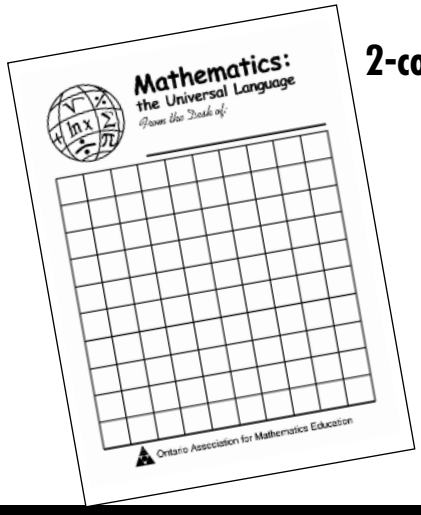
Given the major changes planned in the Statistics Canada learning resources to support education (see

note from Statistics Canada elsewhere in this *Gazette*) various other websites will be picking up some of the resources that may be deleted from www.statcan.gc.ca. I maintain one of these sites, currently named "Joel Yan's Math Resources Using Canadian Data." On this site, you will find a selection of resources to support mathematics teaching from grades 6 to 12, especially the Mathematics for Data Management (MDM4U) course. Here you will find useful datasets, lessons, and exemplary data-analysis projects developed by students for their MDM4U courses.

I encourage teachers to check out **www.teacherweb.com/ON/Statistics/Math/**, especially the new folder "Datasets from the StatCan Math Page". This contains many of the most used microdata file subsets that have been available on the Statistics Canada Data page for mathematics teachers. To be kept up-to-date on these resources, you may wish to add yourself to the Newsflash list. To do this, go to the "What's new?" page, then scroll down, click "Subscribe to This Newsflash," and follow the instructions. ▲

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▲ In the Middle – Developing Our Mathematical Senses: Seeing And Hearing Math, Using Various Technology Tools

CAROLINE ROSENBLOOM
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Caroline Rosenbloom is currently on secondment as an Elementary Coordinator in the J/I Midtown Technology Option and Mathematics Instructor in the pre-service program at OISE/UT. She has enjoyed working with intermediate students and teachers as a middle school teacher, Mathematics Instructional Leader for the Toronto District School Board, and Education Officer at EQAO. Caroline is co-author of "Investigating Mathematics Using Polydrons."

I've had an incredible summer vacation with my family in Amsterdam and Israel. Our excursions were exciting opportunities for us to influence our kids' view of mathematics. We photographed situations that capture ways to look at our world through a mathematical lens. As we strolled through the local streets, seeing historical sites or hanging out by the beach, if we chanced upon a mathematical moment, I would snap a photo and challenge my family with,

"What mathematical questions can you ask?"





Once they got the hang of it, they started taking their own photos and listing mathematical questions they had come up with on their own. This is the kind of thinking I want my own students to have. Not only do I want them to learn the math, but to see the relationships and connections in math around them.

Mathematics needs to be learned through inquiry. Students need to ask questions before they can answer questions. When students generate their own mathematical questions, they become interested in finding the answers themselves (or with a partner).

We also want students to see, hear, or touch math. By exposing our senses to the mathematics around us, we can make those connections and see relationships from the point of view of our personal experiences.

By modelling and providing tools in September, you set the expectations and the stage for a mathematical environment in your class. The use of technology is a growing part of mathematics pedagogy. It allows students to express their ideas in different ways, provides collaborative opportunities, and aids teachers in demonstrating math beyond the classroom.

Evidence of this can be found in a report by Project Tomorrow. This report demonstrates that middle school girls engage more in school when education involves the use of social network tools. By incorporating these tools into our programs, we are not only trying to use skills that adolescents have, but utilizing these abilities to socialize in a mathematical context.

Mirela Ciobanu, a middle school teacher at Samuel Hearne Middle School in the Toronto District School Board, sets high standards for mathematical thinking at the beginning of her program. Right at the start of the year, she introduces the three-part lesson, has students working in math pairs, and has students recording their work on laptops.



Students also use an online whiteboard called **Twiddla** to simultaneously post and share solutions in class. Mirella poses a math problem for groups of students to work on. Each group is given a space on Twiddla to record their solution. Everyone can see the different solutions as they congress their findings. Twiddla is a great tool to use in lieu of chart paper. Teachers can annotate student work as students explain their thinking. The work can be saved as an image, and then shared electronically.

Students in Mirela's class also share their thoughts, reflect on their homework solutions, and provide constructive comments about others' solutions through Kidblog. Parents are able to see what students are working on. They can see the mathematical thinking and language that has developed throughout the year.



Pinterest is a great tool that can be used to communicate images that demonstrate relationships in mathematics. I have recently started to use this tool to share photos that encourage students to see things through a mathematical lens.



Each photo challenges students by asking, "What mathematical questions can you ask?" Looking at these photos and generating a list of questions helps students develop their questioning skills. Students develop a sense of looking at math in different ways.

YouTube has an incredible library of videos. You can hear some mathematics through Vi Hart's **Doodle Music**.

Mathematics learned through inquiry provides students with opportunities to discover mathematical relations to other math concepts, to the world, and to themselves.

Here are some other ideas you can do to develop those inquisitive minds. Students can:

- use digital cameras to take photos, or can search for photos and
- generate a list of mathematical questions
- use a flip cam or photo booth to record themselves solving a mathematical problem
- use Comic Life to create a math puzzle or problem for other students to solve
- use Bitstrips to create a mathematical narrative
- compose mathematical patterns, using Garage Band
- find mathematical connections seen in TV or movies, art, music, nature
- create a mathematical performance as seen on www.mathfest.ca
- create a math game based on the concepts learned

Enjoy as you start your own mathematical journey with your new class!

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▲ Call for Manuscripts

The *Ontario Mathematics Gazette* is inviting manuscripts for all grade levels.

Instructions for submission of manuscripts are found on page 1 of any *OMG*.

Contact the Editor for further details.

▲ Fields Institute – MathEd Forum Notes

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Stewart Craven presently teaches courses for the Continuing Education Department and the Transition Years Program at York University. He is an active participant in the Fields Mathematics Education Forum.

Not unlike the many preceding years of the MathEd Forum, the meetings for 2011/12 have been diverse in topic, thought provoking for its participants, and valuable for the improvement of mathematics education across the province. Sadly, this past year marks the passing of our colleague, Margaret Sinclair. First and foremost she was known for her love of life and her care and compassion for all of those who were touched by her. Her career spanned elementary and secondary teaching, secondary school administration, and professorship in the Faculty of Education at York University. Some of the areas where Margaret made significant contributions include submissions to the Ministry regarding future teacher preparation, the College Math Project (a study that looked at how students performed as related to their high school courses), development of a B.Sc./B.A major in Mathematics for Education at York, research focused on junior teachers' understanding of geometry, and investigations about the use of technology in the mathematics classroom. She will be missed.

The last meeting for the year on April 28 focused on the mathematics of finance. The speakers included Desheng Dash Wu (University of Toronto): preparing a mathematician for a financial practitioner position; Nenad Radakovic (OISE, University of Toronto): teaching risk literacy within the secondary school curriculum: justifications and challenges; and Matheus R. Grasselli (McMaster University, Fields Institute): the role of mathematics in the in the ongoing financial crisis which began in 2008. The focus for the afternoon panel and general discussion was directed toward the place for financial literacy and/or the mathematics of finance in the secondary school mathematics program.

The program for the first meeting in September has

not been finalized, but the October meeting will be held at York University in honour of Margaret Sinclair. All are invited. Meeting agendas are posted at **www.fields.utoronto.ca/programs/mathed/forum/index.html**. ▲

Abacus: A Compilation

Volume 46 – This resource for teachers includes the concepts of addition, subtraction, multiplication, and division of whole numbers and decimals, alternative algorithms, mental math strategies, manipulatives, and links to literature. It will be useful for exploring these aspects of teaching and learning personally or with a team of teachers. Includes one two-coloured bead string.

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Volume 47 – This resource for teachers includes the concepts of equality and relational thinking, algebra as generalized arithmetic, algebra as functional relationships and expressing generality using multiple representations. With links to manipulatives and literature, it will be useful for exploring these concepts personally or with a team of teachers.

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▲ MB4T... Mathematics by and 4 Teachers

ANN KAJANDER
EMAIL: ann.kajander@lakeheadu.ca



Ann Kajander is an experienced classroom teacher currently teaching mathematics and mathematics education at Lakehead University. Her research interests relate to teachers' enhanced learning of mathematical concepts. She and her classroom-teacher colleague Tom Boland are currently writing a new book for teachers called Math Without Memorization: Big Ideas in Modelling and Reasoning for Elementary Teachers and Other Learners.

In this new column, mathematical ideas as needed and thought about by teachers, and further developed for teachers are offered. Readers are encouraged to submit ideas for future columns!

Areas and Volumes – The Power of Modelling

Problem solving, as we now think of it in teaching and learning, involves the capacity to solve new problems—those for which we *do not* have a formula or method readily at hand. As exciting as such a learning mode can be, this paradigm puts the onus on teachers to be prepared for the unknown—including unpredictable student responses to problems and tasks. Responding to unexpected student queries and methods can be a challenge! In my experience, there never seems to be an end to the wonderful ideas students construct in a rich math environment; however, solutions which are partially correct or contain a conceptual flaw (meaning a misconception or unclear concept mixed in with ideas that are understood well) in otherwise insightful work can be especially challenging for teachers to unpack.

The purpose of this column generally is to provide an opportunity to explore some of the specialized mathematical models and concepts that can be useful to teachers to support conceptually based student thinking and learning. Conceptually based learning might be thought of as learning that originates in student exploration of concepts—rather than the provision of

methods by the teacher. Unfortunately, opportunities for teachers to develop such specific-to-teaching mathematical concepts are still not as readily available as they should be. It is telling that, according to our research (e.g., Kajander, 2010; Kajander and Holm, 2011), elementary teacher candidates tend not to understand such specialized models and concepts as well as they would like to when they enter teacher-preparation programs. We have found that a modelling approach (Lesh and Doerr, 2003) is a particularly helpful way of supporting the mathematical development of both new teachers and more experienced teachers alike.

In the discussion to follow, the conceptual underpinnings (the understandings and models that give these concepts meaning) of length, area, and volume will be explored. Consider the following problem, which appeared on an EQAO test a few years ago:

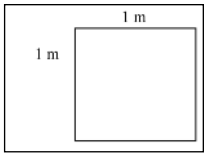
> Elda has a rectangular piece of paper with an area of 0.12 m^2. She cuts this piece of paper into small rectangles, each with an area of 200 cm^2.
>
> What is the maximum number of these small rectangles that Elda can cut?
>
> a) 6; b) 12; c) 24; d) 60. (EQAO, 2009, p. 12)

This problem was reported to have caused a number of queries as to whether the "correct" answer was missing from the responses. (Take a moment to think about how the problem might be solved, if you have not seen it before.) I was curious about how pre-service teachers would respond to a similar question, and so I constructed the following task for use in my pre-service classroom:

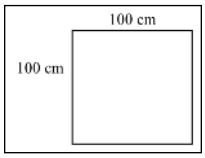
> A rectangle has an area of 0.20 square metres. At most, how many rectangles of area 400 square cm could be made to fit in it?
>
> Draw a model and use it to explain your reasoning.

Many pre-service teachers asked if there was a "typo" in the question. Some simply answered "0." Those who did not draw a model were typically lured in to (mis)applying an algorithm, and tried to multiply the square metres by 100 to calculate square centimetres—yielding the final answer of "none." Let's look at how a model immediately makes this problem clearer. It should be noted that when tackling a new problem, it can be hard to know what sort of model is best. Often several attempts are needed to find a useful model. To follow is one possible starting point.

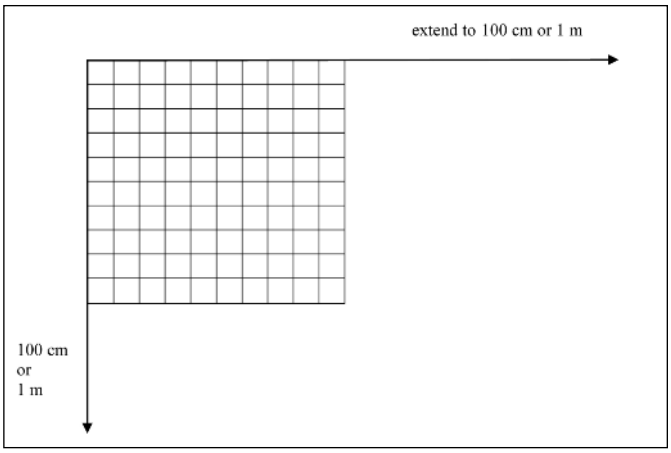
For the example just given, we might begin by simply constructing a simple model of a square meter:



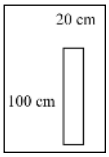
Next we might refine the model to add in the centimetres:



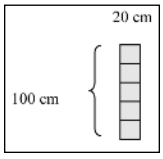
This new model might be enough to remind us that the number of square centimetres is the area, which we find by *multiplying* the dimensions of the region. Or, to create a more detailed and accurate model for students, it might help to create a one square metre region on the floor, using four metre sticks. After creating the square, placing a square decimetre (the "hundreds" piece or "flat" from a set of base-10 blocks) in the corner helps us visualize the remaining area in square centimetres. Suddenly we are able to visualize the many square centimetres in our square metre model—the "100 times 100" – or 10 000—square centimetres in a square metre become "visible" to us.



Returning to the sample problem, we see that students could use such a model in multiple ways to help them solve the problem. For example, for the "how many 400 cm^2 rectangles in 0.2 m^2" problem, there are a number of ways in which students could represent the 0.2 m^2. One way might be to draw a region that is 1 m by 0.2 m (we can also think of the 0.2 m as 1/5 m or 20 cm).



Thinking in centimetres, we see this region is 100 cm by 20 cm, or 2000 cm^2. If we draw our 400 cm^2 rectangles as 20 cm by 20 cm (of course, there are other ways), we can fit in 5 of them:

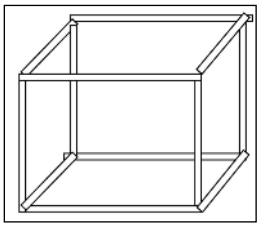


Other arrangements of both the 2000 cm^2 and

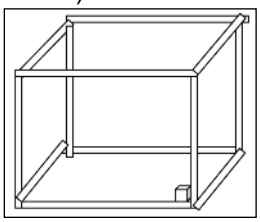
400 cm^2 are also possible, and exploring and comparing other student models and methods helps students visualize flexibly.

Knowing how and when to help students construct appropriate models is an important component of teachers' own professional mathematics knowledge. While it is important not to tell students what method or procedure they should use for a given problem, as this robs them of the opportunity for mathematical thinking, it *is* important that students be provided with the right tools—such as knowledge of models—to solve the problems they are asked to do. The ability to construct an area model to help with area conversions is one such important tool.

Let's go one step further and think about volume conversions, using a similar modelling approach. With the square metre still constructed on the floor, using the four metre sticks, how might we now model a *cubic* metre? Perhaps students could help out in the construction, each of them holding yet another metre stick vertically, at each corner or vertex of the square base. Then four more metre sticks could be used to form the top four edges of our *cubic* meter—perhaps we could fit in four more student helpers around our model—or even just imagine the top four edges:



Placing a thousands cube (or "block") from a set of base-10 blocks in the corner of our metre stick cube model might make the number of cubic centimetres in a cubic metre come to life in a similar way as with the square centimetres:



We might already know that there are a thousand cubic centimetres in the thousands cube. And now we see that there are *a lot* of thousands cubes in that cubic metre! So there are even more cubic centimetres in a cubic metre... in fact, there are 100 x 100 x 100, or 1 000 000 of them!! (Can you think of other ways to calculate this? For example, we could start by finding the number of thousands cubes in the cubic metre.) The metre stick "cube" helps us visualize that there are a million cubic centimetres in a cubic metre. Wow!

Sometimes I get asked, "When should I take away the manipulatives?" or "Shouldn't students eventually be able to do these calculations using a procedure?" The important idea is that the ability to construct a suitable model is one of the most important mathematical tools we can help children develop. Indeed, professional mathematicians often discuss problems with one another using gestures and sketches. However, once students have internalized a model for a given concept, they will eventually be able to conjure it in their mind. When they are able to visualize mentally, they will naturally drop the manipulatives or drawn models because it is simply faster to use a mental image. The key point, then, is to let the *students* decide when to use a physical model, and when to simply mentally visualize. Both are powerful.

Sometimes students do come to our classrooms with misconceptions developed previously. Students who confuse, for example, 8 cm and 8 cm^3, should be encouraged to spend more time using visual models. Teachers might also be aware of the conceptual distinction between additive reasoning (for example, 8 cm is a *length* of 1 cm + 1 cm + 1 cm + 1 cm + 1 cm + 1 cm + 1 cm + 1 cm), and *multiplicative reasoning* (for example, 8 cm^3 is a volume of 2 cm by 2 cm by 2 cm). Early and continued experiences with models can help avoid the development of such misconceptions, or provide students a way to work through them.

Teachers can also guide classroom conversations more effectively by making use of *horizon content knowledge* (Zaskis and Mamalo, 2011), that is, knowledge of where a given concept might be heading, and how ideas connect to one another. As an illustration, a student who asks about the connection between the m^3 notation and exponents, for example, might be encouraged to reflect on that connection. A term such as 1 m^3 describes a volume equivalent to a cube of dimension 1 m by 1 m by 1 m, while the term x^3 can be thought of, among other ways, as representing a volume equivalent to x units by x units by x units. Teacher knowledge of *degree*—which might very roughly be described as the largest number of multiplications of a variable in an expression (which could be given by the largest exponent)—would help students make such connections. Giving students a model or visual basis for ideas early on, and supporting their theories and observations with discussion, can add meaning and power to later use of algebraic notation. The teacher's mathematical knowledge can be the key to such important early insights.

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▲ VISUAL REASONING K–12

MARIAN SMALL
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Marian Small is Professor and Dean Emerita of the University of New Brunswick. She works with many K-12 Ontario teachers on questioning in the math classroom and focusing on big idea. She is the author of many resources including Good Questions: Great Ways to Differentiate Instruction Mathematics Instruction and Big Ideas from Dr. Small: Creating a Comfort Zone for Teaching Mathematics.

Reasoning is an important element in mathematics learning, whether the learner is in Kindergarten or in Grade 12. Visuals, either with or without accompanying words, can be extremely powerful tools for reasoning and explaining (Nelson, 1993); they are powerful not only because they help us understand more quickly, but also because sometimes they lead us to be more general than when we use specific numerical examples. This can be true at any grade level.

Consider Primary Math:

Example 1:

Suppose you want students to explain why the order of addition does not matter.

Student 1:

One student might use several examples, e.g., show that

$$4 + 3 = 7 \text{ and } 3 + 4 = 7$$
$$5 + 9 = 14 \text{ and } 9 + 5 = 14$$
$$2 + 8 = 10 \text{ and } 8 + 2 = 10$$

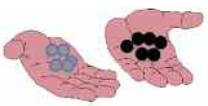
Student 2:

Another might simply state: If you put things together, you can put them together any way you want.

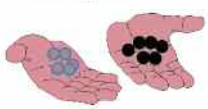
Student 3:

A third student might set up this more persuasive visual argument.

Sandi holds some grey counters in one hand and some black ones in the other.



Then she switches which counters are in which hand.



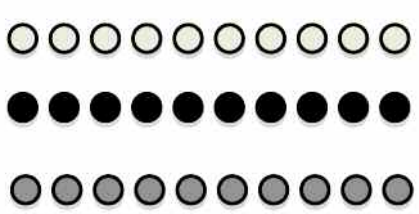
But since she has the same counters, she knows that it doesn't matter in which order you add.

Example 2:

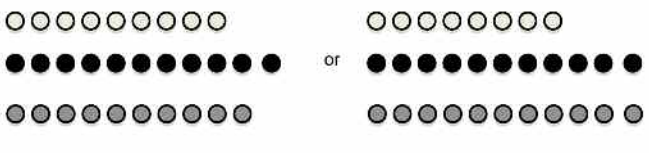
You want students to prove that if there are three (positive) numbers that add to 30, at least one of them is 10 or more.

A visual argument might look like this:

Start with 10 + 10 + 10, which is 30. In this case, at least one of the numbers is 10 (in fact, all of them are), as required.



If you trade some of the white counters for black and/or grey ones, there will be fewer than 10 white, but more than 10 of either black or grey or both.



The same thing would happen if you had traded some of the black or grey instead of the white.

No matter how the counters are organized, there are at least 10 counters of one colour.

Consider Junior Math:

Example 1:

You want students to prove that there is a pair of whole numbers with a sum of 100 and a difference of 8 and only one such pair.

A student in a higher grade, who has learned algebra, might write:

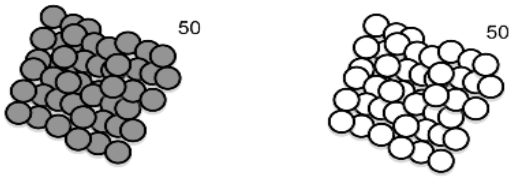
$$x + y = 100$$

$$x - y = 8$$

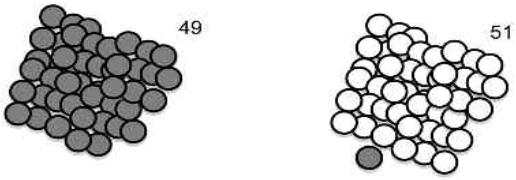
and solve the system of equations.

But the problem is accessible to junior students in a variety of ways, including visual reasoning.

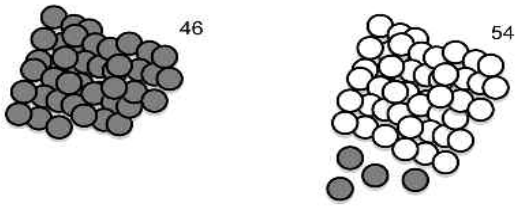
A student might create two piles of 50 counters each, for a total of 100 counters.



The problem is that the piles have equal numbers of counters and we need one pile to have an extra 8. The student begins moving 1 counter at a time, noticing that if he moves 1 grey counter to the white pile, the difference in pile sizes becomes 2 (not 1).



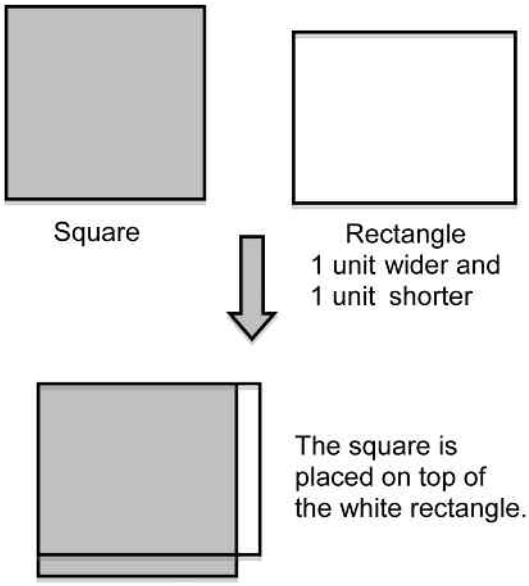
Continuing to move one grey counter over at a time, the student sees that when he or she has moved 4 grey counters over, the difference in pile size is 8. The two numbers are 46 and 54.



It is clear from the diagram that if more grey counters were moved to the white pile, the spread in pile size would increase, and would no longer be 8. So there is only one solution.

Example 2:

You want students to prove that the area of a square is always 1 greater than the area of a rectangle that is 1 unit longer, but 1 unit less wide. For example, a 6 x 6 square has area 36 square units, while a 7 x 5 rectangle has area 35 square units, 1 square unit less.



There is a thin grey rectangle at the bottom of the square that does not sit on the rectangle. There is also a thin white rectangle on the side that is not covered by the square. The thin grey rectangle is 1 unit longer than the thin white rectangle is long; that's the extra 1 unit in the square.

Consider Intermediate Math:

Example 1:

You want students to prove that the sum of two multiples of 3 is a multiple of 3. Although algebra and the distributive principle can be called on, i.e., [$3x + 3y = 3(x + y)$], a visual model can also be used.

Any multiple of 3 can be represented as the area of a rectangle with width 3 and length a whole number.



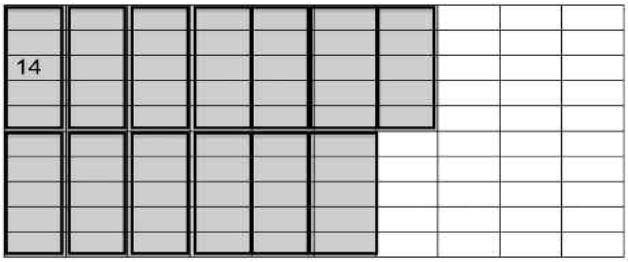
The sum of those multiples of 3 can be represented as the area of a rectangle with width 3 and a whole number length. It is, therefore, a multiple of 3.



Example 2:

Sixty-five percent of the students in a school participated in a read-a-thon. There were exactly 182 participants. How many students are in the school?

Although a student could set up a proportion in the form $\frac{65}{100} = \frac{182}{x}$, a diagram could also be useful. A student could draw a hundred chart to show 65%.



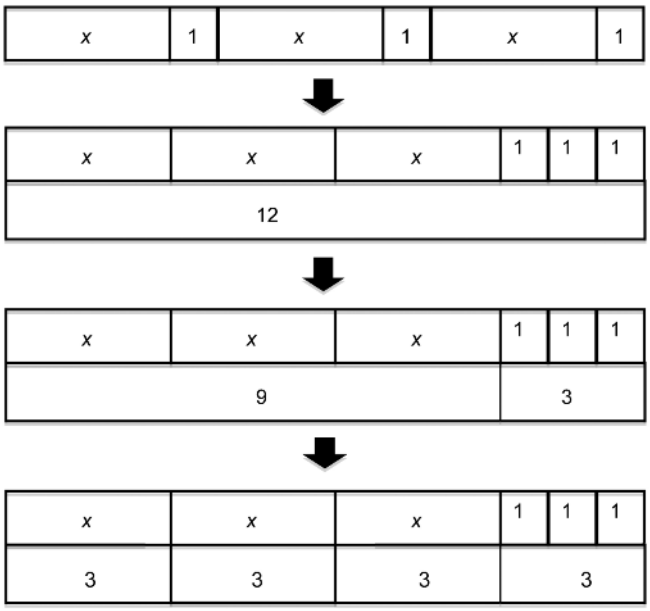
The grey section is worth 182, so each of the 13 groups of 5 squares represents 14 (182 ÷ 13) participants. That means the white section represents 28 + 28 + 28 + 14 = 98 non-participants. The total number of students is 182 + 98 = 280.

Consider Secondary Math:

Example 1:

You want students to solve $3(x + 1) = 12$.

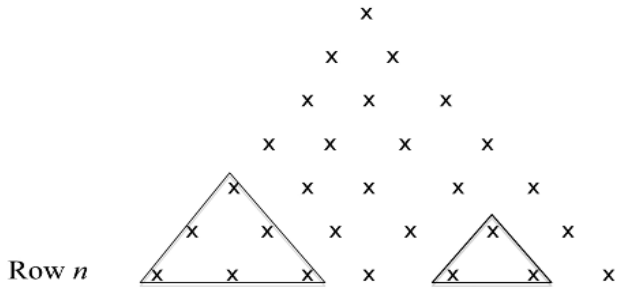
They might use a visual interpretation, where the first diagram shows 3 groups of $x + 1$ and the second diagram rearranges those pieces and shows that the total value is 12.



It is clear that $x = 3$.

Example 2:

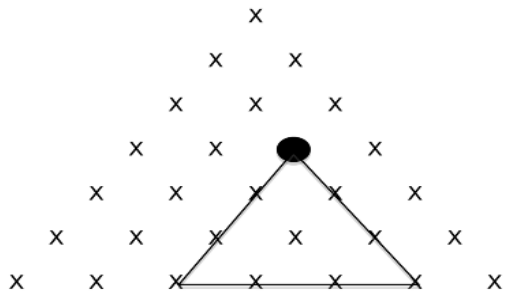
You want your students to prove that ${}_nC_2$ is $1 + 2 + \ldots + (n\text{-}1)$.



$_nC_2$ is the total number of ways you can choose two items from Row *n*, since that row has *n* items in it.

Any pair of items in Row *n* can be uniquely related to a single *x* in the diagram, specifically the vertex of the "equilateral triangle" of which it is base, so counting the number of possible item pairs in Row *n* is the same as counting the number of vertices of equilateral triangles there are.

It turns out that each item in Rows 1 to Row (*n*-1) is the vertex of one such triangle. For example:



Therefore, $_nC_2 = 1 + 2 + \ldots + (n-1)$.

Summary

In summary, it should be clear that no matter what the grade level, visual reasoning has a place. Not only does it support the so-called visual learner, but it supports all students.

The reasoning suggested by the visual is often more general than using specific examples, so this approach also supports our interest in helping students learn to generalize.

References

Proofs without words. (n.d.) Retrieved from **www.mathoverflow.net/questions/8846/proofs-without-words**

Nelson, R.B. (1993). *Proofs without words.* Washington, DC: Mathematical Association of America. ▲

▲ Anyqs II: The Wrath of Khan

NICO ROWINSKY
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Nico loves teaching, learning, and writing about math. He has been a teacher with TDSB for over ten years.This is his second article about Dan Meyer.

Webspace, the final frontier. These are the stories of the math voyageur, Dan Meyer. His five-year mission: to explore strange new videos, to seek out new life and civilizations in the math classroom, to boldly go where no math textbook has gone before.

Perhaps this intro might seem too dramatic for a math teacher from California on a crusade to spread his love of math, but the fact remains that since September 2006, Dan Meyer has been putting his experiences all on the line (and all online) for teachers around the world to read.

His blog, with heading "dy/dan - less helpful," captures a lot about the Dan Meyer philosophy. Clean and simple, a minimalist aesthetic that transfers over to his lesson design and in his video posts, where the students are ultimately inspired to ask the questions. Sharing online is what he does best.

Every idea, video, or picture that he feels will inspire student questions will be posted on his blog or Twitter feed. Attaching #anyqs prompts his readers to submit questions that immediately come to mind.

"Piloting my curriculum w/#anyqs so I know if it's any good before it hits a student's desk. That's huge," writes Dan (on Twitter) justifying his use of the #anyqs tag. And it's not just him who benefits. His modest 6580 followers take full advantage of his posts to start the conversation of what works and what doesn't.

Any Question Is a Matter of Perplexity

"Perplex them!" It's one of Dan's battle cries in workshops, and along with "be less helpful," the messages are starting to become clear. Dan's Perplexity Sessions[1] continuously harp on these themes. Give the

[1] *perplexity.mrmeyer.com*

students enough to start themselves and give them as little help as possible along the way.

Consider a classic math problem. A boat travels against the flow of a river. Given some relevant information, calculate the time of travel from A to B.

How does Dan handle this "River Boat" problem? Simple. First he films himself climbing a flight of stairs. Then descending a similar flight. Then climbing a flight of the UP escalators, and descending a flight of the DOWN escalators. He clips all of these together in one 30-second video[2] and it ends with him beginning to climb the DOWN escalators. That's right, he's going against the current.

In 30 seconds, he has taken a question out of a textbook, framed it in a context that all students—or at least those who have visited malls with escalators—can relate to, and finally created a cliffhanger that begs the viewer to ask some questions. In this case: How long will it take this boat called Dan to climb this river of escalators?

The questions don't need to be written. The students will come up with them on their own. Or at least that's the intention of the first step of the lesson. The rest of the lesson sort of falls into a familiar framework.

Step 1: Show the video and ask the students for questions that perplex them.

Step 2: Ask the students to guess the answer, and ask for an upper-bound, and lower-bound guess.

Step 3: Ask students to define information they will need to answer the question.

Step 4: Give them a pile of information (lengths, heights, depths, times… measurements, measurements, measurements).

Step 5: Give them time to work.

Step 6: Reveal the answer by playing the rest of the video.

If you combine some of these steps, you can see the makings of his "Math Story in Three Acts."

But it's not all smooth sailing for this enterprising young mathematician. There are issues that arise during the lessons. But with Dan's open approach, these issues are also posted on his blog. He confronts them right there, "On Screen." The issues are then either addressed or are further discussed by other teachers in the comments section of the blog. It's the fact that he fully embraces his position as a math educator, with all its failures and achievements, that makes it helpful to other educators.

Another 30-second video attempts to show the difference in water usage between a shower and a bath. In the video, titled "Shower V. Bath," Dan enters in both in a T-shirt and shorts, and even dons a pink shower cap.

> "At a certain point, you say to yourself, 'Hell, I was never gonna run for city council anyway,' and you put on a pink shower cap and you make a math problem.'"
>
> ~Twitter post embracing being a math educator

Each step of the way, he searches for opportunities to make the ordinary textbook problem into a real situation that perplexes students. And although he'll be the first to admit that these videos "doth not a complete curriculum make,"[3] it does seem like an approach that moves away from the lecturing end of the spectrum.

Sal Khan and the Lecturing End of the Spectrum

The Khan Academy, created by Salman Khan in 2006, is a collection of online video lectures that lets users access them for free. Its initial focus was mathematics, but it has branched out into other fields of study, although mathematics still seems to dominate their selection of videos. Oh, and it's big. How big? Khan Academy has received financial support from Google, personal endorsements and financial support from Bill Gates, was featured in Wired Magazine in the August 2011 issue, and Salman Khan's TED.com talk has had over 830 000 views.[4] It has also developed a platform to motivate learners by giving them virtual badges allowing "level-ups" and providing teachers with tracking interfaces. Essentially the gamification of learning.

The creation of the Khan Academy is arguably a massive positive contribution to the world of math students and educators. However, to Dan Meyer, the idea that everyone can learn fractions simply by watching a 10-minute video is problematic.

"Math should be developed in an environment where you can dig in, mess around, and play with the numbers."

Although Dan probably does not want to be labelled a critic of the Khan Academy, perhaps merely a contributor to constructive criticism, some of his comments have raised a few eyebrows.

---

[2] It's actually 31 seconds; see for yourself, ***blog.mrmeyer.com/?p=7649****.*

[3] From a tweet on Sept. 12, 2011.

[4] The same video on YouTube is over 1 million views.

The drama (read Wrath) started when Salman Khan went on the Colbert Report and argued that instead of "one-size-fits-all" lectures, teachers can now (using his academy) assign students a video to watch and that will help free up time and provide more human interaction with students. Dan's questioning response on his blog, paraphrasing Khan, was as follows:

"Don't give one-size-fits-all lectures to students. Instead, send them all home with the same ten-minute video.

Someone needs to tighten up that message."

The post created a big debate in the comments section of Dan's blog. Sixty-seven comments in total were posted over the next 24 hours, culminating in Dan's comment:

"Well, I'd say this thread has played itself out."

But it hasn't. The debate continues. Numerous articles, blog posts, and even the Wired[5] article has noted both drawbacks as well as praise for the Khan Academy.

Earlier this school year, one of Dan's posts brought up a revealing fact about a test run of the Khan Academy in Los Altos School District. The report showed that the Khan Academy did not show a statistically significant improvement in results. Dan posted the article on his blog, commenting,

"Seriously, I am puzzled. I would have guessed Khan's approach to lecture and practice would juice achievement scores on standardized tests, while tanking students' enthusiasm for math. Clearly, I need to keep a loose grip on my assumptions."

In anticipation of the aforementioned wrath of comments, Dan altered his commonly open and inviting comments section by closing them, comically saying,

"Comments closed in advance. I don't need another food fight."

The debate over the Khan Academy model will likely settle to a reasonable equilibrium of compromise from both sides. But for now, allow me a few more Star Trek references before the antagonism goes away.

A Twitter Wormhole

Some might say that there is little trouble one could get into in 140 characters. But brevity is prone to misunderstanding. Case in point, the October 24 post by Dan that some "retweeters" took the wrong way.

The original post had a link to an Info Graphic titled "When Am I Going to Use This?" The poster (an advertisement to a U.S. college) was a scatterplot of sorts, showing NUMBER OF MATH CONCEPTS on the x-axis, and on the y, MEDIAN ANNUAL SALARY of several occupations. Scattered throughout were jobs ranging from Police & Detectives, to Athletes & Coaches, and Lawyers on top. The point the graphic was trying to make was, "Hey kids, math is important." The point Dan was trying to make by showcasing it on Twitter was, "Hey, this poster does not inspire mathematical thinking." Despite his attempt to expose the college's failed attempt at inspiring mathematical thinking, some of Dan's followers took it as an effective example of how to inspire students.

One re-tweeter wrote that this was:

"making math homework matter!"

A frustrated Dan could only reply,

"Agh! No!"

And, "I've failed" was another response to RT that read,

"Why do you need to learn a little math? Gr8 image to share!"

And, "Someone help me," as yet another RT read,

"When Am I Going To Use This? The answer… (and a link to the graphic was attached)."

He almost gives up on a fourth misunderstanding

"Oh come ON. Not even @GothamSchools read my last post beyond the shiny infographic bauble. Bloody hell."

Thankfully his readers came to his assistance, posting a giant WHO CARES over the poster for all to see the futile efforts of the U.S. college.

The risks of using Twitter as a conduit to promote a healthier math classroom played itself out right in front of his eyes. Intending to travel through web space to discover a place of enlightened mathematics teachers, the Twitter wormhole had warped his message and left him stumbling for clarifications. But that hasn't stopped him.

Still Boldly Going

During a workshop late last year, he posted a video of a World Record-breaking Giant coffee cup[6] and how it was made. He tweeted:

"Hey friends, what's the first question that comes to mind off this short video?" and added the hashtag #anyqs.

And right there, live in the workshop, he saw

[5] Boldly titled, "How the Khan Academy Is Changing the Rules of Education," August 2011 Issue.

[6] See for yourself: ***www.threeacts.mrmeyer.com/hotcoffee/***

responses popping up from math educators (and random Twitter followers) from around the world. How much coffee will it hold? How tall is the cup? What is the diameter?[7] Boldy posting an #anyqs during a live workshop shows how much faith Dan still has in the process. In a recent post, Dan still sings the praises of the use of Twitter to test out ideas:

"If you aren't piloting your work on Twitter, you're missing out. #anyqs"

And so continues his journey in search of perplexing questions, avoiding the asteroid belt of lectures left in webspace by Khan (recently featured on *60 Minutes*) and reinterpreting textbook questions into his own 30-second video. Perhaps he'll soon have his own published collection of #anyqs. Let's see… what could it be called?

How about, *#anyqs: The Search for Perplexity*

Or maybe *#anyqs: The Mathematical Voyage*

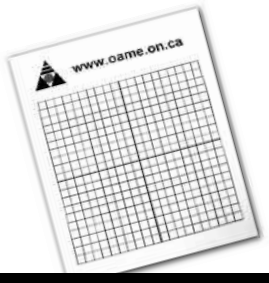
Or let's try *#anyqs: The Undiscovered Lessons*

Regardless, a current collection of most #anyqs can be easily found online. Also, Dan's latest venture, **101qs.com** is seeking to collect and sort through user-submitted videos and images for the sake of determining their "perplexity index." And for those who were lucky enough to have set their coordinates for the OAME 2012 conference this past May, Dan Meyer's keynotes were as entertaining as they were insightful. ▲

[7] For those interested, it was 7 ft. tall by 7 ft. wide.

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▲ Statistics Canada Note to Ontario Mathematics Educators and Teachers

Sent by the Communications Division at Statistics Canada to inform stakeholders that the Education Outreach program will end in June 2012.

FORWARDED BY
MARY TOWNSEND
CHIEF, EDUCATION OUTREACH
STATISTICS CANADA
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Statistics Canada's Education Outreach program will end in June 2012. The majority of the resources that were developed through the Education Outreach Program will continue to be available online through partner organizations.

The Census at School program will be jointly managed by the Statistical Society of Canada (SSC) and the Canadian Mathematical Society (CMS) and will remain accessible at the same address: **www.censusatschool.ca**.

Several learning resources, such as E-STAT, Statistics: Power from Data and the Canada Yearbook Historical Collection will remain on Statistics Canada's website via the "Students and teachers" link in the left-hand menu of the home page. This will bring you to a dedicated page that has been created to house links to archived information (for the discontinued resources) and links that will redirect you to the appropriate websites (for the resources that have been transferred to other organizations).

Please note that E-STAT will remain accessible until June 30, 2013, but its contents will no longer be updated. It will be discontinued July 1, 2013.

Since February 2012, all self-serve standard products—including CANSIM and census data products—are available free of charge on the Statistics Canada website. An open licence agreement has also been adopted to encourage re-use of the information. We trust that you will continue to rely on this wealth of information.

Statistics Canada encourages you to start using the free CANSIM database and to visit the census portal for timely information. ▲

▲ TEAMS PRESENTS OAME 2013 – THINK BIG: Reflecting on the Big Ideas in Mathematics Education

MARY CARD
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Mary Card, Curriculum Leader of Mathematics at Riverdale Collegiate Institute in Toronto (TDSB), is a Director of the TEAMS Chapter of OAME and one of the co-chairs of the Promotions Committee for the 40th Annual OAME Conference "Think Big". Mary was the recipient of a 2003-2004 Prime Minister's Award for Teaching Excellence (Certificate of Achievement) and the 2003 Queen's University Alumni Achievement Award. She continues to enjoy teaching Mathematics to 21st century students.

A Mathematical Opportunity to reflect and to celebrate HOSTED BY the TEAMS chapter of OAME

The Directors of the TEAMS (Toronto Educators Association for MathematicS) Chapter of OAME invite you to attend OAME 2013 THINK BIG: Reflecting on the Big Ideas in Mathematics Education. This conference will provide a mathematical opportunity to not only reflect on the Big Ideas in Mathematics Education in the twenty-first century, but also to celebrate the 40th Annual OAME Conference! The theme of the conference, THINK BIG, aligns with the current research into the Big Ideas in Mathematics Education, and the THINK BIG Conference will provide an opportunity for mathematics educators to work together to explore and reflect.

The OAME 2013 Conference will be held May 2–4, 2013 at Seneca College (Finch Avenue & Highway 401) in the Big City of Toronto. The eight strands that we will think about at the conference are: Big Ideas in Mathematics, High-Yield Instructional Strategies, Problem Solving, Reasoning and Proof, Learning in the Digital Age, Professional Learning Models, Assessment and Evaluation, and Sponsored Product Presentations. Workshops in these strands will be presented at all levels including Primary, Junior, Intermediate, and Senior. There will also be a pre-session on May 1, which involves a public research lesson study at a local Toronto school.



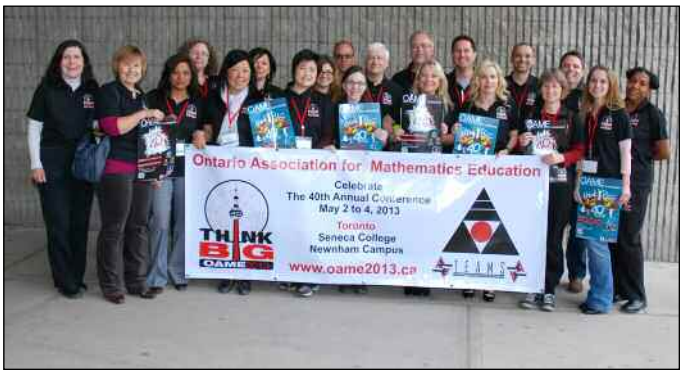
Stephen Lewis

We are honoured that the Toronto Catholic District School Board Education Director, **Bruce Rodrigues**, will give an opening address preceding the Thursday keynote and that the Toronto District School Board Education Director, **Chris Spence**, will give an opening address preceding the Friday keynote. The keynote speakers for this 40th OAME 2013 Conference are **Stephen Lewis**, Chair of the Board of Directors of the Stephen Lewis Foundation, Distinguished Visiting Professor at Ryerson University in Toronto, and a former Canadian Ambassador to the United Nations; **Annie Kidder**, Executive Director of People for Education; and **Bob McDonald**, the host of *Quirks & Quarks* since 1992, and one of Canada's best-known science journalists.



Annie Kidder

The featured speakers at the OAME 2013, Think Big Conference will be **Tess Miller**, Assistant Professor, Faculty of Education at the University of Prince Edward Island, who will speak on "Large- Scale Assessment: 5, 10, 15, 20… What's it worth?"; **Ron Lancaster**, Lecturer



in Mathematics Education, OISE, University of Toronto, who will speak about "The Housekeeper and the Professor by Yoko Ogawa: Using Fiction and Film to Teach Mathematics"; **Marian Small**, Author and International Professional Development Consultant for Improved Mathematics Education, who will speak about "Big Ideas with a Laser Focus"; **Jason Brown**, Professor of Mathematics and Statistics at Dalhousie University, who will speak about "A Hard Day's Math: The Connections Between Mathematics and Music"; **Daniel Ansari**, Canada Research Chair in Developmental Cognitive Neuroscience, University of Western Ontario, who will speak about "Disorders of the 'Mathematical Brain': Developmental Dyscalculia and Mathematics Anxiety"; **Cathy Bruce**, Associate Professor, Trent University, School of Education & Professional Learning; and **Ruth Beatty**, Assistant Professor at Lakehead University (Orillia Campus), teaching pre-service Primary/Junior candidates, who will speak about "Prioritizing Visual Models to Enable Exploration, Understanding, and Communication."



Bob McDonald

Social events that are planned for this conference include a cocktail hour on May 2nd, a banquet at the Donalda Club on May 3, and theatre and sightseeing packages, which will be available on both May 2 and May 3. In addition, since the OAME 2013 THINK BIG Conference is the 40th OAME conference and the Big 4-0 OAME Conference returns to the Big City of Toronto for the first time since 2005, the Directors of TEAMS are holding a "40" contest. There will be one grand prize of full registration to the conference, a $315 value, and ten runner-up prizes of $50 off the conference registration. To enter the contest, please visit **www.oame2013.ca/contest/** and submit your answer to one of the two questions posted.

The Directors of TEAMS are looking forward to an exciting conference, where everyone is encouraged to THINK BIG. For further information, please visit our website at **www.oame2013.ca**. ▲

▲ Important EQAO Research Study: Counting the Grade 9 Math Assessment Toward Final Course Marks

KATIA COLLETTE
EQAO

Earlier this year, EQAO released an important research bulletin that looks into the relationship between students' awareness that their result on EQAO's Grade 9 Assessment of Mathematics will count toward their final course mark and their motivation and achievement on the assessment. Check out this new research bulletin, and many others, in the "EQAO Research" section of EQAO's website, **www.eqao.com**.

Counting the EQAO Grade 9 Mathematics Assessment Toward Course Marks Makes a Difference in Student Results

This research investigated the impact on student results of teachers' counting the EQAO Grade 9 Assessment of Mathematics as part of students' final course marks and of students' awareness of whether the assessment will be counted. ▲

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▲ Sally Strange – And How She Learned to Stop Worrying and Love Grade 7 Math, A Novel

NICO ROWINSKY
E-MAIL: mr_row@hotmail.com

This piece is a preview of a novel written by Nico Rowinsky, a Toronto District School Board math teacher for over ten years. Chapter 1 can be found in the March 2012 Issue. The book is represented by Ali McDonald at The Rights Factory: **alison@therightsfactory.com** *+1.647.226.7328.*

Chapter 2 – It's a Trap

wednesday september 16th

"What do we have next?"

I have no idea.

"I have no idea," I say again, out loud this time, echoing my confusion to Lindsay.

She's clearly as lost as I am with this new schedule. Winona is not like Maplewood at all. In Grade 6, there was no rotating. We just stayed with Ms. Russell the whole day (except for gym). In Grade 6, there were no numbered days, just normal days. In Grade 6, we didn't have 9 different teachers, 7 different textbooks, and 3 different floors; just one teacher, one notebook, and happiness.

"It's day 7. We have math." Confident, friendly, and likely correct since it's Chin.

Lindsay and I stop ourselves from reaching for our timetables—mine folded somewhere comfortably in my bag—a piece of paper with Day 1 to Day 7 across the top, and Period 1 to Period 8 along the side. The last two days, we've had math first period. Today we have math last. Don't ask me to explain why!

Our class makes its way to math, which is in Room 13 (lucky us). I'm in class 7-1 and there are four Grade 7 classes at the school: 7-1, 7-2, 7-3, and 7-4. We each have different homeroom teachers, but we all see Mr. Rowe for math.

Evan is leading the way, but today, he's not his usual charming self. And by charming, I mean annoying, in a cute way. He's actually surprisingly quiet. He is the first to get to the room and the first to sit down. I hope Mr. Rowe doesn't pick on him today. He sits in the back so maybe Mr. Rowe won't notice. I look back to give him a quick glance from my desk, just to make sure, but he's not looking up.

Mr. Rowe collects the homework to be marked and answers a few concerns about the whole "explain why" question. I'm not exactly sure what he means, but basically he is saying, *If you can't explain it, then you don't know it.* I guess I don't know it.

He settles the class and starts the lesson with, "I need your help."

Hmm. Help? I don't think so Mr. Rowe, I really don't think so. It's a trap. There is work nearby and we all know it.

As he walks around the room, dropping off small plastic bags to each group, he continues, "The bags are full of different shapes. Yes, they are colourful. And yes, they look like toys," and half jokingly he adds, "but please don't steal them."

The half-joke gets half-chuckles out of half the class.

"These are called pattern blocks. And I want everyone to take 3 of the red-shaped... umm... shapes out of the bag, please. Everyone, except Evan."

Well, so much for not picking on him. Mr. Rowe's already made his way to the back near Evan.

"Evan is going to help me with something else. You guys, in groups, what I want you to do is see if you can figure out how you can make those 3 red shapes come together," dramatic pause, "to make one triangle." He repeats it one more time, for the benefit of the non-listeners (not me).

We start playing with our new-found toys, while Evan and Mr. Rowe look like they are plotting something at the back of the class.

3 red shapes need to come together as a triangle.

Well, the first problem is that there's six of us in this group and that's too many people for those cute little red shapes. Apparently, Arial has already considered this and is getting some more from the bag so that her, Chin, and me can work on this together. Okay, here's what we came up with. And it wasn't easy.

First we did this with two shapes:

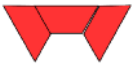


and added the third shape to get this...



nope, not a triangle.

Then we tried: 

and added the third to get… 

nope, no good.

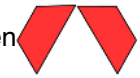
Then:  uhh…  no… 

more no and.. 

what the heck is that?

Ok. Soooo, this isn't working yet. But I have Arial on my left and Chin on my right; I'm in good hands as far as math is concerned. Meanwhile, it looks like Mr. Rowe and Evan have their hands full moving some desks at the back of the class, including Evan's. They've shifted his table towards the centre of the room.

Arial stops us for a sec and slowly begins to move the blocks with some purpose, like she's unlocked some ancient biblical secret. She's got something.

First,  then  then the third 

"Triangle?" she says what we're thinking.

Skeptical, but nodding approval, Chin says, "Looks like a triangle to me," and he points at the space between the red shapes. The three of us stare at it, afraid to move what seems to be the solution to the ancient three-red-shapes-make-a-triangle puzzle.

The others in our group, distracted by our lack of movement, are now looking over to see what we've got. We're not hiding it, but they don't look too convinced we've got the right answer. Mark says rudely, "That's not it." He's a little immature and usually hangs out with Andrew who, like him, still acts like they're in Grade 6.

It doesn't matter. Arial's hand is already up, seeking approval from a higher power. "We'll see." She's quick.

Looking at the recent furniture rearrangement, Mr. Rowe saw all that He (and Evan) had made, and thought, it was very good. Then He notices Arial's hand and makes His way over to our desks to take a look. While staring down at *our* creation, we look up...

"Nice," comes the word from above.

"Is that the answer?" Arial looks for more on our behalf.

"Usually these kinds of problems need a different kind of thinking to get to the solution. And you guys have done that." Circling his fingers over our shapes, he says, "That's a unique solution."

We're smiling. "See!" I defend against Mark's earlier attack with the classic See! in his direction. I miss Grade 6.

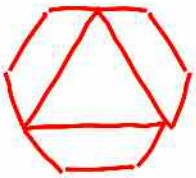
Mr. Rowe makes his way to the front and, "OKAAAAY CLASS, someone has solved the problem."

The class reacts immediately to Mr. Rowe's commandment. All the groups are searching for the triangle, looking to see who's solved it.

"Arial, care to share what your group has found?" All heads turn our way.

"Ya," she pauses to consider how to share, still not wanting to move our solution. "Can I draw it on the board?"

"Ya, sure," he turns to get the marker as Arial gets up. Mr. Rowe hands her a red dry-erase marker as she approaches the whiteboard. She draws a rough but effective copy of our solution.



And she motions along the inside to showcase the triangle we've made.

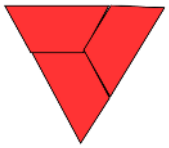
"Good Arial," he takes the marker as she heads back to us triumphantly. "So, let me re-cap—." He quickly puts the lid back on the red marker. Someone in the back of the class laughs aloud. Mr. Rowe continues, "The problem was to make a triangle. And although this is—kind of triangular, I can't say that it is a triangle. These three red trapezoids—they're called trapezoids by the way—can come together to make one triangle. But this," he points to the board, "is not the solution."

The class is now confused. He said someone had solved it. He said Nice. He said we're unique. He said—

"Evan!" as he crosses his arms and leans back on the front ledge. The class turns to see a now enlightened Evan step on a chair, then step onto the top of the tables

they've just moved together. We are all staring at him like he's crazy. Mr. Rowe watches proudly. I guess he must have noticed Evan wasn't feeling good today so he let him off the hook. But I don't get this. *What's he doing on the tables?*

Finding his balance after ascending his own mountain, Evan starts, "I have the solution." And with as much pride as Mr. Rowe, he announces, "I'm standing on it!" Our focus readjusts to look at the three tables they've just secretly arranged. It was a *trap*. Evan's table, previously located by the wall, is a giant trapezoid. And they've put three of them together to make the solution.



Coooooool. I've never seen the desks like that.

Evan gets down and Mr. Rowe takes over. He explains we're going to be working with trapezoids and parallelograms today. I wouldn't say the class reacts with moans or even groans, but there was definitely a sense of disapproval at those two words. It sounds very geometrical, so official. Just as he starts talking and begins to write on the board, I sneak a glance at the clock. 2:31. Another half hour left. OK, a nicely timed washroom break and I'll get through the rest of the class. Plus, I'm looking forward to after school. Volleyball tryouts!

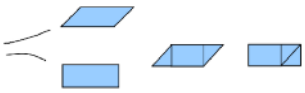
Last week we had tryouts on Tuesday, Wednesday, and Thursday. On Friday, we had a scrimmage because the coaches wanted to see us actually play in a game situation. Evan and Gavin stayed to help the coaches and watched us play. 41 girls tried out, including Lindsay and Arial (although Arial was there for fun and has never been very good at volleyball). Lindsay is okay. Lindsay and I were both on the Maplewood Grade 6 team, but there are a lot of girls trying out at Winona who didn't go to Maplewood. There was one girl who really stood out. Sol.

Sol went to Hillscrest Public School and she's really good. They say she came to Canada from South America last year. Not many of us Maplewooders knew about her until we came to Winona. She's in class 7-4 with a group of other Hillscrest girls. Lindsay used to go to Hillscrest, but changed to Maplewood in the middle of Grade 4 (she got into some mess with this girl Franny, who's also in 7-4).

From 41 we were cut down to 19 girls for this week. Lindsay and I made the first cut, but Arial didn't. The coaches say they are taking 12 girls for the team and the final list will be up Friday morning. Sol will make the team for sure. I'm not sure about us. I can't wait 'til Friday.

Mr. Rowe continues his lesson on parallelograms, the area of parallelograms, and how a parallelogram has the same area as a rectangle. *What?* This is dragging on a bit too much.

Okay, the diagram makes sense. A parallelogram can be cut and made into a rectangle.

Same area: 

I get it, but I'm not paying attention anymore. It's 2:42. People are asking some questions. Perfect time to raise my hand. Smiling.

Mr. Rowe, having finished with the last question (I'm totally zoned out, so I don't even know what the question was, let alone what he just answered), calls on me. I'm still smiling. "Yes, Sally?"

"Um, can I go to the washroom?" *Keep smiling, keep smiling.*

He looks up at the clock, looks back at my smiling face—I think I've got him. He frowns—I think I've got him—and says, "Okay, but make it quick." *YES, I GOT HIM!*

Making it quick is clearly an expression that's up for interpretation. If I really had to go, making it quick might not be an option. Having to go takes as long as it takes. Sometimes, there's no making it quick. But, since I *don't* have to go, I can make it any way I want. I consider all this while walking towards the washroom. I pull out my cellphone (hidden carefully from the view of teachers) to check the time. Cellphones are treated like weapons by teachers. It's almost as bad as having a knife or gun.

In the washroom I do everything except actually go. I look at the mirror, I look at the time on my cellphone, I chat with Tasia, who's in the washroom, likely wasting time like me, and I send a text to Evan to make sure he's OK. He'll get the text after school, but whatever. Mostly though, I am thinking about volleyball. There are two tryouts left and then Friday—BANG, the list! I'm nervous about the final cut, excited about today, and 2:49—*shoot, I gotta head back.*

I like getting back and seeing 2:50 on the clock. Once it's 2:50, I feel like the day is pretty much done. Less

than 10 minutes is easy. More than 10 minutes—well, that depends.

I hurry my last few steps before going in. I can see Mr. Rowe is at his desk working. The whole class seems to have their heads down, working. *What'd I miss?*

I quietly make my way back to my seat and look at Chin, "What'd I miss?" He wastes no time in nodding a couple of times towards the board, encouraging me to read.

IF YOU CAN GET THIS DONE IN CLASS, YOU WILL HAVE NO HOMEWORK TONIGHT!!!

2:51. Nine minutes.

DRAW 3 DIFFERENT-SHAPED TRAPEZOIDS. SHOW HOW EACH SHAPE CAN BE DOUBLED TO MAKE A LARGER PARALLELOGRAM.

What? I read it again, but it's no use. I don't get the first sentence. "Different-shaped trapezoids"? If it's a trapezoid, that's the shape. How can it be different? The answer can't be far, but Chin seems to be hugging his homework like a 1st grader who's just found a lost toy. I look left to Arial's sheet. She's a lefty and not as attached to her work as the big baby beside me. It looks like this:



Not getting it, I quickly look up. 2:54.

I look back across the table to the others, but it's more of the same. Mark is making weird, I'm thinking noises. I clearly missed something, but it's no time to panic. I come up with a quick plan. I'll write something down. I can draw some shapes. Make it look real. Hand it in. And I'm off to volleyball. Easy as that.

I start by copying some of the shapes that are on the board already. 2:56 — volleyball is close.

I continue by copying down the whole question, making it really neat. 2:58.

I'm nervous about today's tryouts, but even more nervous about Friday.

I wish I could just fast-forward the next two days and see the final cut. 2:59.

That would be so cool if I could just turn the page in my life and it would be Friday.

3:00 bell.

Chapter 3 – Making the Cut

friday september 18th ▲

▲ OAME AWARDS, 2012

CHERYL GEOGHEGAN
EMAIL: geoghec@rainbowschools.ca



Cheryl Geoghegan is Chair of the Outreach Committee, which includes the OAME Awards Committee.

The OAME Awards were presented at the Annual Conference's Banquet, held at the stately Renaissance Event Venue in Kingston, Ontario. The banquet followed the second day of the OAME 2012 Conference, held at St. Lawrence College in Kingston.

Award for Exceptional and Creative Teaching in Secondary Mathematics

Sandra Fischl has very recently retired from a career that brought her to the fore in the creative and engaging use of technology in her mathematics classroom, and beyond. Her knowledge in the use of interactive whiteboards, response systems, e-learning objects such as CLIPS and GIZMOS, and her implementation of differentiated instruction and assessment strategies, were instrumental in effecting a remarkable gain in student achievement. Sandy has also been the driving force in her school board, as well as at a provincial level, in the professional development of teachers as she shared, and continues to share, the wealth of talent that makes her so exceptional.



As a teacher at Port Hope High School, Sandy's teaching assignment included all sections of Grade 9 Applied Mathematics. Her classroom was widely recognized as a place where students could explore mathematics fearlessly, as they engaged in activities that were designed to be appealing and interactive, while giving students the opportunity to grasp the true relevance and meaning of the mathematics at hand. With a consistent, fair management style and an evident passion for her subject, her students entered each day into a climate that encouraged participation and peer sharing, hands-on exploration, and the support to move forward in their conceptual thinking that has been described as "formidable." The results of this are evident

not only in the success of her students—whether measured in pass rates or EQAO scores—but also in the anecdotal evidence from those she has taught. Sandy guided her students to successes that stood as a benchmark for her school and her board.

In 2010, Sandy's classroom was featured in a video clip for the Ministry of Education's EduGAINS website, as a model of practising engagement and differentiation of instruction in a mathematics classroom. Before this, however, Sandy was already well-known for her contributions to the professional development of teachers in many jurisdictions of the province, and elsewhere. She was responsible for the development of TIPS student workbooks and authored articles related to the use of technology in the classroom. She has presented workshops and participated in study groups with her own board (Kawartha Pine Ridge District School Board), as well as Queen's University, Trent University, University of Toronto, and the BETT Conference in London, England, to name but a few. She is certified as SMART™ Technologies instructor, and has won numerous awards including OSSTF's Certificate of Excellence.

As Sandy embarks on retirement, she nevertheless continues to offer her expertise to her colleagues. She maintains a website where she selflessly shares her work with us all, and looks to support the education community at large with the great store of creativity and experience that she spent a career developing. Her generosity comes from the heart, as she is a most giving and humble individual… part of what makes her truly exceptional.

Award for Leadership in Mathematics Education



Pat Margerm has been described as "one of the best ambassadors for mathematics in the province." Now retired from the Toronto Catholic District School Board, Pat's days speak of anything but retirement. She is one of the province's most recognized names in mathematics leadership, from her involvement in numerous initiatives and programs, through her role as instructor in various capacities, and ultimately as one of the Ministry of Education's provincial coaches in connection with the Gap Closing Strategy. Her resumé almost reads like that of three people combined!

Pat's passion for mathematics education has found its way into many channels over the years. Following her time as a classroom teacher and resource teacher with the Toronto Catholic District School Board, Pat moved into the role of teaching teachers. She worked as the Course Director in York University's Consecutive Program, and led programs in research and development with respect to effective teaching practice. Her commitment to meeting the needs of all learners, especially in regard to mathematics learning, was evident in her wide involvement in professional development workshops, courses, and projects. She has modelled justice, fairness, and social responsibility as she collaborated on working equity into a mathematics context, as part of her involvement with the Model Schools for Inner Cities project. Furthermore, Pat fosters the strengths of individual coaches through her work as a provincial coach, thus enabling the ripple effect of spreading effective teaching practices to a broad swath of learners in our province.

Pat continues to take part in action research projects aimed at supporting teachers with collaborative inquiry and lesson development. She continues to be a leader in promoting the collaborative inquiry model in connection with her role as Instructional Leader for Additional Qualifications courses, among other settings, and continues to work as an author of articles and materials to support countless colleagues in this area. She has been described as "an extremely talented and inspirational leader," one who encourages those around her to remember always the learner within themselves. She imparts, as a great leader would, the reminders to "observe, wonder, and listen." With Pat, the true quality of leadership lies in the ability to model the message, and inspire. She continues to do just that.

Kenneth D. Fryer Secondary Award

This award is given to a mathematics department that fosters collegiality, teamwork, and excellent classroom teaching. This department must also contribute to the overall development of students and demonstrate leadership in the mathematics education community. This year, recognition goes to the math department at **West Carleton Secondary School** for exactly that. Located in Dunrobin, Ontario, this math team has worked tirelessly to incorporate innovative technological practice into their classrooms in an effort to meet the varied needs and learning styles of their students. Although the school lies in a rural setting, with 100 percent of their students coming to school by bus, the staff—headed by Diane Mes—have worked to bring the



world to their students when they enter their classrooms, and have done so successfully. The teachers in this department have strived to make mathematics learning a part of the broader world of community and life-long learning.

Each teacher in the department displays commitment to the whole, as is evident in the high degree of collaboration that goes into creating a unified and consistent group of professionals. As a group, they recognize that helping students succeed goes beyond just the teaching. Each teacher is involved in numerous aspects of school life and professional involvement, whether it is special education advocacy or Relay for Life organization; whether it is working with EQAO or leadership camp. Yet the cohesive nature of the staff brings these elements together to make them, as one member expressed, "greater than the sum of our parts."

As a department, the teachers of West Carleton Secondary School work to create engaging and challenging lessons and assessment pieces for students. They place great importance on differentiated instruction as an element of their commitment to meeting the needs of all learners. These teachers are extremely giving of their time, ensuring students have access to extra help every lunch period, and communicating with parents/guardians by telephone or through their websites, to ensure all partners in education have access to what is needed. Positive results are seen, particularly in the data around Applied-level courses, garnering attention in their jurisdiction from those wanting to know what they are doing right!

With the elements of collegiality, commitment, and teamwork, this is a secondary mathematics staff that is making a difference, and although the department is obviously finding motivation from within, they are aware their students clearly benefit. That is also a great motivator for teachers who truly care! ▲

▲ Need Statistics Canada Data? Learn To Use Cansim!

ANGELA McCANNY
STATISTICS CANADA
www.statcan.gc.ca



Angela McCanny is mathematics educator with extensive experience in using Statistics Canada data in the classroom. Over the last six years, she has visited more than 500 classrooms, helping teachers find exciting resources for teaching the math and social sciences curriculum.

CANSIM is the Canadian socio-economic database, now free for all users. With more than 43 million time series, CANSIM contains data on almost all aspects of Canadians' lives. CANSIM data are continually updated when new data are released.

For a good introduction to using CANSIM, view Tutorial: Introduction to the New CANSIM Interface at **www.statcan.gc.ca/about-apercu/video/cansim-eng.html**.

1. Go to **www.statcan.gc.ca** – English (or français)
2. In the right menu bar, find Popular picks – select CANSIM.

Activity to teach yourself to use CANSIM: Try to find the median age of the male and female population from 1971–2011. Instructions will be in italics within brackets throughout.

Two search options in CANSIM

1) Search box:

a) In the Search CANSIM box, type in your search term or table number (perhaps found during a previous search). Click Search. *(Type population.)*
b) Select the table by clicking on the table number at the right. (051-0001)

2) Browse by subject:

a) Scroll down to the bottom of the page to the box Browse CANSIM by:
b) Click on the Subject tab. Select whichever subject would contain your topic. *(Population and demography)*
c) Select the appropriate subtopic. *(Population estimates and projections)*
d) Select the table by clicking on the table number at the right. (051-0001)

Note: Experiment with search terms and browse through the tables to find the right combination of topic and frequency (i.e., annual, monthly, quarterly). You will need to read carefully.

Display and modify the data table:

The initial view of the data table appears on the Data table tab. This view displays a subset of the data available in the complete table. (You will see the population of Canada by age group for the last five years.)

You may change the options, such as including data for other provinces or changing other displayed characteristics, using the Add/Remove data:

a) Click on the Add/Remove data tab, and then select your desired items or options at each step. (Click +Expand at right to see the full list of items.)
(Step 1: no change
Step 2: Uncheck Both sexes; check Males and Females
Step 3: Uncheck All; check Median age
Step 4: 1971–2011
Step 5: HTML table, time as rows)

b) Click Apply in the final step at the bottom of the screen.

Note: If you want to change your selections again, click on Add/Remove data (don't use Back to original table unless you are ready to lose the selections you have just made).

In some data tables, you can change the displayed data to show percentage change, averages, or sums, or change the frequency of the retrieved data. For example, if the initial view is monthly, you may be able to convert it to annual or quarterly data.

a) Click on the Manipulate tab, and under Frequency of output data will be: choose annual (average) or quarterly (average) or whatever suits your data. (No change–Data as retrieved)

b) Click Apply.

Statistics Canada assumes responsibility for the quality of data as retrieved with the frequency unchanged. Clients take responsibility for any manipulations made to the original data.

Ready to graph or analyze? Download the data:

a) Verify on the Data table tab that you have selected all the data you need; click the Download tab. (For the download, change back to Data as retrieved, on the Manipulate tab.)
b) Choose Option 1 – Download data as displayed in the Data table tab.
c) Select the data output format type: time as rows
d) Select the file format: CSV for English or SCSV for French (CSV comma-separated values)
e) Click Download data.
f) On the page Alternative format, click on the link Download file from CANSIM.
g) In the File Download box, click Save (not Open) and select a location to save this file.

Note: To download data correctly, the computer's regional settings must be set to the same language as the data table. You can verify the current regional settings from the Control Panel.

Import CSV or SCSV files into software:

Spreadsheet program (i.e., Excel/OpenOffice Calc)

a) Open the spreadsheet program.
b) From the File menu, click Open.
c) Locate the saved file. Click Open.
Note: If the file is not visible, change the File Type to All Files.

Make the spreadsheet ready for graphing:

d) Jot down table number and title listed in the first few rows for future reference.
e) Rename the columns, if needed.
f) Delete rows above column labels.
g) From the File menu, click Save As.
In the Save In box, select a location to save the file.
File name: Rename, if desired. File type: Change to Excel workbook or OpenOffice Calc (.ods)
h) Click Save.

Import into Fathom dynamic statistical software

a) Open Fathom.
b) From the File menu, choose Import, then Import from File.
c) Locate the saved file. Click Open.
Note: If the file is not visible, change the File Type to All Files (*.*).

Make the collection ready for graphing:

d) Double-click on the collection. Jot down the table number and title listed in case 1 for future reference.
e) Click on the arrow at the bottom of the case card until data appears. Rename the attributes, as suitable.
f) Check the first few and last few cases and delete any cases which do not contain actual data.
(Right-click on the case and choose Delete inspected case.)
g) From the File menu, click Save As.
In the Save In box, select a location to save the file.
File name: Rename, if desired.
h) Click Save. ▲

8 PAGE INSERT ELEMENTARY MATH

ONTARIO ASSOCIATION FOR MATHEMATICS EDUCATION

Abacus

Volume 51 • Number 1 • September 2012

COLLECTING AND ORGANIZING DATA

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A Publication of the OAME/AOEM

ABACUS CO-EDITORS



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The main purpose for collecting and organizing data is to gather information in order to answer questions. Such questions are derived often from curiosities about a phenomenon, dilemmas about a situation, or observations that require further investigation. When students collect and organize data, they have an opportunity to learn more about themselves, their environment, issues in their school or community, topics in various subject areas, and so on. Learning activities should help students understand the processes that are involved in formulating questions, seeking relevant information, and organizing that information in meaningful ways.

In this issue, the Research Summary and problems (with multiple solutions) focus on strategies for collecting and organizing quantitative and qualitative data and determining mean, median, and mode where appropriate. The problems we provide are designed for use within a three-part, problem-solving lesson.

LINKS TO LITERATURE AND MANIPULATIVES



PAT MARGERM is an independent literacy and mathematics consultant with more that 30+ years of K to 8 teaching experience with students, in-service and pre-service teachers. She is a long-time OAME board member and works in various aspects of mathematics education.
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Welcome to a new school year and a new math focus for the ABACUS. According to the curriculum expectations, students in grades 2 to 8 collect data through surveys. How does a grade 8 survey compare to a grade 2 survey? In this issue, we will use a picture book to try to collect more meaningful survey results than "what is your favourite ice cream?" Also, we will look at a few ways to collect data using manipulatives.

RESEARCH SUMMARY – COLLECTING AND ORGANIZING DATA

Collecting and Organizing Data

According to Small (2010), "people collect data in order to solve problems. School activities should support this idea – that data collection is a tool that can help solve problems, rather than an end in itself. There should always be a purpose for collecting data, and the method of data collection should suit that purpose" (p. 169).

When students collect and organize primary data, they are compelled to make decisions that are required at different steps of the data management process. Randall (2005) describes a big idea about data collection; that is, "some questions can be answered by collecting and analyzing data, and the question to be answered determines the data that needs to be collected and how best to collect it" (p. 20). Some questions associated with collecting and organizing data are as follows:

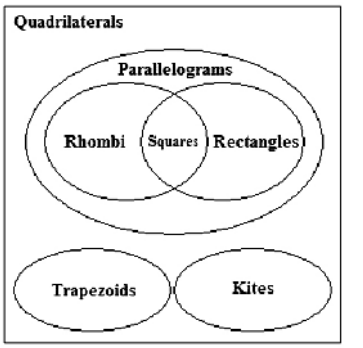
- What do we need to find out? What question needs to be answered? (e.g., How can the school improve on reducing school waste?)
- What kind of data needs to be gathered in order to answer the question?
- Where and from whom should data be collected? (e.g., from direct observations of a situation; asking a set of focused questions to grade 3 students, teachers, or parents)
- How will data be collected? (e.g., carrying out an experiment, taking a poll, paper-pencil or electronic survey)
- How will data be recorded? (e.g., responses directly on a paper-pencil or electronic survey, tallies)
- What ways could data be organized so that it provides answers to a question? (e.g., similar items in categories, by order of data value (least to greatest)) How can we sort and classify objects using multiple attributes (colour, size, texture, thickness)?

Sorting and Classifying Data

According to Small (2010), sorting and classifying data is a prerequisite to many forms of data collection and display.

Venn Diagrams are tools for sorting by multiple attributes in different parts of one of the circles. The example of a Venn Diagram shows the inclusions and intersections of the various types of quadrilaterals.

Carroll Diagrams are cross-classification charts that enable sorting of two or more attributes with a focus on one attribute at a time.



Prime	Not prime
2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53	1, 4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20, 21, 22, 24, 25

	Prime	Not prime
Even	2	4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26
Not even	3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41	1, 9, 15, 21, 25, 27, 33, 35, 39, 41, 45, 49

Mean, Median, and Mode

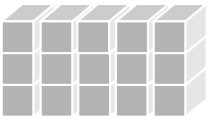
Randall (2005) describes the big idea about data distribution; that is, "there are special numerical measures that describe the center and spread of numerical data sets." Small (2010) explains that there are three measures of central tendency or averages: mean, median, and mode." The best descriptor of the centre of a numerical data set is determined by the nature of the data and the question to be answered. Outliers affect the mean, median, and mode in different ways" (p. 21) and the size or the range of data.

- ***Range*** of a set of numbers is the difference between the highest and lowest numbers in the set. The range measures the spread of a set of data. However, range is frequently not a good measure of distribution because it includes the extremes and it can give us a distorted picture of the data.

- ***Mean or arithmetic mean*** describes a set of data by indicating the value that would result from putting all the values together and distributing them evenly. It is calculated by dividing the sum of the numbers by the number of members in a set. It is conceptualized as making equal groups or as a balance point where there is an equal number of data above and below the mean. Students need to use different sets of numbers and data and apply the algorithm to calculate the mean and see the relationship between the range of data and the value of the mean.
- ***Mode*** of a set of numbers is that number that occurs most frequently in the set. If there is not a more frequent number then there is no mode; if there are two numbers with the highest frequency, then the set of data is bi-modal. Because an extremely high number or an extremely low number can distort the picture of the tendency shown by the mean, the mode is frequently used.
- ***Median*** of a set of numbers is the middle number of the set when the set is ordered. To determine the median, order the numbers from least to greatest, and choose the position of the middle score. For example, if the numbers are 14, 6, 8, 8, 5, 12, 34, 23, and 2, then arrange them as: 2, 5, 6, 8, 8, 12, 14, 23, and 34. Then locate the middle position; in this case the fifth position is the number eight. If the total number of numbers is even, add the two numbers in the middle position and divide them by 2. That will be the median for that set of numbers.



Distribute the cubes evenly to get the mean of 3.



Common Errors, Misconceptions, and Instructional Strategies

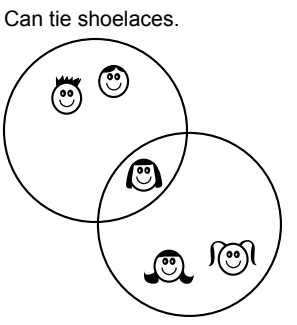
Batanero, Godino, Vallecillos, Green and Holmes (2006) report on Russell and Mokros' categories for classifying students' misconceptions about averages as being: the most frequent value or mode; the reasonable value, the midpoint and an algorithmic relationship. They suggest these strategies to support students in understanding the meaning and appropriate use of mean:

- Given different data sets, choose a central value (i.e., mean, median, mode) that best represents or summarizes the data.
- Build a data set that has a given mean, mode, and/or median. Have the students notice when and what sorts of data sets have the mean, mode, and median are close yet different in value.
- Examine what happens to central values (i.e., mean, median, and mode) when you add extreme data values to the sets.

LINKS TO MANIPULATIVES: USING EVERY DAY OBJECTS AND PHOTOS

A. How can we help each other? (Using student photographs in early primary).

- Organize the pictures of the students in response to the following questions on a Venn diagram: Can you tie shoelaces? Can you zip zippers?
- Tape the photographs on the Venn diagrams and post it near the coats and shoes.
- Students can refer to the Venn diagram to see who to ask for help when it is time to get dressed.



B. What are the students in our school doing to the environment?

- Young students investigate litter in the schoolyard by collecting litter and categorizing it in the classroom.
- Older students may investigate issues with litter in the schoolyard, by mapping or counting the litter in various locations and/or at various times of the day.

LESSON LEARNING GOALS

Identify the curriculum expectations (MOE, 2005) grade-before and grade-after the grade specific learning goal:
- to see the development of mathematical knowledge, skills, and strategies across grade levels
- to determine the mathematical focus of the problems chosen for Before (activation), During (lesson), and After (Practice) parts of the lesson

BEFORE (Getting Started) – 5 to 10 minutes

- Activating students' mathematical knowledge and experience using a prompt or problem, related to the math in the lesson problem.
- Includes student responses to highlight ideas and strategies.

AFTER (Consolidation) – 20 to 25 minutes

- Teacher selects 2 or more solutions for class analysis and discussion in a sequence (1st, 2nd, 3rd, etc.) based on mathematical relationships between the solutions and the lesson learning goal.
- Students (authors) explain and discuss their solutions with their classmates.
- During whole class discussion, solutions are organized (often re-organized) to show mathematical elaboration from one solution to the next and towards lesson learning goal.
- Teacher mathematically annotates (math terms, math symbols, labelled diagrams, concise explanations) on and around solutions to make mathematical ideas, strategies, and models of representation explicit to students.

Comparing Central Measures Problem A:

Which set of data has a mean, median and mode that are equal? Explain.

Group 1: 5, 5, 5, 5, 5, 20 Group 2: 6, 2, 5, 8, 5, 4

Group 3: 1, 8, 1, 8, 6, 8

Solution 1:

Group 2 -> the range is the smallest and the numbers are pretty evenly spaced; 4 data points above 5 and 4 below 5; mean is 5; median and mode are 5

Solution 2:

Not Group 1 -> 20 is an outlier that throws off the mean by making it bigger than median and mode; Not Group 3 - median, mode, median are different

Group 2:

Range->8-2=6
Mean->30/6=5
Median is 5
2,4,5,5,6,8
Mode->5

Group 1:

***Range**->20-5=15*
***Mean**->45/6=7.5*
***Median** is 5*
5,5,5,5,5,20
***Mode**->5*

How did you calculate the median value?
- order the data from least to greatest value
- count in from lowest and highest value towards the middle until locate the middle data value

***Mode** - the data value that appears most often in a set of data*
Australia – seven 1000s
Canada – six 1000s

***Range** - difference between the highest and the lowest values in a set*

DURING (Working on It) – 15 to 20 minutes

- Understanding the problem – Teacher asks, "What information from the problem are we using to make a plan to solve it? Explain." Teacher records in a list below the problem, the information that students identify.
- Students solve the problem on chart paper (landscape) with markers (visible for class discussion) in pairs or small groups.
- Teacher circulates to record different student solutions, in addition to the ones the teacher anticipated.

Conserving Energy Problem B:

Students were asked to score, from 0 to 1000 about the importance of conserving energy? Who cares more?

BrthPlce	Sex	Conserving Energy
Victoria	Female	101
South Australi	Male	101
New South Wa	Female	208
South Australi	Female	302
South Australi	Male	362
New South Wa	Female	391
Queensland	Male	391
Western Austr	Male	444
New South Wa	Male	503
Victoria	Male	521
New South Wa	Female	740
Montenegro	Female	769
Victoria	Female	781
Queensland	Female	1000
New South Wa	Male	1000
New South Wa	Female	1000
Victoria	Female	1000
New South Wa	Female	1000
Victoria	Female	1000
New Zealand	Female	1000

Province	Gender	Conserving Energy
Ont.	Male	5
Ont.	Male	5
Man.	Male	241
Ont.	Male	316
Man.	Male	380
Ont.	Female	422
Ont.	Male	439
Ont.	Female	481
Ont.	Male	529
N.S.	Female	679
Ont.	Female	711
Ont.	Male	738
Ont.	Female	797
Ont.	Female	872
Sask.	Male	1000
Ont.	Male	1000
Alta.	Female	1000
Ont.	Female	1000
Alta.	Female	1000
Alta.	Male	1000

Solution 1:

Range
- *Australia: 1000 – 101 = 899*
- *Canada: 1000 – 5 = 995*

Australian's spread of data 899 is smaller than the Canadian's at 995.

Both have a mode of 1000 so mean and median will be close. So students in both countries care about conserving energy.

Solution 2:

Mode
- *Australia: 1000*
- *Canada: 1000*

Median
- *Australia: (521 + 740) / 2 = 630.5*
- *Canada: (679 + 711) / 2 = 695*

695 > 630.5 so Canadians care more about conserving energy.

Range and Mode Comparisons

Mode and Median HowComparisons

Note: Mathematical annotations include mathematical vocabulary, symbols, elaborations of mathematical details from solutions, labels describing the method/strategy, and questions to further thinking. All annotations are records of students' mathematical discussion.

Grade 5	Grade 6	Grade 7
compare similarities and differences between two related sets of data by determining measures of central tendency (i.e., mean, median, and mode)	demonstrate an understanding of mean and use the mean to compare two sets of related data	determine, through investigation, the effect on a measure of central tendency (i.e., mean, median, and mode) by adding or removing a value or values

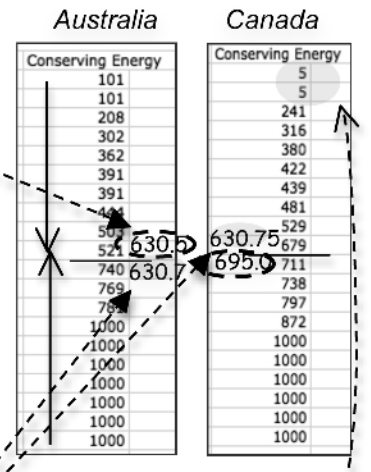
How are the median and mean different?

Median *- data value separating the upper half of the data from the lower half of the data*

- *Middle value between 521 and 740 -> (521+740)/2=630.5*
- *Middle value between 679 and 711 -> (679+711)/2=695*

Arithmetic Mean *- sum of the numbers divided by the size of the collection*

- *(202+510+753+835+1024 +2290+7000)/20 = 12614/20=630.7*
- *(10+557+802+920+1208+ 1449+1669+6000)/2 = 12615/20 = 630.75*



Outliers- *data values much lower (or higher) than rest of data*

- *5, 5*
- *creates greater range of data (1000-5=995)*

AFTER (Highlights/Summary) – 5 minutes

- Teacher and students revisit the student solutions for key ideas, strategies, and models of representation that are related to the lesson learning goal.
- Teacher lists key ideas, strategies, and models of representation separately, so the students can see how the mathematical details from their solutions relate explicitly to the lesson learning goal.

Highlights/Summary:

Compare "*Averages*" *(Measures of Central Tendency):*

- ***mode*** *- data value that occurs most often (e.g., both data sets, 1000 occurs 6 or 7 times; mode is 1000)*
- ***median*** *- data value separating the upper half of the data from the lower half of the data; middle data value in a set of ordered data from least to greatest (e.g., 630.5 and 695)*
- ***(arithmetic) mean*** *- sum of the data values divided by the number of data (e.g., 630.7; 630.75)*
- ***outliers*** *- data values much lower (or higher) than the rest of the data values (e.g., 5, 5 in Canada data)*

AFTER (Practice) – 5 to 10 minutes

- Teacher chooses 2 or 3 problems, similar to the lesson problem for student to solve in pairs as a scaffold and individually.
- Problems could vary by number (choice, size), problem contexts, or what is unknown or needs to be solved
- Students are asked to solve these problems using a strategy different from the one they used for the lesson problem.

Conserving Energy Problem C:

Compare Canadian males' and females' concerns about conserving energy.

Province	Gender	Conserving Energy
Ont.	Female	422
Ont.	Female	481
N.S.	Female	679
Ont.	Female	711
Ont.	Female	797
Ont.	Female	872
Alta.	Female	1000
Ont.	Female	1000
Alta.	Female	1000
Ont.	Male	5
Ont.	Male	5
Man.	Male	241
Ont.	Male	316
Man.	Male	380
Ont.	Male	439
Ont.	Male	529
Ont.	Male	738
Sask.	Male	1000
Ont.	Male	1000
Alta.	Male	1000

"Averages"

- *Arithmetic mean*
- *Mode*
- *Median*

Solution 4:

Country	Range	Mean
Australia	899	630.7
Canada	995	630.75
Country	**Median**	**Mode**
Australia	630.5	1000
Canada	695	1000

Students in both countries care about conserving energy. The numbers suggest that their concern about conserving energy is above average (600 compared to 500).

Solution 1:

Median

- *Females: 797*
- *Males: 439*

797>439; so females care more about conserving energy than males.

Solution 2:

Arithmetic Mean

- *Females: 6962/9 = 773.6*
- *Males: 5653/11 = 513.9*

773.6>513.9; so females care more about conserving energy than males.

Solution 3:

Calculating Mean:
Australia ->12614/20= 630.7
Canada ->12615/20= 630.75

Students in both countries care equally about conserving energy.

Mean Comparisons

Mode, Median and Mean Comparisons

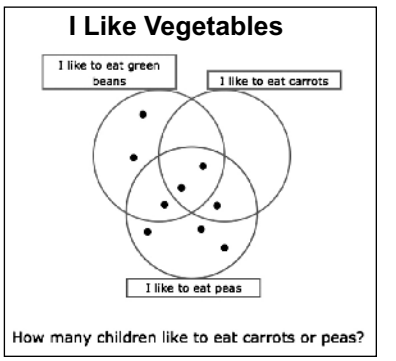
Note: A classroom board is longer proportionally than these 2 pages. Due to the space constraints on these pages, the mathematical annotations are recorded above the solutions with arrows, rather than on and around the solutions.

LET'S DO MATH – VENN DIAGRAMS

Before (Getting Started)

Venn and Vegetables Problem:

How many children like to eat green vegetables?



Solution 1	*Solution 2*
Nine people like green vegetables as all the dots are in the green bean circle or the peas circle.	Six of the dots are in the green (beans and peas) circles and three like carrots

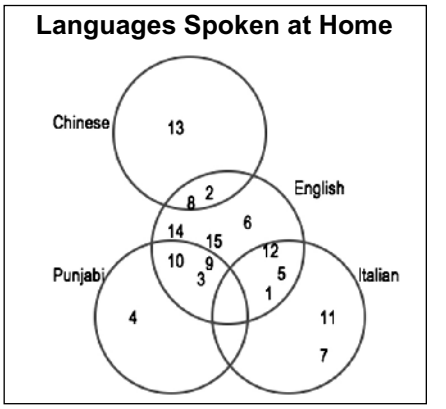
During (Working On It)

Venn and Languages Problem:

Fifteen people were asked the question, "What languages do you speak at home?" Their responses are shown by their number in the Venn diagram. What does the Venn diagram tell you about the survey results?

After (Consolidation)

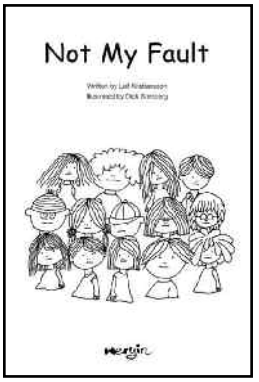
Solution 1	*Solution 2*
11 out of 15 of the children speak English at home. 3 speak two languages – 3 English and Italian and 3 English and Punjabi.	Most speak English and one more language at home. One speaks only Punjabi and one speaks only Chinese. Two speak only Italian. Three speak only English.



Coordinating Discussion for Student Learning:

Why might solution 1 be chosen first for student discussion followed by solution 2?

- *Solution 1* presents information about those that speak English and those that speak 2 languages at home.
- *Solution 2* reports on different information – the number of people who speak a single language at home and that most speak English at home.



LINKS TO LITERATURE

Not My Fault by Leif Kristiansson is a book that is very powerful in its deceptively simple illustrations and text. Each child in the group offers an excuse for not taking responsibility for stopping the bullying of a child. The end of the book has powerful images of global strife.

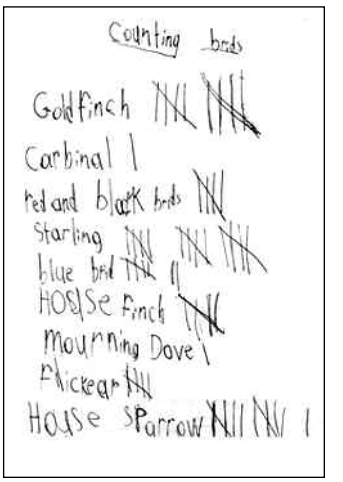
- Create a page of the excuses and ask the students to organize them into categories (e.g., blaming the bullied child, blaming others). Compare the categories used to sort the excuses.
- Use the page of excuses and collect data about which excuses students have heard being used. Which excuses are most common?
- Do we have a bullying problem in our class? Do we have a bullying problem in our school? How could we conduct a survey to find out? Would responses vary by gender, grade, age (adults and students)?

LET'S DO MATH – ORGANIZING AND TALLYING

Before (Getting Started)

Counting Birds Problem:

Marcus created the tally sheet when he went bird watching. What does is tally chart say about his sightings?



Solution 1

Marcus saw 9 different kinds of birds. He saw the most Starlings and only 1 Morning Dove and 1 Cardinal.

Solution 2

Birds	Numbers	Birds	Numbers
Goldfinch	10	House Finch	5
Cardinal	1	Morning Dove	1
Red and Black bird	5	Flicker	5
Starling	15	House Sparrow	11
Blue Bird	7		

During (Working On It)

Causes of Bullying Problem:

Students were asked, "Why do you think some kids bully others?"

Here are some responses:

trying to be cool, insecure, having family problems, trying to be cool, trying to be cool, having family problems, jealous, looking for fun, insecure, angry, trying to be cool, having family problems, trying to be cool, angry, looking for fun, trying to be cool, having family problems, jealous, trying to be cool, insecure, trying to be cool, insecure, jealous, trying to be cool, trying to be cool, insecure, having family problems, trying to be cool

What are the issues that cause students to bully others?

After (Consolidation)

Solution 1

	Tally	Numbers
Trying to be cool	𝍸 𝍸 |	11
Insecure	𝍸	5
Having family problems	𝍸 ||	7
Jealous	|||	3
Looking for fun	||	2
Angry	||	2

I read the reasons one at a time and made a tally mark for each. Then I can see that trying to be cool and having family problems were the two biggest reasons people thought that others bullied.

Solution 2

There are 30 answers. Trying to be cool is the most then insecure and having family problems. The last two are sad but no reason to bully and the first idea that it is cool is just wrong!

Coordinating Discussion for Student Learning:

Why might solution 2 be chosen first for student discussion followed by solution 1?

- *Solution 2* presents the general information about the excuses that occur most often.
- *Solution 1* presents the data in an organized tally chart so all excuses can be seen and their frequency identified.

NEXT STEPS FOR YOUR PROFESSIONAL LEARNING APPLICATION TO YOUR CLASSROOM

- Try these data collection and organization problems yourself first and then offer them to your students.
- To suit your students' learning needs, vary the ways that you have students engage with data collection and organization, as well as determining mean, median, and mode (e.g., use concrete materials, such as centicubes or interlocking cubes, counters; organize data using TinkerplotsTM (dynamic data software) or ExcelTM spreadsheets).
- Think about how the context of the problem evokes the use of data collection and organization (e.g., who cares about conserving energy by country and then by gender).
- Practise noticing the different strategies and the breadth of the mathematics your students use in their solutions. Take note of the number and kinds of solutions that show strategies for analysing data (e.g., central measures, graphs)
- Think ahead of the kinds of mathematical annotations you will record on and around the student solutions to make explicit key mathematical ideas, strategies for collecting and organizing data as the students discuss and analyze each other's solutions.

SUGGESTED READINGS

Some readings used in the Research Summary and in the development of the problems and solutions are listed below:

- Batanero, C., Godino, J. D., Vallecillos, A., Green, D. and Holmes, P. (2006). Errors and difficulties in understanding elementary statistical concepts. *International Journal of Mathematics Education in Science and Technology, 25*(4), 527-547
- Chapin, s., Koziol, Al, MacPherson, J., & Rezba, C. (2002). *Navigating through data analysis and probability in grades 3-5.* Reston, VA: NCTM.
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- Graham, A. (2006). *Developing thinking in statistics.* Thousand Oaks, CA: Sage Publications.
- Schaughnessy, J. M. (2006). *Research on students' understanding of some big concepts in statistics.* Reston, VA: NCTM, 77-98.
- Sheffield, L. et al. (2002). *Navigating through data analysis and probabiityi in prekindergarten-grade 2*. Reston, VA: NCTM.
- Small, M. (2010). *Big ideas from Dr. Small, Grades K-3*. Toronto: Nelson Education.
- Small, M. (2010). *Big ideas from Dr. Small, Grades 5-8*. Toronto: Nelson Education.

CLOSING

Our next issue will focus on key notions of organizing data to construct and read graphs, how teachers need to know these notions for teaching, as well as, the ways that teachers can use these mathematical ideas and pedagogical strategies within a problem solving-based teaching and learning lesson framework.

CALL FOR *ABACUS* SUBMISSIONS

Send your teaching strategy ideas, problems and student solutions to the *Abacus* Co-Editors via email kathkubo@rogers.com or mkestell@sympatico.ca