8-PAGE INSERT ELEMENTARY MATH

Ontario Mathematics Gazette

OAME – ONTARIO ASSOCIATION FOR MATHEMATICS EDUCATION

AOEM – ASSOCIATION ONTARIENNE POUR L'ENSEIGNEMENT DES MATHÉMATIQUES

Vol. 51 #2
December 2012

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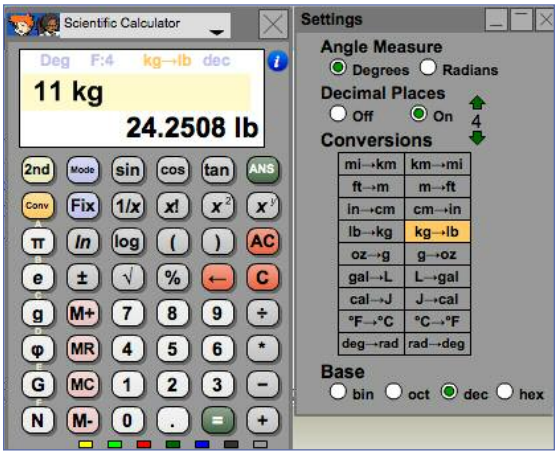
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See In the Middle – The Art of Mathematics: Fostering a Love of Math





See Communication Technology



See What's the Problem? To the Max!

$$f(x) = \frac{2x^2 - 8x + 17}{x^2 - 4x + 7} = \frac{2x^2 - 4x + 7}{x^2 - 4x + 7} = 2 + \frac{3}{x^2 - 4x + 7}$$

$$f(x) = 2 + \frac{3}{x^2 - 4x + 7} = 2 + \frac{3}{(x - 2^2) + 3}$$

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TEAMS

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Submission of Articles

The *Ontario Mathematics Gazette* (*OMG*) is looking for news items, articles, and good ideas that are useful to mathematics teachers and mathematics teacher educators. We are seeking submissions, preferably from mathematics teachers K–12 and other mathematics education professionals, that describe innovative and creative approaches to mathematics teaching.

Please keep in mind the following criteria when making submissions to the *OMG*:

- The ideas/activities must be of interest to the readership.
- The ideas/activities must be fresh and innovative.
- The mathematics content must be appropriate for the readership.
- The mathematics content must be accurate.
- The article must be well written and easily understood.
- The article and its ideas must be free of sexual, ethnic, racial, or other bias.
- The article must not have been previously published, nor should it be out for review by other publications.
- The article must be original.

Articles must be word-processed in MS Word, double-spaced with wide margins, not exceeding 10 numbered pages of text, and prepared according to the *Publication Manual of the American Psychological Association, Sixth Edition*. Figures and diagrams should be drawn by computer, if possible, or drawn in black ink in camera-ready form. Embedded images must also be submitted separately in jpeg or tif format. Proof of the photographer's permission is required, and for **photos of students** under the age of 18, the written permission of a **parent or guardian is required**.

You must submit **one complete copy** of your article, embedded with any tables, figures, and captioned photographs or graphics, to the Editor, Immaculate K. Namukasa, along with **separate files for each of the text, graphics, and/or photographs**. Please e-mail all files to Immaculate K. Namukasa at **immaculate.namukasa @oame.on.ca**.

Your name should not appear anywhere in your article, including websites, so that your article can be sent out for blind review. Your name, full mailing address, and e-mail address must be included on a separate sheet. Upon review, you will be notified as to whether your article has been accepted for publication (as is, or pending minor or major revisions) or rejected.

The Editor reserves the right to edit manuscripts prior to publication. Once an article is published, it becomes the property of OAME.

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The views expressed or implied in this publication, unless otherwise noted, should not be interpreted as official positions of OAME.

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Submission of Advertisements
Advertisements for publication in the *Ontario Mathematics Gazette* should be sent to **Robert Sherk** at the above address. Courier is recommended to avoid possible delays. Deadlines for advertisements are January 23 for the March issue, April 1 for the June issue, July 1 for the September issue, and October 1 for the December issue.

Full-page advertisements are to be on 8.5" by 11" paper with a minimum of 0.5" margins and single sided. Each advertisement should be camera ready and colour advertisements should have no bleeds.

Advertising Rates
Advertising rates are available by telephoning, e-mailing, or writing to the Advertising Manager.

▲ Editor's Message

IMMACULATE KIZITO NAMUKASA
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Immaculate Kizito Namukasa is an associate professor at the University of Western Ontario. Her current research interests in mathematics education include mathematical thinking and activity, mathematics teacher education, non-routine problem solving and learning environments, complexity research framework, and critical mathematics education. Namukasa is also interested in globalization and internationalization of schools and schooling in general.

Mathematics teachers and educators know so well that it is one thing to be encouraged to teach in ways which align with new visions of mathematics and yet another issue to have access to resources needed for teaching mathematics in helpful ways. Several articles in this issue share exemplary resources. One special feature, in particular, is about CLIPS. It illustrates how to use new tools now available within CLIPS. Jarvis, in the article entitled *RE4MUL8*, presents a newly created, open-source, Ontario-grown, so to speak, resource with a video database of teacher lessons and interviews. The article on scholarly activities shares examples of activities that teachers in their role as scholars, teachers-as-scholars, engage in. The fourth article, by Watson, is a follow-up of an article published in the March 2012 issue on Foundation, a tool for learning algebra. The *Gazette*'s regular contributors once again offer inspiring ideas that support teaching mathematics in helpful ways, as well as report on activities of meetings at which teachers and educators explore ways of promoting new visions of teaching mathematics. Because *Gazette* articles share resources, showcase exemplary ideas and lessons, and present teachers' experiences, I see their major role as continuing to occasion shifts in what teachers and educators in Ontario know, feel, and believe about learning, teaching, and assessing mathematics.

In the March 2012 edition of the *Gazette*, Sarina shared in detail how a number line may be used to explain minus times minus equals plus to middle-grade students. In my classes with teacher candidates, we do

a scavenger hunt activity in selected approved textbooks for middle grades, to identify contexts and materials used when teaching integers. The contexts are also well summarized by three students in a CLIPS video on integers (available at **oame.on.ca/CLIPS/**). Etaf Khan, a teacher candidate in a class I taught in 2006, shared how she made sense of integer multiplication. For her, *minus times a minus equals plus* made perfect sense with the analogy of multiplying emotions. Multiplying emotions? Teacher candidates in the class and I wondered about where Etaf was going with this analogy. But we followed her analogy closely, and before long, the class agreed on using the emotion of love and, its opposite, hate to explore how binary products of love and its opposite hate mirrored multiplication of integers: The product *I love to love equals*… yes, it equals *love*. People love to love and this means they love. When a student loves to love mathematics, he or she loves mathematics. But what would the product of love and hate be? We all quickly tried this product out: If a person *loves to hate*, as in those very rare cases, then he or she… hates: *I love to hate* equals *hate*. And surprisingly, *I hate to love* also equals *hate*. *I hate to hate* equals *love*. *I hate to hate mathematics* equals *I love mathematics*.

In the same class on teaching integers, I do introduce another context that appears to do more justice to treating integers as abstract entities: The activity involves extending number patterns, albeit to the unconventional left side. It involves students completing shrinking patterns (skip counting) to the left side, as shown in the table below. Students receive a worksheet with only the first quadrant of the table completed. They then complete all the rows, first by going from right to left, and then the columns from top to the bottom. After all shrinking patterns are completed in the table, the next step is to encourage students to observe the patterns each of the four quarters (quadrants) of the table. In the top right quadrant, students notice that the numbers are all positive, and in the bottom left quadrant, the numbers are all positive. And in the remaining two quadrants, the numbers are all negative. By adding a row at the extreme bottom and a column at the extreme left in a table displayed on a screen, SMART Board™, or chart paper, a teacher may guide students to notice that the numbers in the table represent products of integers. Thus, positive numbers times positive numbers, products located in the top right quadrant, would always yield positive products. The quadrant on the top left

Multiplication of Integers – Extending patterns

← Skip counting backwards in each row													Times
+5		-25	-20	-15	-10	-5	**0**	**+5**	**+10**	**+15**	**+20**	**+25**	
+4		*-20*	*-16*	*-12*	*-8*	*-4*	**0**	**+4**	**+8**	**+12**	**+16**	**+20**	
+3		-15	-12	-9	-6	-3	**0**	**+3**	**+6**	**+9**	**+12**	**+15**	
+2		*-10*	*-8*	*-6*	*-4*	*-2*	**0**	**+2**	**+4**	**+6**	**+8**	**+10**	
+1		*- 5*	*-4*	*-3*	*-2*	*-1*	**0**	**+1**	**+2**	**+3**	**+4**	**+5**	Skip counting downwards in each row
0		*0*	*0*	*0*	*0*	*0*	**0**	**0**	**0**	**0**	**0**	**0**	
-1		+5	+4	+3	+2	+1	0	-1	-2	-3	-4	-5	
-2		+10	+8	+6	+4	+2	0	-2	-4	-6	-8	-10	
-3							0	-3	-6	-9	-12	-15	
-4							0	-4	-8	-12	-16	-20	
-5							0	-5	-10	-15	-20	-25	
X		*- 5*	*-4*	*-3*	*-2*	*-1*	0	+1	+2	+3	+4	+5	

represents positive numbers times negative numbers, the bottom right quadrant represents a negative times a positive, and the fourth one, a negative times a negative. Students may then write down the four rules for multiplying integers that would have emerged from this patterning activity.

The analogy of emotions whose products mirror integer multiplication draws from the affective and motivational (as opposed to the cognitive) domains. The affective and motivational domains are increasingly referred to in conversations about learning. The role of affect in learning mathematics cannot be ignored. Like integers on a number line, what affect—especially enjoyment, attitudes, comfort levels, confidence, perceived ability (self-efficacy), and beliefs—says about usefulness of mathematics can be represented on a number line, a spectrum showing the negative, unhelpful, deactivating affect on one side, and positive, helpful, activating emotions on the opposite side. A student's comfort at and confidence in his or her ability to solve mathematics problems, for instance, is a helpful, activating emotion toward mathematics learning. On the other hand, mathematics teachers know very well that extreme anxiety at doing mathematics is a deactivating, unhelpful emotion, as it lowers students' level of engagement when learning mathematics; students do not feel motivated to even try.

For practice, beliefs about mathematics—its learning, teaching, and assessments—have direction (e.g.,

negative, neutral, or positive) and magnitude (e.g., high, moderate, or low), as can be shown on a number line. For instance, a belief that mathematics learning is enhanced by activities that build upon students' experiences is a positive, helpful, activating teacher belief about learning mathematics. On the negative side, the belief that mathematics learning is being able to get the right answers quickly is not helpful. Other helpful beliefs about math include beliefs that mathematics is predominately a tool used to discern patterns to help us understand the world around us, and that knowing how to solve a mathematics problem is as important as getting the correct solution. A report on the performance of 15-year-olds at the PISA international examination had this to say about promoting positive mathematics affect and motivation among learners: "Students with high levels of student motivation and mathematics confidence and low levels of mathematics anxiety outperformed students with low levels of students motivation and mathematics confidence and high levels of mathematics anxiety" (Bussiere, Cartwright, & Knighton, 2004, p. 46).

What a great opportunity, especially at this time of the calendar year, to review and reflect on ideas and resources that endeavour to support new visions of teaching mathematics via improving students' confidence, comfort, and motivation at learning mathematics.

Reference:

Bussiere, P., Cartwright F., & Knighton, T. (2004). *Measuring up: Canadian results of the OECD PISA study. The performance of Canada's youth in mathematics, reading, science and problem solving. 2003 first results for Canadians aged 15.* Ottawa: Human Resources and Social Development Canada; Council of Ministers of Education, Canada; and Statistics Canada. (Statistics Canada catalogue number 81-590-XIE – number 2). ▲

NOMINATIONS FOR OAME/AOEM BOARD OF DIRECTORS 2013–2014

Each year, elections are held for a President-Elect, a Vice President (3-year term), and two Directors (up to a 3-year term) of OAME/AOEM. OAME/AOEM members running for Director will declare their intent to work on either Grades JK–6 or Grades 7–12 issues based on their personal experience, expertise, and interests.

Nominations for these positions are to be submitted by 4:00 p.m., January 15, 2013, to:
Connie Quadrini, Past President,
CEC, York CDSB
320 Bloomington Rd., W., Aurora, ON L4G 3G8

Include the nominee's qualifications with the Nominations Form. A nominee must be a member of OAME. By March 2013, the Nomination Committee will publish a slate of candidates on the website, including qualifications, for consideration by the membership. Voting will be on the website, and is open to all OAME/AOEM members. The election results will be released at the Annual General Meeting of the OAME/AOEM in Toronto, May 2013.

NOMINATION FORM

I nominate:_____

for the position(s) of:

☐ President-Elect ☐ Vice-President ☐ Director

If for Director, area of personal experience/expertise, interest:

☐ Grades JK–6 or ☐ Grades 7–12

Nominee's Address: _____

_____

E-mail: _____

Telephone: Home (_____)_____

Work (_____) _____

Nominator's Name : _____

Nominator's Address: _____

_____

E-mail: _____

Telephone: Home (_____)_____

Work (_____) _____

▲ President's Message – Mathematicians Stand On Each Other's Shoulders



CATHY HALL, PRESIDENT
E-MAIL: cathy.hall@oame.on.ca

"Mathematicians stand on each other's shoulders."
- Carl Friedrich Gauss -

Those of you who know me well are only too familiar with the fact that I'm a huge fan of Gauss, even 150+ years after his demise! I often relate stories of this mathematician and his accomplishments during PD sessions with teachers. As well, I have witnessed some of the most powerful "Aha!" moments with students as they investigate his patterns.

Sometimes called "the prince of mathematics," Carl Gauss was a prodigious child, at the age of three, informing his father of an arithmetical error in a complicated payroll calculation and then following up by stating the correct answer. In elementary school, when his teacher assigned the problem of summing the integers from 1 to 100 to his students, thinking it would be the ultimate busy work, Gauss thwarted his teacher, as he often did, by immediately writing down the correct answer on his slate. How did he arrive at the correct solution of 5050 so quickly? By cleverly spotting that the sum was built upon an easy pattern... 50 pairs of numbers, each pair adding up to 101!

A few years later, Gauss demonstrated a method for constructing a heptadecagon, using only a straightedge and compass. Soon afterwards, he proved that every number is the sum of, at most, three triangular numbers. While not a stellar example of the perfect mathematician, since he often left his work unfinished or backed up by vague proofs, he was nonetheless a flexible and creative thinker, who was always willing to build upon the knowledge of others in his field, by "standing on their shoulders," so to speak.

In many ways, I see Gauss and his work as a model for our mandate in OAME... and especially so this year, when our focus is OUTREACH. The obvious links between triangular number imagery and our association's logo aside, we should not be too hasty in discounting the power of the triangle in getting the job done. After all, we are all familiar with the structural integrity of the triangle when used in applications such as bridge building and construction, as well as the grounding properties of the base of a triangle. In the case of OAME, this strong base is provided by our members and our ability to build upon each other's strengths.

The quotation I chose as the jumping-off point for this article is one of the more obscure, but in my opinion, one of the more astute quotations attributed to Gauss. As your president this year, I know just how important it is that we do, indeed, stand on each other's shoulders in our quest to help the students and teachers in this province reach the pinnacle of mathematics achievement. As we engage in building strong professional learning communities within our schools, our boards, and our regional chapters, I hope that we take the time to dig ever deeper into the math we are teaching and the delivery models for doing so. It might seem to represent a huge risk at times, but if we keep in mind that we have a safety net in the form of those colleagues forming the base of the pyramid, then it will give us the courage to climb onto their shoulders and reach higher than ever before.

If you think back to my President's Message in the September *Gazette*, you will remember that I spoke about a restructuring of the OAME Board of Directors. This three-year transitional process has now begun, and as it moves forward, will result in a pared-down governing body that will rely more fully on the input of chapters and their Chapter Reps, who will make up the majority of the Board of Directors. Our 15 regional chapters are, after all, the strength of our organization, and this restructuring will highlight that fact. Chapters will, in a sense, represent the shoulders of our provincial math association and, as such, will carry more weight, both literally and figuratively. I firmly believe that our chapter associations are the key to a successful OAME outreach initiative. Too often, I think we take for granted the countless hours our Chapter Councils and Chapter Executives put in on our behalf. I would like to take this opportunity to thank all the individuals who are the driving force behind their chapter initiatives. Although we may not always do a very good job of saying so, I want to assure you that your President and Board of Directors appreciate all your hard work and leadership in support of Ontario's math community!

Outreach can have a myriad of different meanings, depending on the context and the initiative itself. However, the basic premise always remains the same.

Outreach is simply the process of engagement with individuals and organizations by other entities or individuals, with the primary purpose of serving as a resource and a support group. Unlike sales or marketing, which are one-way and focused on immediate actions, such as highlighting a product or making a purchase, outreach is driven by two-way engagement and is not focused on immediate outcomes. Rather, it is aimed at creating and sustaining mutually beneficial and sustainable relationships. This encapsulates the basic premise of our work within OAME as we strive to support our teachers and their students. Right now, one of our main initiatives is preparing ourselves for a provincial curriculum review in mathematics. Your OAME Curriculum Committee has been working diligently to develop and fine-tune a process by which our members can become involved in providing informed feedback on where we're coming from and where we see ourselves going in math instruction in the future.

In my previous President's Message, I urged you to be proactive in this regard by taking a close look at not only your own grade expectations in math, but also at the big picture from Kindergarten to Grade 12. The goal here is to focus on connections, disconnects, gaps, and conceptual verity in the current math curriculum. You will be happy to hear that we are starting to roll out some workshops where you will have a chance to share your insights and feedback in this regard. In fact, by the time this issue of the *Gazette* comes out, you may already have had a chance to become involved in one of these workshops, since our initial target audiences will be participants at chapter mini-conferences and the Fall Leadership Conference. Some chapter associations are also planning to run forums, where this will be the working topic. We hope to be sharing some of our results and garnering additional feedback at OAME 2013 in May. Our ultimate goal in this outreach initiative is that of gathering insights from our membership and using them to shape "our OAME voice" on this topic.

A key component of outreach is that the groups providing it are often not stationary, but mobile. In other words, they are meeting those in need of outreach services at locations convenient to the target audience. This can be a bit tricky, given the logistics of our chapter boundaries. However, we are making a concerted effort to reach more of our members by putting in place some new PD approaches. In my own chapter, we have already "hit the road" to present a mini-conference for some of our out-of-town members who find it difficult to make it to the city for our regular offerings. We were pleasantly surprised by the outcome of our experiment, in that we had a good cross-section of teachers, coaches, and administrators join us for this event. The atmosphere was super-charged as folks interacted and did hands-on investigations into how their students make sense of algebraic relationships across the grade continuum. The discourse was rich, the problem solving intense, and the thanks we received made all the travel and organizational work worthwhile.

While an actual roadshow may not always be a viable option because of the cost, time, and personpower involved, many chapters are finding that they have been getting similar results using technology to draw their members together in a virtual workshop setting. We would like to hear about your chapter's outreach initiatives—or your ideas about how OAME can better serve your needs. To this end, we will be sending out an e-survey via ThoughtStream, a process that requires very little time or effort on your part (or ours!), but one that allows you to provide us with meaningful feedback on the topic at hand. Watch for it in your Inbox near the end of the month.

I'll close now... but not before leaving you with a small "outreach" homework assignment. I want you to do a bit of hunting to find something about Gauss that might make you want to join me as one of his biggest fans. It should be something that intrigues you enough to share it with someone else, whether that someone be a fellow colleague, your students, a family member, or even a stranger during your travels. You might be surprised at how accessible some of his findings are, to even the youngest children! I'd be happy to hear about any "Aha!" moments, if you care to share them with me. I'd also love to receive pics of any literal interpretations of Gauss's opening quotation. For that, you'll need to be not only a flexible math thinker, but also a well-coordinated and flexible climber and human pyramid builder. Good luck with both challenges! ▲

▲ Call for Manuscripts

The *Ontario Mathematics Gazette* is inviting manuscripts for all grade levels.

Instructions for submission of manuscripts are found on page 1 of any *OMG*.

Contact the Editor for further details.

OAME/AOEM AWARDS FOR EXCELLENCE IN MATHEMATICS EDUCATION



The purpose of the Ontario Association for Mathematics Education's Awards is to formally recognize individuals and educators who demonstrate an outstanding contribution to mathematics education in Ontario. The successful candidate of each award enhances the learning environment, providing opportunity for students to do, see, hear, and touch mathematics in a profound and meaningful way. If you know someone who qualifies for this honour, please nominate her/him/them for one of the following awards.

OAME AWARDS

OAME/AOEM Life Membership Award recognizes an educator who has contributed in a significant way to OAME/AOEM; demonstrates outstanding leadership in mathematics education; and has accumulated ten or more years of membership in OAME/AOEM.

Award for Outstanding contribution to OAME/AOEM and Mathematics in Ontario recognizes an individual who has made a significant contribution to OAME/AOEM, but who is not necessarily an educator or a member of OAME/AOEM.

Award for Exceptional and Creative Teaching in Elementary Mathematics recognizes an exceptional and creative elementary teacher who demonstrates excellence in mathematics education and contributes to the overall development of students.

Award for Exceptional and Creative Teaching in Secondary Mathematics recognizes an exceptional and creative secondary teacher who demonstrates excellence in mathematics education and contributes to the overall development of students.

Award for Leadership in Mathematics Education recognizes an educator who has demonstrated leadership by contributing in a significant way to the development of mathematics teachers and enhancing mathematics education in Ontario.

Secondary School Department Award recognizes a secondary mathematics department which fosters collegiality, teamwork, and excellent classroom teaching; contributes to the overall development of students; and demonstrates leadership in the mathematics education community.

Elementary School Staff Award for Exceptional and Collaborative Mathematics Teaching recognizes an elementary school (a school accommodating any or all of Grades JK–8), whose staff fosters collaborative and excellent mathematics teaching; contributes to the overall development of student learning of mathematics; and demonstrates leadership in the mathematics community.

OAME/AOEM Awards Nomination Process

1. Complete the Award Nomination Cover Page appropriate to the specific award, found at **www.oame.on.ca** or contact the Executive Directors for an electronic version (**EDs@oame.on.ca**). You may include a one-page letter of recommendation.
2. Provide up to three letters of support, of no more than two pages each. These supporting documents may be gathered from a variety of sources to reveal a full picture of the nominee. The support documents will provide precise information about the nominee's contributions as they relate primarily to the specific award.
3. Include the nominee's Curriculum Vitae of up to five pages. (C.V. not required for Secondary School Department award and Elementary School Staff award)
4. Send the completed cover page and supporting documents by January 15, 2013 to: OAME Executive Directors, 1652 Chester Dr., Caledon Village, ON L7K 0W6, fax: 519-316-0123, or email: **EDs@oame.on.ca**

Incomplete or inaccurate information may cause a nomination to be withheld from consideration.

DEADLINE: Nominations are due by January 15, 2013 to the Executive Directors (contact info above).
The awards will be presented at the Annual OAME Conference in May 2013.

▲ Technology Corner – Twitter – My Math

MARY BOURASSA
E-MAIL: mary.bourassa@oame.on.ca



Mary teaches mathematics at West Carleton Secondary School in Ottawa. She is a strong advocate for the appropriate use of technology in the classroom. She has presented workshops internationally, authored mathematics resources, is a past VP of OAME and a Past President of COMA. An award-winning teacher, Mary continually strives to learn new and better ways of helping students learn and love mathematics.

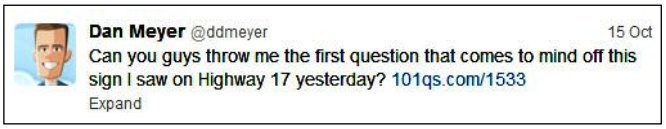
For many years, I ignored Twitter. I never saw the need to join, as I didn't realize the potential benefits. Then I went to one of Dan Meyer's sessions at OAME 2012 and decided that it would be interesting to read his tweets. So I signed up and started my Twitter education. Now Twitter is more important to me than I could ever have imagined. The benefits stem from my new Personal Learning Network (PLN).

Creating a PLN is an ongoing journey. I am getting better at finding good people to follow. I follow many math teachers, but also many educators who never tweet or post about math. Both are valuable resources, as they provide links to great activities, ideas to build on, and thought-provoking posts that sometimes turn my thinking about education upside down.

For those of you who do not use Twitter, I will briefly explain how it works. Tweets are message that are 140 characters or less. Your followers see them. If you keep your account public, then anyone can follow you. You see the posts of those you follow. You can retweet someone else's message. For example, if Vi Hart (@vihartvihart) posts a link to her latest video on hexaflexagons, you can spread the math by retweeting it so that your followers will also get the link. You can also add hashtags to your messages, indicating your tweet's subject, such as #mathchat, #edtech, #3act, or #oame2013. Hashtags are also used for chats that occur at a pre-set time, such as #edchat, which happens every Tuesday at noon and 7 p.m. eastern time.

Although I continually work to improve my teaching in all my classes, my current focus is my MFM2P class. The Grade 10 Applied students need to be active in order to remain engaged, and they need to see the real-world connections in the math that they do. The ideas I am finding through Twitter are mostly for that class, although they can be used for others. Many of the teachers I follow in the U.S. teach Algebra in Grade 8, yet what they do works really well for 2P.

Some of my math PLN follows (no pun intended). They are all awesome! Although I have never met most of them, I enjoy reading their tweets. I will admit that this sometimes feels wrong, as I am lurking—reading a conversation going on between others—but they know it is public, so I remind myself of that fact. These folks all share freely. They post ideas, activities, lessons, and advice. Need inspiration? Ask your PLN!

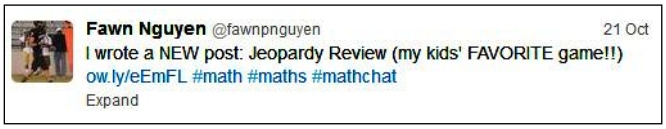


Dan Meyer (@ddmeyer), famous for his three-act lessons, Dan has numerous resources online. I have used the cup-stacking activity to explore slope and *y*-intercepts with my 2P class. It was great! In groups, kids need to determine the number of Styrofoam cups needed to reach as high as the top of my head. Each cup has a lip, whose height represents the slope. The rest of the cup is the fixed height. Great questions follow, making them really think about the two elements of a linear equation. Dan has a Google Doc of three-act lessons. He is also the keeper of 101 Questions (**www.101qs.com**), a site dedicated to potential Act 1 pictures and videos. It prompts you to enter the first question that comes to your mind. The pictures and videos are accorded a perplexity score based on the number of people who post questions versus the number of people who skip it. Dan continues to make teachers think about how to engage students and how we can change what we do for the better.



Andrew Stadel (@mr_stadel) also posts many three-act lessons. The latest one I used for my 2P class was his basketball shot three-act. Act 1 is a video of him shooting 1-point and 2-point shots, with the clock counting down from 60 seconds, but it stops partway through. Act 2 shows the total number of shots made and the total number of points. Students have to figure out how many

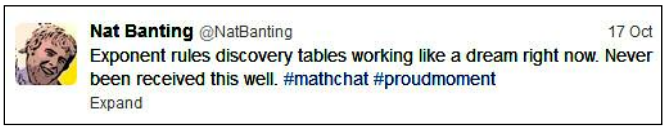
of 1-point and 2-point shots he made. Act 3 shows the entire 60 seconds, with the number of shots and points increasing along the way. I gave this to my 2P class on the first day of solving linear systems and let them run with it. I got equations (so awesome!), tables, and graphs out of the groups, and they understood what the solution meant. Great start to the unit! Andrew also has an estimation site to help improve students' sense of estimation. There is a new one every day. We go there for the last few minutes of class time, and I keep track of the best estimators, who are not necessarily the best math students. There will be a prize forthcoming at the end of the month.



Fawn Nguyen (pronounced "win," so the hashtag #nguyening is quite popular with her tweeps) (@fawnnguyen) posted an excellent activity she took from NCTM, where students are given polynomial expressions shown algebraically, in words, using tables and graphically. They have to cut them out and match them up. This activity was the starting point of solving equations in 2P and was stretched out over two days because it was so rich and provided so many connections for students. Fawn also has fun review activities and games that her students play, that can be incorporated at almost any level. I get new ideas from her every week—now that's nguyening!



Michael Pershan (@mrpershan) posts math mistakes that students commonly make. The discussions that follow about why students are making these mistakes and how we can overcome them are very important. Michael also has very interesting posts on problem solving, among other math topics, on his blog. He was also the winner of the #MTT2K contest (there's a hashtag for you to check out!).



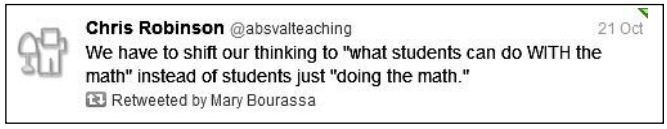
Nat Banting (@NatBanting) is a fellow Canadian who is working on project-based learning. He has excellent resources, such as Trigonometric Mini-Golf which gives students a very open-ended task and lets them learn as they work through it. He uses the Guess Who game with numbers instead of people to encourage students to think about the characteristics of numbers. He has a sprinkler task that is fantastic. And there is much, much more. Nat posts what works and talks about what didn't work as well as planned, and how tasks can be improved or extended. Great resources here!



Megan Hayes-Golding (@mgolding) organizes the Global Math Department. The group's online description is, "We are math teachers who share what we've learned, 'cause we don't want our classes to suck the energy from students. Professional development among friends, not just colleagues. Fun! Immediately useful! Interesting!" and more importantly, "We want to be better teachers. We share freely. We are always supportive." Every week, Megan organizes some type of PD, usually on Tuesday evenings. The latest have been on three-act lessons, standards-based grading (a big thing in the U.S.), and desmos.com. Can't thank her enough for all her time and energy, helping us all be better teachers.



Renee (@approx_normal), once you get past all the crazy posts (which are very entertaining), has some good thoughts on pedagogy. She made me think about how best to teach quadratics when she blogged about her success teaching it in a different order from previous years. This idea of sharing what you did, whether it worked well or not, is very powerful.



Chris Robinson (@absvalteaching) is another in the three-act lesson crew. Clearly he has a young child, as his latest three-act lessons involve Mr. Potato Head and blocks. He also thinks and posts about pedagogy—lots of thought-provoking questions. I will never stop learning or trying to improve with folks like Chris around!

There is so much out there that it can be overwhelming. Want math facts? Try @maanow. Want math history? Try @OnThisDayInMath. Want other great activities and lessons? Try @mathalicious, @jamestanton, or @Maths_Master. Use TI technology?

Follow @TICalculators. Use desmos.com? Follow @desmos. Use a SMART Board™? Follow @SMART_Tech. Love TED talks? Follow @TED_ED. You get the idea.

Outside of math, there is also much of value on Twitter. Will Richardson (@willrich45) is a huge proponent of twenty-first-century schools and makes me think every time he blogs. His book, Why School?, is definitely worth the read (available on Kindle). He pushes me to rethink what I do in the classroom and how I do it. Other twenty-first-century learning gurus include the Couros brothers (@gcouros and @courosa), along with Patrick Larkin (@patrickmlarkin), but there are so many who post on this subject that you should find your own to follow. You can learn about flipping your classroom, from those who do and those who did and no longer do. You can learn about problem-based learning. You can learn about the use of technology in schools. You can learn about the introverted students in your classroom and what works best for them. Tony Baldasaro (@baldy7) regularly posts on the subject, and I have learned much through him about my students and myself, as I am an introvert myself. I also follow some whose opinion I disagree with, some whose tweets irk me, but also make me examine my own thinking and reasons for doing things the way I do.

Twitter is also the great leveller. It doesn't matter who you are, where you are from, or how long you have been teaching, you can contribute. It connects you to educators from around the world, who all want to be more effective teachers. It gives you immediate access to resources, ideas, and opinions that will keep you growing as an educator, and this will all benefit your students. ▲

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▲ What's the Problem? To the Max!

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Shawn is the head of Mathematics, Technology, and Family Studies at Cairine Wilson Secondary School in Orleans, Ontario. He is a past and present member of the OAME Board of Directors and is currently Editor-in-Chief of Crux Mathematicorum with Mathematical Mayhem.

Welcome back, problem solvers. Last time, I left you with the following problem:

Determine the maximum possible value of the function.

$$f(x) = \frac{2x^2 - 8x + 17}{x^2}$$

This problem appeared as part of problem M472 in the February 2011 issue of *Crux Mathematicorum with Mathematical Mayhem*, and the solution appeared in the November 2011 issue. You can check out more problems at the *Crux Mathematicorum with Mathematical Mayhem* website, **http://journals.cms.math.ca/CRUX/**.

Ah yes, maximizing functions, that calls for Calculus! (Trumpets blare and a hero appears, the sun glinting off her pencil….) Differentiating the function yields,

$$f'(x) = \frac{(x^2 - 4x + 7)(4x - 8) - (2x^2 - 8x + 17)(2x - 4)}{(x^2 - 4x + 7^2)}$$

$$= \frac{2(x-2)(2x^2 - 8x + 14 - 2x^2 + 8x - 17)}{(x^2 - 4x + 7^2)}$$

$$= \frac{-6(x-2)}{(x^2 - 4x + 7^2)}$$

Thus, as $x^2 - 4x + 7 = (x - 2^2)$, we have $f'(2) = 0$, with $f'(x) > 0$ for $(x) < 2$ and $f'(x) < 0$ for $x > 2$. Hence, there is a local maximum at $x = 2$, and so the maximum value of the function $f(x)$ is $f(2) = 3$.

As always, when you have a nut to crack the best thing to do is to grab a sledgehammer! Now don't get me wrong, I love Calculus. It is one of the greatest achievements of the human race, but just because you can use Calculus, it doesn't mean you should.

Let's back things up a bit and think in terms of material that was covered in MHF4U. We can rewrite our function of the form

$$f(x) = \frac{2x^2 - 8x + 17}{x^2 - 4x + 7} = \frac{2x^2 - 4x + 7}{x^2 - 4x + 7} = 2 + \frac{3}{x^2 - 4x + 7}$$

Now, clearly if we want to maximize $f(x)$, we need to minimize $x^2 - 4x + 7$ (and ensure it is non-zero). Since we know that $x^2 - 4x + 7 = (x - 2^2) + 3 \geq 3$, we know that

$$f(x) = 2 + \frac{3}{x^2 - 4x + 7} = 2 + \frac{3}{(x - 2^2) + 3}$$

is maximized when , so the maximum is $f(x) = 3$.

We get the same results with some "lesser" tools, but we may come away with a better sense of why the function behaves as it does. If we take things one step further and back up to MPM2D, we can see that

$$f(x) = \frac{2x^2 - 8x + 17}{x^2 - 4x + 7} = \frac{2(x-2) + 9}{(x - 2^2) + 3} = \frac{2(x-2) + 3 + 3}{(x - 2^2) + 3}$$

Even without polynomial division, a Grade 10 student (with a bit of ingenuity) can use the tools at hand and come up with a solution to the problem. I plan to use this problem with my Grade 10 class next semester (and my Calculus class this semester). I am interested to see what they come up with.

Jack LeSage sent me a nice, non-Calculus solution to the problem similar to the second one above. Jack also suggested some generalizations. A nice problem for teachers or students is to make up a problem similar to the one above.

If we note that our final form of our function is

$$f(x) = a + \frac{k}{(x - h^2) + k}$$

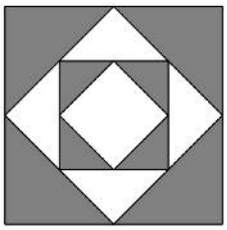
where, to make the solutions nice, a, h, and k are integers, with $k > 0$ (why?). In this case, our maximum is $f(h) = a + 1$. We can then "disguise" the problem by putting it in the form of the original problem, with a little algebra. You can then do another type of problem such as

$$f(x) = a + \frac{bk}{(x - h^2) + k}$$

where now the maximum is $f(h) = a + b$, so the sky is the limit. Students could create problems for each other and then solve them to get some practice of the necessary skills and to help them internalize the relationship between the algebraic form and the solution.

Now for your homework.

Tell me everything you can about the diagram to the right. (Feel free to send me your thoughts.)



Until next time, happy problem solving! ▲

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▲ Not-Quite-So-Elementary Mathematics: The Mathemagic of Trees

LYNDA COLGAN
E-MAIL: colganl@educ.queensu.ca



Lynda Colgan's career has included roles as a classroom teacher, a university professor, and newspaper columnist. Her contributions to mathematics and its teaching have been recognized through awards such as the Marshall McLuhan Foundation Distinguished Teacher Award. Lynda always exhibits a passion for mathematics and views her professional mission as dispelling the myth that math is the bad guy.

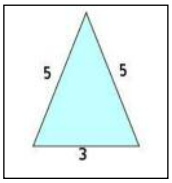
I'll never forget the day I came home from elementary school to find my mother sitting on the verandah, quietly crying. A stoic Scot, I'd never seen her cry before, and the sight was unsettling. "Did something happen to Dad," I asked, "or to one of the dogs?" "No," she whispered, her eyes welling with tears, "but we are going to lose a good friend very soon." By then, I was crying too. "Who is going to die, Mom?" "Our beautiful, old Elm tree," she said, sadly. "It has Dutch Elm disease, so they'll be coming to cut down our stately lady in the next few days."

At the time, I must confess, I did not understand my mother's anguish over the felling of the tree she called her friend. It took many years, but I now "get it." I, too, would be heartbroken to lose one of the grand old maple trees that share my home. No matter the season, I love my trees and the gifts they bear. They provide luxurious shade in the summer and act as natural air conditioners. They shelter nests and offer themselves up as gymnastic equipment to local squirrels. They play jokes—tickling the windows with their leaves like a child ringing the doorbell and then running away to hide—only to be still when one falls for the trick and answers their beckon. They are accurate predictors of rain, turning the underside of their leaves skyward when blue skies will soon change to grey. They decorate the landscape, their leaves changing from deep burgundy to deep green and then to gold, as spring changes to summer and then to autumn. They are simultaneously symbols of strength, standing tall and proud in the face of fierce winds, and reminders of the wisdom of flexibility, bending, but not bowing to that which they cannot control—like ice and heavy snow—so that they will not break.

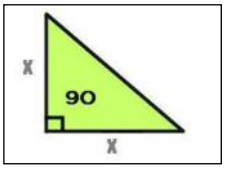
At a time in our history when global warming, pollution, and disease threaten many species of animals and plants which are key to our own sustainability, it seems appropriate to examine the *mathemagic* of trees. Doing so may give each of us a new lens through which to look at that bright red maple leaf on Canada Day and deepen our commitment to becoming the caretakers not only of the trees on which grow our national emblem, but the elm, birch, pine, and too many others to catalogue here.

Many questions come to mind when one looks at a majestic, old tree. How tall is it? How old is it? How large is its canopy? How many leaves does it have? What does it add to the air that we breathe? Mathematics holds the key to finding each of these answers.

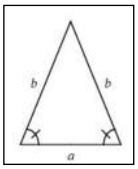
Although there are many sophisticated instruments for precisely measuring the height of a tree, there are also a number of simple and reasonably accurate methods that can be integrated into elementary mathematics lessons to determine the approximate height of backyard, schoolyard, or parkland trees.



To apply the first method, which is attributed to Native Americans, we need to use geometry—specifically, the definition of an isosceles triangle. An isosceles triangle has the property that two of its sides have equal length, as illustrated.

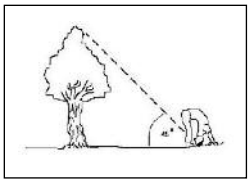


For our application, we will be considering a special type of iscosceles triangle: the right isosceles triangle, which has two equal sides and a 90-degree angle.



The fact that isosceles triangles have two equal sides, results in the fact that the two angles opposite the equal sides are also equal. Since the sum of all angles in a triangle must be 180 degrees, in a right isosceles triangle, the other two equal angles must sum to 90 degrees, with the result that each angle has a measure of 45 degrees.

We are now ready to compute the height of a tree, using the Native American method.

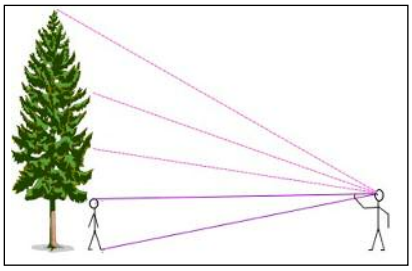


Source: **http://nrich.maths.org/2434**

Walk far enough from the tree to find the point where you are just able to see the top of the tree from the upside-down position shown in the diagram. The distance from this spot to the base of the tree is approximately the height of the tree.

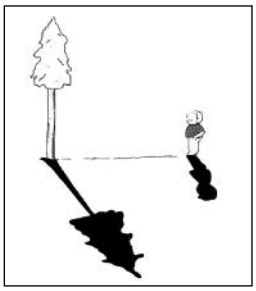
Why does this method work? For an average adult (who can bend over in such a way), the angle that is formed as the person looks through his or her legs is approximately 45 degrees. The angle between the tree trunk and the ground is fairly close to 90 degrees. Using what you know about the angles and sides of a right isosceles triangle you can use the definition and properties to conclude that the height of the tree and the distance from the tree to the person is about equal.

If you would like to check the accuracy of your measurement, you can try another practical method. For this strategy, you will need a partner.



Source: **http://nrich.maths.org/2434**

Ask your partner to stand at the base of the tree. Now walk away from the tree, holding your pencil at arm's length, between your thumb and forefinger. When you reach the point where the top and bottom of the pencil coincide with the top of the head and the bottom of the feet of your partner, stop. Next, find the height of the tree in "pencil lengths." Accurately measure the height of your partner and multiply this by the number of pencil lengths you counted for the height of the tree. This is worth doing several times from different distances and working out an average to improve the accuracy.



Source: **www.fws.gov**

You can also use shadows and the principles of similar triangles to determine a reasonable measure for the height of a tree. First, measure your own height and the length of your shadow. Then, measure the length of the tree's shadow, using the same units. Use ratios to determine the height of the tree, i.e., $\frac{\text{tree's height}}{\text{tree's shadow}} = \frac{\text{your height}}{\text{your shadow}}$.

There are a number of reliable methods for computing the age of your tree. The first is to count and examine its growth rings by examining a horizontal cross-section from the trunk of the tree.



Source: **http://drwile.com/tree_rings.jpg**

Tree growth rings are formed from the centre outwards; the ring closest to the centre is the oldest, and the ring closest to the bark is the youngest. Wide rings indicate good growing conditions, while narrow rings indicate a poor growing season. While the rings are a year-by-year record that reflects the climatic conditions in which the tree grew, counting them requires either that the tree be cut down or core-boring into the trunk (a method that can be very harmful to a tree, since the boring leaves a hole in the tree through which insects may enter); therefore, arboriculturists or arborists (those who study trees) have developed non-invasive methods.

One reasonably accurate method of determining the age of a tree begins with finding the circumference (or girth) of the tree, since we know that as trees grow, the trunk gets thicker—the older the tree, the greater its girth. On average, trees increase their girth by 2.5 cm a year.



Source: **www.gardenbuilders.co.uk**

Measure 1 metre up the trunk from ground level. Wrap your tape measure around the tree at this height and record the distance around the trunk. To calculate the age of your tree, divide its circumference by 2.5. For example, if your tree measures 1.5 m around its trunk, convert the measurement to centimetres by multiplying by 100.

$$1.5 \times 100 = 150$$

Now divide the measurement in centimetres by 2.5 (since on average, trees increase their girth by this amount annually).

$$150 / 2.5 = 60$$

This means your tree is about 60 years old.

Many scientists are studying the tree canopies (leaf cover) of cities by comparing satellite photographs of an area taken a number of years ago with recent images. Sadly, in most cases, in most cities, the area of the tree canopy is much smaller because of development—a fact that has enormous impact on our local ecosystem, including water and air quality, temperature, noise pollution, and wildlife.

To determine the canopy of your tree, you will need eight small sticks (Popsicle sticks work well). Next, imagine the tree's leaf cover as a clock. First, place sticks in the ground at the edge of the leaf cover at 12:00, 6:00, 3:00, and 9:00. Then place sticks halfway between each of those four sticks. When all eight sticks are inserted into the ground, use string to create a large circle around all the sticks, and then measure the length of the string. The length of the string is the circumference of the tree canopy, from which you can calculate the area of shade.

Why is your tree's canopy important? Researchers say that shade trees can lower summertime temperatures in cities, significantly reduce air-conditioning bills, and trap greenhouse gases responsible for global warming. Did you know that a city lot with 30 percent plant cover provides the equivalent cooling necessary to air-condition two moderately sized houses 12 hours a day in the summer? Trees also play a key role in the water cycle.

We can tell that water has evaporated from a mud puddle when it dries up. We can see water in its condensed form by looking at clouds. We can see and feel precipitation in the form of rain and snow, and we can see our breath on a mirror and feel it against our hand, but usually we cannot see the water plants give off.

To do this simple and simply amazing experiment, we need a reasonably accurate count of the number of leaves on your tree. Because counting individual leaves would be impossible, we obtain a good estimate by first counting the number of leaves on a small twig from the tree; then counting the number of twigs on an average branch; and, next calculating the approximate number of leaves on a branch. As a last step in the estimation process, we count the number of branches on the tree, and multiply the number of leaves on a branch by the number of branches on the tree.

To measure your tree's transpiration rate (i.e., how much water it gives off), put a small, clean plastic bag over a small branch and close it off tightly with a twist-tie. When you check the bag the next day, you will find that there is water inside. You may be able to get an accurate measure of the volume of water by using measuring spoons. Since you know the number of leaves that were enclosed in the plastic bag and the volume of water they produced, as well as the total number of leaves on the tree, you can calculate the quantity of water the tree gives off in a 24-hour period. If you compare the amount of water produced by a deciduous tree, like the maple, in comparison to a coniferous tree, you should find that the former produces about four times as much water daily as the latter. If you have a vegetable garden, try the experiment with other plants—according to some experts, one stalk of corn gives off about 4 litres of water a day through its leaves.

These are but a few examples of the ways in which we, as elementary mathematics teachers, can bring life to topics like isosceles triangles and concepts such as similarity. There is no doubt that a brainstorming session among peers could generate countless other applications having interest and integrity, for as Ralph Waldo Emerson once said, "The wonder is that we can see these trees and not wonder more." ▲

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▲ In the Middle – The Art of Mathematics: Fostering a Love of Math

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TWITTER: @mathplay_net
PINTEREST: http://pinterest.com/mathplaynet/where-s-the-math/



Caroline Rosenbloom is currently on secondment as an Elementary Coordinator in the J/I Midtown Technology Option and Mathematics Instructor in the pre-service program at OISE/UT. She has enjoyed working with intermediate students and teachers as a middle school teacher, Mathematics Instructional Leader for the Toronto District School Board, and Education Officer at EQAO. Caroline is co-author of Investigating Mathematics Using Polydrons.

Talk about math and mathematical experiences can lead to discussions of tools and techniques, of time spent repeatedly solving questions, all of which look like samples given in the text. All of us, as mathematics educators, wonder why mathematics is not discussed as something to love, experience, and explore; something that is beautiful and challenging and exciting. *A Mathematician's Lament*, by Paul Lockhart (2008), explores this situation, drawing analogies between "traditional" mathematics education and a world in which music education is taught in an uninspiring and rote way.

He begins his story with a musician who wakes from a terrible dream. In the dream, this musician finds himself a student in a society where music education is mandatory and taught with a strong focus on "basics" first.

> Since musicians are known to set down their ideas in the form of sheet music, these curious black dots and lines must constitute the "language of music." It is imperative that students become fluent in this language if they are to attain any degree of musical competence; indeed, it would be ludicrous to expect a child to sing a song or play an instrument without having a thorough grounding in music notation and theory. Playing and listening to music, let alone composing an original piece, are considered very advanced topics and are generally put off until college, and more often graduate school.
>
> As for the primary and secondary schools, their mission is to train students to use this language—to jiggle symbols around according to a fixed set of rules: Music class is where we take out our staff paper, our teacher puts some notes on the board, and we copy them or transpose them into a different key. We have to make sure to get the clefs and key signatures right, and our teacher is very picky about making sure we fill in our quarter notes completely. One time we had a chromatic scale problem and I did it right, but the teacher gave me no credit because I had the stems pointing the wrong way.

This situation is obviously absurd from the point of view of inspiring great musicians. Everyone is able to explore and appreciate music, even before memorizing notes and their written representation. Even more disturbing, in the analogy, is the extent to which basics are studied to the exclusion of motivation. So why is mathematics different?

I believe we have improved our mathematics pedagogy by differentiating instruction to meet different learners' needs. We have also thought of ways to provide a variety of tools to represent mathematics in many ways. But are students exposed to and enjoying mathematics for its intrinsic beauty, patterns, and hidden truths?

Since September 2009, I have been seconded to OISE, hoping to inspire the next generation of mathematics educators. You can imagine my excitement when my teacher candidates approach me, asking how they can further their background in mathematics, wanting to inspire their students to love mathematics, as they have been inspired. So how do we do this for students in the classroom? How do we foster a love of math?

Mathematics is interesting when it is seen taking its natural place in a story, a work of art, a video, or weaved into our everyday lives. Some of the resources below provide great connections between math, art, statistics, and real life. They provide a great way to add inspiring mathematics to our lessons, without requiring that students first "learn the notes."



Field Trips

- Take a walk around the city and create a **math trail** for your students. Then have them create their own math trail! Ron Lancaster, Senior Lecturer at OISE/UT, has designed many math trails to help you get started. Math trails are a great way for students to see geometric designs and patterns in nature and architecture in their very own neighbourhoods.
- Visit the **Museum of Mathematics** in Manhattan, New York. Opening day is December 15, 2012! An exciting new museum that is all about math!
- Visit the **Bridges Virtual Museum**. This is the website for the Bridges Organization, which oversees the annual Bridges conference on connections between art and mathematics.

Videos

- **Math and Escher Art** – *Inspirations* is a short movie inspired by Escher's works. It is a strange vision of what Escher's studio could look like.
- **Numberphile** – This is a great website that has short videos about numbers and all sorts of other math-related things.
- **George Hart** – George is a mathematician, computer scientist, and sculptor. You can see many of his cross-disciplined art pieces in his videos.



Podcasts

- **BBC Radio4: More or Less Behind the Stats** – Tim Harford investigates numbers in the news. Numbers are used in every area of public debate. But are they always reliable? Tim and the *More or Less* team try to make sense of the statistics which surround us.

So how do I incorporate this into my math lessons? To change the pace, I sometimes introduce a short mathematical video or work of art, like those listed above, to the beginning of class. Students discuss what they liked and disliked about the work. We explore what they were clear and confused about. They raise mathematical questions, some of which we explore. Others we leave for further thought and investigation, understanding that interesting questions, by themselves, are a very important part of mathematics. We include parents/guardians by sending home a link to a video or photo and have students describe their discoveries, investigations, and open questions to their families.

Mathematics can help us model and explain global trends, optimize procedures, calculate and represent questions, and much more. But let's help students see that the process of thinking in different ways, and the art of mathematics, is just as beautiful and valuable as reading a classic novel or listening to their favourite song. As math educators, we need to expose our students to this beauty and see this as an important part of our math lessons.

> "The mathematician does not study pure mathematics because it is useful; he studies it because he delights in it and he delights in it because it is beautiful."
>
> ~ J.H. Poincare

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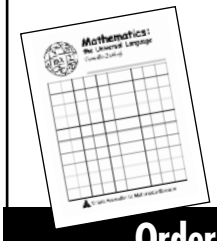
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▲ Directors' Dialogue

ANN MICHELE STENNING
E-MAIL: a.stenning@tvdsb.on.ca

Minutes from the Board of Directors' Meeting October 12–13, 2012, Toronto, Ontario

Board Meeting, Friday, October 12, 2012

Cathy Hall, OAME's president, began the board meeting by noting the broad depth of our knowledge. Members were reminded of changes to the structure of OAME, with a reduced number of board directors. The organization is mindful of how to keep costs in check and to look for efficiencies. Susan Petri's resignation was announced due to work commitments, and new board members were introduced. These announcements were followed by group discussions on ways to promote OAME.

Board reports followed. The NCTM report was recorded in the agenda and board members heard from OAME's 2012 conference co-chair, Bill Lundy. Bill noted that the banquet on Friday was well received and that Kingston financially supported the event in exchange for promoting the city.

As a final activity, board members considered the concept of equality within the curriculum from Grades 1 to 12. Members looked at curriculum documents to identify how equality is introduced. For example, teachers use phrases such as "equal to," "simplify," "solve," "find the missing number," and "balance" to describe equality. We were encouraged to wonder whether if more teachers used the word "balance" throughout the grades, perhaps more complex examples of equality would be easier to understand. Members noted that "equality" can take on different representations, such as an equation, table of values, and graph. Many also noted that equality exists in an expression as it is simplified and in an equation as it is solved. The feedback collected on equality throughout the grades will be summarized in preparation for curriculum review.

Board Meeting, Saturday, October 13, 2012

To begin the meeting, the board meeting's agenda was passed and the minutes from the February board meeting were approved. The financial report for 2011–2012 and proposed budget for 2012–2013 were reviewed. Board members subsequently voted on motions.

Anne Yeager noted that the recent OMCA meeting focused on current resources for math educators, coordinators, and coaches. OAME's annual and leadership conference dates were announced, and resources from the LNS, Math GAINS, CLIPS, financial literacy, from Paul Costa, Gizmos, and TIPS were previewed.

The OTF report was presented by Bill Otto. He noted that a large amount of work on financial literacy has been released. Future plans will streamline the financial resources in an effort to manage educators' implementation of the Ministry of Education's (MOE's) vision. The mental health initiative developed by the MOE is moving into the schools to support students.

Bill Otto noted that the College Math Project (CMP) is now completed. As a result of findings from the CMP, math funding will expand to first- and second-semester courses to support students in college. The College Student Achievement Project (CSAP) will replace the CMP.

Nominations for OAME awards are coming up.

In the afternoon, Bill announced some reminders on behalf of the National Film Board (NFB). All teachers can use the subscription service to the NFB. They can go online to obtain a subscription. If a school board is registered, the subscription is covered by the board.

The communication committee announced that OAME is now on Facebook and Twitter. OAME can be found @OAMEcounts.

The outreach committee asked board members to consider hiring a person to secure contracts for a venue on behalf of OAME's annual conference.

Anne Yeager chaired the committee that looked at the seven guiding principles in Mathematics Education K–12. She indicated that the committee will link the guiding principles to current resources that will implement the seven principles in the classroom.

EduGAINS is a mathematics resource inventory for teachers, which links curriculum expectations to resources. The document is being developed and a release date has not been announced.

Before adjourning, board members were asked to help themselves to materials that promote the 2013 annual conference in Toronto at Seneca College.

Written Reports

Chapter Reports

The CHAMP Fall Event 2011 provided another great PD opportunity, with a selection of workshops geared to K–12. The evening was capped with a keynote titled "Marvelous Mathematics for All Ages" by George Gadanidis from Western's Faculty of Education.

The OMO regional playdowns took place in three boards in our region last May. CHAMP ended up sending three

teams to the provincial playdowns hosted by the South West Ontario Association for Mathematics Education at St. Clair College in Windsor. Although we did not win the individual or team medals, CHAMP Team #3 won the Scavenger Hunt and a great time was had by all participants.

Last Spring, CHAMP held math contests for students in Grades 9–11. Close to 20 schools participated in this opportunity for students to enjoy more math. We plan to run the contest for yet another year.

On October 23, 2012, CHAMP is hosting its "Fall Event: Problem-Based Learning Across the Grades." This event will host two keynote speakers: David Zimmer and Ron Lancaster.

The executive committee is currently involved in the early stages of planning for the provincial conference in 2014. The conference organizing committee has had several meetings, and we are excited to be playing host for the first time since 1991. Any members interested in volunteering may contact the either of the Chairs: Paul Alves or Dwight Stead. We hope to see all OAME members at the conference in 2014 at the DoubleTree Hotel in Toronto.

ISOMA is busy planning their two major events of the year: their annual conference and their Math Olympics. ISOMA would like to thank Beth Johnston and her colleagues at Greenwood College for hosting yet another very successful ISOMA Math Olympics in April 2012. The day was filled with fun math challenges, and all participants expanded their mathematical knowledge. Congratulations to the ISOMA team, consisting of students from Havergal College and Crescent School, who achieved gold overall at the 2012 Ontario Math Olympics! ISOMA is proud of their accomplishments. We would like to recognize Karen Mackasey (Havergal College) and Kee Ip (Crescent School) for their time and great effort in preparing the ISOMA team for the OMO. We look forward to the events Brad Stephens and his colleagues at Holy Trinity School are planning for the 2013 ISOMA Math Olympics this upcoming April.

ISOMA is also starting to plan their annual Math Conference. This year, the event will be held at Bishop Strachan School on January 9, 2013 from 1:30 p.m. to 6:00 p.m. The theme of this year's conference will be "Project-Based Learning." The conference will begin with a project-sharing opportunity by grade level, followed by two sessions centred around problem-based learning and concluding with a keynote speaker. We are very excited to have Carmel Schettino as our keynote speaker. Carmel is currently a teacher at Deerfield Academy and a doctoral student at the University of Albany. Her research interests include pedagogical and instructional methods centred on problem-based learning; gender equity and social-justice issues in mathematics education; discourse in the mathematics classroom; and technological implications for teaching mathematics. With 20 years of teaching experience, Carmel has consulted for a number of schools and presented for the NCTM with respect to Problem-Based Learning in the Secondary Mathematics Classroom. More details, including a copy of the program and a link to the registration page, will be available on the ISOMA website (**http://chapters.oame.on.ca/isoma/**) by mid-November. Please contact Sheila Barclay (**sbarclay@bss.on.ca**) for more details.

MAC2 had a very successful year this year. We hosted a Spring conference entitled "Social Justice and the Math Classroom." It was well attended. Students from both the local Public and Catholic boards were invited to attend the Math Olympics competition. The students had an exciting and fun-filled day doing math activities. We sent two teams of students to the OMO. We will be hosting our Fall mini-conference within the next two weeks. It is entitled "Bringing Math to Life," with a plenary session "Laughs and Math" by Don Fraser. We have invited parents, guardians, and educators to attend. We also are looking forward to offering professional development at our future meetings. This school year brings with it some changes in the NOMA executive. We wish some of our former members all the best in their new professional roles, and we look forward to welcoming new members who bring energy and enthusiasm to NOMA. Traditionally, NOMA has hosted a conference in the Fall; however, with the local Numeracy PA days being held in the Fall, we are shifting our conference to the Spring. It will be a busy Spring, as we will also be hosting our regional OMO playdowns at that time. We continue to work toward developing stronger connections with district school boards, the faculty of education, and all of the communities in the NOMA region.

NWOAME has a busy year planned. We are kicking it off on Wednesday, October 24 with an evening presentation by Dr. Cynthia Nicol. Dr. Nicol's presentation will focus on "Culturally Responsive Mathematics" (based on *Tluuwaay 'Waadluxan Mathematical Adventures: For Gina Waadluxan Tluu: The Everything Canoe* (Nicol & Yovanovich, 2010) and Investigating Culturally Responsive Mathematics Education in Haida Gwaii islands (Nicol, Archibald, & Baker, March 2010). We have also planned a February evening event featuring Dr. Ed Doolittle as speaker. Dr. Dolittle will speak about Aboriginal math education and learning. All events will be promoted to both members and non-members, with a focus on increasing membership. As well, NWOAME will be hosting our annual Pi Day celebrations in city schools and

our regional math Olympics in April. We are looking forward to a busy year.

The principal focus of QSLMA this past year was for "OAME 2012: con.networks" conference to take place with few, if any, mishaps, in addition to making a profit. There were a few dark days when we had paid for promotional items before we had realized any revenue, which left us concerned about our solvency. However, we prevailed in the end. The vast majority of attendees enjoyed themselves and received some benefits from the conference sessions offered.

The Fall 2011 mini-conference attendance level was as expected. David Zimmer was the feature speaker capping a series of Saturday-morning workshops. The 2012 mini-conference has been moved to Saturday, February 2, 2012, with Cathy Bruce and Ruth Beatty as the scheduled feature speakers.

New president Melissa Baker has organized teams to implement this mini-conference in addition to other professional activities.

WOMA continues to experience a strong turnout at the mini-conferences. The host school is located close to major highways and there are SMART Boards™ in every classroom. The mini-conference begins with snacks and drinks along with a keynote speaker, and dinner to wrap up the evening, for only $10/person. The last mini-conference was well attended, in large part due to Marian Small, the keynote speaker. In light of the current work environment, the possibility of hosting a Fall mini-conference is still being debated.

Other Reports

NCTM: Since launching its "Agenda for Action in the 80s," NCTM has provided mathematics educators with several groundbreaking publications that have defined a vision for change in mathematics education. Although mathematical content is important in all of these publications, they also focus on the significance of changing traditional approaches to how we teach mathematics—change that is critical to offering every child the opportunity to achieve his or her maximum potential for learning mathematics.

Keeping this in mind, NCTM has started a new daily electronic email newsletter called NCTM SmartBrief. This is a free resource that allows you to learn more of what's happening in the mathematics community. Current articles are chosen by knowledgeable educators, then summarized. You are given direct links to the original source, to follow up and learn more. NCTM SmartBrief is found on the NCTM website. Many people who go to the NCTM website for SmartBriefs or "illuminations" find ideas that teachers can use in their classrooms. In one of the recent SmartBriefs, a teacher used sticky notes to engage students to give feedback on lessons. The teacher gave a sticky note to each student in the class and asked them to give an example of work they had learned in the lesson that day. As students exited the class, they placed the notes on the door. After collecting the notes, the teacher had a clearer view of what the students had learned. It was a nice way to get instant feedback.

Other teachers responded with more ideas around how they used these notes in their classrooms. Ideas included giving "way to go" notes to students who did well during that class. This is specific praise and reinforces good behaviour.

For students to learn mathematics, they must be engaged in doing it. This is not always easy to do, and often teachers may not know how, or whether or not, students are really engaged and learning. A member of NCTM posed the following question to a class of students: "What engages you as a student?" Here are the top ten answers:

1. Working with peers
2. Working with technology
3. Connecting what we do in school with what we see and do outside of school
4. Knowing that the teacher really loves what he or she does
5. Allowing movement in your classroom. We are allowed to get out of our seats to do math.
6. Visual material provided by the teacher
7. Choices that the teacher provides, e.g., on assignments, homework questions
8. The teacher knowing us as people—our names, what we do outside of school, etc.
9. The teacher mixing up your approaches to teaching, varying the lessons
10. The teacher is human and has a sense of humour

NCTM is also currently looking for articles to publish in their journals or yearbook. These articles can include assessment: how you do it and how you view it; technology tips: explain how technology has improved student learning; and favourite lessons: create an informal, personal narrative of no more than 600 words to describe your favourite lesson. Check out the NCTM's website for details about writing articles for them. ▲

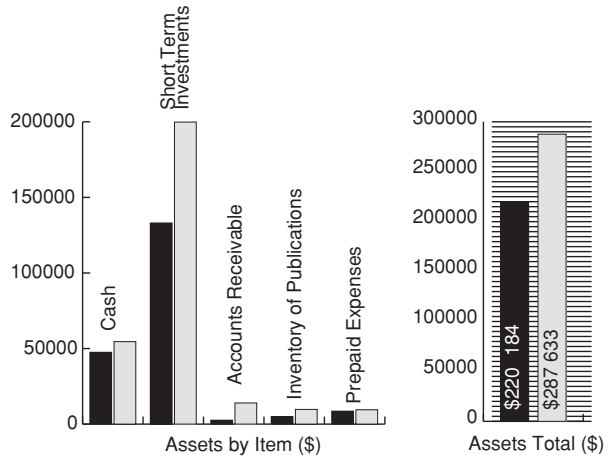
OAME Financial Statement 2011-12

Notice to Readers

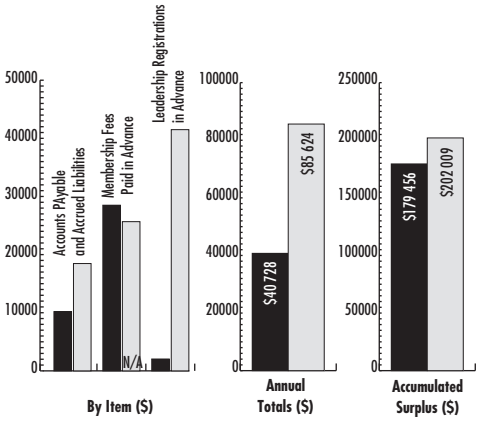
On the basis of information provided by management, I have compiled the balance sheet of the Ontario Association for Mathematics Education (Association Ontarienne pour L'Enseignement des Mathématiques) as at August 31, 2012, and the statement of operations and operating fund balance for the year then ended. I have not performed an audit or a review engagement in respect of these financial statements and, accordingly, I express no assurance thereon. Readers are cautioned that these statements may not be appropriate for their purposes.

London, Ontario, August 31, 2012, PHILIP J. GRIFFIN, Chartered Accountant, Licensed Public Accountant

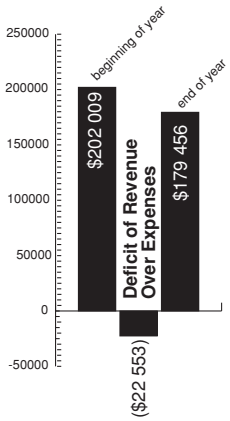
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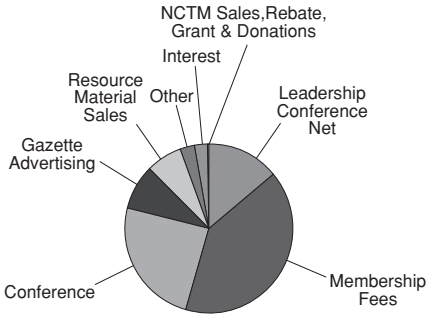
Liabilities and Fund Balance



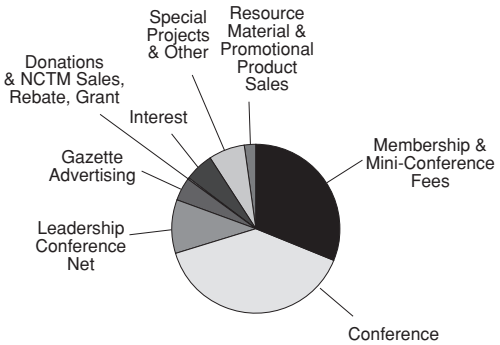
Operating Fund Balance



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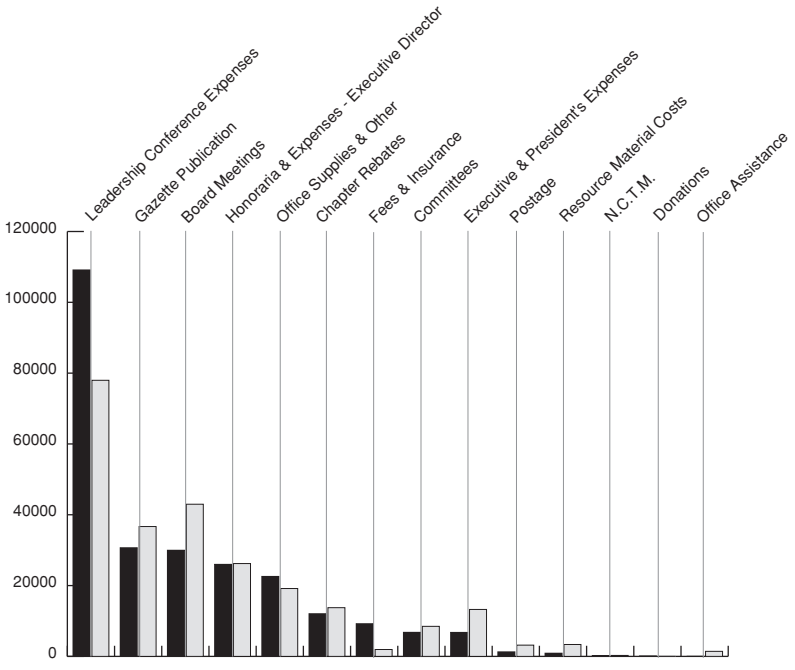
2012 ($124 023)



2011 ($164 548)

2012/2011

Expenditures



▲ Fields Institute – MathEd Forum Notes

STEWART CRAVEN
E-MAIL: numeratecitizen@mac.com



Stewart Craven presently teaches courses for the Continuing Education Department and the Transition Years Program at York University. He is an active participant in the Fields Mathematics Education Forum.

Our first meeting in September was an unqualified success. The theme, "Engaging Students in the Mathematics Classroom," attracted a large number of teachers at all levels (elementary to post-secondary), as well as a number of teaching candidates from various faculties of education.

Dave Medd (Centennial College) led off with a presentation entitled "Spicing up the Bland – Making Standard Math Material a Little More Engaging." Dave showed how snippets of videos and references to pop culture—along with healthy doses of humour—are key to engaging students in the classroom. "Motivating Students in a Large Life Sciences Math Classroom" was the topic addressed by **Miroslav Lovric** (McMaster University). Miroslav believes that in order to captivate the interest of today's students, it is critical to employ the Internet and real-life situations using real data (e.g., the employment statistics of life sciences graduates or medical school acceptance rates). **Doug Henrich** (Halton District School Board) entitled his talk "Practical Ethics: Developing a Social Conscience." In his abstract, Doug writes, "Ethics is an important aspect of all students' lives. It is difficult to present ethics to students in a manner that both engages them and provides practical strategies that they can apply both inside and outside the classroom. This is especially difficult when they may not even know what Ethics is or [when students] confuse ethics with morality. This discussion outlines an approach that has proven successful for high school Computer Engineering students in Grades 10, 11, and 12." **Judy Mendaglio** (Peel District School Board) then wrapped up the morning session with the topic, "Use of Blogs in Math Classes."

The afternoon session began with **Ron Lancaster** (University of Toronto) speaking about "Engaging Students with Digital Content." He described his presentation by stating, "We can touch the lives of our students and engage them in deep mathematical thinking by using digital content such as photos, videos, and newspaper articles as the basis of the questions we ask and assign to our students." Our last presenter for the day was **Ann LeSage** (University of Ontario Institute of Technology). Ann discussed "Becoming Critical Consumers of Children's Literature for Teaching Mathematics." While Ann acknowledged the motivational benefits of using children's literature to learn mathematics, she made the point that not all books with mathematical themes are effective tools for teaching.

Our next meeting will be held at York University on October 27. This meeting will be dedicated to memory of **Margaret Sinclair**. Each presenter has known and worked with Margaret over the years and will highlight aspects of her research. The Speakers List includes **Lyndon Martin** (York University), **Walter Whiteley** (York University), **Laurel Schollen** (Seneca College), and **Ami Mamolo** (York University). There will also be a panel featuring Pat Rogers (Wilfred Laurier University), Stewart Craven (York University), Kathy Kubota-Zarivnij (Toronto Catholic District School Board, York University), and Pat Margerm (Toronto District School Board, York University). ▲

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MB4T... When Is "Close" Good Enough?: The Circle Area Formula

ANN KAJANDER
EMAIL: ann.kajander@lakeheadu.ca

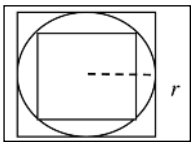


Ann Kajander is an experienced classroom teacher currently teaching mathematics and mathematics education at Lakehead University. Her research interests relate to teachers' enhanced learning of mathematical concepts. She and her classroom-teacher colleague, Tom Boland, are currently writing a new book for teachers called Math Without Memorization: Big Ideas in Modelling *and* Reasoning for Elementary Teachers and Other Learners.

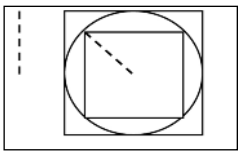
An intriguing aspect of the curriculum is that even though we like to think we have mathematics well ordered to progress from more concrete and simple ideas to more sophisticated and abstract ones, there are places where this organization is not linear. In particular, a problem-based pedagogical stance often prompts questions of "why" something is true, and the answers may not be straightforward. If we are hesitant to draw upon more sophisticated ideas to answer students' questions—either because these ideas are deemed "too hard" for them, or possibly even due to our own uncertainty—we risk reverting to a rule-based learning mode... and giving non-answers such as "because it just is." I have found that young children are indeed able to handle quite sophisticated mathematical ideas as long as these ideas are treated visually and intuitively, rather than based on formal computations and proofs. The classroom development of the area of a circle is one such topic where intuitive ideas about higher-order mathematical ideas such as limits come into play.

The Ontario Mathematics Curriculum contains the Expectation "determine, through investigation... the relationships for calculating the circumference and the area of a circle, and generalize to develop the formulas" (Ontario Ministry of Education, 2005, p. 113). Hence, it is no longer sufficient to simply "tell" students the formula for the area of a circle, and indeed this formula can and should be derived by students themselves via provision of the right learning context. However, there are some subtleties in the development of the circle area relationship, and these will be the topic of the current column.

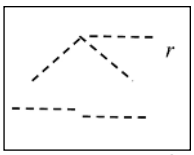
Prior to the specific derivation of the circle area formula, conjectures can be made about the relationship between the radius and area of a circle, using patterning. For example, students who are given input/output charts showing (whole number) radii, together with resultant areas of these circles, may well be able to conjecture that the area seems to be about three times the radius squared in each case. Prior to doing such a patterning activity, it may be interesting to explore a diagram such as this:



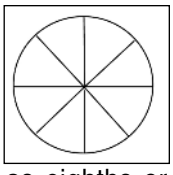
In the diagram, the circle area seems to be less than the area of the outer square, but more than the area of the inner square. You might want to draw in some more circle radii to help you see the dimensions of the outer square, and the inner square, in terms of the circle radius r more clearly:

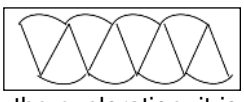


Drawing multiple copies of the original radius length allows us to measure the area of both squares in terms of *r*. For example, the side length of the outer square is 2 x *r*. Hence, the area of the outer square is (2 x *r*) x (2 x *r*), so it makes sense that the contained circle area is something a bit "less" than 4 x *r* x *r*. It is a bit more complex to find the area of the inner square in terms of *r*. If we choose to avoid using Pythagorean theorem, we can look at the inner square as being composed of two right-angled triangles. Each triangle has base 2 x *r* and height *r*. Using the triangle area formula (½ x *bh*), we get the result of *r* x *r* as each triangle's area. Since there are two such triangles in the inner square, the square inside the circle has area 2 x *r* x *r*. This analysis may help to conjecture 3 x *r* x *r* as a rough estimate of the pattern relationship in the input/output chart of radii and circle areas—an approximate amount between 2 x *r* x *r* and 4 x *r* x *r*. (Also see Kajander (2007) for other explorations related to investigating circle areas concretely.)



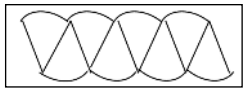
After such initial explorations, students are ready to explicitly derive the circle-area formula, and for this they should be given appropriate manipulatives. Often textbooks suggest using fraction circle pieces such as eighths or twelfths. Students may be asked to rearrange the fraction wedges from the circular shape into a shape that "looks" like a rectangle, or is "close" to a rectangle:





While this activity is a great start to the exploration, it is not appropriate to immediately accept the idea that a shape

such as this actually is a rectangle. Again, if we do "tell" students to make this assumption, or ignore the issue completely, we are dangerously close to returning to a rule-based methodology. Students are well aware that this "humpy" shape is not a rectangle. Saying it is "close" is hardly good enough. Students may well recall that when they tried to convince us previously that when a certain fraction piece was "close" to something they wanted to model, they were told to go back and find the "exact" sized piece. They may wonder why "close" is suddenly good enough now.



This dilemma can be easily solved by moving to more flexible concrete materials. If students are given circles drawn on paper, with lines illustrating a small fractional unit such as eighths or twelfths, they can cut these wedges out with scissors to produce the previous "rectangular" wedge arrangement. Next, we can ask how we could make this shape look "more" rectangular. Students will soon come up with ideas.

By cutting the fraction wedges along each centre radius, each can be cut in half.

Rearranging them again into the "rectangular" shape makes it clear that, even after one set of cuts, we are much "closer" to a rectangle. Yet another set of cuts makes it even clearer.



Now we are ready for a whole-class discussion about an important and powerful mathematical idea!

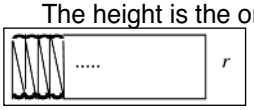


The teacher may want to introduce the idea that one of the beauties of mathematics is that we can repeat or "iterate" a mathematical process over and over. Sometimes the quantities involved get "smaller and smaller" and sometimes "bigger and bigger." As a side exploration, in order to get an initial sense of the "smaller and smaller" idea, we might ask students to type a single-digit whole number into their calculator, and then keep on dividing it by 2, over and over. What happens? What if they kept doing this for hours? What if a computer was doing it? Students can share in a discussion that "eventually" the quotient will be as close to zero as they like. Such a concrete exploration might give students a strong intuitive notion of the powerful idea of a mathematical limit, which is the concept at work in the circle area example, even if they don't yet understand limits in a highly theoretical and rigorous way.

Let's return to the circle area task, with our newly developed idea of a limit. After a few rounds of cutting the paper wedges, we can imagine a computer doing the same thing, over and over. In mathematics, we can make the wedges as small as we like—just like when we were dividing numbers by 2 with the calculator. Eventually, the wedges will be so narrow, they will almost look like vertical lines arranged in parallel. Students can use their imaginations to visualize the initial curved wedge shapes getting "closer and closer" to a rectangle. How close? Well, as close as we want! The rectangle is the "limit" of the process of narrowing (halving) and rearranging the wedges.

We can now complete the derivation of the circle area, using the rectangular shape. First, we need to find the dimensions of such a rectangle compared to the original circle. By checking the pieces in the original circle arrangement, we can see that the height comes directly from the circle's radius:

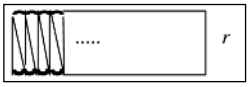
The height is the original circle radius.



The base also comes from the original circle. Looking at the original eight-wedge circle makes it more clear that the base of the rectangle is only *half* of the original circumference (the other half of the circle circumference forms the top edge of the rectangle). Since the circle circumference is 2 x π x r, half of it is just π x r. (We assume the circumference formula concept has already been studied at this point.) So the rectangle base is π x r.

Returning to our "rectangle," we write the dimensions:

So the area is base x height or π x r x r.



π x r

By multiplying the base and height of the rectangle, we see our area is thus π x r x r^2 Sometimes we write this as πr^2. However, many teachers prefer to use the π x r x r version of the formula because they are aware that their students find it more accessible to use without the exponents. Knowing that π is about 3 aligns well with the prediction earlier that the area would be between 2 x r x r (or $2r^2$) and 4 x r x r (or $4r^2$).

Such a classroom exploration can make the fine points of the derivation of the circle area formula seem more reasonable. As well, discussing powerful mathematical ideas like limits early in students' learning makes these same ideas more assessable later when they are reintroduced more formally.

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▲ RE4MUL8: Creating an Online and Mobile Technology Support for Grades 7–8 Teachers of Mathematics

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Dr. Daniel Jarvis is an Associate Professor in the Schulich School of Education, Nipissing University, North Bay, Ontario. *He would like to thank the participating teachers and coordinators/coaches who took part in the math lessons and subsequent debriefing interviews, which now form the basis of the RE4MUL8 Project teacher resource.*

The main purpose of this brief paper is to present the mathematics education community with a newly created and freely available resource for coordinators and for teachers of mathematics, entitled RE4MUL8. The project involves a web/mobile platform featuring videos of classroom lessons and teacher interviews, Twitter/Facebook news feeds, and links to other mathematics resources.

Background

In 2003, the Ontario Ministry of Education (OME) commissioned a mathematics education resource document entitled *Targeted Implementation and Planning Supports: Grades 7, 8, and 9 Applied Mathematics (TIPS)*[1] (Consortium of Ontario School Boards, 2003). As a doctoral candidate at that time, I was among the approximately 40 teachers, coordinators, and university faculty who were brought together to write and develop the TIPS materials. TIPS featured a number of different components (e.g., research summaries, problem-based lessons, planning templates, posters, downloadable files) and was well received in the province (Jarvis, 2006). Based on positive feedback from the field, TIPS was then revised and expanded two years later (2005) to match the then newly revised Grades 1–8 Ontario curriculum (Ontario Ministry of Education [OME], 2005); to offer completed Grades 7–8 and Grades 9–10 Applied-level lessons; and, to address specific areas such as English Language Learners, learners with special needs, and split-level classrooms. Since that time, TIPS has continued to expand in content and has been incorporated into the OME website, Math GAINS[2], and as such, now forms an important archival part of a much larger support network of useful resources and activities for Ontario teachers.

Having interviewed a group of 64 participants in 2003–2005 including teachers, administrators, coordinators, and OME representatives, regarding the TIPS resource development and implementation for my doctoral research study (Jarvis, 2006), my findings highlighted some of the perceived professional learning needs of Intermediate Division[3] mathematics teachers. *Choice, voice,* and *ongoing support* emerged as three main characteristics of quality professional development models. First, mathematics teachers tended to prefer the opportunity to *choose which type of professional learning they require* at any given stage in their career (e.g., self-selected individual or collaborative action research projects); second, teachers required a *means of sharing classroom results/information* with other mathematics teachers both within and beyond their school and board (e.g., division meetings, PD/conference presentations, online learning systems); and, third, mathematics teachers desired *ongoing professional development* (e.g., multiple sessions throughout the school year, and often involving an online component which allows for increased accessibility and social networking/support).

Abrahamson (2004) differentiated between what he called "creative destruction" and "creative recombination," the latter term referring to the repeated use of existing quality initiatives and projects in new configurations, as opposed to the complete abandoning of former ideas simply for the sake of change (cited in Fullan, Hill, & Crevola, 2006, p. 14). The videotaping of Grades 7 and 8 instructors implementing TIPS-based lessons, and then being interviewed following each lesson, sought to likewise "creatively recombine" these existing quality resources which had already been used in varying degrees throughout Ontario. But first, I would like to share a few words about the thinking, listening, and writing that have led to the creation of the new project.

Middle Ground

Several years ago, I submitted a paper to the *Gazette* entitled *Thinking Outside the Rectangular Prism: Fostering Problem-Based Mathematics Learning* (Jarvis, 2008). In

[1] *Ontario Ministry of Education TIPS for Revised Mathematics (TIPS4RM) Resource:* ***www.edu.gov.on.ca/eng/studentsuccess/lms/tips4rm.html***
[2] *Math GAINS (Growing Accessible Integrated Networks) is accessible at* ***www.edugains.ca/newsite/math2/index.html***.
[3] *In Ontario, Canada, the Intermediate Division refers to students in Grades 7–10.*

this work, I presented a problem-based approach/checklist and made the case that any teacher who regularly implements these kinds of activities would automatically be covering the seven Ontario Curriculum Math Process expectations (i.e., problem solving, reasoning and proving, reflecting, selecting tools and computational strategies, connecting, representing, and communicating) within their mathematics program.

I recently asked my Bachelor of Education pre-service teachers to write a short essay regarding their mathematics teaching experiences involving reform methods during their school practicum placements. With their permission, I here share a few of their insights regarding the collaborative learning aspect of reform-based teacher practice:

> **Teacher 1:** One other thing that I saw while teaching math that interested me was how enthused a classroom could become about a problem if they did it collaboratively, while it was equally impressive to see how deadened the class could be at the prospect of a worksheet. Like I said earlier, I didn't learn this way. I did worksheet after worksheet growing up and I knew how boring they were. So, this group work was new to me and I got enthused along with them as they worked to solve a problem. I did worry when it was time to test the students at the end of the unit on measurement because so much work had been done in groups, and all of a sudden, it was time to work alone, but they all did well.

> **Teacher 2:** Students had the option to work alone, in partners, or small groups. I did caution them that they should each know how to solve the problems, and that group work did not mean one person does all the work and you copy the answer. It was very beneficial for the weaker students to be paired up with stronger students to work their way through the problems and talk it out. I learned that many students learn best while working in groups as opposed to individually for math. That was one of the big adjustments that I had to make as a teacher because I was always used to math being quiet pencil-and-paper work.

While these collaborative strategies are learned and implemented by many new teachers during their formal preparation in faculties of education, it is often harder for experienced teachers in the field to move toward this perhaps very different approach (i.e., collaborative structures and rich problem-based lessons) to mathematics teaching and learning.

A colleague of mine and I crafted another article for the *Gazette* entitled *"Messy Time" Transition to Reform-Based Mathematics Teaching and Learning* (Jarvis & Franks, 2011), in which we presented the results of some case-study research, wherein teachers were asked to describe the "messy" process of moving from a traditional teaching model to a more problem-based approach. They were also asked to comment on how they were coping with trying to plan and implement a "balanced instructional" program that would feature both types of classroom mathematics activities (i.e., memorization/practice/mental estimation skills, as well as more problem-based learning (PBL) with group work/manipulatives/technology/etc.). One teacher highlighted a perceived polarity of practice:

> **Teacher 3:** Do you have to do a problem every day to be teaching through problem solving? No. You don't have to be doing a problem every single day, but I think it's hard to feel like you're in between. I think people feel like you're either here or there.

Time required to prepare PBL lessons, particularly when one is first beginning to use this approach, was cited as being one of the main challenges in striking the curricular "balance," as noted by this educator:

> **Teacher:** I find that it's taking me more time to prepare because I have to rethink the way I did stuff. So, I'm trying to do more stuff where it's more verbal and fewer paper-pencil tests. So, it's because it's new—I know the math program, but to find new ways to do it. We have a lot of good resources, but it's to go through the resources and find what I'm looking for, for what I want to teach, in a different way—I find that quite time-consuming.

With the advent of new technologies (social media, smartphones/mobile devices) and the rapidly growing use of these tools in society, I was curious to know how these kinds of tools might be used by pre-service (soon-to-be) and in-service (novice/experienced) teachers in terms of facilitating and supporting problem-based mathematics planning and teaching in Ontario schools. An online video database of teacher lessons and interviews would allow Intermediate Division mathematics teachers the ability to easily view lessons and interviews to which they could relate, and from which they might possibly draw inspiration and ideas for changing and sustaining practice. Mobile device options would allow teachers even fuller access, as well as providing a mechanism for receiving math news updates.

Foreground

To *reformulate* means "to formulate again, or in a different way; to alter or revise; to reformulate plans." At its heart, the RE4MUL8 project advocates for changed thinking (teacher beliefs) and related pedagogy (teacher practice). Six different district school boards in Ontario were approached with an invitation for a mathematics coordinator and a teacher (i.e., an effective, reform-based Grade 7 or 8 teacher, recommended by the coordinator/board) to become involved in the project. Six sets of coordinators/teachers did volunteer (one site eventually involved four teachers/coaches), and when all research ethics approvals had been secured, filming finally began. In each case, a teacher was asked to deliver a problem-based lesson that was tied to one of the Targeted Implementation and Planning Supports for Revised Mathematics (TIPS4RM) *Continuum and Connection* topics (fractions; integers; patterning to algebraic modelling; perimeter, area, and volume; proportional reasoning; and solving equations and using variables as placeholders), and then to further take part in a post-lesson debriefing/interview (usually with the coordinator if he or she was in attendance and wanting to participate) that was audio- and video-recorded. Interviews were then transcribed by a research assistant, and these were returned, along with the completed video vignettes, to participants for final checking.

In 2010, with funding from Nipissing University and a Schulich STEM Initiatives grant, over 22 hours of video footage (two cameras) was edited for preparation and inclusion in the 20 project videos. A subscription to VIMEO Plus was purchased in order to upload completed video clips to a secure, web-based server/channel for storage and access. Twitter and Facebook accounts were established for micro-blogging of math-related issues/events at the local, provincial, national, and international levels (see Figures 1 and 2).



Figure 1. *RE4MUL8 Twitter account for sharing math-related news.*



Figure 2. *RE4MUL8 Facebook page for sharing math-related news.*

The video footage links and Twitter/Facebook components were then merged into the web-based resource, using an *edublog* (see Figure 3), which was in turn linked to a newly registered Canadian domain name for easy access (**www.re4mul8.ca**).



Figure 3. *RE4MUL8 teacher resource available at* ***www.re4mul8.ca****.*

The RE4MUL8 resource presently features an introductory video, seven TIPS-inspired lessons, and seven debriefing interviews, often conducted with both the teacher and the coordinator/coach. Further, in five thematic video compilations, teachers and coordinators/coaches share insights on the following topics: co-teaching, problem-based learning (PBL), "messy time" transition to reform-based teaching, math coaching, and the Ontario Ministry of Education's CLIPS math teaching resource. Links to the two above-mentioned *Gazette* articles, the OME MathGAINS site, the MATHeMATRIX, the Twitter/Facebook feeds, and the new web app for mobile device access (e.g., smartphones/tablets) are also available via links from the index page.

Please be encouraged to visit and share this new resource (online and/or using the web app). The insights provided by both the new and by the more experienced teachers and coordinators/coaches are informative, honest, sometimes quite humorous, and timely. I trust they will inspire Intermediate Division teachers to adopt,

or to become increasingly adept with, problem-based teaching practices in mathematics classrooms. For those who are involved in the crucial work of supporting teacher practice within a city or board jurisdiction, may the informative comments of your counterparts shared in the interviews likewise serve as a challenge and reaffirmation of the great work that is already taking place in Ontario. The reflective interviews and themes may prove to be particularly useful in teacher professional development sessions, since they represent a wide range of teacher comfort/progress with PBL methods. Feedback relating to the resource is welcomed and can be emailed directly to the author.

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▲ Scholarly Activities Of Mathematics Teachers

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Scholarship of Teaching and Learning

As mathematics educators at post-secondary institutions in Alberta, we have witnessed shifts as the expectations of teachers' professional practice continues to move out of the closed doors of the classroom and enter into public spaces for shared reflection and inquiry (Shulman, 2004; Smith & Smith, 2006). The teaching in post-secondary institutions is undergoing a similar shift in an effort to place teaching on par with other scholarly endeavours (Boyer, 1990). In our work with pre-service and in-service teachers, we believe we could both learn from and contribute to the Scholarship of Teaching and

Learning (SoTL) movement in post-secondary institutions by understanding the ways in which teaching in schools might be considered scholarly. This paper describes our efforts to understand what forms of scholarly activities teachers of mathematics engage in, and make recommendations for professional learning opportunities. We believe that by sharing our results, teachers could consider ways to enhance teaching and learning in their practice, and begin to see their own teaching as scholarly.

It is important to note that scholarship of teaching is not synonymous with excellence in teaching (Hutchings & Shulman, 1999). Rather, teaching becomes scholarly when one engages in reflection and inquiry, studies, documents, and analyzes issues of concern, with the purpose of enhancing student learning and improving practice. The scholarship of teaching occurs when evidence-based inquiry is shared, is subject to critique, and contributes new knowledge on teaching within a discipline (Hutchings, 2002; Hutchings & Shulman, 1999; Kreber, 2001).

The words *scholar, scholarly,* and *scholarship* are typically only associated with academe, and the work of elementary and secondary school teachers is only occasionally referred to as scholarly (Diezmann, 2005; Hatch, 2006). Yet, the origin of "scholar" is from the Latin schola, or its equivalent "school" (*Oxford English Dictionary*, 2011), and the SoTL movement shares goals applicable to teaching at all levels, including improving student learning and enhancing the practice and profession of teaching (*Carnegie Academy for the Scholarship of Teaching and Learning*, n.d., para. 2).

In order to better understand how school teaching can contribute to the view of teaching as scholarly, we designed a study to identify the range of scholarly activities that educators in school mathematics engage in, and the perceived impact of these activities on their practice and professional communities. In particular, the study focused on the three forms of activity identified as "scholarly" according to the SoTL literature:

1. Engagement with the literature on teaching and learning to improve practice
2. Critical inquiry into and reflection on practice
3. Communication of results to contribute to the scholarship of teaching and learning (Richlin, 2001; Trigwell, Martin, Benjamin, & Prosser, 2000).

Similar to the context in Alberta, government and professional organizations in Ontario are emphasizing the importance of teacher inquiry and professional growth. Indeed, the Ontario Association of Mathematics Education's guiding principles (OAME, n.d.) show a commitment to supporting teacher professional growth and inquiry into teaching and learning mathematics. We believe that this context provides an opportunity for teachers to engage in scholarly activities of reading literature, inquiring and reflecting on practice, and sharing results. This article presents positive aspects of current teachers' scholarly activity and prompts us to consider how this can be enhanced and facilitated within our classrooms and schools.

Our Study

We asked currently or recently active Canadian school teachers who were members of professional or research associations, authors of articles in research or professional journals, and/or had engaged in graduate studies focused on mathematics education to complete an online survey, and then invited volunteers to participate in semi-structured interviews. Sixty participants completed the online survey that consisted of both closed and open-ended questions to collect demographic information and data on the three scholarly activities of reading, inquiring, and disseminating. Telephone interviews with a subset of 14 self-selected survey participants occurred approximately two months after the close of the survey. During the interview, participants were asked to clarify and expand upon their survey responses.

In general, the survey participants were primarily female (72 percent) over the age of 45 (50 percent) with more than 15 years of teaching experience (43 percent). Two-thirds of the 60 participants were employed as classroom teachers (67 percent) at the elementary (31 percent), junior high (24 percent), or high school (26 percent) level, with the balance of them teaching at more than one level (19 percent). Participants identified themselves as curriculum leaders (17 percent), consultants (17 percent), school administrators (10 percent), and coaches (5 percent), often in addition to being classroom teachers. The 14 interviewees from this sample represented a similar demographic in gender, age, experience, teaching position, and leadership roles.

Engaging with Literature

On the survey, we asked what resources participants used to inform their practice of mathematics teaching and learning (see Figure 1). In the follow-up interviews, we asked further about their use of literature they had listed on their survey. The most frequently mentioned sources for literature on the survey included online sites, print teaching resources, professional journals, and classroom textbooks.

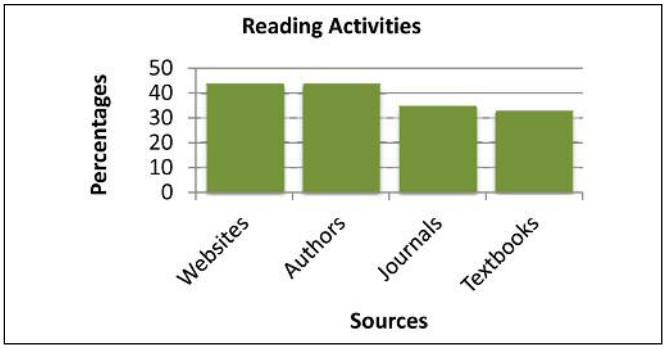


Figure 1: *Sources of literature referred to by participants*

Online sites and print teaching resources were identified by 44 percent of participants. The websites mentioned most frequently were activity sites or question-and-answer forums for both teacher and student audiences (e.g., virtual manipulatives, applets, homework or test help) and sites with lesson ideas for teachers. In a follow-up interview, one participant identified an online National Council of Teachers of Mathematics (NCTM) position statement on the use of calculators as helpful because "it's backed by research, and you know, you can share it with parents and you know you can trust it." Two interviewees referred to TED.com, a site devoted to inspiring speeches across all domains, and one stated, "It makes beautiful connections to other subjects." The participants also referred to a large range of print teaching resources, usually by listing specific book titles or by responding "anything by" particular authors, including Marilyn Burns, Marian Small, John Van de Walle, and Grayson Wheatley, to name a few. Most of the books identified were teaching resources in which authors mediated current theories of learning and provided activity-based applications. Only one participant referred to literature about mathematics content, specifically in the area of probability, number theory, and the history of mathematics.

Print journals were listed by 35 percent of participants, specifically National Council of Teachers of Mathematics (NCTM) professional journals and the provincial mathematics council journal, were mentioned most frequently. These journals are peer-reviewed, and articles are primarily applications of research or shared teaching experiences. Only one research journal was listed by one participant. Two people interviewed said, "I used ideas from *Teaching Children Mathematics*. I'd flip through those every once and a while" and "A co-worker of mine had some activities that he pulled from an older copy of the *Mathematics Teacher* and I'd seen a couple of things from articles that were in there that I thought were really good."

Finally, authorized textbooks were identified by one-third of survey participants. Not surprisingly, most of the participants referring to their textbooks as a key resource for teaching were teaching junior high and senior high school mathematics.

The follow-up interviews provided insight into the literature teachers identified as informing their practice. The sources were originally selected based on recommendation, usually by colleagues in their school district or by a leader of an in-service workshop. Unlike SoTL, teachers did not appear to seek out literature to inform them of particular issues of student learning. Instead, the recommended resources were often taken up, discussed, and critiqued collaboratively. Participants began to see their own teaching practices differently and become more attuned to particular issues of student learning within their classes. Through their readings with others, together, they addressed the broad goal of challenging taken-for-granted notions of what it means to teach and learn.

Inquiry into Practice

Survey participants were asked to complete a series of four-point Likert scale questions (i.e., from "not significant" to "very significant") to determine the participation rate and the perceived impact of common professional activities. We selected activities that potentially provided opportunities for inquiry and reflection on student learning (see Table 1). Over three-quarters of participants had participated in mathematics workshops, collaborative planning, study groups (e.g., Professional Learning Communities), and mathematics education conferences. The two professional activities that teachers rated as most significant on their practice of mathematics teaching were post-secondary coursework and collaborative planning or lesson study.

Table 1 *Participation Rate and Perceived Impact of Professional Development Activities*

Professional Development Activities	Participation Rate (%)	Perceived Impact (4-point Likert scale)
Post-secondary coursework	37	3.5
Collaborative planning/Lesson study	88	3.3
Self-initiated classroom research	46	3.2
Mentoring student/beginning teachers	68	3.2
Study groups	79	3.2
Conferences	75	3.1
Peer coaching/mentoring	70	3.1
School-based inquiry projects	68	3.0
Participant in research studies	21	3.0
Summer institutes	40	2.9
Workshops	93	2.9
Government curriculum/ assessment activities	65	2.9
Received coaching/mentoring	51	2.7
Online professional development programs	54	2.2
Preparation for math contests	26	1.8

In follow-up interviews, most participants mentioned inquiring collaboratively with colleagues as being particularly meaningful. The inquiry usually had a practical aspect such as preparing lesson plans or marking provincial examinations, but conversations around these practical tasks caused them to reflect on their practices.

Participants also spoke about the ways that student understandings of mathematical ideas spurred them to inquire into and improve their practice: "I started making some changes, and especially in the mental math aspect. You know, started incorporating some mental math activities and saw a significant change in student attitude, in student response to mathematics, in engagement, in just an overall impact that was basically initiated a 'wow' for me." Several participants pointed to observation of teaching, coupled with conversation, as being meaningful to their practice.

One important finding that became apparent in the interviews is that the impact of scholarly activities was significant, but only when integrated and shared through conversation with colleagues to examine their own practice.

Communication of Results

The third area of scholarly activity examined the ways in which participants communicated the results of their inquiry into practice with others. In an open-ended question, survey participants were asked to describe how they share their experiences in and knowledge of mathematics with others through presentations, publications, online dissemination, or other forums (see Figure 3).

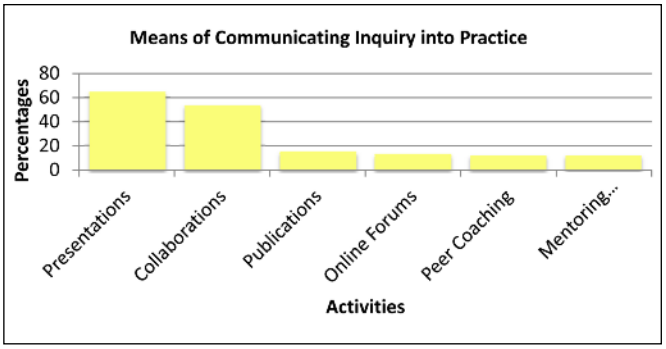


Figure 2

While most interview participants acknowledged that presentations at teacher conferences were important, they consistently talked about how their presentations fostered further collaborations that had even greater impact on their teaching practice. One interviewee stated, "I've worked with small groups of teachers, have done some in-servicing at other schools... When teachers have seen a presentation, often they'll email me and they'll ask me for more, and I invite them to come to my classroom... And that's more powerful even than doing a session at convention." Another said, "I just presented an [interactive whiteboard] thing on Friday... I'm getting to the point where I could probably do something at a teachers' convention."

The emphasis in SoTL literature for communicating results is to contribute new knowledge to the scholarship of teaching and learning. The participants in this study actively denied that they were presenting new knowledge to the field. Instead, they spoke of extending or transforming teaching ideas located in teaching resources they had been using—often the very resources that had been recommended to them by colleagues and identified as a key resource. For example, one participant developed a project for Kindergarten teachers that incorporated a variety of centre ideas from an authorized textbook. Over half of the participants in the study indicated that they shared their inquiry into teaching and learning through

presentation and collaboration. Only one interview participant mentioned publishing her work in a research journal.

A key finding related to the activity of communication was that our participants did not perceive knowledge as something owned by an individual and then disseminated; instead, literature, ideas, and practices spread through local networks where they were taken up, elaborated, and enfolded back into local communities of practice.

Contributions of Teachers to the Scholarship of Teaching and Learning

The results from our study suggest that the teachers engaged in all three forms of scholarly activities, including reading current literature on teaching and learning within mathematics education, inquiring into their own practice and into the practice of colleagues through collaboration, and publicly communicating their ideas both in formal and informal ways. However, we found several differences in the scholarly activities of the teaching participants compared to the activities described in the SoTL literature.

First, the teachers' activities did not follow a reading–inquiry–dissemination process to investigate a pre-specified issue in learning. Teachers became more attentive to issues of learning through their inquiry activities; inquiry was not a result of an issue of student learning. Here, the three forms of scholarly activity were described as an integrated and ongoing process, rather than as a linear process.

Second, the sharing of results of inquiry was generally not perceived as an effort to contribute broadly to teaching and learning, but as a collaborative opportunity within their local communities of practice. In fact, all three forms of scholarly activity appeared to stem from and contribute to collaborative relationships with other teachers. It was the interactions among teachers within their "communities of conversation" (Shulman, 1993, p. 6) that allowed them to analyze aspects of their own practice which previously had been unexamined.

Third, the initiation of scholarly activities was typically not self-generated. Instead, the interview participants made clear that involvement in a project, presenting at a conference, or writing for a journal was typically initiated by colleagues, whereby a fellow teacher or administrator prompted them to take on a leadership role.

Indeed, given our recruitment for this study, many of our participants were considered leaders in their schools and districts. Their responses suggest their paths were not purposeful or predictable. With further study, we believe their stories may provide insight into understanding the identity of mathematics education leaders, particularly in the elementary school. One unanticipated finding was that the elementary teachers we surveyed and interviewed did not view themselves as experts. Many were initially quite uncomfortable with teaching mathematics, but became leaders in spite of negative personal experiences, mostly because of positive professional experiences. In this integrated process, they framed themselves as teachers with an ongoing desire to learn, as scholars devoted to both teaching and learning, who engaged in active inquiry and communicated their knowing with others. This points to the need for further research to investigate what teachers' views of collaboration are and how we can facilitate such scholarly activities within professional development settings.

Implications for Teachers

In this study, teachers' inquiry into practice was impacted most by participation in contextual, long-term lesson study with colleagues. Rather than attending one-time workshops, we suggest that teachers look for professional learning opportunities that are devoted to the collaborative development of lessons. We recommend that conference committees include study groups as part of the annual conference to provide teachers with opportunities to begin such study initiatives. We envision that this would be a space where teachers who are interested in the same topic could meet for a time of sharing and collaboration. We believe that study groups within teacher associations could facilitate a connection between teachers with similar passions.

Teachers are the experts in knowing what teaching and learning issues exist in their classrooms. We encourage teachers to attend to the issues by finding forums through which they can share not only their concerns, but opportunities to inquire into, investigate changes, document what they have learned, and share it with others. The value of collaboration was strongly emphasized in all forms of scholarly activities. Most teacher leaders in mathematics in our study considered sharing their ideas because of encouragement from colleagues. While this is an important avenue through which to identify leadership, we believe that teachers play a role in recognizing their own scholarly work in schools and in being willing to share this work with others.

A scholar is someone devoted to learning, someone who acquires knowledge through focused inquiry, and communicates knowing with others (Andresen, 2000). Throughout this study, teachers provided strong evidence that they were scholarly. We believe that teachers' professional knowledge has the potential to make essential contributions to our understanding of teaching and learning if viewed as a scholarly act.

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▲ Foundation: A Computer Tool That Builds Expressions for Computation

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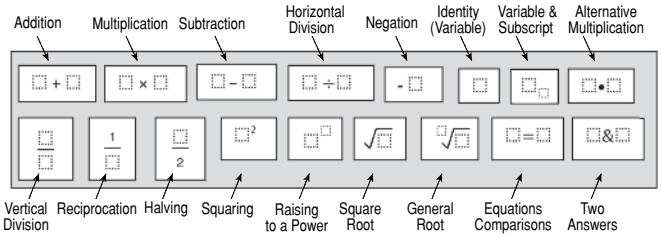
Don is a retired, but formerly: a teacher for secondary and post-secondary Mathematics; a magazine co-editor; the creator of a computer language self-teaching system; the Director of Lakehead University Computing Services; author of: "The Board Member's Companion". Currently working on a teaching aid that executes Mathematics notation directly on computers. Mathematics (Oxford), Computer Science (London).

Algebraic notation is a problem-solving tool. It represents real-world problems in a form that can be manipulated to create solutions. This paper focuses upon student difficulties with algebraic notation. It makes a case for addressing these difficulties by building with the 'Foundation' notation (Watson, *Ontario Mathematics Gazette*, March 2012), currently under development.

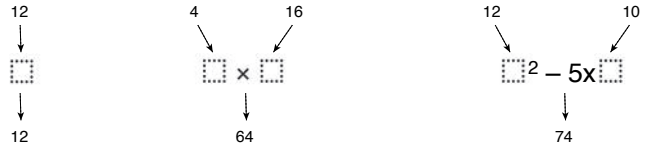
Foundation

Foundation was designed originally as a means of enabling mathematical notation to be its own computer language. Mathematical expressions (e.g., $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$), when built with Foundation's simple functions, become computer calculation programs. This is an easy and intuitive application that is useful from Grade 1 to Grade 12. In the younger grades, it could improve student understanding of addition, subtraction, multiplication, and division.

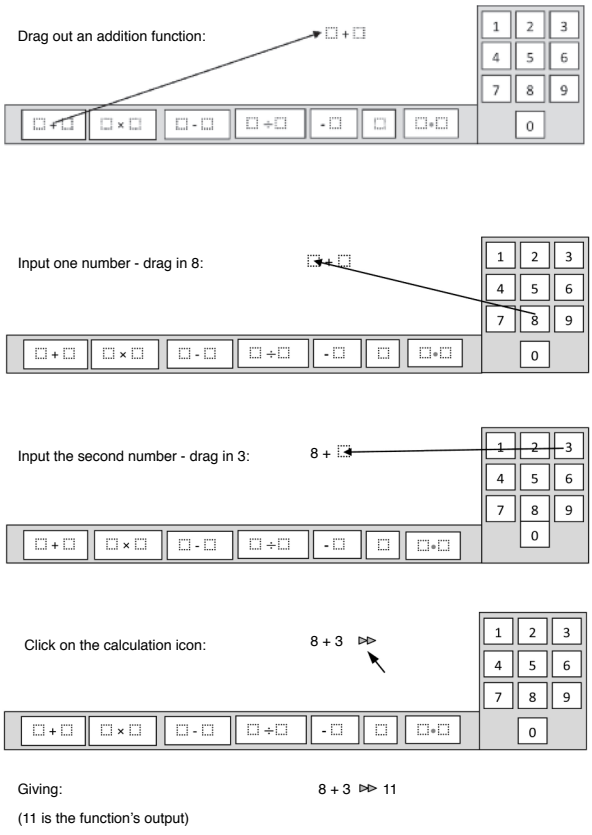
Foundation has another major application. It enables arithmetic to provide a preparation for algebra that is not possible with arithmetic alone. As a result, students can be equipped to cope with algebraic manipulation when it is introduced. It is this preparation for algebra, using the Foundation functions below, that will be discussed in the following paper.



This toolbar shows the basic Foundation functions used in elementary school. Whereas variables (and numbers) are the building blocks of Algebra, functions are the building blocks of Foundation. "□" is Foundation's variable. Each "□" is a "doorway" into the function, through which any number or expression can be dragged or keyed. Variables, functions, and expressions become easy-to-understand processes that accept input and produce output:



From the perspective of the Grade 1 student, these Foundation entities could all be considered number machines. Below is a sequence of step-by-step snapshots for a simple calculation. The toolbars underneath and to the right are abbreviated to save space:



Representation

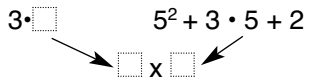
In arithmetic, the basic unit to be processed is the number. In algebra, it is the expression. This significant conceptual leap is taken soon after algebraic notation is introduced, when algebraic variables are used to represent not only numbers, but expressions and variables. For example:

The expression "3*g* + 2*k* + 4*g*" has two "*g*" expressions that need to be combined. Strictly speaking, this involves two steps:

First exchange the expression "2*k*" with the expression "4*g*" by using the commutative rule that "*a* + *b* = *b* + *a*," with "*a*" and "*b*" representing the expressions "2*k*" and "4*g*." This brings the "3*g*" and "4*g*" side by side.

Then add "3*g* + 4*g*" by using the distributive rule "*ac* + *bc* = (*a* + *b*)*c*," where "*c*" represents the variable "*g*."

Foundation offers the opportunity to prepare for this conceptual leap. Like algebra, Foundation processes anything – numbers, variables, or expressions, which can be easily dragged into variable "doorways."



In this example, the Foundation variables (the two "□" s) accept these expressions to produce:

$$3\bullet\square \times (5^2 + 3 \bullet 5 + 2)$$

This enables the order of teaching to be reversed. Understanding of arithmetic expressions as entities can be learned first, and the understanding of Algebraic expressions can be built upon this background. As a result, it is possible to delay the abstraction of letter variables until students are more mature.

Generality

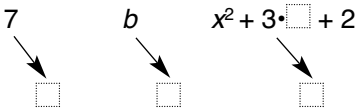
Arithmetic is limited by its lack of generality. When generality is required, it must be provided by something outside arithmetic:

a) Algebraic variables (letters) are introduced as a means of providing generality for problem solving.

b) Arithmetic formulae, like "Area = Length × Width," represent numbers with words, a level of abstraction equivalent to representing them with letters.

c) Arithmetic rules are expressed through algebra. For example, the distributive rule is defined as "*ab* + *ac* = *a*(*b* + *c*)."

Since arithmetic and algebra intermingle in this way, it is not possible to separate arithmetic from algebra. Consequently, it is not possible for arithmetic to provide a complete preparation for algebra. However, Foundation has the generality to provide both separation from and preparation for algebra with its variable "□". Any number, any letter, or any expression can be dragged into and be accepted by a variable:



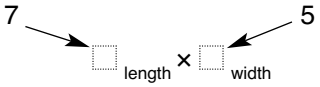
Giving: 7 *b* $x^2 + 3\cdot\square + 2$

This generality can be used to address mathematical problems in different ways as early as Grade 1, without using algebraic variables. One question could be: "If '3 + □ = 10', what number needs to go into the box as input to make this true?" Another question could be: "What number needs to go into the number machine '3 + □' as input to give an output (value) of 10?" Potential answers could be tried by dragging numbers into the variable and calculating. For the younger grades, conceptual understanding can be supported with the use of Foundation's coloured tiles to represent numbers on the screen, which students could also represent through manipulating actual tiles.

The addition of subscripts to variables enables arithmetic formulae to be defined without algebra. For example:

Area is: $\square_{\text{length}} \times \square_{\text{width}}$

Subscripts are merely labels and take no part in the calculation. Numbers can be dragged into the Foundation variables:



And the calculation icon touched: 7× 5 ▷▷ 35
(The area 35 is output)

The rules of arithmetic can be illustrated without algebra by using specific arithmetic examples. For example, the distributive rule could be written:

3•4 + 5•4 = (3 + 5)•4 ▷▷

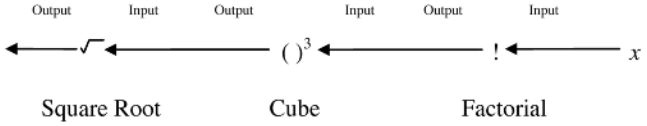
Calculate: 32 = 32

With Foundation, it is easy to keep changing the numbers and observe that the equality consistently holds, which prompts the question: "Why?" It provides an opportunity for "learning by inquiry." Here the usual examples can be used to show that the rule fits the real world. For example, a 3•4 rectangle, when placed beside a 5•4 rectangle, becomes an 8•4 rectangle.

Thus, if Foundation notation were used while arithmetic is studied, arithmetic could be separated from algebra; mathematical truths could be learned "by inquiry," and a sound conceptual base could be built for future algebraic manipulation.

Creating Expressions with Foundation

Arithmetic and Algebra would both be simpler if all operators had only one variable. An example would be $\sqrt{(x!)^3}$. This could be represented by a one-line diagram, in which the output from each operator is fed as input to the next:



If $F(x) = \sqrt{(x)}$, $G(x) = x^3$ and $H(x) = x!$, then $\sqrt{(x!)^3}$ can be seen to be the composite function: $F(G(H(x)))$. This same composite function is created by Foundation when functions are successively dragged into the variable of another function. For example: *x*

Dragging *x* into □!: creates: *x*!

Dragging *x*! into □³: creates: $(x!)^3$

Dragging $(x!)^3$ into $\sqrt{\square}$: creates: $\sqrt{(x!)^3}$

Consequently, dragging expressions into function variables creates composite functions. However, it also achieves a second objective. It programs the computer. Dragging *x* into □! directs the computer to pass *x* as input to □! Dragging *x*! into □³ directs the computer to pass the output from *x*! as input to □³. Dragging $(x!)^3$ into $\sqrt{\square}$ directs the computer to pass the output from $(x!)^3$ as input to $\sqrt{\square}$.

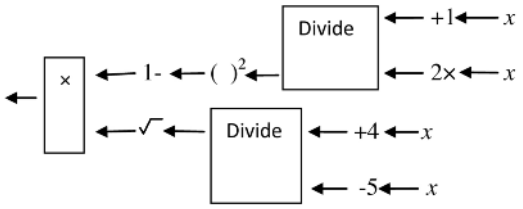
To learn more about the expression $\sqrt{(x!)^3}$, a value such as 3 can be given to the variable and the computer program executed with the stage icon (▷) as the execution trigger. The program executes one stage at a time, showing how the expression processes:

1) Calculate the factorial: $\sqrt{(3!)^3}$
2) Pass the output from the factorial as input to the cube: $\sqrt{(6)^3}$
3) Pass the output from the cube as input to the square root: $\sqrt{216}$

Result: 14.697

Two Variable Functions

Operators that operate on two numbers (e.g., "+," "-," "×," and "÷") create more complicated expressions. An example might be: $\sqrt{\frac{x+4}{x-5}}\times\left(1-\left(\frac{x+1}{2\times x}\right)^2\right)$. A diagram to represent such an expression requires more than a single line:



Students do not get practice in understanding or working with expressions of comparable complication while studying arithmetic alone, but they would if Foundation were used. Apart from a variable change from x to "□," the expression looks identical:

$$\sqrt{\frac{\square+4}{\square-5}}\times\left(1-\left(\frac{\square+1}{2\times\square}\right)^2\right)$$

The benefit of using Foundation here is twofold. First of all, the full expression can be built on the screen by dragging expressions into variables. To build the expression:

Drag 1 into an addition function:

□ + □ to give: □ +1

Drag 2 into a multiplication function:

□ x □ to give: 2 × □

Drag 2×□ and □ +1 into a division function:

$\frac{\square}{\square}$ to give: $\frac{\square+1}{2\times\square}$

Drag $\frac{\square+1}{2\times\square}$ into a squaring function:

□ ² to give: $\left(\frac{\square+1}{2\times\square}\right)^2$

Drag 1 and $\left(\frac{\square+1}{2\times\square}\right)^2$ into a subtraction function:

□ - □ to give: $1-\left(\frac{\square+1}{2\times\square}\right)^2$

Drag $1-\left(\frac{\square+1}{2\times\square}\right)^2$ into a multiplication function:

□ × □ to give: □ × $\left(1-\left(\frac{\square+1}{2\times\square}\right)^2\right)$

This completes half of the expression. The other half can be created in the same way. The method is equally valid as a means of creating algebraic expressions once Algebra has been introduced. For example:

$\sqrt{\frac{x+4}{x-5}}$ $\quad$ $1-\left(\frac{x+1}{2\times x}\right)^2$

□ x □

Giving: $\sqrt{\frac{x+4}{x-5}}\times\left(1-\left(\frac{x+1}{2\times x}\right)^2\right)$

It is important to recognize that the purpose of this exercise is not just to produce the complicated expression, which could be created in advance and downloaded from a library when needed. Its main purpose is to teach the structure and meaning of expressions.

The second benefit of using Foundation is that it builds a computer program as well as a mathematical expression. If 7 is input to all of the variables, we get:

$$\sqrt{\frac{7+4}{7-5}}\times\left(1-\left(\frac{7+1}{2\times 7}\right)^2\right)$$

The calculation icon can then trigger the computer program, giving the answer: 1.579

The teacher also has the option of using the stage icon, which triggers the computer to execute the calculation one step at a time.

Process and Object

Algebraic notation's greatest value may well be the opportunity it provides for the manipulation of expressions containing letter variables, but these letters are solely abstract objects. Sfard (1989) defines them as a ***structural*** concept. In a previous paper, Watson (2012) quotes Sfard defining a second conception of a mathematical entity, the operational process: "It would be of little or no avail to introduce a new mathematical ... [entity] by means of its structural description....In the classroom, therefore, the operational approach should precede the structural" (Sfard, 1989, p. 152, as cited in Watson, 2012). It would therefore appear to be valuable to introduce the concepts on which algebra is based as operational processes while arithmetic is being studied, something easily accomplished using Foundation.

In Foundation, variables, functions, and expressions are all processes–the manner in which Sfard (1989) suggests that new concepts should be introduced. This would provide students several years of experience using variables, expressions, and functions as operational processes before they become structural objects in algebra.

Functions

A function, one of the most important concepts in

mathematics, is generally not introduced until a student is well into the structural approach of algebra in high school. Arithmetic lacks functions and thus the capability to provide the preparation necessary to understand functions at a higher level of mathematics. In university undergraduate courses, this may well be a contributing cause of the "…weak function understandings… (of) many high-performing undergraduates (e.g., students receiving course grades of A in calculus)" (Carlson & Oehrtman, 2005, para. 2). However, Foundation is built from functions. Instead of first encountering them in high school, students have the opportunity to begin understanding functions as number machines in Grade 1. A more formal approach to functions can be added after this approach is fully understood. Twelve years of school experience with functions would give students considerable preparation for post-secondary studies.

Conceptual Steps

When a student begins algebraic manipulation in the older grades, he or she normally faces multiple major concepts at the same time, including: the change to expression as the basic component, structural complexity, expression manipulation, and complete abstraction with letter objects. Through using Foundation, it is possible to rearrange the sequence of conceptual steps into an order that allows new concepts to be introduced one at a time.

Primary Grades

Foundation was originally intended for the senior grades before its value in preparation for algebra was recognized. It now appears from initial piloting in a Grades 1/2 classroom that its value begins as early as Grade 1. Anecdotal reports from the teacher suggest that the method enabled students to achieve a deeper understanding of concepts in both the Algebra and Number Sense and Numeration strands. Students were then able to apply the concepts in a problem–solving context. In addition, they were able to make connections between the two strands and between different operations (e.g., repeated addition and multiplication as equal expressions: $3 + 3 + 3 + 3 = 4 \times 3$). The ability to drag expressions around the screen helped students to see them as whole entities, rather than a sequence of individual parts. They were successful in understanding functions in terms of input and output. Thus, Foundation has the potential to become a consistent and expanding asset throughout all school grades.

Conclusion

Mathematical expressions built with Foundation look identical to traditional mathematical notation. In intermediate expressions, Foundation variables, algebraic variables, and numbers are side by side. Thus Foundation could be considered to be part of mathematical notation, rather than a tool from outside it. Its inclusion would furnish mathematics with a new capability – a direct link between mathematics and computing that turns the computer into a mathematics machine. With the help of this mathematics machine, the study of arithmetic could equip students to better cope with algebra.

The facilities already implemented in Foundation could make a significant contribution to the teaching of arithmetic and algebra. They are also a solid base upon which high school mathematics could be built. The factorial, polynomial, trigonometric, exponential, and logarithmic functions have already been implemented with Foundation. Facilities for sequences, vectors, series, limits, tables, graphs, and early calculus will be in place later this year. As the system continues to evolve, the manipulation of expressions and fractions are planned for the Spring of 2013.

In the primary/junior grades, Foundation is not only useful as a preparation for algebra in the intermediate/senior grades; it could also help directly in the teaching of the primary-/junior-grade curriculum. Thus, it offers the opportunity for students to use the same mathematical learning facility consistently from Grade 1 to Grade 12.

It is impossible to demonstrate an active resource in a static paper. The Foundation website at **www.FoundationNotation.com** contains a short video that shows Foundation at work. The website also contains Foundation itself for teachers to try.

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What's New with CLIPS?

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Greg is from the Simcoe Muskoka Catholic District School Board. Greg is also the current OAME webmaster.

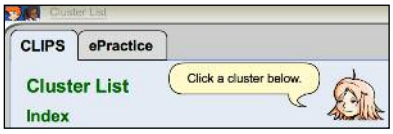


Ross is from the Near North District School Board. Both Greg and Ross are currently on assignment with the Ontario Ministry of Education as Provincial Math Leads, working on the CLIPS project, in conjunction with their colleague, Agnes Grafton, from the Brant Haldimand Norfolk Catholic District School Board.

Have you visited the MathCLIPS.ca website lately? Over the last year there have been a number of changes and updates to the site. Here are just a few you should check out.

MathCLIPS and ePractice activities are now accessible from the same URL.

Whether you visit using the CLIPS website address (**MathCLIPS.ca**) or the ePractice site (ePractice.ca), you can switch back and forth between CLIPS and ePractice activities.



What are CLIPS?

CLIPS (Critical Learning Instructional Path Supports) activities are clusters of interactive activities that support students with challenging-to-teach-and-learn math concepts. CLIPS activities are based on research and developed by Ontario teachers using a three part lesson design. The dynamic visuals, variety of representations, and narrated instructions appeal to students with multiple learning styles.

What are ePractice activities?

ePractice activities provide further practice and feedback to support the J/I and I/S Gap Closing print resources.

Several of the tools available along the right side of the CLIPS wrapper have been updated.



1. **Camera icon now available to take screen shots.**

   The camera icon allows you to take a screenshot of all or a portion of the current CLIPS activity screen. As a teacher, you may find this useful to create an image to use in a test, an assignment handout, or activity blog post. Students might use this to document their progress in an activity, or to save a snapshot for an electronic journal or portfolio.



2. **Wrench icon now gives access to standalone CLIPS tools.**

   The tool pod has been completely redesigned and now houses a broad set of tools. Some are electronic versions of common classroom manipulatives, such as the Fraction Strips tool, and others are newly designed specifically for CLIPS, such as the Hops on a Line tool. There are also links to a few select high-quality web tools. Each CLIPS tool can be used in its own dedicated browser window, or can be downloaded and used directly from the desktop, or when offline. Help videos and PDFs are being compiled to assist teachers and students with the use of various tools. See the next issue of the *Gazette* for more details about the CLIPS tools.

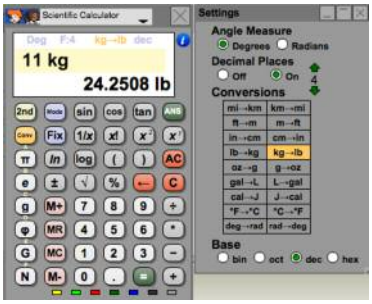


3. **The CLIPS calculator has had a major facelift and added functionality**.

   The CLIPS calculator gives students and teachers access to a basic calculator, a scientific calculator, and a graphing calculator. All the calculators have been redesigned to have a common look and feel. Select a calculator from the drop-down menu at the top, beside the logo of the CLIPS mathcots. For those who are looking for the previously available conversion calculator, you will note that those functions have been incorporated into the scientific calculator.

Scientific Calculator

Check out the updated scientific calculator, and click on the new blue Mode button to bring up the Settings pod. You can fix the number of decimal places to display in your answer, change the angle measure desired, or even switch from binary, octal, decimal, or hexadecimal number systems.

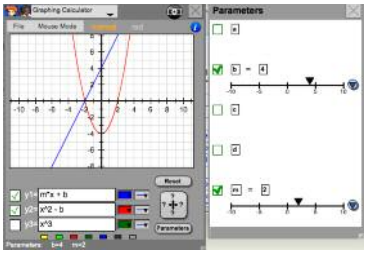


To perform a conversion, select the conversion type from the Conversions panel, then enter a number in the calculator and click the orange Conv button. (2nd + Conv will use the opposite conversion, e.g., in the example above, 2nd + Conv would calculate the conversion of 11 lbs. to 4.9895 kg.)

Did you know that every time you hit the = key in the Basic or Scientific calculator, the answer that is calculated is also stored on your computer's clipboard? You can paste the answer directly into another program, using Edit $\frac{\text{tree's height}}{\text{tree's shadow}} = \frac{\text{your height}}{\text{your shadow}}$ Paste.

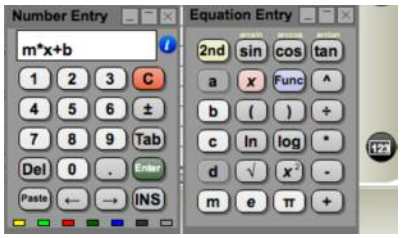
Graphing Calculator

The graphing calculator has received a number of exciting updates over the past year. The biggest update is the addition of a new Parameters pod, accessible from the Parameters button at the bottom of the calculator.



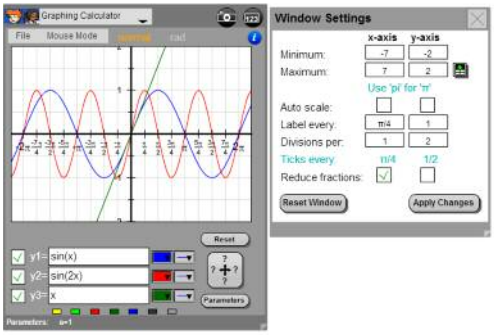
You can define up to five different parameters to use in the equations panel. Click the checkbox to make the parameters active, then use them as part of the equations *y*1, *y*2, or *y*3. Drag the triangle sliders in the parameters pod to change the values, and see the results in the graph window. Note that the value of the parameters is displayed in the status bar at the bottom of the graph pod, which is very useful when saving screenshots.

There is a special camera icon available inside the graphing calculator. It works identically to that on the CLIPS wrapper, but it has a few "presets" already programmed into it: You can capture the graph window itself, or the whole graphing calculator (which will include the graph window, equations, and the status bar).

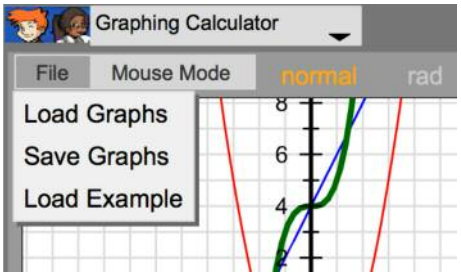


Are you using an interactive whiteboard and having trouble with keyboard entry? CLIPS has its own Num Pad, which can be accessed directly from the right side of the wrapper. Click on the [123] icon to bring up the Number Entry Pad to enter numbers into any text boxes in a CLIPS activity and in the graphing calculator. An additional Equation Entry pad is available when using the graphing calculator, which allows you to enter parameters, operations, and functions into the fields that allow them.

For example, you can click on the Windows Setting Icon (the honking big one with all the question marks on it) and get very fine control over how the scale is displayed. Use the entry pads to set the horizontal scale to $\frac{\pi}{4}$. Notice how nicely the multiples of pi are displayed.



After creating a lovely set of graphs, perhaps with some carefully chosen parameters, wouldn't it be nice if you could save that work? Now you can, using File → Save Graphs.



A small XML file is created with your settings. To restore the equations, parameters, and graph, choose File → Load Graphs. You can make your XML file available to others as a starting point for their exploration. If you go to File → Load Example, you can see some stored examples. If you have a good idea for a sample, please forward the saved XML file to the authors and we might publish it!

The CLIPS calculator is designed to work online, offline, and embedded in Interactive White Board activities. Now that it has a wider range of functionality (pun in ten did), including the ability to capture screenshots, create beautifully scaled trig graphs, and save your work, isn't it time you and your students made it your calculator of choice? ▲

▲ Communication Technology

FRANK MARINGOLA
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I am a Communication Technology teacher. Currently I am with the Toronto Catholic District School Board and the Simcoe-Muskoka Catholic District School Board. I am interested in inter-disciplinary potential of Math and Communication Technology to engage learners.

Communication Technology is a subject that has the possibility to promote the Geometry and Spatial Sense strand. Students in Communication Technology have the potential to own the material, take responsibility, and become self-motivated. Students are "digital natives" and increasingly are in class with multiple technologies: e-readers, phones, laptops, tablets. Digital projectors are tools to expand curriculum visually, an expedient avenue to students' learning styles. Digital projectors are stepping stones to students' attention and for engagement. Displaying images of hexagons in class, such as a turtle shell, can lead to students becoming motivated to take over the learning process, and expand the pathways to learning mathematics by looking at shapes and patterns. Connections can be made between prior learning and new material. These experiences help students connect mathematics with their everyday world.

Pictures can clarify mathematical vocabulary that is new to a student, who, for instance, is a newcomer to Canada, or to ESL/ELD students. Revisiting prior learning to introduce it differently, such as with the photograph of a turtle shell, attends to geometric growth. Communication Technology offers that opportunity by showing prior learning or new learning in a new format. Connecting to the world around them, by seeing geometry outside the classroom, shows students geometric growth in a different manner. Instead of reading about it, they visualize what they have been taught. Geometry is ubiquitous in our world; it is visible in animals, nature, formations such as crystals and gems, and even in the solar system. Geometric and Spatial Sense can be appreciated, and once the connections are made to its use in architecture, carpentry, and science, students can find mathematics a subject they no longer fear or loathe.

Reflection in plenary on the shapes visible on turtles, and then reflection in private can help students move through the levels required in the curriculum. Growth in geometric reflection should be observed throughout the year. Geometry is a gift to us and Spatial Sense is our reflection of thanks. Appreciating our surroundings, our world, is central to our math growth. Without mathematics and fluency in it, we cannot become.

Students are best prepared for the world when they have made personal connections to their learning. Lessons that are based on reflection and on learners' cultural heritage arouse curiosity. Hexagons are universal and can be located in any culture. Religions from around the world employ the hexagon in their architecture, symbolism, or other form of expression.

Communication Technology opens up this geometric knowledge and presents it vibrantly. Photoshop software is a tool to explore geometric activities, from editing photographs to manipulation. Animation software can allow students to draw and animate shapes. Video production and editing offers learners multiple opportunities to practise their spirit of enquiry. ▲





▲ Book Review: *Counterexamples in Calculus*

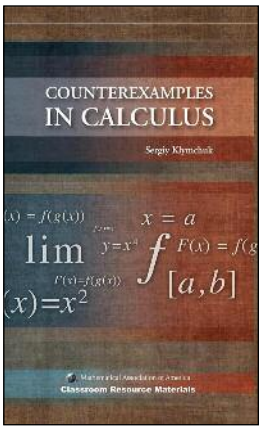
REVIEWED BY TIM SABALD

Klymchuk, S. (2010). *Counterexamples in calculus*. Washington, DC: Mathematical Association of America.



Tim is an assistant professor at the Schulich School of Education at Nipissing University and a vice-president of OAME.

This short book provides numerous examples that highlight the importance of finer details in both calculus and some topics that precede calculus. For teachers in Ontario, there are examples that are relevant to Advanced Functions (MHF4U), Calculus and Vectors (MCV4U), as well as Advanced Placement courses. There is additional content that will be of interest to college- and university-level calculus instructors. This additional content is an interesting way of refreshing the high school teachers' memory about some of the finer details of single variable calculus.



The book provides a set of 91 statements without answers and then the same set of statements with answers. This facilitates two distinct approaches to reading the book: as a challenge to independently develop counter-examples, or as a validation of one's teaching, with an emphasis on the details. As a secondary school teacher reviewing the book, I combined the two approaches and challenged myself for the parts I teach and took the validation approach to the parts I do not teach.

An example that is relevant to advanced-function teachers is: If a function is 1:1 and has an inverse on a particular interval, then we might wrongly think that it is strictly increasing or strictly decreasing on that interval. A counter-example is provided that shows that the function can be both increasing and decreasing within the interval. The key detail, in this instance, is an erroneous assumption regarding continuity. (The reader is left to ponder what the counter-example might be.)

Another example is the suggestion that if $f(x)$ and $g(x)$ are both discontinuous at the same point $x = a$, then $f(x) + g(x)$ will also be discontinuous. A counter-example is provided that fits quite nicely into the combining functions section of the advanced functions course. A related suggestion for calculus is that if $f(x)$ and $g(x)$ are not differentiable at $x = a$, then the sum $f(x) + g(x)$ may incorrectly be assumed to also not be differentiable at $x = a$.

With limits, a clear but important counter-example is a continuous function which has a limit as it approaches infinity through the integers, but does not have the limit within the real numbers. Symbolically:

$$\lim_{n \in I \to \infty} f(n) = A \quad \text{does not imply that} \quad \lim_{x \in R \to \infty} f(x) = A$$

Another calculus example is that a function that is continuous and decreasiwng on (a, b) has a non-positive derivative on (a, b), which is shown to be false, using an example where the derivative is undefined for a point in (a, b). Examples such as these allow a fine tuning of one's instruction to include some of the finer details. It is also food for thought about instruction regarding domains, and lends itself to teaching critical thinking about the conditions required for different theorems.

Having seen books of counter-examples before, I was concerned that there might be repetitive use of some strange examples. This was not the case, as the instances where a function was used more than once were beneficial because of the unusual example that was being used. As one example, a function is provided that demonstrates that if the first derivative of a function is zero at a global maximum, i.e., $f'(a) = 0$, then in any small neighbourhood of the point, the particular function is both increasing and decreasing! Slight variations of this function are used in a few solutions, but serve to familiarize the reader, rather than detract from the book.

Overall, the collection is a useful resource and a worthwhile read for anyone teaching at the Grade 12 university level. Some examples will be of interest to Grade 11 university teachers, but it is unclear to what extent they might include the counter-examples. Certainly the book would be a good resource for a high school math department and food for thought for math specialists. ▲

▲ 2012-2013 OAME Board

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Abacus

Volume 51 • Number 2 • December 2012

ORGANIZING AND READING DATA

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A Publication of the OAME/AOEM

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We collect data from the real world and analyze them in order to find out something about our world. In fact, data are often collected to answer questions and support or challenge hypotheses and arguments. However, because the amount of accumulated data can be overwhelming. It must be organized. The average (or mean), the median and the mode are three ways to organize data as measures of central tendency. Data can also be arranged and presented in charts, graphs or tables. Different charts are used for different purposes. From these charts, graphs or tables, a quick look can determine the answer to many questions. As you read data, ask yourself, What is the author's motivation for producing the diagram or chart? How does it support or refute an argument? What is its main message? Therefore, reading data is way to gather information in order to answer questions, to see relationships between sets of data and to raise further questions. So, when students read data, they have an opportunity to learn more about themselves, their environment, issues in their school or community, topics in various subject areas, and so on. In this issue, the Research Summary and problems (with multiple solutions) focus on strategies for reading quantitative and qualitative data. The problems we provide are designed for use within a three-part, problem-solving lesson.

LINKS TO LITERATURE AND MANIPULATIVES



PAT MARGERM is an independent literacy and mathematics consultant with more that 30+ years of K to 8 teaching experience with students, in-service and pre-service teachers. She is a long-time OAME board member and works in various aspects of mathematics education.
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Venn diagrams, graphs, tables, and charts are tools that can be used in both mathematics and literacy contexts. Students can use these tools to write reports in subjects such as science, health, social studies, history, and geography. In addition to crossing subject areas, these tools also connect strands of the math curriculum. Students can use blocks to create patterns, then collect data about the number of blocks in the pattern, organize the data into a table of values, graph the data and then generalize the relationships algebraically. Students can collect data about the dimensions and areas of rectangles, organize the data in a table, graph the data, and generalize the relationships in measurement formula. Manipulatives such as found materials, fraction circles (for circle graphs) the Hundreds Carpet™, and pocket charts can be used to represent data graphically. In this issue we will look at the use of linking cubes. We will also look at using linking cubes to respond to some inquiries based on some ideas in two picture books.

RESEARCH SUMMARY

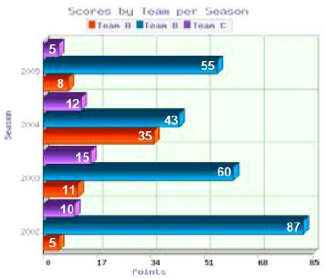
RESEARCH SUMMARY

Randall (2005) explains that “some questions can be answered by collecting and analyzing data, and the question to be answered determines the data that needs to be collected and how best to collect it. Data can be represented visually using tables, charts, and graphs. The type of data determines the best choice of visual representation” (pp. 20-21). An effective graph should communicate at a glance some overall information about the data. Graphs typically used in elementary classrooms, include:

Questions Completed

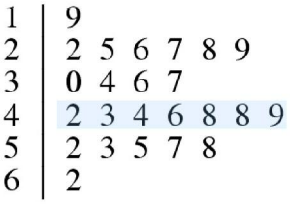


pictograph



bar graph

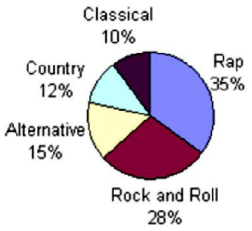
Number of Tags Collected



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2	2 5 6 7 8 9
3	0 4 6 7
4	2 3 4 6 8 8 9
5	2 3 5 7 8
6	2

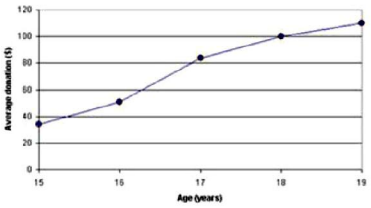
stem and leaf plot

Music Preference



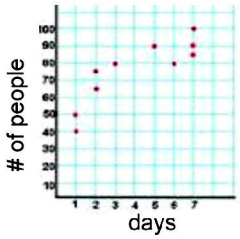
circle graph

Donations for Flood Relief



line graph

People Attending Show



scatter plot

When looking at the information contained within a graph, it is useful to consider: the title of the graph, the labels on the axes, the scale, or scales, used, the source of the data the size of the samples used (if appropriate). Ask students why this information is important to consider. More specifically, Small (2010) notes that students should be able to read individual specific details of the data from graphs, as well as work with multiple details. The kinds of conclusions that students might draw involve comparisons among data, such as which category has the greatest or least value, or which category has more data values than the other (i.e., pictographs, bar graphs, stem and leaf plot, circle graph). Line graphs and scatter plots are read to see trends and relationships between the variables on each axis.

Instructional Strategies

Russell (2006) explains that “at all grade levels, students need support to interpret their data representations in a way that helps them relate the data collected back to the context and to the actual event or information the data represent. Teachers can help students move away from reading graphs as if they are a collection of symbols towards using data to answer questions (pp. 27-28). Some teaching strategies include:

- having students develop their own representations from data they collected so that they experience connecting the symbols they use in familiar contexts
- developing with students, questions about comparing groups so that students have a reason to characterize each set of data as a whole
- encouraging students to notice where data is concentrated (e.g., About how much of the data set is part of that cluster? Is it more or less than half of the data? About what fraction of the data is it?)
- carrying out quick, fifteen minute data reading activities, asking questions like, “What do the data we collected and represented tell us about what we wanted to know?”

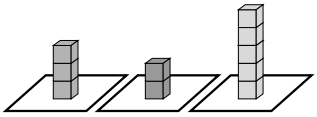
Whitin (2006) suggests these strategies for developing students’ as critical readers of data and measures of central tendency, such as:

- prompting students to question numerical information to consider and expose the intentions implicated in the data
- connecting data collection and analysis to the interests and background of the students, as they can be better critics if the data is tied to familiar contexts
- have students practice creating categories of the data to become more aware of the limits of their visual display
- asking questions like, “Who benefits from using the data in this way?” as it underscores the idea that people use data for a reason
- involving students in gathering data from different samples (e.g., adults, children; older children, younger children), so that they can compare the results. How are the data sets the same and different? What accounts for these similarities and differences?

LINKS TO MANIPULATIVES: LINKING CUBES

Linking cubes can be useful in making the transition from concrete graphs (K—1) to bar graphs, and for collecting discrete data (grade 2).

The class could consider an inquiry, such as, “What colour are your mittens?” Students can create a concrete graph with their mittens or respond to this survey by choosing a linking cube to represent the colour of their mittens. Posing questions such as which colour is most common can be extended to consider reasons for each frequency. (e.g., why don’t many parents buy white mittens for their children – too difficult to keep clean, they are harder to see in the snow if they come off). Students can extend this inquiry to investigate if there is a colour of mitten that is most common in the lost and found. Students can also collect data to investigate the changes in the number of mittens in the lost and found over time, compare the number of mittens to the number of gloves, collect data about where mittens are lost (inside the school and outside of the school) and compare the sizes of the mittens that are lost.



“How many pockets do you have?”, is a question that has appeared frequently in lesson plans. The teacher may ask students how they can keep track of the pockets as they count them. Students can put one linking cube in each of his/her pockets and then count the cubes. A variety of questions could be posed: are there differences in the number of pockets on clothes made for boys/girls or are there differences in the number of pockets on clothes in different cultures?

LET’S DO MATH

Before (Getting Started)

Data from Frequency Tables: Problem A

What conclusions can you draw from the data in this frequency table?



Solution 1

24 people answered the survey.

The highest number of people like snowboarding and that was about 20%.

20% of the people like sports with balls.

Solution 2

9 people like winter sports (snowboarding and hockey)

That is 9/24 about 38%.

Equal numbers of people picked skateboarding, soccer, and other.

The least popular were drama and badminton.

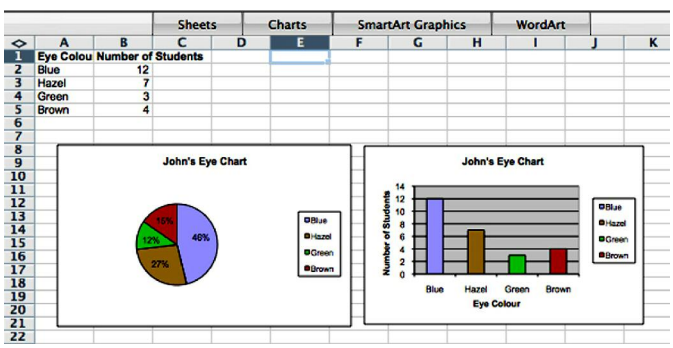
During (Working On It)

Frequency Tables and Graphs: Problem B

This survey was done in Australia. Data about people’s eye colour is presented in three forms:

- A frequency table (in EXCEL)
- A circle graph
- A bar graph

Which presentation of data is more useful?



LESSON LEARNING GOALS

Identify the curriculum expectations (MOE, 2005) grade-before and grade-after the grade specific learning goal:
- to see the development of mathematical knowledge, skills, and strategies across grade levels
- to determine the mathematical focus of the problems chosen for Before (activation), During (lesson), and After (Practice) parts of the lesson

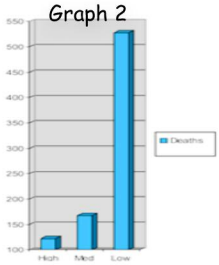
BEFORE (Getting Started) – 5 to 10 minutes

- Activating students' mathematical knowledge and experience using a prompt or problem, related to the math in the lesson problem.
- Includes student responses to highlight ideas and strategies.

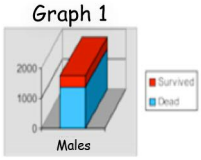
Perceptions of the Titanic Tragedy

Four Titanic survivors made graphs to represent their reactions to the event. Which comment matches which graph?
Person A: "Lots of people were saved."
Person B: "Most of the people who died were poor."
Person C: "The males were brave."
Person D: Working on ships is not safe."

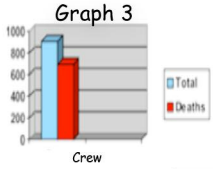
Graph 2



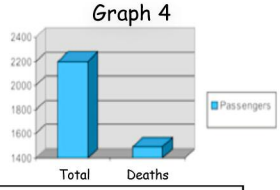
Graph 1



Graph 3



Graph 4



Solution 1

I think Graph 4 matches Person A comment. It shows the total passengers compared to the number who died.

Solution 2

Person A – Graph 4
Person B – Graph 2
Person C – Graph 1
Person D – Graph 3

DURING (Working on It) – 15 to 20 minutes

- Understanding the problem – Teacher asks, "What information from the problem are we using to make a plan to solve it? Explain." Teacher records in a list below the problem, the information that students identify.
- Students solve the problem on chart paper (landscape) with markers (visible for class discussion) in pairs or small groups.
- Teacher circulates to record different student solutions, in addition to the ones the teacher anticipated.

Oh! They Built the Ship Titanic Problem

How are the tables the same? Different? Analyze the data in the table and write statements about what you find. Support your statements with data.

Data Organized by Economic Status and Gender						
Economic Status	Number of people on the Titanic			Number of deaths resulting from the sinking of the Titanic		
	Male	Female	Total	Male	Female	Total
High Income	180	145	325	118	4	122
Middle Income	179	106	285	154	13	167
Low Income	510	196	706	422	106	528
Other	862	23	885	670	3	673
Total	1731	470	2201	1364	126	1490

Data Organized by Economic Status and Age						
Economic Status	Number of people on the Titanic			Number of deaths resulting from the sinking of the Titanic		
	Adult	Child	Total	Adult	Child	Total
High Income	319	6	325	122	0	122
Middle Income	261	24	285	167	0	167
Low Income	627	79	706	476	52	528
Other	885	0	885	673	0	673
Total	2092	109	2201	1438	52	1490

AFTER (Consolidation) – 20 to 25 minutes

- Teacher selects 2 or more solutions for class analysis and discussion in a sequence (1st, 2nd, 3rd, etc.) based on mathematical relationships between the solutions and the lesson learning goal.
- Students (authors) explain and discuss their solutions with their classmates.
- During whole class discussion, solutions are organized (often re-organized) to show mathematical elaboration from one solution to the next and towards lesson learning goal.
- Teacher mathematically annotates (math terms, math symbols, labelled diagrams, concise explanations) on and around solutions to make mathematical ideas, strategies, and models of representation explicit to students.

-the first two instances and in the third statement chose incorrect numbers to compare
- true that all the children who died were from low income families
-not all the children from low income families died (that would be 52/79=66%).

Solution 1

Same	*Different*
-People who died -Same totals -Rich, middle poor	*-One by gender -One by age*

We do not know who the "others" were – maybe workers.

Solution 2

-More men died than women 1731 – 470 = 1261
-More low income men died than high income men 422 – 118 = 304
-"All the low income children died 52/52."

Identifies Similarities and Differences

Absolute Differences

Note: Mathematical annotations include mathematical vocabulary, symbols, elaborations of mathematical details from solutions, labels describing the method/strategy, and questions to further thinking. All annotations are records of students' mathematical discussion.

Grade 4	Grade 5	Grade 6
-read, interpret and draw conclusions from secondary data presented in charts, tables and graphs (including stem-and-leaf plots and double bar graphs)	-compare similarities and differences between 2 related sets of data using a variety of strategies	-demonstrate an understanding of how data from charts, tables and graphs can be used to make inferences and convincing arguments

AFTER (Highlights/Summary) – 5 minutes

- Teacher and students revisit the student solutions for key ideas, strategies, and models of representation that are related to the lesson learning goal (anticipated success criteria)
- Teacher lists key ideas, strategies, and models of representation separately, so the students can see how the mathematical details from their solutions relate explicitly to the lesson learning goal.

1364/1490
ratio of number of women compared to total passengers

126/1490
ratio of number of men to total passengers

-easy to read chart that compares the correct numbers for two specific cases and draws a conclusion

Highlights/Summary:

Compare similarities and differences between 2 related sets of data by:

- *identifying similarities and differences*
- *calculating absolute differences in the data values of the sets of data*
- *calculating relative differences (using percents) in the data values of the 2 sets of data*
- *comparing specific cases to support argument*

AFTER (Practice) – 5 to 10 minutes

- Teacher chooses 2 or 3 problems, similar to the lesson problem for student to solve in pairs as a scaffold and individually.
- Problems could vary by number (choice, size), problem contexts, or what is unknown or needs to be solved
- Students are asked to solve these problems using a strategy different from the one they used for the lesson problem.

Solution 3

1238 more men died than women: 1364 to 126

$\frac{1364}{1490} = 92\%$

$\frac{126}{1490} = 8\%$

All the children who died were from low income families died 52/52.
And 66% of the children from low income families died. $\frac{52}{79} = 66\%$
If you are poor. don't take your children on boats.

Solution 4

Both tables show the numbers who died on the Titanic; one shows the data by gender and the other by age.

Comparisons: number died / number on board

	Males	*Females*
High income	$\frac{118}{180} = 66\%$	$\frac{4}{145} = 3\%$
Middle income	$\frac{154}{179} = 86\%$	$\frac{13}{106} = 12\%$
Low income	$\frac{422}{510} = 83\%$	$\frac{106}{196} = 54\%$

"Middle and low income males should be careful when deciding to sail on a cruise ship. More than 9/10 of those on the Titanic died."

Relative Comparisons

Compares Cases and Draws Conclusions

Who Uses Computers at Home? Problem

Compare the data about computer use at home. What conclusions could you draw from the information in the charts?

Raw Data

	Sex		
UseComp	Female	Male	Total
Yes	45	45	90
No	2	8	10
Total	47	53	100

Percentage Data

	Sex	
UseComp	Female	Male
Yes	96	85
No	4	15
Total	100	100

Solution 1

Chart 1 shows the same number of males and females using a computer at home; Chart 2 says more women do - about 4 X as many men as women do not use computers at home.

Solution 2

Compare the number of women who use computers at home to the total number of women who responded to the survey →45/47 = 0.957 = 96%; For men 45/53 = 0.849 = 85%
Percent numbers (on the right) mean the whole population; about 10% more females use a computer at home than males.

Note: A classroom board is longer proportionally than these 2 pages. Due to the space constraints on these pages, the mathematical annotations are recorded above the solutions with arrows, rather than on and around the solutions.

Anticipating Student Responses:

Solution 1

From the frequency table, I can see that 26 people answered the survey question.

The circle graph and the bar graph show a majority has blue eyes.

The circle graph suggests almost 50% of the people have blue eyes. I like the circle graph best. The bar graph just shows numbers like the frequency table.

Solution 2

All three representations show that blue eyes are most common. I do not think that would be true if we considered data from Ontario.

The circle graph shows the whole as 100% of the people and that really emphasizes that about 50% have blue eyes and the other 50% have one of 3 colours (hazel, brown or green).

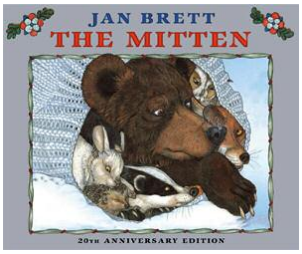
From least to most common: green, brown, hazel, blue.

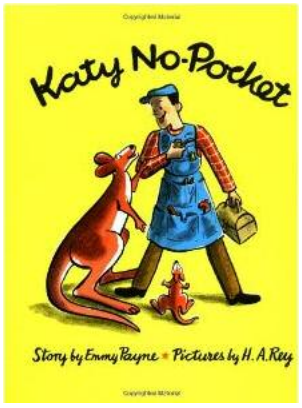
Coordinating Discussion for Student Learning:

Why might solution 1 be chosen first for student discussion followed by solution 2?

- Solution 1 reports on the graphs and the frequency table and the student says s/he likes the circle graph more but does not share reasoning
- Solution 2 talks about the most common occurrence of blue eyes and uses the circle graph to report on the relationships of blue being about half and the other three being the other half. The student refers to the circle being 100% or the whole. This student also comments on the fact that the data is from Australia and that it may be a different picture if it were collected in Ontario.

LINKS TO LITERATURE





The Mitten is a Ukrainian folktale that is available in a few versions as a picture book, one of which has been written by popular author Jan Brett. In this story, Nicki drops his white mitten in the snow, and goes on without realizing that it is missing. One by one, woodland animals find the mitten and crawl in; first a mole, then a rabbit, a badger and others, each one larger than the last. Finally, a big brown bear is followed in by a tiny brown mouse, which is, ironically, when the mitten explodes. Students can inquire into the question, “How big is your mitten?” by using linking cubes to measure the length, the width, and the capacity of their mittens. The cubes can then be organized graphically in order to respond to the original inquiry question (e.g., some materials can stretch to hold a lot of cubes and some cannot).

Katy No-Pocket by Emmy Payne is a classic book that can be used to investigate differences. Mother Katy Kangaroo doesn't have a pocket in which to carry her son Freddy like all the other mother kangaroos but other animals carry their babies in different ways. A wise owl sends Katy to the city to find a solution and a kind carpenter gives her an apron with many pockets. She uses her new pockets to not only carry her own baby but all the other babies too. Students can collect data to inquire about how many pockets are in their clothing (do all coats have pockets?), how do they use their pockets (what is in your pocket?) and how big is your pocket (how many cubes will your pocket hold?).

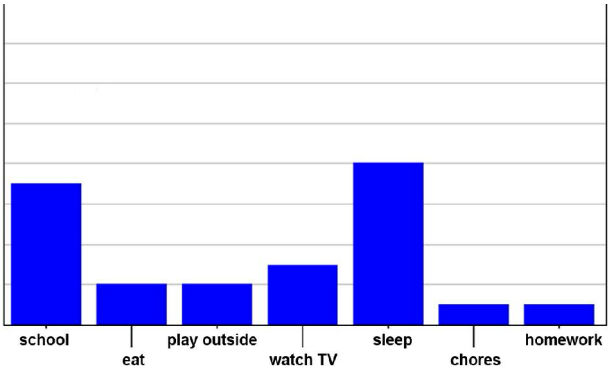
LET’S DO MATH – SCALES ON GRAPHS

Before (Getting Started)

Tracking Time Problem: Problem A

What does this graph tell you about the way students respond to the question, “On average, how many hours do you spend: sleeping, eating, studying, at school, playing outside, watching TV, doing chores, and working on homework?

How many hours do you spend on … ?



Solution 1

It says that students spend the most time sleeping and the next bundle of time at school. They spend the least time doing chores and on homework.

Solution 2

I speculate that each horizontal bar on the graph represents 2 h. Because when I use that idea to count I get 24 h and that is the total number of hours in a day

School	7	Sleep	8
Eat	2	Chores	1
Play outside	2	Homework	1
Watch TV	3	Total	24

During (Working On It)

Household Pets Problem: Problem B

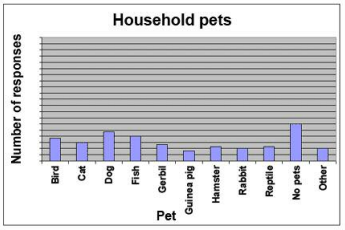
Jayden, Louis, and Gayle gathered the data randomly from 103 students in their school. What do their graphs suggest about the distribution of pets in the households of their peers?

The labels for the pets (on the horizontal axis) are in the same order as the names of the pets in the data table.

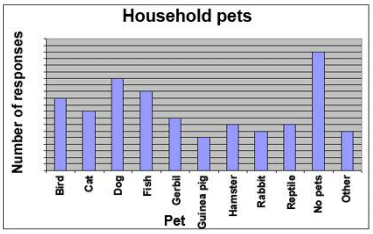
Pet	Number of responses	Pet	Number of responses
Bird	11	Hamster	7
Cat	9	Rabbit	6
Dog	14	Reptile	7
Fish	12	No pets	18
Gerbil	8	Other	6
Guinea pig	5		

After (Consolidation)

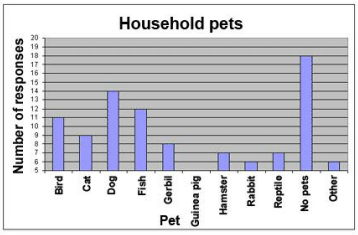
Gayle's graph



Louis's graph



Jayden's graph



Anticipating Student Responses:

Solution 1

Louis and Gayle's graphs look almost the same but Gayle's shows zero Guinea pigs. Her scale starts at 5 and counts by 1s. She should show that there is a break in her vertical scale.

Louis' must count by 1s too but starts at 0 because you can see 5 Guinea pigs counted.

Solution 2

Jayden and Louis did not show scales on their graphs. Louis seems to have counted by 1s. Jayden must have counted by 3s because "No pets" that supposed to show 18 is 6 marks high (18/6=3).

Louis' scale lets the differences show better because there is more differences in the heights when you count by 1s .

Solution 3

You have to have a scale on both axes in order to read information from a graph. However, all graphs show the relationships among the kinds of pets students reported. The most students reported no pets and then the most popular are dogs, fish, and birds.

Coordinating Discussion for Student Learning:

Why might solution 1 be chosen first for student discussion followed by solutions 2 and 3?

- *Solution 1* reports on the problems in two of the graphs – highlighting the importance of recording a scale on both axes.
- *Solution 2* suggests scales for two of the graphs without report or compare the data.
- *Solution 3* reports on the relationships including the highest four categories without commenting on the scales.

NEXT STEPS FOR YOUR PROFESSIONAL LEARNING

Application to Your Classroom

- Try these data problems yourself first and then offer them to your students.
- To suit your students' learning needs, vary the ways that you have students engage with reading different sorts of data and different representations of data, such as data presented in advertisements that compare costs, news articles that show trends in growth or relationships between variables and data collected for local and school community events and issues>
- Think about how the context of the problem evokes the purpose and use of reading data (e.g., progress of litterless lunches in a school; comparing school absences just before the weekend and in the middle of the week; relationship between hours spent on homework and grades).
- Practise noticing the different strategies and the breadth of the mathematics your students use to read data. Take note of the number and kinds of solutions that show strategies for reading data (e.g., identifying greatest and least data value; compare data values among a set of variables; examining the effect of outliers or the addition or the omission of particular data values)
- Think ahead of the kinds of mathematical annotations you will record on and around the student solutions to make explicit key mathematical ideas, strategies for reading data as the students discuss and analyze each other's solutions.

SUGGESTED READINGS

Some readings used in the Research Summary and in the development of the problems and solutions are listed below:

- Chapin, s., Koziol, Al, MacPherson, J., & Rezba, C. (2002). *Navigating through data analysis and probability in grades 3-5.* Reston, VA: NCTM.
- Charles, R. (2005). Big ideas and understandings as the foundation for elementary and middle school mathematics. J*ournal of Mathematics Education Leadership, 7*(3), 9-24.
- Graham, A. (2006). *Developing thinking in statistics.* Thousand Oaks, CA: Sage Publications.
- Russell, S. J. (2006). In G. Burrill and P. Elliott (Eds.), *Thinking and Reasoning with Data and Chance* (pp. 17-29). Reston, VA: NCTM.
- Shaughnessy, J. M. (2006). Research on students' understanding of some big concepts in statistics. In G. Burrill and P. Elliott (Eds.), *Thinking and Reasoning with Data and Chance* (pp. 77-98). Reston, VA: NCTM.
- Sheffield, L. et al. (2002). *Navigating through data analysis and probability in prekindergarten-grade 2*. Reston, VA: NCTM.
- Small, M. (2010). *Big ideas from Dr. Small, Grades K-3*. Toronto: Nelson Education.
- Small, M. (2010). *Big ideas from Dr. Small, Grades 5-8*. Toronto: Nelson Education.
- Whitin, D. (2006). Learning to talk back to a statistic. In G. Burrill and P. Elliott (Eds.), *Thinking and Reasoning with Data and Chance* (31-39). Reston, VA: NCTM.

CLOSING

Our next issue will focus on key notions of reading and comparing graphs to make inferences (e.g., to tell a story, to provide evidence, to construct a message or argument), how teachers need to know these notions for teaching, as well as, the ways that teachers can use these mathematical ideas and pedagogical strategies within a problem solving-based teaching and learning lesson framework.

CALL FOR *ABACUS* SUBMISSIONS

Send your teaching strategy ideas, problems and student solutions to the *Abacus* Co-Editors via email kathkubo@rogers.com or mkestell@sympatico.ca