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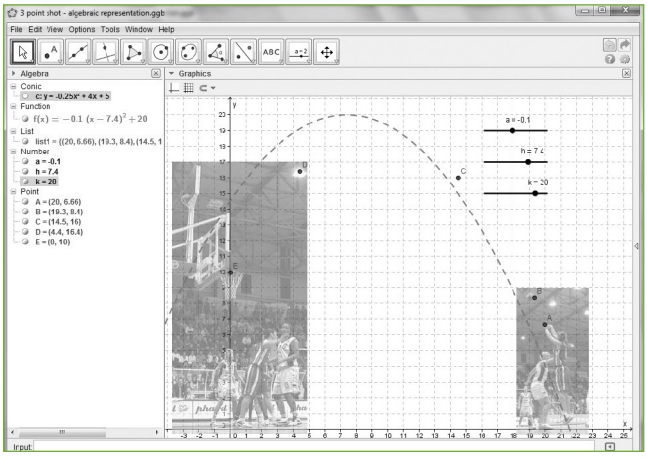
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Vol. 51 #4
June 2013

IN THIS ISSUE

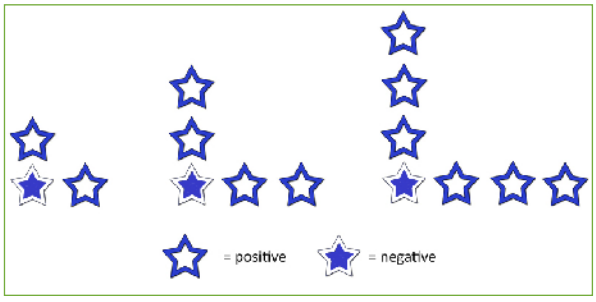
- Editor's Message
- Letter to the Editor: Are Teachers More Than Statistics?
- President's Message
- The Abacus
- Technology Corner – Visual Patterns
- OAME/NCTM Report June 2013
- What's the Problem? Cylinders within Cylinders
- Hey, It's Elementary – I'm Diggin' Those Calculations: What Historians Have Unearthed About Calculating Devices Through the Ages
- MB4T... Always A New Epiphany: From Imperfect Understandings To New Insight
- In the Middle – What is So Special About Numbers?
- Fields Institute – MathEd Forum Notes
- Directors' Dialogue
- What's New with CLIPS? New and Improved Tools!
- Linking Representations Using Geogebra
- Communication In A Blended Math-talk Community: Extending The Boundaries Of Classroom Collaboration

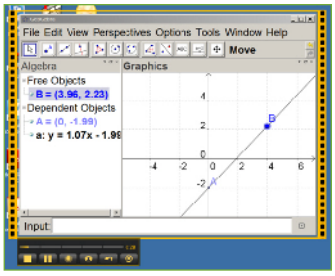
See Linking Representations Using Geogebra





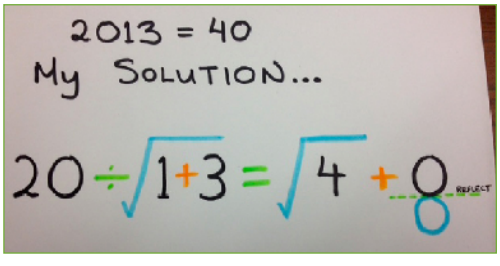
See Communication In A Blended Math-talk Community: Extending The Boundaries Of Classroom Collaboration





See Technology Corner – Visual Patterns

See In the Middle – What is So Special About Numbers?



Submission of Articles

The *Ontario Mathematics Gazette* (*OMG*) is looking for news items, articles, and good ideas that are useful to mathematics teachers and mathematics teacher education. We are seeking submissions, preferably from mathematics teachers K–12 and other mathematics education professionals, that describe innovative and creative approaches to mathematics teaching.

Please keep in mind the following criteria when making submissions to the *OMG*:

- The ideas/activities must be of interest to the readership.
- The ideas/activities must be fresh and innovative.
- The mathematics content must be appropriate for the readership.
- The mathematics content must be accurate.
- The article must be well written and easily understood.
- The article and its ideas must be free of sexual, ethnic, racial, or other bias.
- The article must not have been previously published, nor should it be out for review by other publications.
- The article must be original.

Articles must be word-processed in MS Word, double-spaced with wide margins, not exceeding 10 numbered pages of text, and prepared according to the *Publication Manual of the American Psychological Association, Sixth Edition*. Figures and diagrams should be drawn by computer, if possible, or drawn in black ink in camera-ready form. Embedded images must also be submitted separately in jpeg or tif format. Proof of the photographer's permission is required, and for **photos of students** under the age of 18, the written permission of a **parent or guardian is required**.

You must submit **one complete copy** of your article, embedded with any tables, figures, and captioned photographs or graphics, to the Editor, Immaculate K. Namukasa, along with **separate files for each of the text, graphics, and/or photographs**. Please e-mail all files to Immaculate K. Namukasa at **immaculate.namukasa @oame.on.ca**.

Your name should not appear anywhere in your article, including websites, so that your article can be sent out for blind review. Your name, full mailing address, and e-mail address must be included on a separate sheet. Upon review, you will be notified as to whether your article has been accepted for publication (as is, or pending minor or major revisions) or rejected.

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▲ Table of Contents

Up Front

▲ Editor's Message /Immaculate K. Namukasa 2

▲ Letter to the Editor: Are Teachers More Than Statistics? /Tim Sibbald 4

▲ President's Message /Cathy Hall 6

Regular Features

▲ The Abacus . . . . . .8-Page Insert

▲ Technology Corner – Visual Patterns /Mary Bourassa 10

▲ OAME/NCTM Report June 2013 /Todd Romiens 12

▲ What's the Problem? Cylinders within Cylinders /Shawn Godin . . . . . .

▲ Hey, It's Elementary – I'm Diggin' Those Calculations: What Historians Have Unearthed about Calculating Devices through the Ages /Lynda Colgan 13

▲ MB4T... Always a New Epiphany: From Imperfect Understandings to New Insight /Ann Kajander 19

▲ In the Middle – What Is So Special about Numbers? /Caroline Rosenbloom 21

▲ Fields Institute – MathEd Forum Notes /Stewart Craven 22

▲ Directors' Dialogue /Ann Michele Stenning 23

Special Features

▲ What's New with CLIPS? New and Improved Tools! /Greg Clarke/Agnes Grafton /Ross Isenegger/Markus Wolski 25

Articles

▲ Linking Representations Using Geogebra /Geoffrey Roulet /Jill Lazarus 28

▲ Communication in A Blended Math-Talk Community: Extending the Boundaries of Classroom Collaboration /Geoffrey Roulet /Jill Lazarus 34

Notices

▲ OAME Tote Bag 5

▲ OAME Sticky Graph Pads 7

▲ OAME T-Shirts 13

▲ *Abacus*: A Compilation Volume 46, Volume 47 22

▲ OAME Board Members . . . . . Inside Back Cover

Advertisements

▲ Pearson . . . . . Back Cover

About the Ontario Mathematics Gazette

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Advertisements for publication in the *Ontario Mathematics Gazette* should be sent to **Robert Sherk** at the above address. Courier is recommended to avoid possible delays. Deadlines for advertisements are January 23 for the March issue, April 1 for the June issue, July 1 for the September issue, and October 1 for the December issue.

Full-page advertisements are to be on 8.5" by 11" paper with a minimum of 0.5" margins and single sided. Each advertisement should be camera ready and colour advertisements should have no bleeds.

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Editor's Message

IMMACULATE KIZITO NAMUKASA
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Immaculate Kizito Namukasa is an associate professor at the University of Western Ontario. Her current research interests in mathematics education include mathematical thinking and activity, mathematics teacher education, non-routine problem solving and learning environments, complexity research framework, and critical mathematics education. Namukasa is also interested in globalization and internationalization of schools and schooling in general.

Have you ever wondered about an all-in online learning environment that could support students' online learning? At the very minimum, for me as a mathematics educator, such an environment would support mathematical symbols as well as textual, visual, and audio information. Ten years ago, when I started teaching online, available online environments were mostly text based. This made sharing mathematics solutions online extremely difficult. Today, more forms of information are shared online. Witness the redefinition of the word "text" to also refer to visual, digital, symbolic, and other form of texts.

Learning platforms and software have come a long way. Several types of online software, including those that operate on smart mobile devices, have been invented. Much mathematics information can be shared online. Still, few of these web-based software appear to be developed in close collaboration with teachers, educators, and educational researchers. Consequently, it is not rare for a teacher to find himself or herself integrating more than one online environment to accomplish a single learning task. This was the case last year. When I taught an online course, I found myself supplementing the platform available at my university with three web-based open-source platforms. These platforms allowed me to easily embed videos, to co-create documents with learners, and to share computer screens in real time.

Good design for mathematics learning platforms requires platforms that have capabilities for use of mathematics symbols, geometrical and statistical

constructions, dynamic graphing and constructions, and freehand sketches. Several mathematics learning software such as mathematics apps, games, software and applets are available via the Web. Design that offers room for use of these mathematics software within an online learning environment is still difficult to come by. Researchers attribute this difficulty to computer experts working in isolation from educators. Consequently, interface design principles are given priority over pedagogical goals (Gadanidis, Sedig, & Liang, 2004, p. 295). Also, pedagogical goals for today's students, and those outlined in new curricula, are very different from pedagogical goals of ten years ago, when the design experts were at school learning mathematics.

In this June issue, two articles focus on how the mathematics processes, specifically mathematics communication and representing, may look like in blended online and in-class learning environments. The two articles—"Linking representations" by Roulet and Lazarus, and "Communication in a blended math-talk community" by Lazarus and Roulet—very neatly use examples from high school mathematics. These articles are timely. Many principles shared in these articles are also applicable to elementary school teaching. The article by Clarke, Ross, Grafton, and Wolski illustrates the newest tools available within the Critical Learning Instructional Paths Supports, CLIPS web-based resource for learning mathematics.

Mathematics ideas and tasks to be explored during the summer holidays are suggested in several columns. In the column "What's the Problem?," Godin shares solutions to a rich geometrical problem of nested cylinders. In the "Technology Corner," Bourassa shares dynamic linear and quadratic patterning problems that are accessible at a website. She nicely illustrates how, for Grades 9 and 10, the domains of these patterning problems could involve integers, as well as include the negative quadrants. Colgan in her "Hey, It's Elementary" column, shares selected historical computation methods that fascinate learners. Kajander, in "Mathematics By and 4 Teachers," elaborates on the idea that mathematical reasoning might develop over time when students are prompted to think about mathematical tasks on their own. And Rosenbloom, in her "In the Middle" column, exemplifies interesting mathematics found in everyday activities.

Three columns report about activities that took place during Spring. Romiens summarizes two articles from the NCTM Smart Briefs—a daily publication offering snapshots of mathematics education in the news, whereas Craven shares proceedings from the April Mathematics Education Forum at the Fields Institute in Toronto.

Gazette editorship involves working closely with several authors and regular contributors, whose goals are to improve mathematics learning in Ontario. Regular contributors make my job easier and more enjoyable. For two of these regular contributors, this is their last issue, at least for a while. On behalf of myself and the *Gazette* team, I would like to thank this year's OAME president, Cathy Hall, for her thoughtful and inspiring President's messages. Also, I would like to thank Caroline Rosenbloom for her regular column, "In the Middle." This is Caroline's last issue, as she is taking on other projects this coming year. Caroline has been kind enough to find another dedicated and well-respected mathematics educator, who will keep the torch of "In the Middle" lit. Many thanks to Connie Quadrini for agreeing to assume this responsibility. Congratulations to the new OAME president, Sonia Ellison. The *Gazette* team and *Gazette* reader look forward to your term as president of OAME and to reading your messages in this publication! Happy summer holidays to everyone!

Reference

Gadanidis, G., Sedig, K., & Liang, H.N. (2004). Designing online mathematical investigation. *Journal of Computers in Mathematics and Science Teaching, 23*, 273–296. ▲

▲ **Call for Manuscripts**

The *Ontario Mathematics Gazette* is inviting manuscripts for all grade levels.

Instructions for submission of manuscripts are found on page 1 of any *OMG*.

Contact the Editor for further details.

▲ Letter to the Editor: Are Teachers More Than Statistics?

TIM SIBBALD
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Tim Sibbald is an assistant professor at the Schulich School of Education at Nipissing University and a vice-president of OAME.

Teachers are known to move between schools at all stages of their careers. Some moves are between school boards, but the majority appears to be within school boards. The word "appears" is the first signal that the available statistics may not be representative of the Ontario situation. The U.S. Department of Education (Keigher, 2010) has gathered statistics based on 3.4 million teachers and found that 7.6 percent move each year, somewhat less so in secondary schools than in elementary schools. While reading reports about the interpretation of statistical tables about teacher movement has a high degree of credibility, particularly given the source, there is a lack of detail about the personal side of moving between schools. Some teachers may jest about their willingness to move; however, there is a sense of belonging in a school and a professional purpose that the statistics do not seem to reveal.

Movement between schools as a result of a mandated policy is understandable. This has occurred in Bermuda, but to the best of the author's knowledge, not in Ontario. There were efforts to encourage mixing of employees following amalgamations of school boards many years ago, but they did not go as far as policies obliging teachers to move schools. The amalgamation process created geographically large boards in Ontario (72 altogether) when compared to the United States (Michigan, for example, has 556 districts in spite of a similar population size.). This raises the very real possibility that the U.S. statistics may not properly reflect the movement of teachers in Ontario. The smaller geographical board size increases the chance that a teacher can change boards, without moving his or her home, and this has been noted as a method of increasing pay. There have been some monetary incentives used in some U.S. school districts to influence the movement of teachers, but the practice is rare.

Statistics have pointed to a number of factors that may be involved in teacher movement. However, there is a persistent question of whether the factor caused the movement, was caused by the movement, or if the factor and movement had some unobserved common root cause. Consider, for example, that the rate of movement is higher for at-risk schools. Does this mean that having more at-risk students in the classroom causes teachers to move? It is difficult to reconcile this view with the professional intent most teachers demonstrate to help students. Does a teacher choosing to move cause students (and the school) to become at risk? Given the timelines for transfers and that few teachers do not know they will transfer until late in the school year, it seems very unlikely. Lastly, could it be that there are hidden factors that contribute to both increased numbers of at-risk students and also to teacher movement? There are plausible scenarios, but the statistics do not reveal hidden factors because they were not included. In some cases, such as statistics around influences of the cultural composition of schools, the factor is enormously complex and it is unclear if the U.S. context is a valid source of statistical information that might guide Ontario policies.

What is known, from the U.S., is that there is a higher rate of teacher movement from lower socio-economic schools. The rate is also higher for middle schools and smaller schools. The rate of moving may decline a little with increased career years, but it is not a substantial decline. In terms of career stage, the only finding worth noting is that induction programs for new teachers tend to lower their rate of movement.

An explanation for the various statistical observations has been slow in coming. The most promising theories pertain to career and personal motivations. Examples of the former are movement for leadership opportunities, such as headships, or the quality of relationships between teachers, where one might be inclined to move from a fractious staff, but less inclined to move from a co-operative or encouraging staff. Personal motivations may be as simple as reducing the time it takes to travel from one's house to the school and back, a reduction in commute time that can have a direct impact on time with family. This may be overly simple, however, and it may be that teacher salaries grow in a manner that allows

them, collectively, to migrate to middle-class neighbourhoods and then migrate schools in a manner that would account for the statistical observation of teachers moving from lower socio-economic schools. As the teachers' financial situation improves, they may choose to live, and work, in a neighborhood better matched to their personal status. Consider an Italian finding that teachers in Italy tend to move toward their place of birth (Barbieri, Rossetti, & Sestito, 2011).

The situation is one that is ripe for teaching statistics. Large-scale statistics point to various factors, but are unable to distinguish between correlation and cause. Validity and reliability have to be addressed cautiously because the Ontario situation has unique characteristics making it different from the U.S. and Italy. A qualitative approach can potentially be revealing about the underlying issues that teachers experience as they wrestle with decisions around their movement. This is the approach being taken by an Ontario research project. Teachers who decide to move between schools, in their school board, are being sought and would ideally be available to be interviewed before they move. Further details are available, with no obligations, from the author (tim.sibbald@oame.on.ca). It is hoped that by examining the personal stories, the challenges of teacher movement will be better understood.

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Barbieri, G., Rossetti, C., & Sestito, P. (2011). The determinants of teacher mobility: Evidence using Italian teachers' transfer applications. *Economics of Education Review, 30*, 1430–1444.

Keigher, A. (2010). U.S. Department of Education. National Center for Education Statistics. *Teacher attrition and mobility: Results from the 2008–09 teacher follow-up survey* (NCES 2010-353). Retrieved from **http://nces.ed.gov/pubsearch** ▲

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▲ President's Message

CATHY HALL, PRESIDENT
E-MAIL: cathy.hall@oame.on.ca



As you read this, the school year will be winding down… and so will my term as OAME President. Usually the President's term of office is "bookended" by the Annual General Meeting that is held during OAME's Annual Conference in May. This year we will be changing that tradition due to a constitutional change that will now align our OAME year more closely with the school year. In other words, I've lucked in by grabbing a couple of extra months of wielding the President's gavel!

I know that sounds facetious, but the reality is that my year as President has flown by all too quickly. As I'm sure my predecessors did, I too had a million plans for what I wanted to accomplish this year… and, as I'm sure many of them felt as their terms ended, the months pass all too quickly, leaving us wondering if we've made any impact at all on the math world in Ontario and beyond.

I'm sure many of you are wondering the same thing about the impact you've had on your students this year. The reality of our world as educators is that we are goal setters—the more goals, and the higher we set the bar, the better! When you stop to think about it, you'll realize that we're also some of the best hurdle jumpers in the world, no matter what the obstacle. You only have to think back through the past week, let alone the entire school year, to relive some of those potential detours on the road to success… widely varying student needs, timetabling woes, classroom-management issues, curriculum pressures, ongoing job actions, and lack of networking/PD, to name a few! And yet, you've carried on, determined to make math come alive for your teachers, students, and parents. I want to extend a sincere thank you for your dedication and tenacity during what has been an unusually tough year in many respects… well done!

It takes a lot to keep OAME down! That has been evidenced in the fact that our various regional chapters have maintained the status quo as much as possible during a very uncertain professional climate this past year. They have, for the most part, been able to carry on with their local events (mini-conferences, forums, Math Olympics playdowns, socials). In my own chapter, we have made good on our promise to head out "on the road" with PD events for COMA members and their colleagues in outlying communities. I applaud COMA Council for their creativity and "outside the box" thinking in making this happen! It is my hope that other chapters give this outreach approach a try; I think they will find that the rewards far outweigh any negatives. Not only are we serving the needs of more of our members; we are also raising the visibility of OAME, thereby building our member base and offering another source of quality PD for the province's math educators. I am very excited about some of the plans that are in the works for several chapters this coming fall as they pilot-test a new model for real-time virtual PD in many regional hubs across the province… another in the ever-growing list of online PD opportunities available to you as OAME members!

Of course, this does not diminish the power of face-to-face professional development offerings. One of the best of these events could be found at the OAME Leadership Conference in November. Although registration numbers were lower than usual, those members who were allowed to take advantage of self-directed PD had a chance to celebrate some math success stories from around the province… shared by some of our most respected math leaders, among them Doug Duff, our keynote speaker. It was inspiring to share in his journey as a school administrator instilling a culture of change within his school. By employing his "9 Points of Change," he led his school to impressive successes over the past three years. We also heard from a team of teachers from that same school, who have worked collaboratively to develop innovative teaching and assessment practices for their students. These three individuals—Lisa Cianci, Kevin Gordon, and Bill Nediger—shared their insight into how they and their students had experienced a major paradigm shift in taking ownership of math understanding—and the ensuing success of their "Reaching the Bar" initiative. Conference participants were equally engaged when it came to breakout sessions led by Joyce Tonner (Principal), Robert Cop (Consultant), Liliana Bastianelli (Department Head), and Jennifer Capitani (Instructional Coach). In each of these sessions, the presenters shared their success stories and practical strategies that could be adopted/adapted to bring about similar successes for conference participants. The conference wound up with a Saturday morning session with Connie Quadrini and myself, in which participants drilled down into the depths of the math curriculum from K–12 as they investigated the concept of Equivalence—

how we teach it and how our students come to understand it. All in all, this year's Leadership Conference was a resounding success… I just wish that more people had been able to take advantage of it!

At this time, I would also like to offer a special thank you to some of the behind-the-scenes players who have been there for me and for OAME this year:

- Executive Directors – Fred and Lynda Ferneyhough… Thank you for working tirelessly to ensure that OAME remains the vital professional organization it is. You keep the wheels on the bus turning smoothly!
- Past President – Connie Quadrini… Thank you for paving the way for me, and giving me a road map to follow—and for keeping me from taking too many detours along the way!
- Vice-Presidents – Bill Otto, Paul Alves, Tim Sibbald… Thank you for helping me see the big picture of "the forest and not just the trees"!
- Incoming President, Sonia Ellison… Thank you for your enthusiasm and commitment to "making a difference" for Ontario math teachers. I look forward to helping you make your vision a reality next year!
- OAME Directors and Chapter Reps… Thank you for volunteering your time and for spearheading OAME initiatives within your working committees at our Board meetings throughout the past year. Your expertise and vision serve as OAME's signposts!
- OAME Webmaster – Greg Clarke… Thank you for serving as our IT guru—and for being at our beck and call 24/7 for "OAME math emergencies"!
- The Executive and/or Council of our 15 regional chapters – CHAMP, COMA, GOLDEN, GVMA, ISOMA, MAC^2, NOMA, NWOAME, PRMA, QSLMA, SAME SWOAME, TEAMS, WOMA, Y4MA… Thank you for all the time and effort you devote in working on behalf of your chapter members—and for spearheading and organizing exciting PD events for them!
- Editor of the *Gazette* – Immaculate Kizito Namukasa… Thank you to you and your team, including *Abacus* Editors, Kathy Kybota-Zarivnij and Mary Lou Kestell, for producing such a cutting-edge professional resource!
- QSLMA… Thank you for trying your best to keep OMO alive this year. Unfortunately it was not to be. However, this exciting event will be back next year, hosted by MAC^2!
- TEAMS… Thank you for "OAME 2013: Think Big" — one of the most ambitious and exciting Annual Conferences ever! I'd like to thank Co-Chairs Wayne Erdman and Anthony Meli and their organizing committee, student volunteers, sponsors, and presenters for making this event truly outstanding!
- Financial Literacy Writing Team – Anne Yeager, Project Manager… Thank you to this team of writers for carrying forward OAME's Financial Literacy resources initiative. We look forward to the completion of the resources for the remaining divisions… and to being able to use them with our students and teachers!
- All of our OAME members… Thank you for your ongoing support and dedication in making OAME one of the most vital and well-respected professional organizations in the world! It has been an honour serving as your President!

It's been a challenging year, folks… but we all did our best to "make math happen" and, for that, I think we should give ourselves a well-deserved pat on the back. Enjoy your summer, everyone! ▲

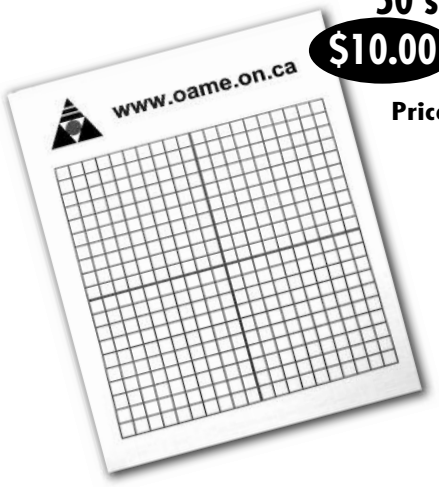
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▲ What's the Problem? Cylinders within Cylinders

SHAWN GODIN
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Shawn is the head of Mathematics, Technology, and Family Studies at Cairine Wilson Secondary School in Orleans, Ontario. He is a past and present member of the OAME Board of Directors and is currently Editor-in-Chief of Crux Mathematicorum with Mathematical Mayhem..

Welcome back, problem solvers. Last time, I left you with the following problem:

> A student has two open-topped cylindrical containers. The larger container has a height of 20 cm, a radius of 6 cm, and contains water to a depth of 17 cm. The smaller container has a height of 18 cm, a radius of 5 cm, and is empty. The student slowly lowers the smaller container into the larger container. As the smaller container is lowered, the water first overflows out of the larger container and then eventually pours into the smaller container. Determine the depth of the water in the smaller container when the smaller container is resting on the bottom of the larger container.

This problem is question #25 from the *2002 Pascal Contest.* The Pascal contest is one of a dozen mathematics competitions administered every year by the Centre for Education in Mathematics and Computing (CEMC) at the University of Waterloo. The University of Waterloo has been creating mathematics competitions for Canadian (and international) students for 50 years. Each year, students from all over Canada and around the world enjoy the challenges presented to them in CEMC contests. For example, this year alone, over 26 000 students wrote the Pascal contest for Grade 9 students, including about 3500 international students. You can check out more contests and other resources at the CEMC website, **http://www.cemc.uwaterloo.ca**.

This problem is nice because, although it was the last problem in the contest, and thus one of the more difficult (only 10 percent of students obtained the correct answer), the math needed isn't that difficult. We will need to use the formula for the volume of a cylinder, but we have to think carefully about how to apply it.

The first thing is to really understand what is happening in the problem. The original problem contained a couple of diagrams, but even then, there are some subtle ideas at work. If you have access to a science lab, it would be worthwhile to perform this experiment to get a sense of what is taking place as the smaller container descends.

If you perform the experiment, you will see that there are three different things that occur:

- First, the smaller container is descending, and the water level in the larger container is rising.
- Next, as the smaller container continues to descend, the water pours out of the larger container.
- Finally, as the top of the smaller container enters the larger container, the water starts to pour into the smaller container.

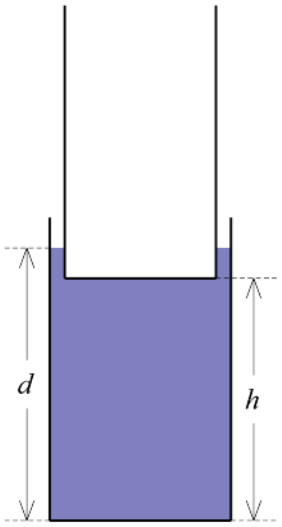
If you gather some data by measuring the depth of the water in the larger container (dependent variable) and the height of the bottom of the smaller container (independent variable), the result is a linear relation.

We can demonstrate the linear relation algebraically. We will let d represent the depth of the water, and h the height of the bottom of the small container, both measured in centimetres. The water is pushed into the shape of a cylinder with a cylindrical indentation in the top (from the smaller container), thus

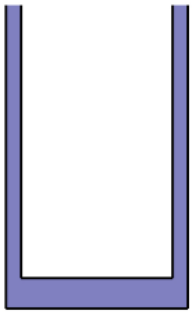
$$\pi(6)^2 d - \pi(5)^2 (d - h) = \pi(6)^2(17)$$

which simplifies to $d = \frac{25}{11}h + \frac{612}{11}$.

This relation is only true for $17 \leq d \leq 20$, which corresponds to $17 \geq h \geq 15.68$. Although this is an interesting relationship, possibly worth pursuing as an example of a linear relationship connected to volumes of cylinders, it doesn't really come into play while solving the problem.



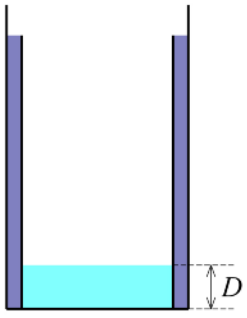
When the water reaches the top of the larger container, it starts to spill out as the smaller container continues to descend. The next important event happens when the tops of the two containers are at the same level. From this point forward, the water will pour into the smaller container.



At this point, we need to calculate the volume of water remaining in the larger container. Using the same ideas as above, we will obtain the volume of water remaining, in cubic centimetres, which is

$$\pi(6)^2(20) - \pi(5)^2(18) = 270\pi$$

Finally, as the smaller container continues to descend the water pours into it until it is at the bottom of the larger container. The diagram below illustrates the result.



Letting D represent the final depth of the water in the small container, we get

$$\pi(6)^2(18) - \pi(5)^2(18) + \pi(5)^2D = 270\pi$$

which, if we solve for D, gives us the correct depth: 2.88 cm.

The official solution describes a nice interpretation that allows you to reach the answer more quickly. I suggest that you check it out, as alternate solutions tend to provide insights into the problem.

Here is another alternate solution that uses some other ideas. If we follow the same thinking as above, we know that we will arrive at the second diagram before the water starts to flow into the smaller container. As we continue to push the smaller container down, the 2 cm thick "disc" of water below the smaller container must have the same volume as the water that ends up inside the smaller container (can you see why?).

Since the volume of a cylinder is directly proportional to the area of its base and also directly proportional to its height, then if we take the same volume, but change the area of the base from 36π square centimetres to 25π square centimetres, we have changed the area of the base by a factor of $\frac{25}{36}$. Thus, we must change the height by a factor of $\frac{36}{25}$ so that the volume remains unchanged. Thus the new height is $\frac{36}{25} \times 2 = 2.88$ centimetres, as we discovered previously.

If you do not have access to a lab to experiment, you may do so with linking cubes and dealing with square-based prisms rather than cylinders. Here is a similar problem that can be done with linking cubes. The larger container is a 4 x 4 x 10 square-based prism, and the smaller container is a 2 x 2 x 9 square-based prism. The level in the larger container was 9 units deep at the start. If you go through the process, the level in the small container at the end should be 4 units.

Now for your homework:

Calculate: $6^2 - 5^2$, $56^2 - 45^2$, and $556^2 - 445^2$. Find and describe the pattern in the questions and answers and prove that the pattern continues. As always, see if you can find other similar patterns.

Until next time, have a great summer and happy problem solving! ▲

▲ Technology Corner – Visual Patterns

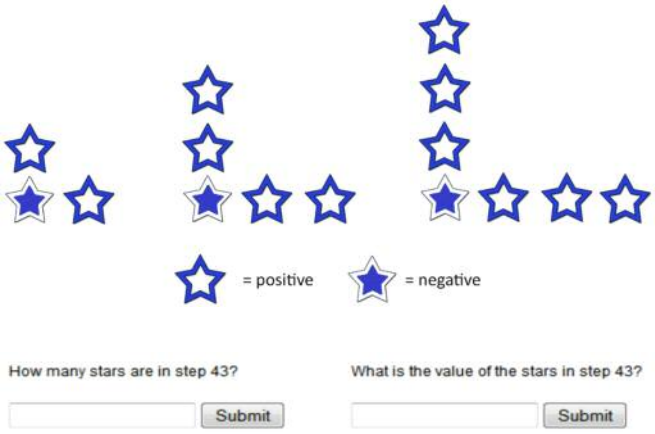
MARY BOURASSA
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Mary teaches mathematics at West Carleton Secondary School in Ottawa. She is a strong advocate for the appropriate use of technology in the classroom. She has presented workshops internationally, authored mathematics resources, is a past VP of OAME and a Past President of COMA. An award-winning teacher, Mary continually strives to learn new and better ways of helping students learn and love mathematics.

Fawn Nguyen (@fawnnguyen) started a marvelous website featuring all kinds of visual patterns (**http://visualpatterns.org/**). She posted many of these patterns herself, but also allows others, including students, to submit their own. The 97 patterns currently on the website include many linear patterns, as well as quadratic, cubic, and exponential patterns. There are also input-output machines, and the latest patterns are three-dimensional and can be rotated—very cool! Here are a few of the patterns that you will find there.

Pattern #11 (The representation has been modified, as the original has red and blue stars that are difficult to differentiate in print.)

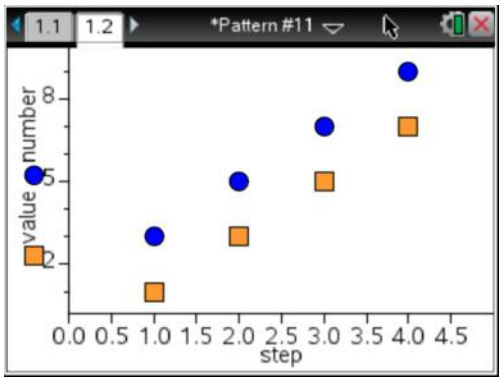


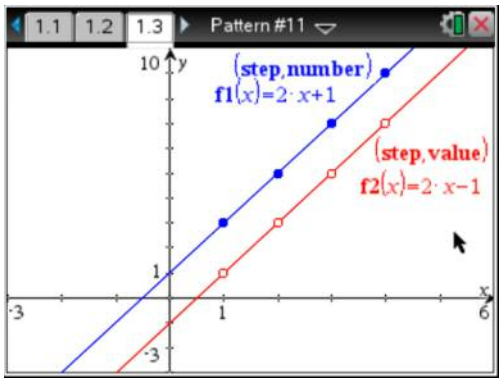
Students can create T-charts or tables of values to represent the number of stars and their value at each of the initial steps. They can then determine the patterns. It may be useful for students to draw the next few patterns, as the process can help them better understand how it is changing from one step to the next.

Step	# of Stars	Value of Stars
1	3	2 – 1 = 1
2	5	4 – 1 = 3
3	7	6 – 1 = 5
4	9	8 – 1 = 6
…	…	…
n	**2*n* + 1**	**2*n* – 1**
43	2(43) + 1 = 87	2(43) – 1 = 85

Once students have the pattern, they can check it online by submitting their answer. If it is incorrect, they will be prompted to try again. If they are correct, a window will pop up with the pattern.

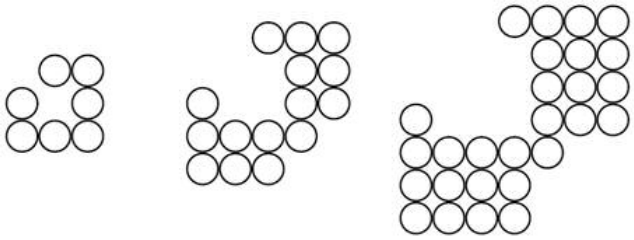




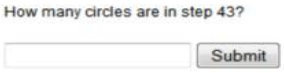


It would be interesting to have students determine

what step 0 would look like. This is very relevant in Grade 9, when students are learning about the slope-intercept form of a linear equation. Could you continue this pattern with negative values for n? If so, what would it look like?



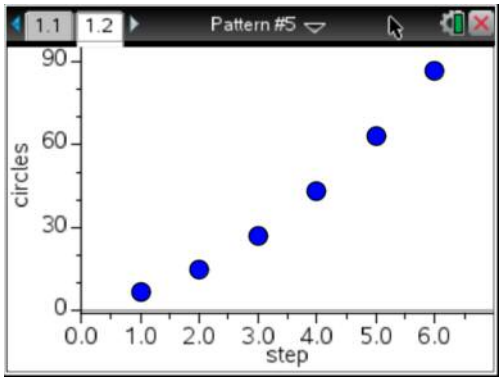
Pattern #5

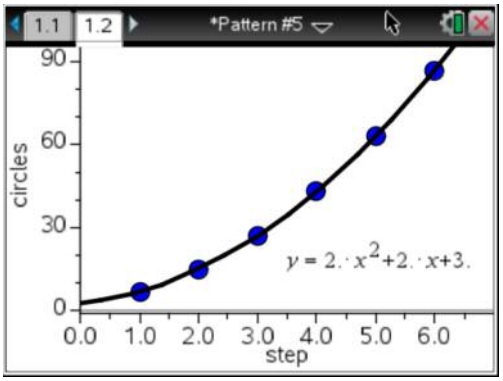


Students can make a table to record the number of circles at each step. They can then work on finding the pattern: let n be the step and C be the number of circles, so $C = 2(n + 1)(n) + 3$. Once students find this pattern, they can check that it works for the next few steps, which also gives them more data for the table. They can then determine the type of relationship by looking at the first and second differences. As the latter are constant, this pattern is quadratic.

Step	1	2	3	4	5	6
# of Circles	7	15	27	43	63	87
1st Differences	—	8	12	16	20	24
2nd Differences	—	—	4	4	4	4

They can plot the data and perform a quadratic regression to get the equation of the curve.





How does this compare to the equation found from the data? Graphing both equations will demonstrate that they are identical, or you can expand the first equation and compare the results:

$$\begin{aligned} y &= 2(x + 1)(x) + 3 \\ &= 2(x^2 + x) + 3 \\ &= 2x^2 + 2x + 3 \end{aligned}$$

They are equivalent. It's important for students to see the same equation in different forms.

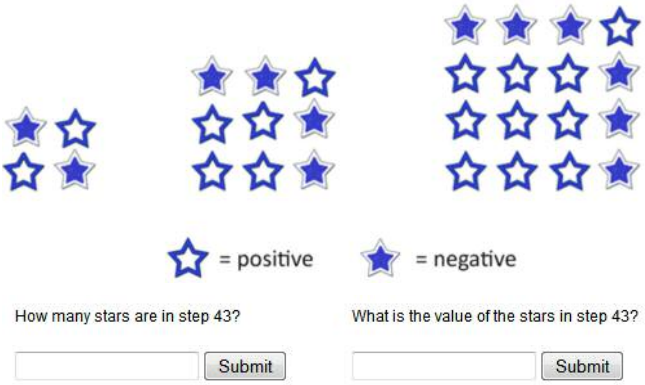
We can now determine the number of circles at step 43:

$$\begin{aligned} y &= 2(43)^2 + 2(43) + 3 \\ &= 3787 \end{aligned}$$



Notice that the pattern shown when the correct answer is submitted is in the same form as the quadratic regression.

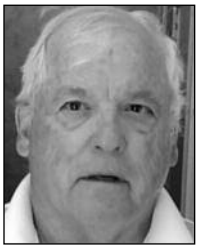
I will leave you with pattern #12, below, which I really like. The original has light and dark green diamonds that I have changed to stars for better visibility. You'll have to go to the visual patterns website to verify that you found the correct pattern!



▲

▲ OAME/NCTM Report March 2013

TODD ROMIENS
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Todd recently retired from teaching K–12 methodology in the Faculty of Education at the University of Windsor. He is a Past President of OAME and a life member, as well as a previous editor of the Gazette*.*

The NCTM "smart brief," which was launched a while ago, is a free line to many different aspects of mathematics education around the world. Each issue gives a précis of different projects and gives links to how you can learn more about them. Here's one that sparked my interest:

"Should Teachers Group Students by Ability?"

Greg Toppo, in his March 18, 2013 *USA Today* article, "More teachers are grouping kids by ability," he noted that "new findings on more than 20 years of research suggest that despite decades of controversy, elementary school teachers now feel fine placing students into ability groups." In the United States, grouping was frowned upon, as it was considered to be discriminatory. While this is an issue, it seems more teachers are finding that the better results achieved make it worthwhile. Critics say that the practice in which students are grouped by ability within a specific class is a human rights issue because it creates lower expectations, while supporters say it helps children improve their skills.

How do you feel? With ability groups somewhat prevalent in high school, with academic and applied courses, perhaps the need has been met to some extent. However, as we all know, these groupings can be far from homogeneous.

Elementary school teachers face a far different situation. There is very little attempt, if any, to group students by ability. On the contrary, the Grade 7 class that I teach on a regular basis has five fairly good mathematics students, ten who generally meet expectations, three with special needs and are on individual programs, and four who do not attend very regularly. The class has 33 students, so you can see the potential problems. Add to this that many do no homework and have little support at home. Also, it is very rare for a student to be held back a grade, no matter what level of achievement he or she has attained, so often their backgrounds coming in are weak.

Perhaps another look at ability grouping should be considered?

Another issue that seems to be getting some new attention is the use of a calculator in the early grades. This topic has been controversial since calculators were made available decades ago. A math specialist in Florida, named Pat Willard, "warns against parents and educators using calculators in the early grades, as they can interfere with students' understanding of fundamental concepts." I have always felt that calculators can be a valuable tool to help students understand concepts as opposed to replacing them. A close look at the Ontario curriculum document for elementary school shows many expectations for the use of calculators in the early grades; however, none of these expectations are about "computation." I believe this is where the problem issues arise. The curriculum expectations where calculators are concerned use the words "explore," "solve problems," and use "appropriate strategies" where calculators are to be used. The word "compute" does not come up.

An important question that should be asked when students use a calculator for computation is, "How do you know the answer on the calculator is correct?" This should lead to more time spent on estimation skills, a topic that does not get its fair share of time in elementary school.

Most of the research involving student use of calculators reveals that they are excellent for exploring number patterns, promoting mathematical ideas about prediction, estimation, and the introduction of decimals. They are also a great help in teaching percents and fractions. In 1991, NCTM recommended that every teacher at every level promote the use of calculators to enhance mathematics instruction by:

- using calculators in computation (large and ugly numbers), problem solving, concept development, pattern recognition, data analysis, and graphing
- keeping current with the state-of-the-art technology appropriate for the grade levels being taught
- exploring and developing new ways to use calculators to support instruction and assessment

The NCTM Curriculum and Evaluation Standards (1989) also recommended that all students use

calculators to:

- explore and experiment with mathematical ideas such as patterns, numerical and algebraic properties
- develop and reinforce skills such as estimation, graphing, and analyzing data
- focus on developing problem-solving processes rather than the computations associated with the problems
- perform tedious computations that often develop when working with real data in problem situations
- gain access to mathematical ideas and experiences that go beyond those levels limited by traditional paper-and-pencil computation.

So why is this still an issue? ▲

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▲ Hey, It's Elementary – I'm Diggin' Those Calculations: What Historians Have Unearthed about Calculating Devices through the Ages

LYNDA COLGAN
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Lynda Colgan's career has included roles as a classroom teacher, a university professor, and newspaper columnist. Her contributions to mathematics and its teaching have been recognized through awards such as the Marshall McLuhan Foundation Distinguished Teacher Award. Lynda always exhibits a passion for mathematics and views her professional mission as dispelling the myth that math is the bad guy.

> *It is unworthy for excellent men to lose hours like slaves in the labour of calculation, which could safely be relegated to anyone else if machines were used.*
>
> ~ Gottfried Wilhelm Leibniz (1646–1716)

By the end of the one-year concurrent education program here at Queen's, many primary/junior teacher candidates are wondering how to put the pieces of the puzzle together with respect to the elementary curriculum. Having experienced Mathematics, Language, Science & Technology, Social Studies, Health and Physical Education, and The Arts as distinct curriculum courses, early career teachers are left wondering how best to integrate the curriculum without compromising the integrity of the content, while simultaneously respecting subject-specific pedagogical practices.

In an effort to support these early career teachers and provide scalable teaching units and lesson plans, I often provide professional learning opportunities such as *Mathematics and Visual Art* (an exploration of the wonders of da Vinci's anthropometric measures, the golden ratio, and the rule of thirds); *Mathematics and Science* (an examination of light, mirrors, and reflections that culminates in the construction of a kaleidoscope);

Mathematics and Music (a workshop about Mozart's *Dice Game*, fractions, and transformational geometry); *Mathematics and Social Studies* (exploring quilt codes from the Underground Railroad, using 9-square and 16-square grids and transformations); and *Mathematics and Drama* (performing *Mathakazam!* and *The 3-D Little Pigs*).

This year, in an effort to inspire beginning teachers to look for connections that are fresh, engaging, purposeful, and fun, I designed a workshop that investigated the aids to calculation through the ages. How did accountants in ancient Egypt tally up totals? What about medieval merchants: how did European traders communicate with the sellers of silks and spices from the Far East? What about Mr. Scrooge: in Victorian times, what tools did crotchety, old Ebenezer use to torment his debtors?

While it is beyond the scope of this article to recreate the entire workshop by presenting a thorough and comprehensive examination of the many tools that historians and mathematicians have placed along millennia-long timelines, what follows is a crash course in some interested highlights from my session about the history of calculators—tools designed to record calculations or expedite them.

Webster's New Twentieth Century Unabridged Dictionary includes the following definition for a word that strikes fear into many a heart: *calculus*.

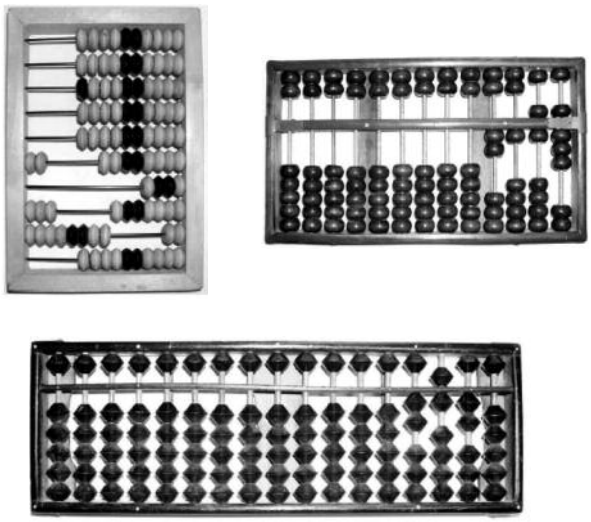
Calculus: n; pl. calculi, calculuses

(Latin, calculus, a small stone or pebble used in reckoning; diminutive of calx, calcis—a stone)

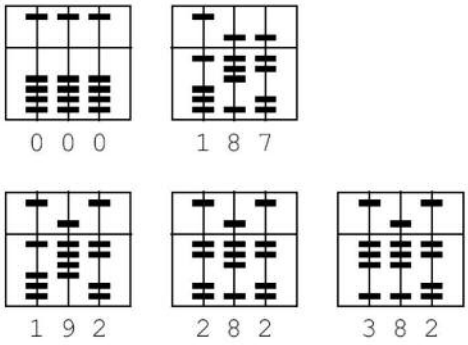
1. *A small stone or pebble*
2. *Any hard, solid concentration or deposit or any part of the body…*
3. *In higher mathematics, (a) a method of calculation; (b) the use of symbols: (c) a method of analysis; …*

Computational aids began with the use of pebbles, and then beads. As early as 3000 BC in China, the beads were strung onto wires; thus was born the first abacus.

There are many kinds of abaci around the world. The beads on the Russian abacus (top illustration) are set to represent 2.38. The beads on the Chinese *Suan Pan* (middle illustration) are set to 2168. The Japanese *Soroban* (third illustration) shows 2620.



Abaci are not restricted to representing numbers or for counting, however. They are powerful computational devices that can be used to aid in calculation. This schematic diagram shows how the beads of the *soroban* are manipulated to add 187 and 195 [add 10, subtract 5; add 100, subtract 10; and, add 100].



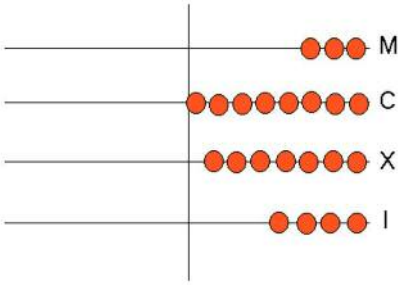
Around 1200 AD, in Italy, medieval counting boards burst onto the scene. It is believed that these boards gave rise to "a counting."

The picture below shows a competition between the counting board and arithmetic done by paper and quill arithmetic. It is from G. Reisch's Margarita Philosophica (1508). The counting board was a simple device for extending the method of adding piles of pebbles to larger numbers, without having to use too many pebbles. The idea is that pebbles on one line stand for just what they are, one pebble, but pebbles on other lines stand for different numbers of pebbles, 10, 100, 1000 pebbles.

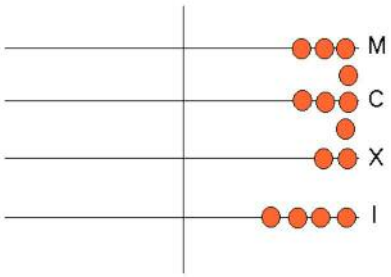
Roman numerals were used with the boards; therefore, the labels on the diagram below represent particular symbols in that number system. "I" represents ones, "X" represents tens, "C" represents hundreds, and "M" represents thousands.



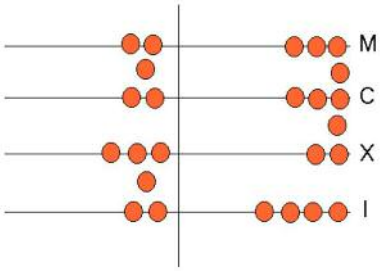
The number shown on the board below is 3874.



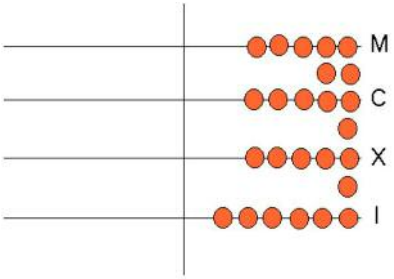
It seems that medieval accountants got tired of needing so many pebbles, and adapted the boards to better suit their practical purposes. By using the spaces between the lines to represent five times the value of the line below, it was possible to use the counting boards more efficiently. For example, a counter placed between the "I" and the "X" lines would represent V (5); a counter between the "X" and the "C" lines would represent L (50); a counter between the "C" and the "M" lines would represent D (500); and a counter above the "M" line would represent MMMMM (5000).



To add MMDCCXXXVII (2737) to MMMDCCCLXXIIII (3874)—the number represented above—you would first represent the other number, MMDCCXXXVII (2737), on the other side of the counting board, like this:

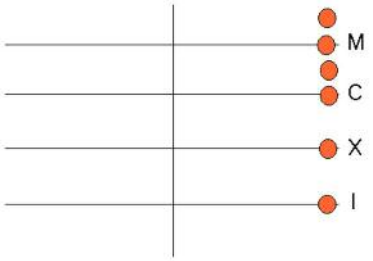


Then you push the counters from the left side over to the right side so that they are all together to represent the answer to the addition problem.



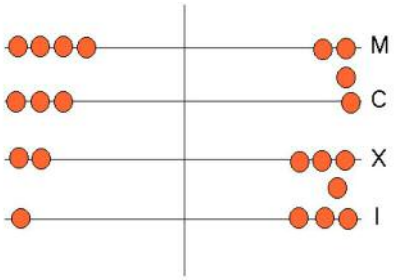
Now the challenge is to express the number, using as few counters as possible. If you look at the "I" row, you'll notice that there are 6 or 5+1 counters, so you remove 5 "I" counters and add 1 "V" counter to the row above. Now you have 2 five counters or 1 ten, so you remove the 5 counters and add one to the "X" row. That gives you 6 counters in the 10 row, so you remove 5 counters and add one 50 counter to the row above. That now gives you 2 counters in the 50 row, which is equal is 100, so you add a counter to the "C" row. The result of this rearrangement is that there are 6 counters in the "C" row, which can be rewritten as one 500 and one 100. Five hundreds are removed from the "C" row and one 500 counter is added to the 500 row. This produces 3 five-hundred counters, which is the same as 1 one-thousand counter plus 1 five-hundred counter. Thus, 2 five-hundred counters are replaced with 1 one-thousand

counter in the "M" row, giving up 6 one-thousand counters. We can still reduce the number of counters by removing 5 thousands counters and replacing them with 1 five-thousand counter above the "M" row.

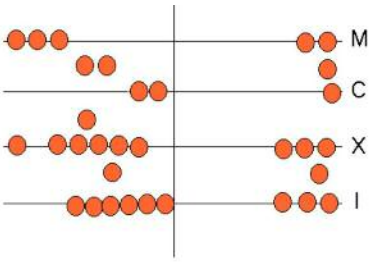


The answer? 5000 + 1000 + 500 + 100 + 10 + 1 or 6611.

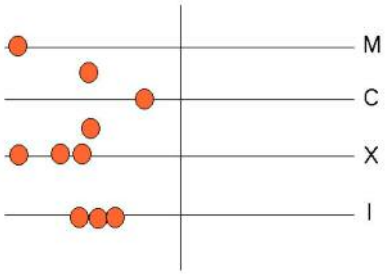
You can also subtract, using the counting board. To subtract 2638 from 4321, for example, set up the counters this way on the board.



Using the same principle, we must prepare the board so that there are enough counters on the left-hand side to allow us to match up counters on the right side and subtract by taking away the same number of counters from each side of the board. This means that we rearrange the counters on the left to give us another configuration that represents 4321. In this case, we have 3000 + 1000 + 200 + 50 + 60 + 5 + 6.



Now the counting board is ready for you to do the subtraction by removing counters in pairs from the two sides until there is nothing left on the right side. What is left on the left side will be your answer.



The final answer is 1683 (1000 + 500 + 100 + 50 + 30 + 3).

Around the same era, finger mathematics, which became very prominent and highly sophisticated methods for representing numbers, large and small, and performing calculations, were popular and widely used. The long period in which "finger reckoning" was popular was followed by another important epoch in which counters and beads were relegated to the past and mechanical tools rose to prominence. One of the most revolutionary tools of this age were Napier's Bones (invented by Scots mathematician James Napier): small rods carved from real bones, wood, ivory, and horn, and engraved with multiplication tables. The bones were efficient and effective tools for multi-digit multiplication and division, and there were sets for computing square and cube roots.

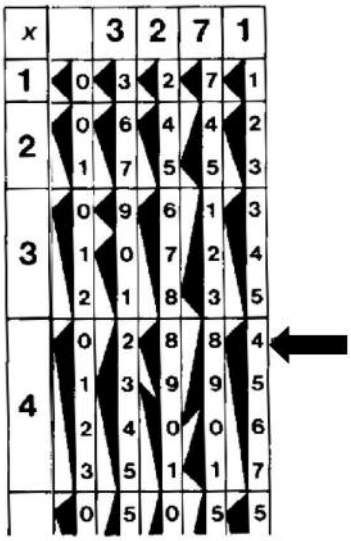




One problem with Napier's bones was that the user is required to perform a number of additions and must keep track of the "carries" that occur during these additions. In the late 1800s, a Frenchman named Henri Genaille invented an improved version of Napier's bones in response to a problem posed by Edouard Lucas. The Genaille-Lucas rulers allow for multiplication and division, without the regrouping required by Napier's bones.

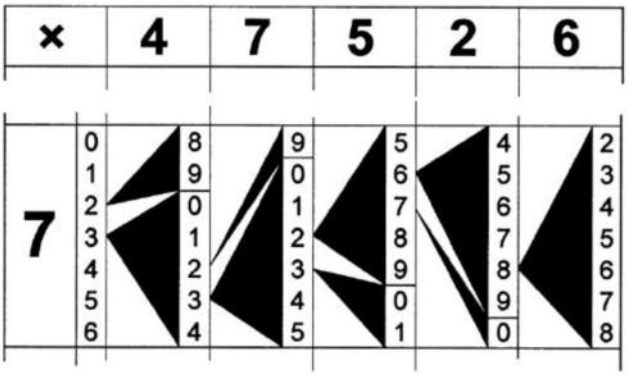
You'll need to cut out a set of the 11 rulers (one index ruler and one for each digit 0 through 9), by cutting the template at the conclusion of this column into vertical strips. Once the rulers are cut, you're ready to do foolproof multiplication just by following the arrows.

To multiply 3271 by 4, select the rulers for 3, 2, 7, and 1, placing them to the right of the index ruler. Then start at the right-hand side, top digit, of the 4th row, and read off the product by following the arrows to the left.



3271 X 4 = 13084

To multiply 7 x 47526, select the rulers for 4, 7, 5, 2, and 6, place them in order to the right of the index ruler, and then follow the arrows from right to left to find the product: 332 682. Note that in the diagram below, only the 7 row is shown.

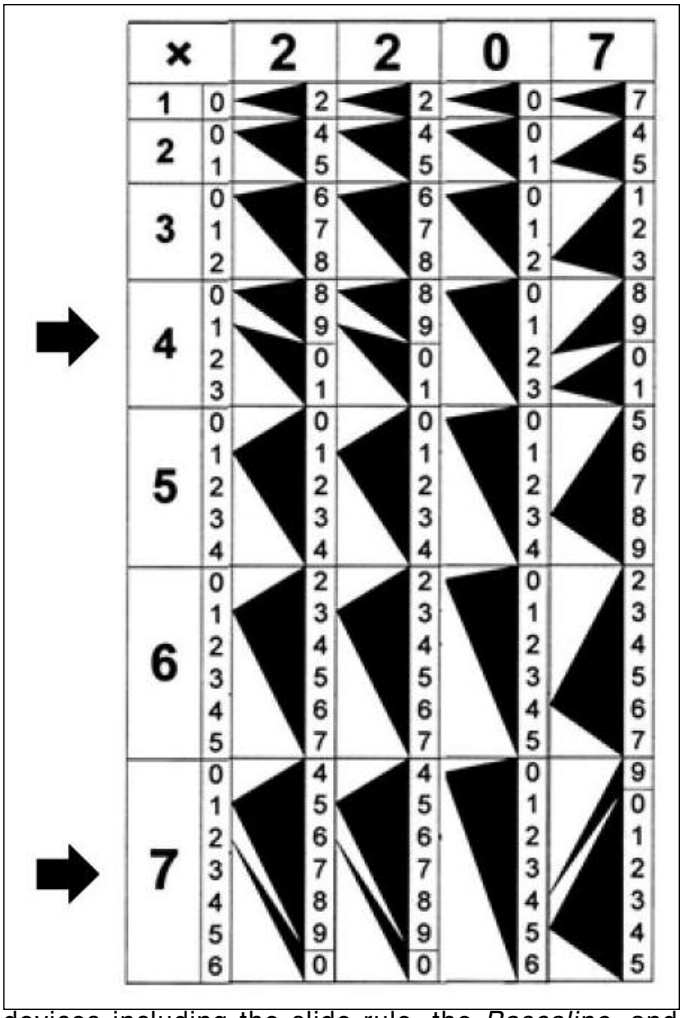


To do a multi-digit multiplication with these magical multiplying rulers, for example, 47 x 2207, begin by selecting the 2, 2, 0, and 7 rulers and placing them to the right of the index ruler.

Read from the fourth row, we get 08828 (by following the arrows from right to left), and for the seventh row, 15449. We must remember that the "4" in 47 really represents 40, so we multiply 8828 by 10 to get 88280.

Add 15449 + 88280 to get the final product: 103 729.

What followed this phase of simple mechanical tools was a time of development of intricate mechanical devices including the slide rule, the *Pascaline*, and Liebniz *Stepped Drum Calculator*, none of which were particularly portable. In fact, the slide rule remained the computational tool of choice until the advent of the handheld electronic calculator in the 1970s.



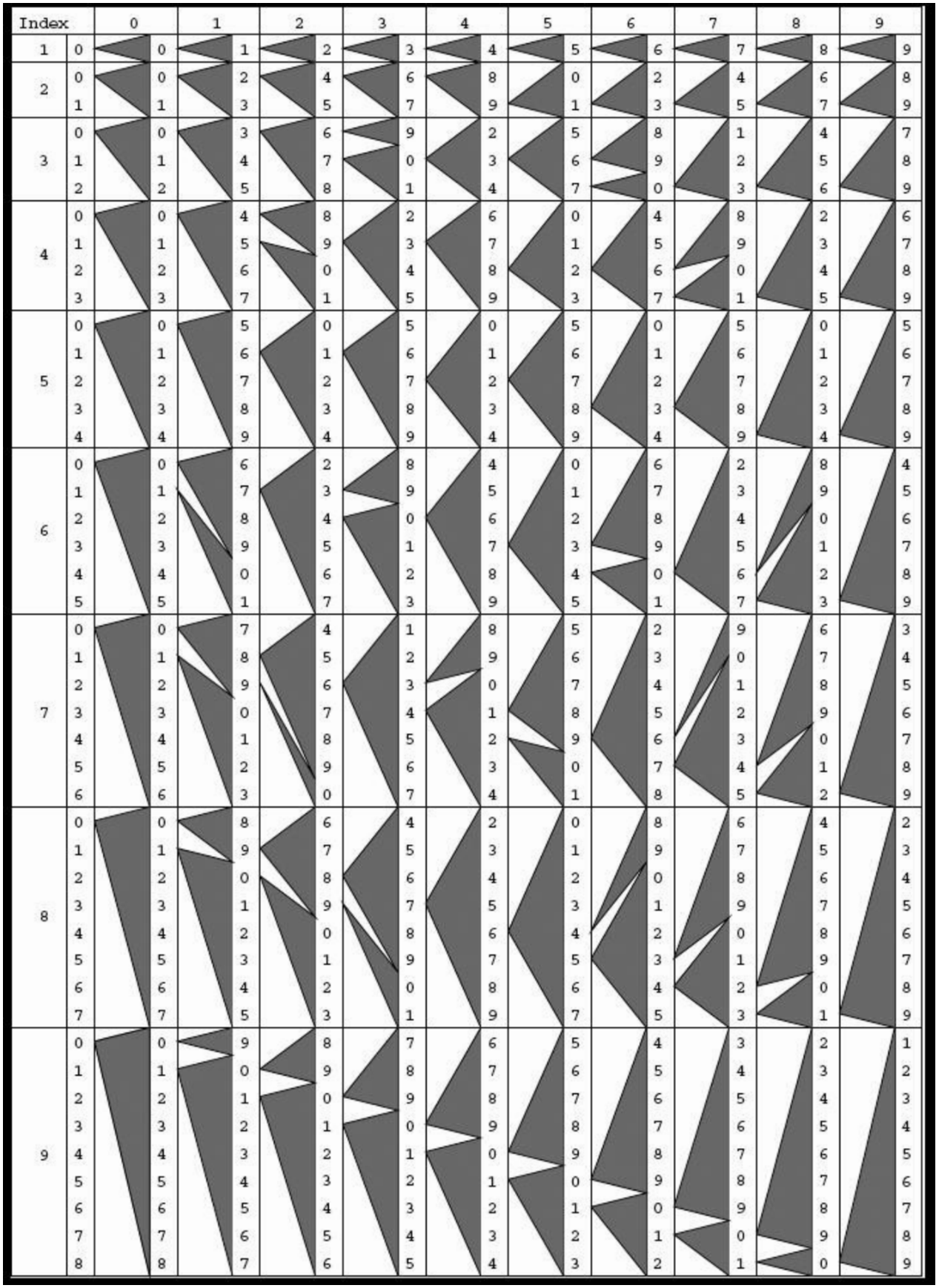
The teacher candidates' reactions to this new workshop were positive. They were encouraged that there were fascinating historic tools that could put a little fun into the *Number Sense and Numeration* strand. As for me, I am delighted that some elementary students may catch the computation bug by learning to add on an abacus, subtract with beads, and multiply with funny-looking rulers… learning and laughing all the way!

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Index
0
1
2
3
4
5
6
7
8
9

▲ MB4T... Always a New Epiphany: From Imperfect Understandings to New Insight

ANN KAJANDER
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Ann Kajander is an experienced classroom teacher currently teaching mathematics and mathematics education at Lakehead University. Her research interests relate to teachers' enhanced learning of mathematical concepts. She and her classroom-teacher colleague, Tom Boland, are currently writing a new book for teachers called Math Without Memorization: Big Ideas in Modelling *and* Reasoning for Elementary Teachers and Other Learners.

As the school year draws to a close, many of us may be wondering what impression we have made on our students during the year, and whether their understandings are sufficient to carry them into the next school year. We might wonder if they will be able to use the concepts from this year to reason further mathematically. In this column, the mathematical process of reasoning, the relationship of reasoning to incomplete understanding, and in particular, how reasoning may develop unexpectedly over time, will be explored initially. Look for a more detailed follow-up discussion of reasoning and proof next year!

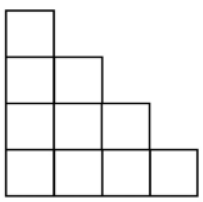
Recently I have been struggling with the ideas of reasoning and proof for our forthcoming (Kajander & Boland) mathematics book for elementary teachers, and wondering "just how much" to say about mathematical reasoning, formal proof, and rigorous arguments. As part of this work, I have been thinking about elementary students'—and their teachers'—mathematical reasoning. In Zack and Reid's (2003; 2004) two-part article about Grade 5 students exploring formulae for pyramids within rectangular prisms, or at least approximations of these with cubes, we are reminded just how much mathematics some young children are indeed able to grapple with. In particular, Zack and Reid talk about "good enough understanding," and in doing so, they remind us of two critically important ideas in mathematics learning. First, the authors make the point that there are often "messy,... tentative and unfinished" (2004, p. 28) components in learning. They further provide examples of students who, after much time had passed and no further intervention, had developed "deep and stable understanding" (2004, p. 28). This is an interesting thought as the school year draws to a close, which might prompt us to wonder whether our students will in fact continue to develop mathematically during the holidays.

In this current column, I want to share a personal mathematical epiphany, as connected to Zack and Reid's (2003; 2004) wonderful work about encouraging challenging reasoning in elementary school. The point of my example is not to necessarily shed light on a particular mathematical idea, but rather, to make the point that no matter how familiar an interesting problem might be, there may always be a new mathematical insight and connection to be made. This idea of ongoing insight relates to Zack and Reid's work in that they remind us that it is not always necessary—and perhaps even impossible—to "finish" any given problem, and that the mind may continue, either consciously or subconsciously, to strengthen our understanding over time.

To use an example, I have been using the "staircase" problem (Kajander, 2007) for many years with pre-service teachers. This problem involves constructing a "staircase" from tiles or cubes, to represent the sum 1 + 2 + 3 + 4 + 5 .... *n*. It is the "sum of the first n natural numbers" problem. In high school mathematics, the method for solving this problem is presented algebraically (for example, by adding the expression twice, once in ascending order, and once in descending order, so that we have *n* sums of (*n* + 1), which must then be divided by 2), which provides the formula for a simple arithmetic series. But what I particularly like about the geometric solution to this problem is that there always seems to be another solution method! If you are not familiar with this problem, you might want to try it yourself first. Here is the problem using a diagram for four steps.

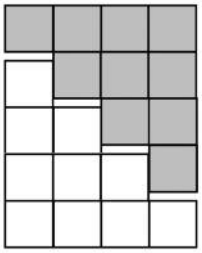
The Staircase Problem

The problem asks us to construct a series of staircases out of tiles or cubes, and look for a way to count the tiles based on the picture number. Here is the fourth staircase (*n* = 4). (The third staircase has only three "steps," and so on.)



One possible solution method for counting the tiles involves creating a duplicate staircase and fitting it (upside down) against the original one. Of course, this doubles the number of tiles, but that can be addressed later.

So here is the fourth staircase "doubled."



Making this same doubling construction with other sizes of staircases allows us to see the connection between the picture number of the staircase pattern (which coincides with the width of the base), and the number of tiles in the rectangle formed from two staircases. If the base of this rectangle is the picture number (say n), then in the rectangle, the height is *one more* than the picture number, or $n + 1$. So the area of the rectangle is $n \times (n + 1)$. To bring us back to the original staircase, we need only divide by 2. A number of geometric solutions are also possible (see Kajander, 2007), and I am continually surprised at the variety of new ones students develop.

While theoretically the staircase problem is a relatively challenging problem suitable for intermediate-level or older students, I have found younger students have also had some success when engaging with it, particularly if we use specific values for the picture number. Using this problem as an example, Zack and Reid's (2004) description of "good enough" understanding provides a way to approach ideas that may be relatively mathematically challenging.

You may have seen the two previous "Math 4 and By Teachers" columns, which illustrated instances in which challenging ideas, the intuitive notions of the concepts of infinity and mathematical limits, crept into topics in the elementary curriculum. And yet, one might argue, such topics are much too advanced for elementary students! The need to draw upon relatively more sophisticated mathematical notions does sometimes occur in mathematics learning.

Perhaps it was the juxtaposition of the ideas from these previous columns about the notion of limits—combined with the work I was doing with pre-service teachers with the staircase problem—all of which was happening in parallel to reading Zack and Reid's articles (2003; 2004), that prompted me to think about the staircase problem differently. A mental image flashed into my mind in thinking about the staircase problem, of the size of the tiles representing the number of steps changing. If the tiles used in the model were getting smaller and smaller, while the number of tiles (steps) was growing, we can imagine the model changing. Take a moment to construct this mental image... what happens to the staircase as this process continues? I suddenly had a mental image of the mathematical *limit* of larger and larger numbers of these smaller and smaller tiles, being exactly the triangle area—and yielding the triangle area formula (half of the base multiplied by the height)! I realized that eventually, when the tiles get small enough and there are enough of them, the staircase problem becomes the triangle area problem!

While this personal epiphany may not be profound, it was a fun mathematical moment for me, given that the staircase problem is a problem I have worked with for years. So as our year draws to a close, let us all take Zack and Reid's message to heart. It is helpful to provide students with rich, challenging problems, and to remember that a "good enough" or incomplete understanding is important and developmentally helpful. It is not necessary to "know" everything about a given problem right away—perhaps it is not even possible. And lastly, let us be reminded that our students will continue to develop mathematically over the holidays, even without our intervention, when they are provided with problems that are worth mulling about.

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▲ In the Middle – What Is So Special about Numbers?

CAROLINE ROSENBLOOM
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Caroline Rosenbloom is currently on secondment as an Elementary Coordinator in the J/I Midtown Technology Option and Mathematics Instructor in the pre-service program at OISE/UT. She has enjoyed working with intermediate students and teachers as a middle school teacher, Mathematics Instructional Leader in the Toronto District School Board, and Education Officer at EQAO. Caroline is co-author of Investigating Mathematics Using Polydron.

Sometimes the most interesting finds in math are the ones we see every day. When they are part of our lives, numbers become more meaningful. We use them to calculate, to help us mark time, and to conceptualize, model, and explain relationships. But math can also be interesting in the abstract, just on its own.

The **Mathematical Association of America's NumberADay** blog is a wonderful site that identifies and explains interesting properties of numbers. For example, here are some interesting facts about the number 88:

88 = 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11 + 12 + 13.

88 is the sixth number that stays the same when written upside down.

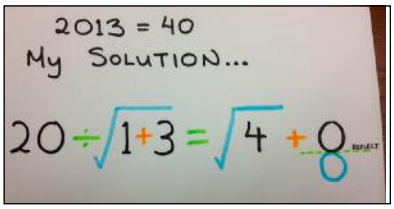
88 is the smallest number such that its square has all its digits twice: $\mathbf{88}^2 = 7744$.



Number Gossip is also another site devoted to numbers and their properties. Did you know that 6 is the only number (except 1) such that the sum of all the primes up to 6 equals the sum of all the composite numbers up to 6 (inclusive)?

What's special about 2013? Though it may not be such an interesting number, Math Palette has identified some interesting facts—it's odd, composite, and its prime factorization is 3 x 11 x 61.

OAME's Annual Conference even ran a math contest for delegates to promote the 40th annual conference in 2013. The "What's up with 2013 = 40?" contest awarded motivated math buffs with free OAME registrations.



Neil Casey, Teacher Candidate in the Midtown J/I Technology Option at OISE/University of Toronto, was a lucky winner, who creatively saw patterns in numbers.

Numbers in Music

What does pi sounds like? Musician Michael John Blake created a mathematical symphony, using pi. He decided the song would be in C, then assigned each note a number: C = 1, D = 2, and so on up through 9. Using those assignments, he played the sequence of pi's digits: 3.14159 through 31 decimal places at 157 beats per minute (314 divided by 2). He played this part on several instruments, and layered them to make a song. You can find his pi song on YouTube.

You can also find *Fibonacci in Music* in Fibonacci Jamming on YouTube. Pretty cool!

So numbers can be found in places we don't expect. I guess that's why investigating them is so interesting. Whether we see numbers through **Number Photography** on Redbubble or Numbers in Nature, math can be seen as beautiful and interesting in its natural form. A great article to read is *Ottawa's hidden math scene: Revealing the numbers behind the city's art and culture*, to see how math is integrated in our world.

Though this will be my last article for *In the Middle*, I continue to look for mathematics around our world and hope you will too! Feel free to follow my blog at **www.mathplay.net**. Together we can continue to find the things that make mathematics fascinating to us.

Mathematically yours, Caroline

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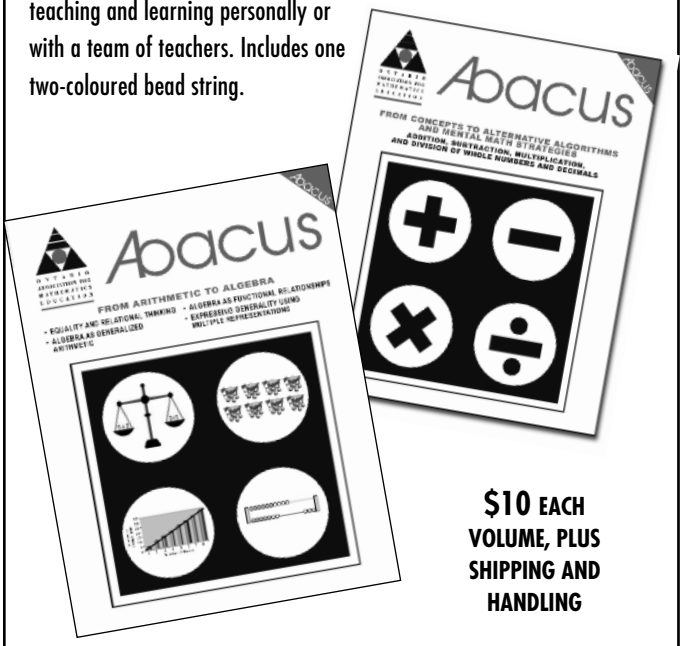
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ABACUS: A COMPILATION

Volume 46 – This resource for teachers includes the concepts of addition, subtraction, multiplication, and division of whole numbers and decimals, alternative algorithms, mental math strategies, manipulatives, and links to literature. It will be useful for exploring these aspects of teaching and learning personally or with a team of teachers. Includes one two-coloured bead string.



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▲ FIELDS INSTITUTE – MATHED FORUM NOTES

STEWART CRAVEN
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Stewart Craven presently teaches courses for the Continuing Education Department and the Transition Years Program at York University. He is an active participant in the Fields Mathematics Education Forum.

Our last meeting of the 2012–2013 school year was held on April 27. The general theme of the session was, "What does math look like outside the classroom?" The following are descriptions of the talks as they appeared in the agenda:

What Happens When Math Gets Home?

Jacqui Strachan (Director of Outreach and Parent Support, People for Education)

Jacqui Strachan talked about her personal experiences with the math her boys brought home from school, and provided her perspective as a member of a parent advocacy group. The presentation will comprise a short talk as well as a Q and A opportunity, during which Forum members asked questions about mathematics in Ontario from "a parent's eye view."

Learning on a Continuum

Carol Carruthers (professor of Mathematics and a coordinator of the Applied Science and Technology Program at Seneca College)

Eight years ago, this educator gave traditional lectures, with students furiously copying notes and then individually practising questions from the textbook in an effort to understand concepts. Fast-forward to the present, where students join a session on tablet PCs; collaboratively listen, watch, and discuss concepts; and with a digital pen, take notes that are saved to a virtual binder available 24/7. The continuum evolves by using web-based applets, WebQuests, portfolios, pre-class assignments, flipped classroom, hybrid learning, and more. The development of this methodology was largely based on student opinion and feedback. The format for this session included a short presentation to illustrate

this evolution, followed by the opportunity for participants to discuss with a student panel how the students believe this pedagogy has benefitted or hindered their learning.

CLARITY

Derrick Driscoll (high school math teacher, London, Ontario)

Derrick discussed CLARITY, an innovative software package that allows teachers to view the performance of every student and make informed decisions about appropriate interventions. Evidence based on the use of formative assessment, in conjunction with the data provided through the use of CLARITY, indicated that over the two-year research period (2010–2012), EQAO performance trends in certain strands and categories showed greater improvement in these classes than in the general School District. A colleague who adopted this approach also garnered similar student gains over this period.

Mathematics Programs at the Ontario Science Centre

Martin Fischer (educator at the Ontario Science Centre)

Science Centres have made a name for themselves by creating rich, hands-on experiences that encourage young and old to experiment and explore the world. In 2011, the Ontario Science Centre expanded its school offerings into the realm of mathematics—using the same approach of wowing the students and sparking curiosity. To date, the focus has been on probability and data management through investigating catapults, bungee jumping, loaded dice, and Galton Boards. The presenter engaged the Fields audience in a discussion about other areas of mathematics that could be explored and "workshopped" in a similar manner.

This was a very interesting meeting. The Fields Institute Education Forum will reconvene in September. ▲

▲ Directors' Dialogue

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Ann Michele Stenning is a mathematics teacher at South Collegiate with the Thames Valley District School Board. She enjoys mentoring new teachers and learning new ways of how to address the various learning styles of her students.

OAME Board of Directors Meeting February 22–23, 2013 Doubletree Hotel, Toronto International Airport

Friday, February 22, 2013

A written report by Todd Romiens, the NCTM representative, announced that NCTM is producing a set of books called *Annual Perspectives in Mathematical Education*. NCTM is seeking contributors for the first issue that is due out in 2014. NCTM provides online activities such as "Dynamic Paper," which creates number lines, grids spinners, and tessellations.

Ontario Math Olympics (OMO) 2012, hosted by TEAMS, ran well. However, concerns about the success of OMO 2013 were discussed. To gauge the level of interest in OMO 2013, feedback was solicited from the boards. By early April, OMO 2013 was cancelled.

Cathy Hall shared highlights from Leadership Conference 2012. The theme was "On the Leading Edge, Making Mathematics Happen." Friday night's speaker, Doug Duff, spoke of his staff's willingness and success at improving mathematical knowledge. Bill Lundy forwarded a cheque for OAME's share of the 2012 conference. The board expressed their appreciation to Bill, Amy, and their committee. Marc Husband, chair of OAME 2013, asked each chapter representative to distribute OAME conference brochure to schools in their area.

Cathy Hall distributed a monograph on proportional reasoning from the Ministry of Education, K–12 Math Working group. Board members were given an opportunity to share their definition of proportional reasoning and to discuss ways to incorporate proportional reasoning when solving problems.

Saturday, February 23, 2013

The board members approved the agenda and the last meeting's minutes. The budget was also approved. Board members discussed how to meet electronically instead of meeting face to face in February. If approved by the board members, implementation of an electronic meeting would occur the following year. Board members discussed an offer to participate in selecting the winner for the Prime Minister's Award. Bill Otto announced that OAME's feedback was requested to develop a countrywide admission test to college. Committee work followed.

The Outreach Committee voted that one electronic OAME membership would accompany each OAME award, with the exception of the Life Membership award. The committee also agreed that the organizing chapter of the annual conference should receive 20 digital memberships to the Gazette. Sonia spoke to the need for chapters to host the OAME conference beyond 2014 and OMO beyond 2013. To encourage chapters to host these large events, Sonia suggested that the same venue be used from one year to the next. This would allow the sharing of information on the venue. The Awards Committee also confirmed that award winners had been selected for the Mathematics Leadership Award, Creative Teaching in Secondary Math, and the Elementary Staff Award. The committee asked that OAME's home page contain a link to the OAME awards and the criteria for each award. The Outreach Committee is looking for chapters to host future OMO events and the annual OAME conference. Interested chapters should contact OAME's executive directors.

The Communications Committee announced that they are looking for tweeters. Look on the OAME home page for an invitation to "like us on Facebook" and "follow us on Twitter."

The Curriculum Support subcommittee discussed ways to reorganize the online resources available on OAME's website. There is also a possibility of adding an interactive piece to the website for members to add comments and feedback.

The Curriculum Review subcommittee announced that they would present professional development on proportional reasoning to the teachers with whom they work. The group also plans to write an article for the *Gazette* on proportional reasoning. Committee members linked ideas between the Paying Attention to Mathematics K–12 document and the Seven Foundational Principles for Improvement in Mathematics. These connections will be posted on the OAME website.

The Strategic Planning and Renewal Committee presented a schedule of the chapters that will select their chapter representative for a three-year term, beginning this Fall.

Chapter Reports

CHAMP has been busy planning the OAME 2014 Conference, meeting on a monthly basis and collaborating electronically. The theme of OAME 2014 is "CHAMPions 4 Change: Mathematics 4 the 21st Century." CHAMP plans on running the 2013 Fall Mini-Conference with a virtual conference option (eConference) at Humber College. OAME has given a grant to CHAMP to support this component. This virtual conference will be the launch for the virtual conference component of OAME 2014. CHAMP continues to meet every first Thursday of the month. For more information, please visit **www.oame2014.ca**.

COMA took advantage of the University of Ottawa's reading week to host an evening Winter Forum event at their SITE building on February 21. Tom Hamza, President of the Investor Education Fund, shared his expertise and some fabulous resources to help educators foster financial literacy in all classrooms from K–12. The event was well attended, and one lucky attendee joined our group of "Pay-It-Forward" winners, receiving up to $500 in expense reimbursements to attend the OAME 2013 conference in May. Looking ahead to the Spring, COMA is preparing to hit the road again, this time heading south of Ottawa to Kemptville, offering valuable professional learning to mathematics educators in every corner of our national capital region.

ISOMA would like to thank Sheila Barclay and her colleagues at the Bishop Strachan School for hosting a very successful Fall/Winter conference. Held on the afternoon of January 9, the event was filled with activities and sessions exploring Problem-Based Learning. Carmel Schettino delivered the keynote address and shared her vast knowledge of problem-based learning. The great planning and organization of Sheila and her colleagues made the event run smoothly.

The next event on the ISOMA calendar is the annual Math Olympics. Brad Stephens and his colleagues at Holy Trinity School are in the midst of organizing the event. Scheduled for the afternoon of April 19, the day will include lots of math fun. We look forward to a great event.

MAC² hosted a Fall mini-conference in October. It

was entitled "Bringing Math to Life," with a plenary session, "Laughs and Math," by Don Fraser. Both parents and educators were invited to attend. A professional development night for teachers, entitled "Mega Math Mystery," was also organized. It involved a four-part activity that used QR technology to solve math problems. These PD nights are an attempt to reach out to current and future MAC2 members. The next PD session for teachers will occur at the end of February or early March.

NOMA has been fortunate to welcome new members to our chapter this year, and we look forward to the infusion of new energy and ideas. Planning for the Spring OMO playdowns will begin very shortly. There are a number of individuals on that committee who will be bringing a fresh perspective and it promises to be enjoyable. As we move toward Spring workshop planning, we continue to be conscious of the parameters within which teachers wish to focus their professional development.

QSLMA's mini-conference had ten presenters, who offered sessions such as "Connecting Patterning and Algebra, Roadmaps or GPS?," "Successful Student Collaboration (with Technology) in the Secondary Math Classroom," "Building Spatial Sense to Help Students Mathematically," and more.

WOMA's Spring conference featured a keynote address by Lynda Colgan, PhD. As the mathemagician on stage, Lynda presented a number of math tricks to reveal the magic in math and the math in magic. The event began with food and refreshments, followed by two professional development sessions. The evening finished with dinner and spellbinding acts by Dr. Colgan. ▲

▲ What's New with CLIPS? New and Improved Tools!

GREG CLARKE, AGNES GRAFTON
ROSS ISENEGGER, MARKUS WOLSKI



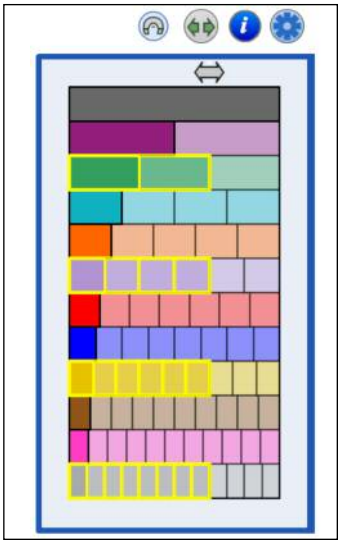
Greg Clarke is from the Simcoe Muskoka Catholic District School Board, and Ross Isenegger is from the Near North District School Board. Both are currently on assignment with the Ontario Ministry of Education as Provincial Math Leads, working on the CLIPS project, in conjunction with their colleague, Agnes Grafton, from the Brant Haldimand Norfolk Catholic District School Board. Markus Wolski is a teacher with the Bluewater District School Board and has been working on the CLIPS project for the past two years.

Last issue, we introduced the CLIPS Tools pod and gave a general overview of how CLIPS tools work. This month, we will focus on some of the tools that have been improved or recently created.

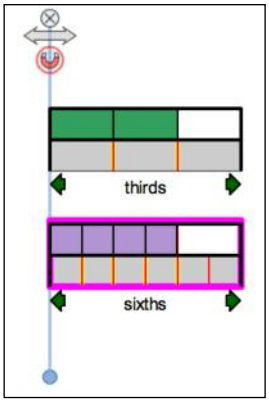
Fraction Strips – New and Improved!

The Fraction Strips tool, which was used as the example in last month's column, has since been updated.

The Equivalence Bar (double grey arrows) can be dragged anywhere within the Fraction Tower. As the bar is dragged, rows of equivalent fractions are highlighted. In the diagram, the equivalence bar was released at $\frac{2}{3}$. Now, you can drag the entire highlighted strip to the workspace.



There are also two new buttons in the top right of the tool.

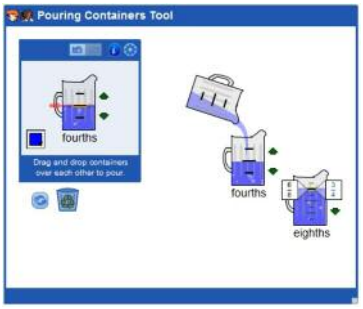


Click on the magnet button to activate the Alignment Bar (see right). Drag the Alignment Bar across the workspace. Fraction Representations that are close to the bar will snap to it so that you are better able to compare them.

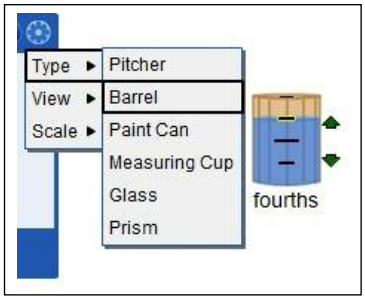
Click on the green arrows button to show the stepper that allows you to change the parts per whole. This feature was added to support use on an interactive whiteboard, as the stepper was designed to appear if you roll over the grey reference bar; however, rollovers do not work so well on an interactive whiteboard.

Pouring Containers – Representing Fractions, Using Volume

Pouring Containers is used to explore fractions, using a volume representation.



Start by designing a container. An assortment of container types is available, and the user can control the amount of liquid in each.



Once you have designed the container, drag it to the workspace and the fun begins! The contents can be combined by dragging one container over another. Liquid is transferred until the pouring container is empty, the receiving container is full, or the liquid in the pouring container has reached its pour bar line. (To turn on the pour bar, click on "View" in the settings). Tick marks on the side of the container will highlight when the liquid level is exactly at the fraction indicated. Hovering over any tick mark will show a label that indicates what fraction of the container the tick represents.

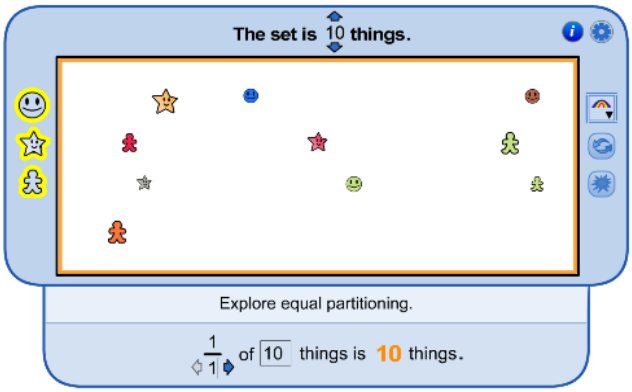
Try using the Pouring Containers tool to model these questions:

$$5 \times \frac{2}{3} \qquad \frac{1}{2} + \frac{1}{3} \qquad \frac{7}{8} - \frac{3}{4}$$

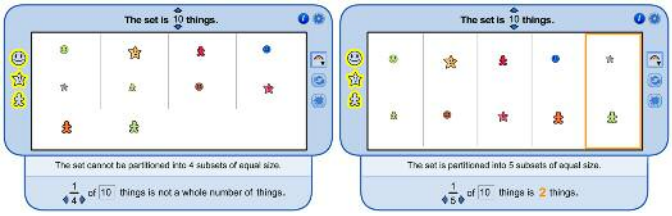
Partitioning Sets by Number – A Brand New Tool for Fraction Multiplication

Hot off the press is a brand new tool to help explore finding a unit fraction of a set of objects.

The Partitioning Sets tool allows you to place multiple objects of up to three different types and a variety of sizes in the workspace. At this point, the objects are draggable, allowing the user to explore.



Next, choose a fraction with which to partition the set. Click the GO button to see the objects dealt out into the partitions in the workspace. The dealing out simulates how many students interact with concrete items when placing them into groups. Will the set of objects partition into subsets of equal size, or not?



Integer Number Line Tool – For Addition and Subtraction of Integers

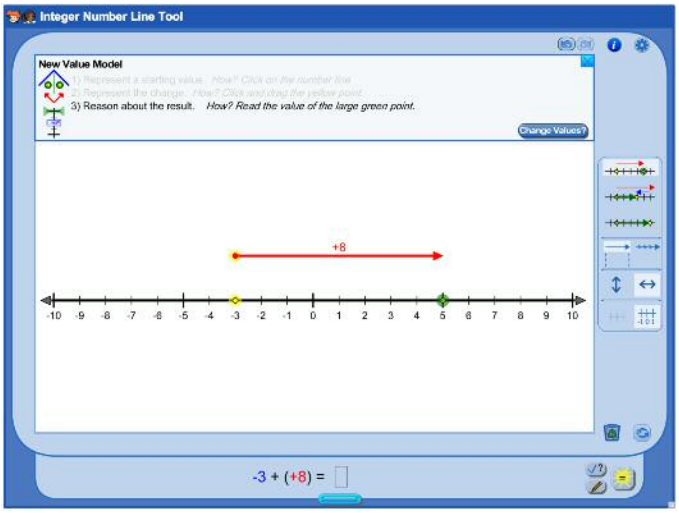
The latest tool has been developed to help students visualize integer addition (and subtraction) along a number line. The tool has a built-in context-sensitive help system in the form of an onscreen character, Linus. You can click to plot points, or drag and drop, to create vectors that represent a change in position. An equation for the model created is displayed below (and can be hidden, if desired). The tool is in its final review at the time of writing, so the final version may vary slightly from screenshots supplied.

Addition can be modelled in two ways, using this tool.

The **New Value model** uses the value of the initial point on the number line as the first integer in the sum.

Subsequent integers are represented by the change vectors drawn on the number line. Consider a context such as this:

> "This morning it was -3 degrees Celsius when I woke up. While I was at school, the temperature rose 8 degrees. What was the temperature at the end of the school day?"



This could be represented by the equation: (-3) + (+8) = +5

Note that in this model, the initial value represents a point on the number line, and the answer also represents a point on the number line, specifically, the new value after a change (or series of changes).

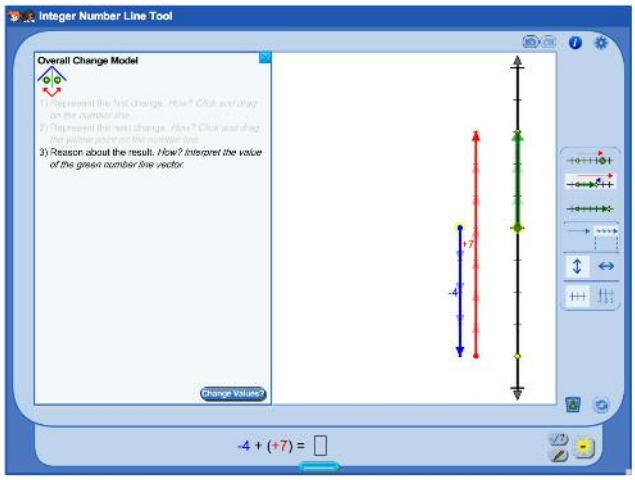
The Number Line tool allows you to model a series of changes. Click and drag from the current end point to a new location on the number line to add another change vector to the model.

The **Overall Change model** often uses an unlabelled number line, since it relies only on the change vectors to represent the integers in the equation. Here is a sample context:

> "The cost of Mathopolis Inc. stock dropped $4 during the morning trading session, but then rose $7 in the afternoon session. What was the overall change in stock price for Mathopolis Inc. at the end of the day?"

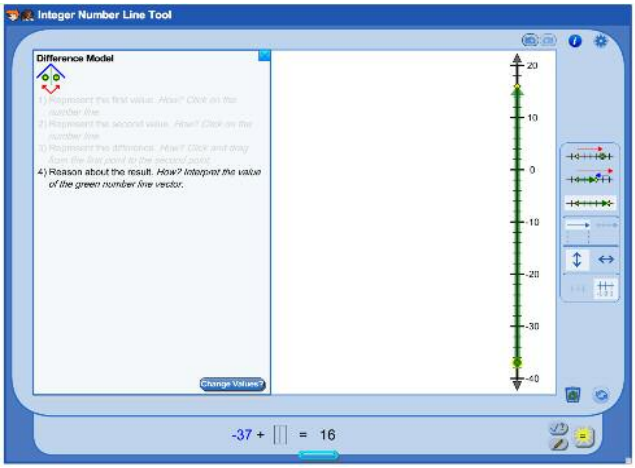
This can be represented by the equation (-4) + (+7) = +3.

Note that in this model, all the values represent changes, and the answer represents the overall change (from start to finish).



The third model available with this tool, the **Difference model**, can be used to represent the difference or change in position between two points on the number line. For example:

> "Thermal, California is located 37 m below sea level. Davis, California is located 16 m above sea level. What is the difference in elevation between the two towns?"



When the Difference model is selected, the equation can be written as a Missing Addend (e.g., -37 + ? = 16) or as a Subtraction statement (e.g., 16 - (-37) = ?).

Clicking the  button allows you to change format of the equation, without changing the equation values (rewriting with brackets, optional + signs, etc.).

For more information about these and other CLIPS Tools, and for ideas about how to use them in your classroom, check out the following wiki that houses all the supports for these tools. This is a work in progress, so if you have a great idea about how to use one of these tools, please consider sharing it with the authors.

http://mathclips.wikispaces.com/MathCLIPS+Tools ▲

▲ Linking Representations Using Geogebra

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Jill Lazarus is currently building on her experiences with teaching mathematics at General Panet High School, in the Renfrew County District School Board, by pursuing a PhD in education at the University of Ottawa. She is interested in a range of areas in mathematics education, including curriculum implementation, communication in mathematics, educational technologies, and assessment.

In secondary school mathematics, representing mathematical ideas and modelling situations generally takes the form of numeric, geometric, graphical, algebraic, pictorial, and concrete representation, as well as representation using dynamic software. Students should be able to go from one representation to another, recognize the connections between representations, and use the different representations appropriately and as needed to solve problems. (Ontario Ministry of Education, 2005, p. 16)

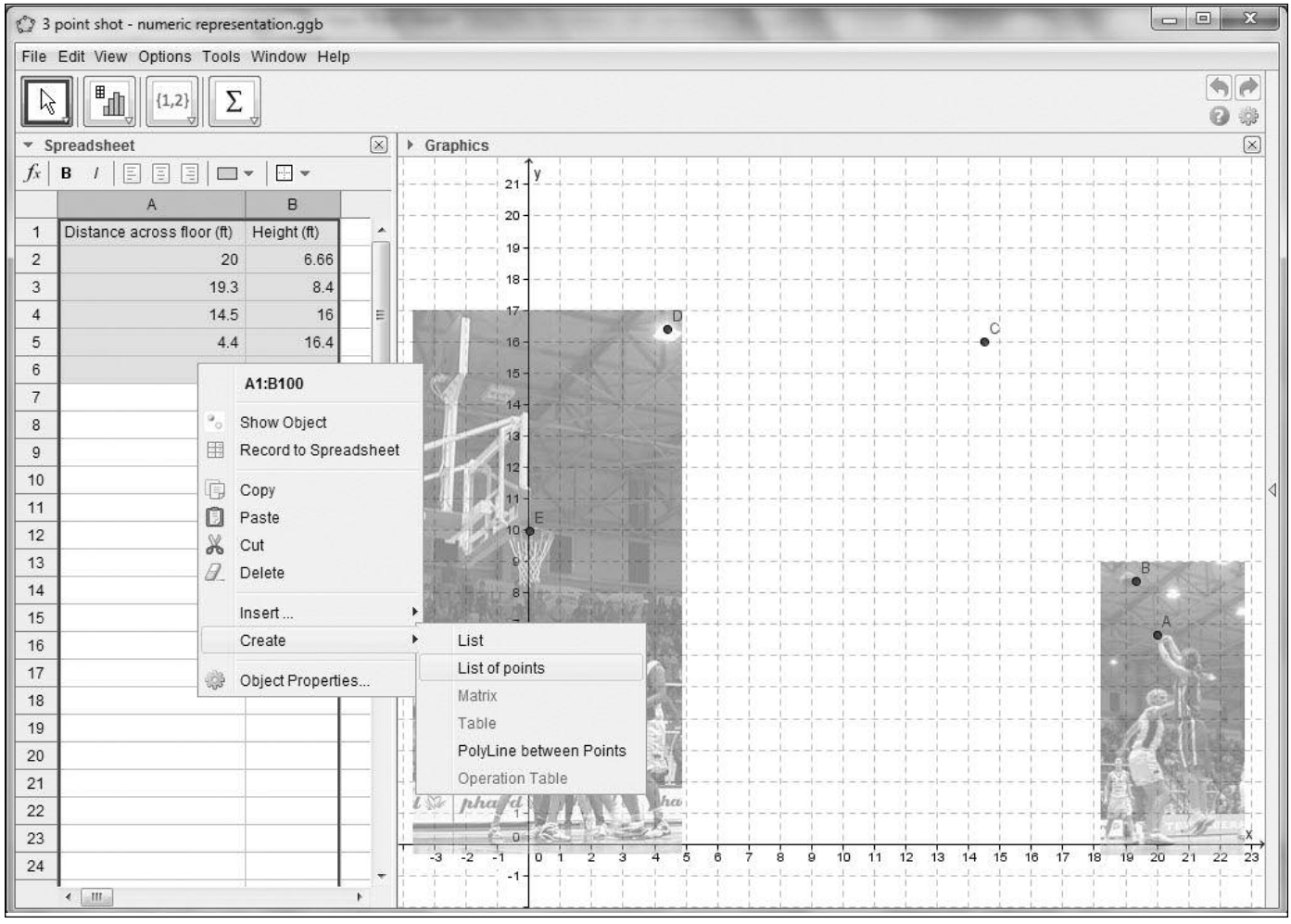


Figure 1: Pictorial & Numeric Representations of Ball Path **http://oame.on.ca/main/files/gazettelinks/Gz51i4RouletFigure1.html**

The quotation above from the Ontario Grades 9 and 10 curriculum document, and similar statements found in the guidelines for Grades 1 to 8 and 11 and 12, identify and expand upon Representing, one of the seven key mathematical processes. GeoGebra (International GeoGebra Institute, 2012a), an example of the "dynamic software" regularly referred to in our curriculum, can play a significant role in supporting students' learning to connect and move between different representations.

Giving an introduction to GeoGebra on paper presents a problem. In GeoGebra, the connections between representations are dynamic; that is, a change in one model (i.e., an algebraic equation) produces corresponding changes in other representations (i.e., a curve). Static images on paper cannot really illustrate this dynamic linking. Fortunately, GeoGebra's easy integration with the Web provides a way to expand the effectiveness of this article.

GeoGebra is written in Java and its **File > Export** option supports the creation of web pages holding applets, presenting the software in any state constructed by the user. A URL has been provided for each image in this article, and following these links will take the reader to active versions of GeoGebra as shown in the pictures. This sharing of dynamic tools is also supported by the fact that GeoGebra is free open-source software that you may download to your computer from **www.geogebra.org**. GeoGebra can operate within your web browser; thus, you can interact with the applets even if you do not want to install the program on your computer. This feature becomes very powerful when you wish to employ a course website to share GeoGebra learning objects with your class. Students can access the applets from any Internet-connected computer at home or in the school or local public library.

In this article, the topic of quadratic relations, found in both the Academic (MPM2D) and Applied (MFM2P) Grade 10 mathematics courses, is used as a context, but the tools and dynamic linking in GeoGebra serve for all secondary school courses and on into the college and university levels. For the elementary school grades, there is a reduced version, GeoGebraPrim (International GeoGebra Institute, 2012b); Prim for primary school, Grades 1–6 in Europe. The activity outlined below, the exploration of the path of a ball when a basketball player takes a jump shot, is not new, and has been conducted in a variety of ways with pencil and paper and supporting information and communication technology (ICT). GeoGebra provides some additional tools to enhance student exploration and aid the teacher's delivery of supporting materials via a course website.

Pictorial and Numeric Representations

Using a photograph (Finizio, 2005) of Italian national basketball team member, Sara Giauro, making a three-point shot and placing it on a grid, we can describe the location of the ball at a number of points as it travels from her hands to the basket. Starting the investigation with a photograph originally from a news report of a basketball game provides a real context for the problem, but also leads to some need to adjust the picture. For this application, the image, contained in a JPEG file obtained from the Web, has been split and mounted on the graph in two parts to compensate for the distance shortening produced by the photographer's telephoto lens. If the origin of a Cartesian plane is set on the floor directly below the basket, and the graph scale is adjusted to reflect the height of the basket rim at 10 feet, the location of the ball at five points along its flight path can be described by the numbers in the GeoGebra spreadsheet shown in Figure 1. Here, column A gives the distance in feet along the floor from Giauro's location, and column B records the height (feet) the ball is above the floor. GeoGebra provides many of the functions found in commercial spreadsheet software, but for this example, all we need to do is select the two columns and use the Create > List of points tool to plot the corresponding points on the graph. Students can, if they wish, rescale the graph and have the embedded photographs adjust by the same factors.

Algebraic and Graphical Representations

The locations of the five sample points and common knowledge would likely lead a class to suggest that the ball follows a parabolic path. If this investigation came somewhere near the end of the Quadratic Relations unit, students might suggest models of the forms $y = ax^2 + bx + c$ or $y = a(x - h)^2 + k$, with x representing the horizontal distance from Giauro in feet and y the height above the floor in feet. By entering, via the **Input Bar**, various examples of either equation, students could work toward values for the parameters. However, a better approach would be to use the **Slider** tool (Figure 2); create three adjustable parameters for a, h, and k; and then input the general function $f(x) = a(x - h)^2 + k$.

As students manipulate the values of the parameters—a, h, and k—the corresponding function expression is shown in the Algebra window, and the associated curve is displayed (Figure 3). With careful

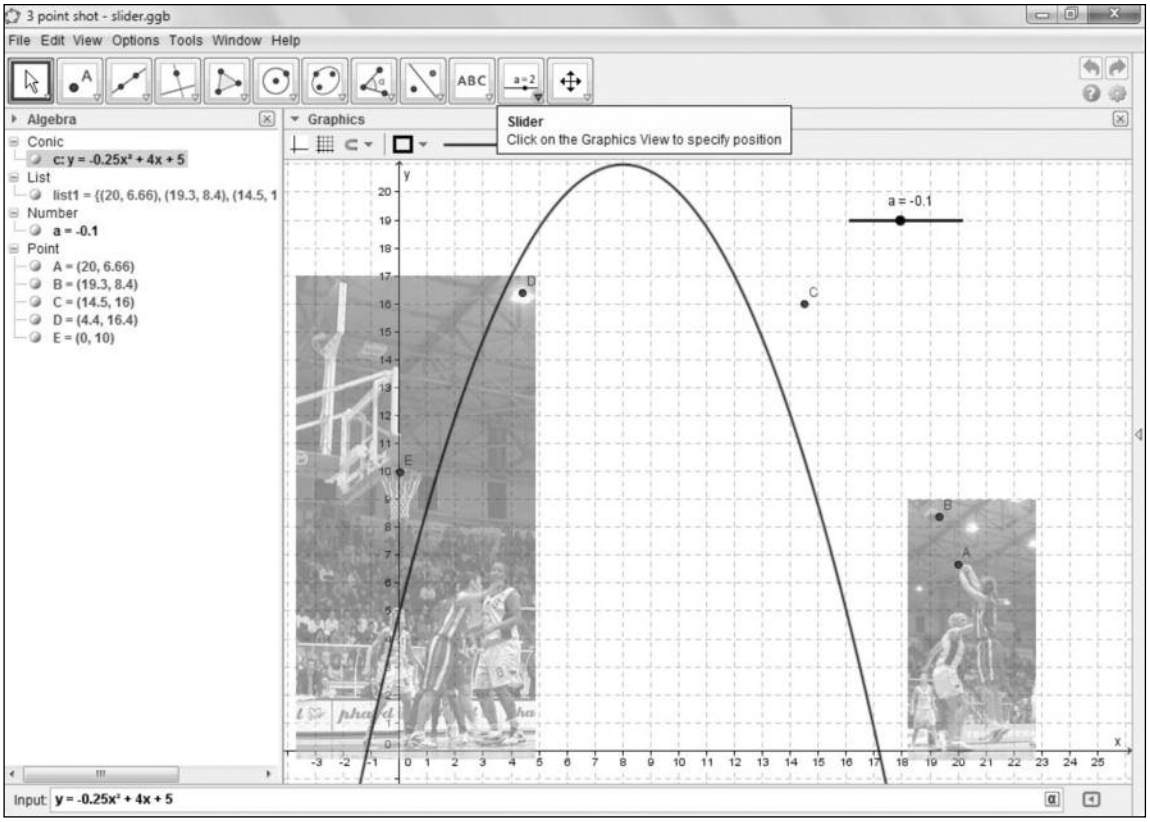


Figure 2: Trying a Quadratic Model URL: **http://oame.on.ca/main/files/gazettelinks/Gz51i4RouletFigure2.html**

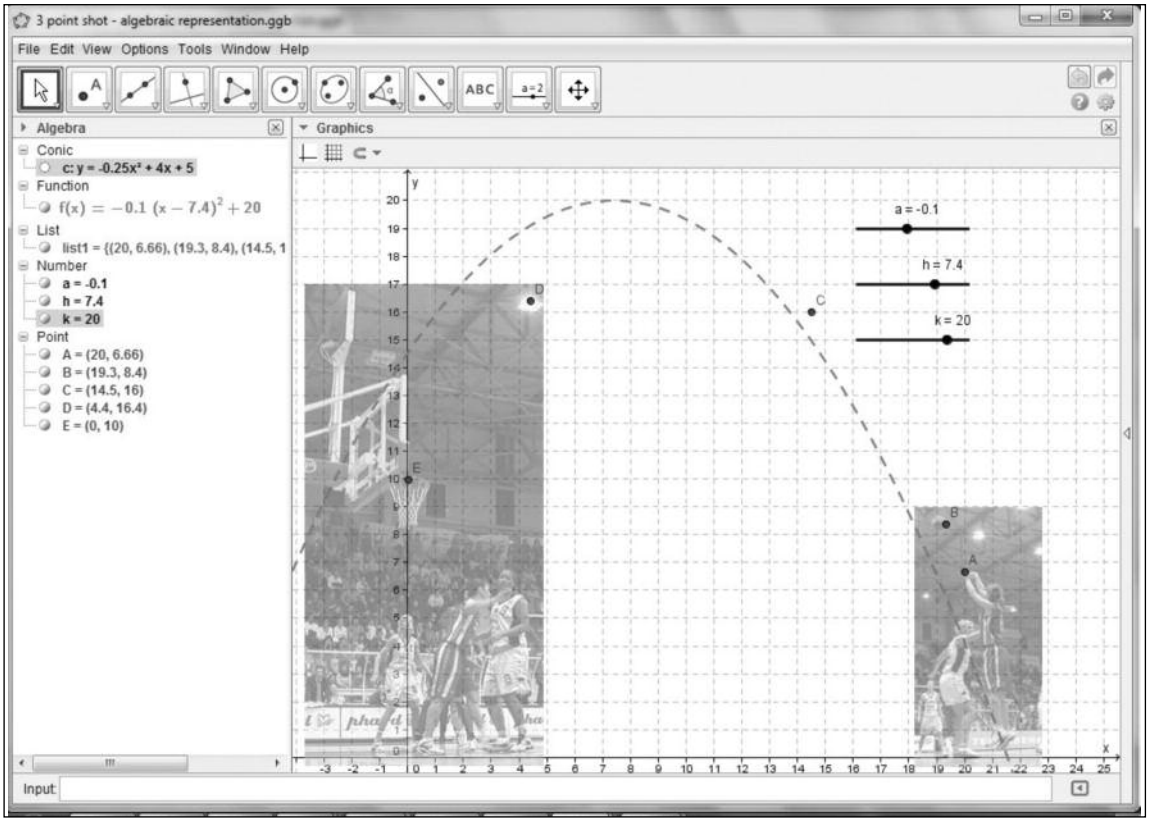


Figure 3: Using Sliders to Explore a f(x) = a(x – h)² + k Model URL:
http://oame.on.ca/main/files/gazettelinks/Gz51i4RouletFigure3.html

adjustment of the parameter values, students can generate a curve that comes close to passing through the five sample points and develop an algebraic model that is something like $f(x) = -0.1(x - 9.25)^2 + 18.75$.

Using the Algebraic Model in the CAS Window

In the photograph, Giauro has driven to the corner and is shooting her three-pointer across the court. Suppose Giauro had made a shot with about the same arc, but had aimed too high. That is, suppose the algebraic model for her shot were $f(x) = -0.1(x - 8)2 + 21$. In this case, the ball would pass over the basket and could be caught by another player on the opposite side of the court (see Figure 4). If you were on the court and wished to catch the ball just above your head at 5.75 feet, how far away from the basket should you stand? Problems such as this can be solved by manipulating the expressions of a model in the Computer Algebra System (CAS) window of GeoGebra (Figure 4). Copy-and-paste brings the algebraic model to the CAS window, and substituting 5.75 for $f(x)$ gives the quadratic equation, $5.75 = -0.1(x - 8)^2 + 21$, to which we can apply the Solve Numerically tool (see Figure 4). The negative root tells us that a player wishing to catch the ball at a point 5.75 feet above the floor must stand 4.35 feet away from the basket across the court from Giauro.

Geometric Representation

The Euclidean loci descriptions of the conics, such as the focus-directrix definition of a parabola, are no longer topics of the Ontario mathematics program, but the Graphics (geometry) view of GeoGebra could be used to briefly introduce the concept via an online learning object. In Figure 5, a free point labelled **Focus**, and a second point **D**, on the *y*-axis, have been added to the coordinate plane. With a line constructed perpendicular to the *y*-axis through point D as the directrix, we can use the **Conic** > **Parabola** tool to produce a curve that students could explore by dragging the points, Focus

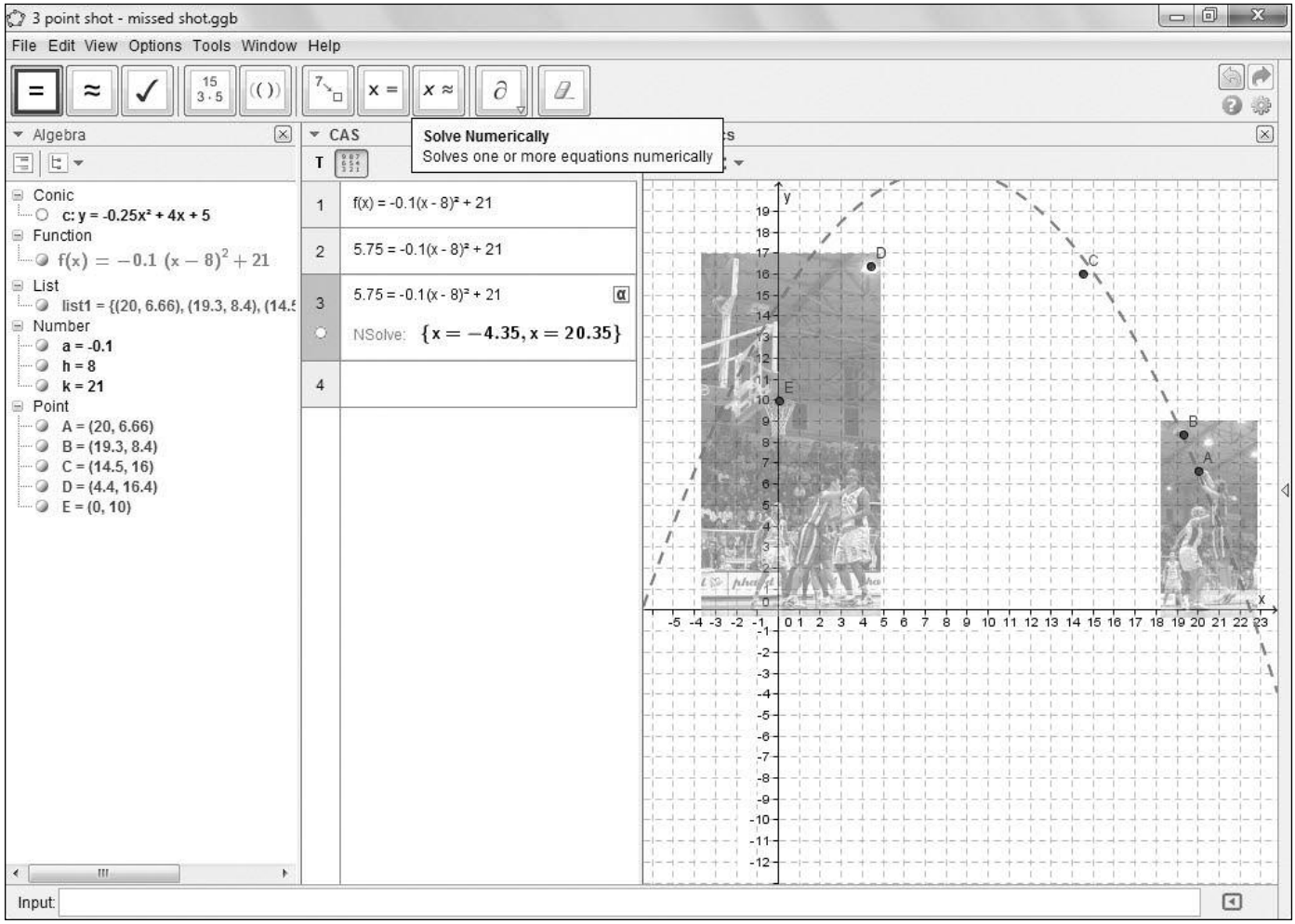


Figure 4: Using the CAS to Solve a Problem

URL: **http://oame.on.ca/main/files/gazettelinks/Gz51i4RouletFigure4.html**

and D. To make the activity more engaging, a moving image of a ball can be attached to the curve, and now the students' task is to develop a curve that takes the ball from Sara's hands to the basket. Using the **File** > **Export** > **Dynamic worksheet** option, a web page (see Figure 5) holding this activity could be generated and mounted on a course website.

Statistical Modelling

The above approach linking numeric, graphical, algebraic, and geometric representations is effective with classes initially exploring the connections between parabolic curves and quadratic functions. Once understanding has been developed, a class might move on to using the statistical modelling tools of GeoGebra to build quadratic algebraic representations needed for problem solving. The initial Spreadsheet view of the five sample points along the path of the ball also provides statistical tools, such as Two Variable Regression Analysis (see Figure 6). Clicking on this tool and selecting a Polynomial model of degree 2 brings up a window that shows a regression model, $y = -0.1x^2 + 1.93x + 9.98$ (see Figure 6). This algebraic representation could then be copied over to the Algebraic and Graphic views of GeoGebra and employed in the CAS view to solve problems such as the player position above.

GeoGebra Collaboration

With GeoGebra available in 62 languages, the software is developing a large worldwide community that comes together on the GeoGebra website (**www.geogebra.org/cms/en/community-info**) to provide mutual advice and support, and share many GeoGebra-based online learning tools (**www.geogebratube.org**). Visit these sites if you wish more information, and follow the URLs presented with each figure to explore in more detail the steps in the constructions above. In addition, the ability to export GeoGebra applets and embed them in wiki and course-learning

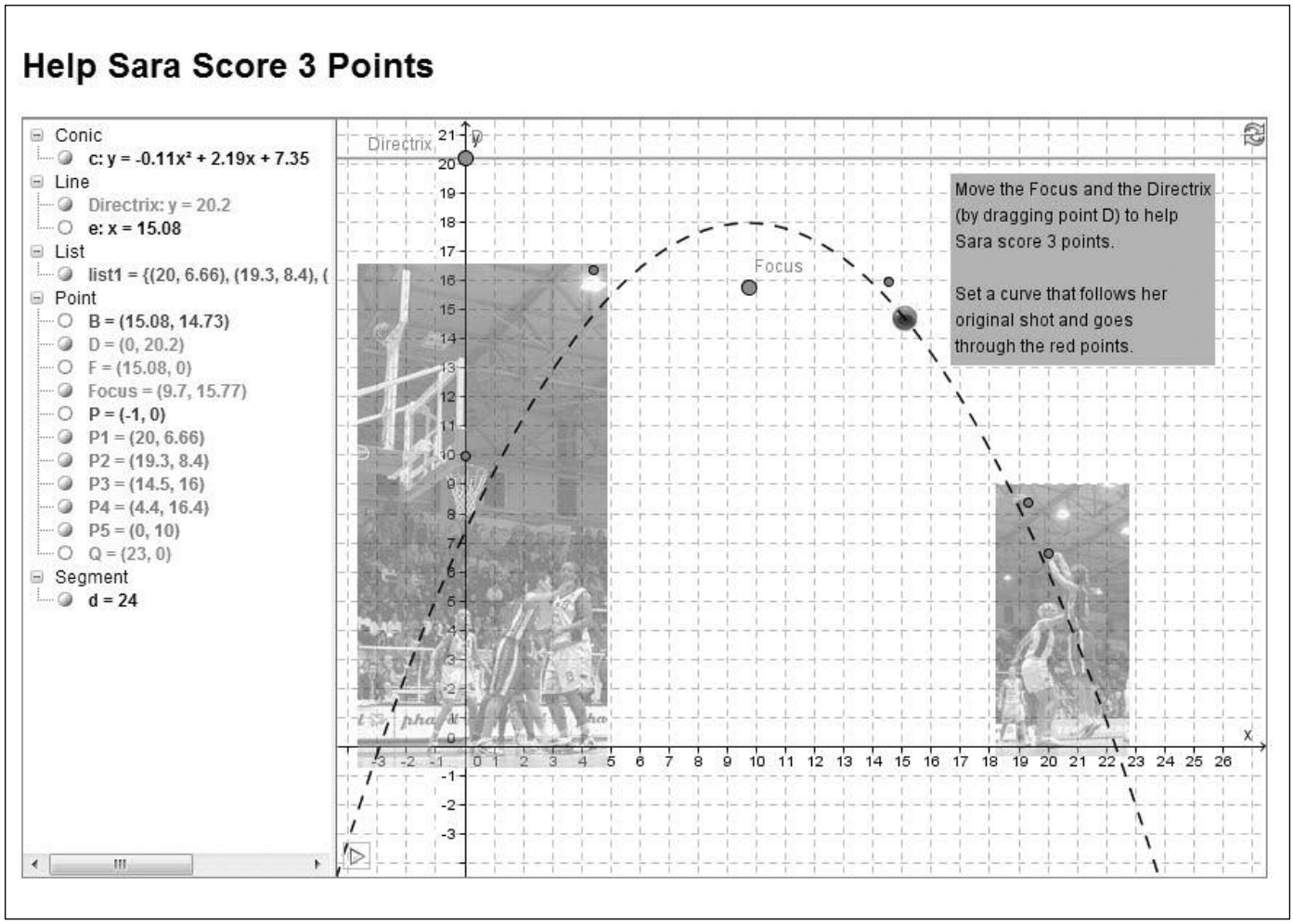


Figure 5: An Online Learning Object for Exploring the Focus-Directrix Parabola Definition
URL: **http://oame.on.ca/main/files/gazettelinks/Gz51i4RouletFigure5.html**

management system (LMS) pages can aid in the development of a classroom mathematics learning community. Students can post their GeoGebra work along with explanations of their thinking, and others in the class can interact with the applets and comment on the work (Lazarus & Roulet, 2013, this issue).

References

Finizio, M. (Photographer). *Three point shoot by Sara Giauro (Phard Vomero Napoli) during FIBA Europe Cup Women Finals 2005.* (2005). [Photograph]. Naples, Italy: Creative Commons (cc-by-sa-2.0-it). Retrieved from **http://en.wikipedia.org/wiki/File:Three_point_shoot.JPG**

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International GeoGebra Institute. (2012b). *GeoGebraPrim* (Version 4.0.41.0) [Software]. Available from **www.geogebra.org/webstart/4.0/GeoGebraPrim.jnlp**

Lazarus, J., & Roulet, G. (2013). Communication in a blended math-talk learning community: Extending classroom boundaries. *Gazette, 51*(4), 34-40

Lazarus, J., & Routlet, G. (2013, March 13). Supporting Collaborative Mathematics Using a Wiki, GeoGebra, and Jing. *In Math Future*. Retrieved from **http://mathfuture.sikispaces.com/SupportingCollaborativeMath**

Ontario Ministry of Education. (2005). The Ontario curriculum, Grades 9 and 10: *Mathematics, 2005* (revised). Toronto: Queen's Printer for Ontario. Retrieved from **www.edu.gov.on.ca/eng/curriculum/secondary/math910curr.pdf**

Note 1: Resources to assist you and your students while working with GeoGebra can be found at **http://wiki.geogebra.org/en/** and **http://queensgeogebra.pbworks.com**.

Note 2: Using the URLs listed with the figures in a web browser or clicking on the links embedded in the PDF version of this article should automatically load the GeoGebra applet. If this does not happen on your computer you may need to go to the GeoGebra download site at: **http://www.geogebra.org/cms/download**, and download and run the Java Webstart program. ▲

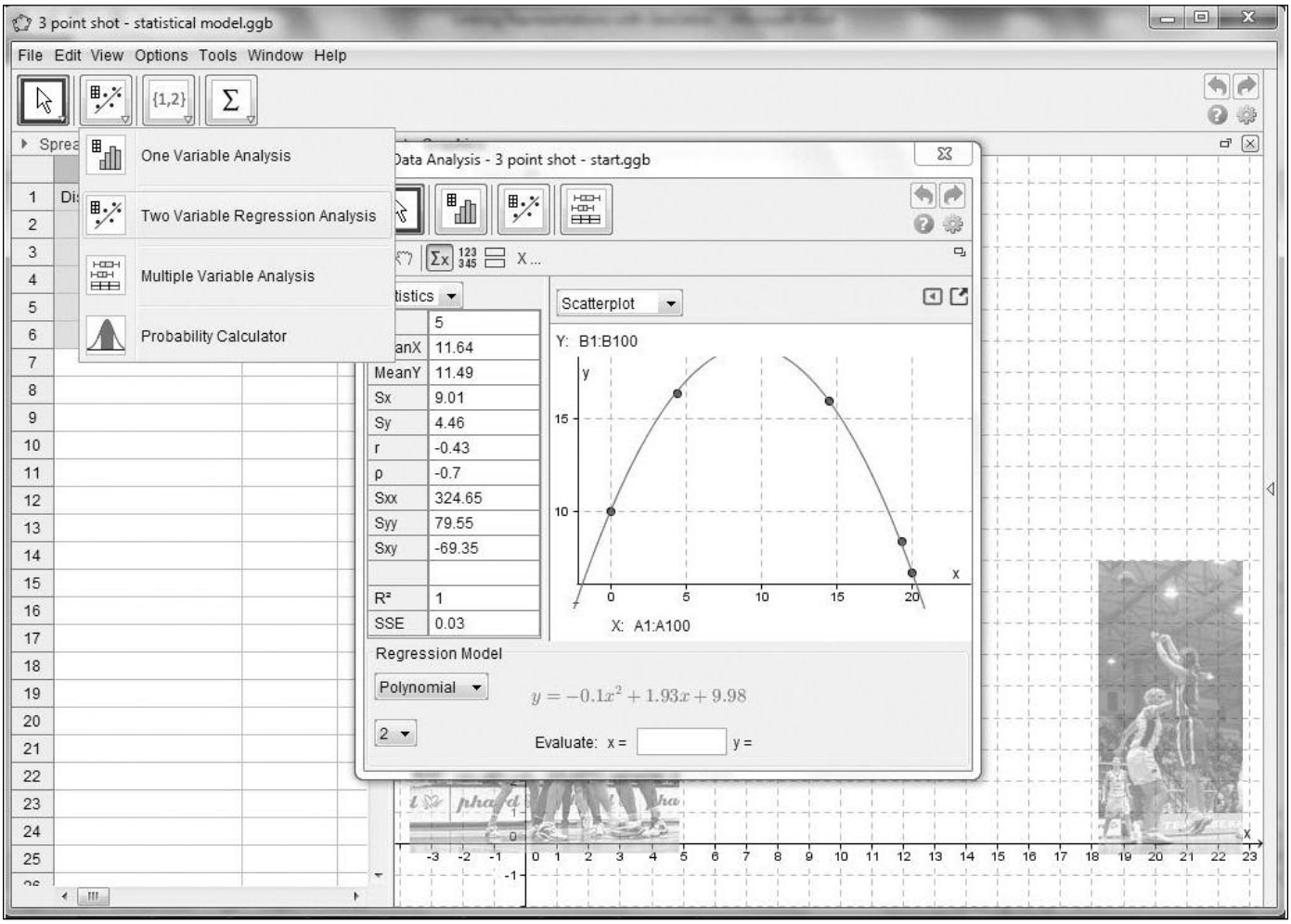


Figure 6: Using Two Variable Regression to Generate an Algebraic Representation
URL: **http://oame.on.ca/main/files/gazettelinks/Gz51i4RouletFigure6.html**

▲ Communication in a Blended Math-Talk Community: Extending the Boundaries of Classroom Collaboration

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Like the National Council of Teachers of Mathematics (2000), the Ontario Ministry of Education (OME, 2005) emphasizes that communication is an essential process in learning mathematics. The Ontario Mathematics Curriculum does not expand on what communication looks like in an online environment, perhaps because online learning was less prevalent in 2005 and 2007, but the Ontario Ministry of Education does identify: (i) *what* it means to communicate in mathematics, and (ii) *how* communication should be supported. These two components are fundamental in any mathematics classroom. In this article, we focus on what mathematics communication may look like and how it can be supported in a blended collaborative mathematics learning environment. In particular, we describe how a wiki, dynamic mathematics software, and screencast videos were used to support communication in the classroom and online in the Principles of Mathematics (MPM2D), Grade 10, Academic course.

Communication in a Blended Collaborative Learning Environment

Communication, according to the Ontario Mathematics Curriculum guidelines, is "the process of expressing mathematical ideas and understanding orally, visually, and in writing, using numbers, symbols, pictures, graphs, diagrams, and words" (OME, 2005, p. 16). With the technological tools that we employed, students were able to express and record their thinking in all of these forms. Furthermore, dynamic mathematics software allowed students to interact with graphical, algebraic, and numeric representations in mathematics (see Roulet & Lazarus, 2013).

In addition to changing the way students express their thinking in mathematics, we considered the impact of blended learning in a classroom environment, where students are actively contributing to mathematics learning. Instead of using the wiki and video screencast technologies to communicate information to students, students were involved in online collaboration. This approach reflects thinking in the Ontario Mathematics Curriculum about how communication should be supported in a mathematics learning environment. In particular, according to the Ontario Ministry of Education, "effective classroom communication requires a supportive and respectful environment that makes all members of the class comfortable when they speak and when they question, react to, and elaborate on the statements of their classmates and the teacher" (OME, 2005, p. 16).

The learning environment envisioned in the curriculum guidelines reflects components of a math-talk learning community. The math-talk framework, which was supported throughout the implementation of this project, is referenced in teacher resource documents that are provided by the Ministry of Education (OME, 2008a, 2008b). According to Hufferd-Ackles et al. (2004), in a math-talk learning community, there is a shift from the teacher asking questions to *students* and the *teacher* posing questions. Students are expected to *explain* their thinking and to elaborate on their peers' thinking. There is a shift away from the teacher as a *source of all mathematical ideas* to student ideas also influencing the direction of lessons. Students also take more

responsibility for learning for themselves and for each other (Hufferd-Ackles, Fuson, & Sherin, 2004). As we demonstrate below, blending technologies and bringing the collaboration to an online environment provided students with alternative ways to communicate their thinking. The blended approach allowed for the math-talk community to be extended across time and space.

Facilitating the collaborative learning that is emphasized in a math-talk community requires careful consideration. Delich, Kelly, and McIntosh, for example, acknowledge that online collaboration requires "both structure and flexibility" (2008, p. 18). Structure can be provided, for example, by giving clear instructions. Flexibility may be promoted "by being willing to move in new directions that emerge during the collaborative exchanges" (Delich, Kelly, & McIntosh, 2008, p. 18). These authors also emphasize that "teaching and learning still relies on people—expert learners and beginning learners—more than technology" (2008, p. 18). Simply creating a wiki and telling students to interact is unlikely to lead to meaningful collaboration (Lavin, Beaufait, & Tomei, 2008).

Combining Tools to Facilitate Blended Collaboration

Integrating technological tools provides a range of possibilities for facilitating blended collaboration in mathematics. In the approach presented here, "blended" means offline classroom activities along with asynchronous online sharing. Online collaboration was supported with a wiki, dynamic mathematics software, and screencast software. The three free online tools and our approach to integrating them to facilitate a blended math-talk learning community are outlined below.

A Wiki for Collaboration

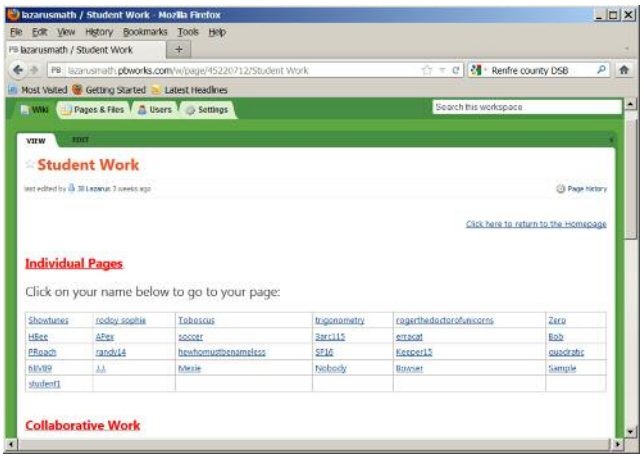


Figure 1: Wiki links to student pages

A wiki, in this case PBworks (**http://pbworks.com**), provides a medium for supporting student interactions. Wikis are "collaboratively editable websites that can be used for various purposes" (Lavin, Beaufait, & Tomei, 2008, p. 391). The collaborative editing feature allows for the aspects of communication identified in mathematics curriculum documents: questioning, reacting, and responding to each other's statements. As Lavin, Beaufait, and Tomei acknowledge, typically, "where a team of people is working on creating a knowledge base of some kind, a wiki implies engagement with information and ideas more than people, though of course it was ultimately people who produced those ideas" (2008, p. 295). Thus, by blending our math-talk community, students worked together in the classroom in addition to interacting with information and ideas online. Although PBworks is a free tool, and in the past we used the free version, for this project we chose to pay to upgrade, which allowed us to keep some pages public and other pages closed to the teacher and students in the class (see Figure 1). Our approach to controlling page access is explained in more detail below.

Representing and Interacting with Representations, Using GeoGebra

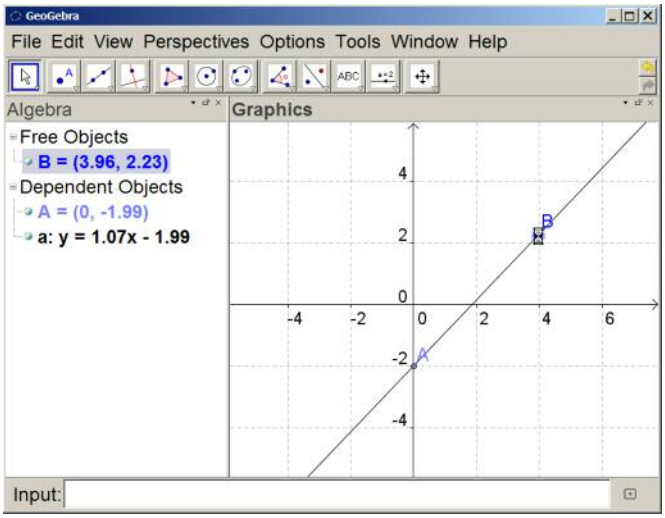


Figure 2: GeoGebra construction

Dynamic mathematics software, GeoGebra (International GeoGebra Institute, 2012), allows students to represent their thinking graphically, algebraically, and numerically (see Figure 2). In GeoGebra, which is free and may be used online or offline, the representations are linked so that in addition to communicating in these forms, students can explore and describe the interactions between representations (see Roulet & Lazarus, 2013). In terms of online sharing, GeoGebra

constructions may be embedded into wiki pages as applets so that students can post a dynamic sketch; someone else can access and work with it, and then she or he can re-post. Thus, since the wiki alone did not allow for easy mathematics representations, embedding GeoGebra applets was important for supporting communication using mathematics notation.

Screencast Videos Using Jing

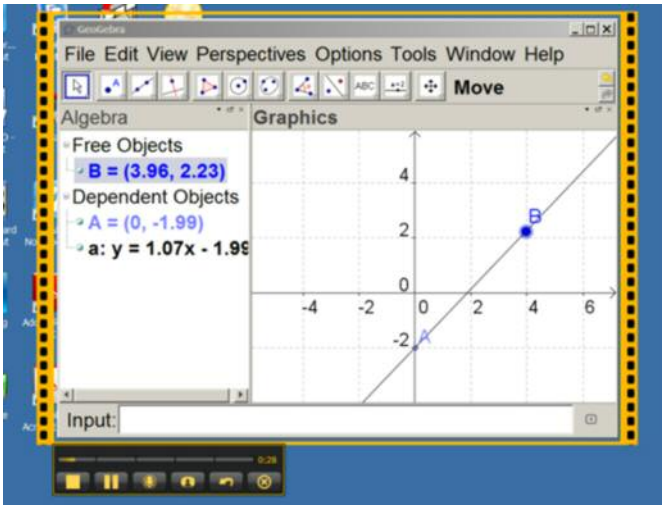


Figure 3: Screencasting a GeoGebra construction with Jing

With Jing (**www.techsmith.com/jing.html**), free screenshot and screencast software, students may record their screens and voices as they explain their thinking (see Figure 3). This reduces the need for text and provides a YouTube-like experience for students. To reflect that students are an important source of mathematical ideas in the math-talk community, videos posted to the wiki were primarily student generated rather than teacher generated or online videos that presented information to students. For more information on the software and technical details involved in online mathematical collaboration, see the Collabmath wiki at **http://collabmath.pbworks.com**.

Implementing a Blended Math-Talk Learning Community

To provide structure and flexibility, we carried out this project in a series of stages. This was essential because students had limited or no prior experience with the technologies. According to the Ontario Mathematics Curriculum, when students lack prior knowledge of the technologies that are being employed, "it is important that teachers introduce their use in ways that build students' confidence and contribute to their understanding of the concepts being investigated" (OME, 2005, p. 14). To maintain this position, we carried out the integration of technologies in six stages: (i) Setting Up the Environment, (ii) Posting Mathematics Text, (iii) Embedding GeoGebra Applets, (iv) Embedding Jing Videos, (v) Commenting on Each Others' Work, and (vi) Combining Skills.

Stage 1: Setting Up the Environment

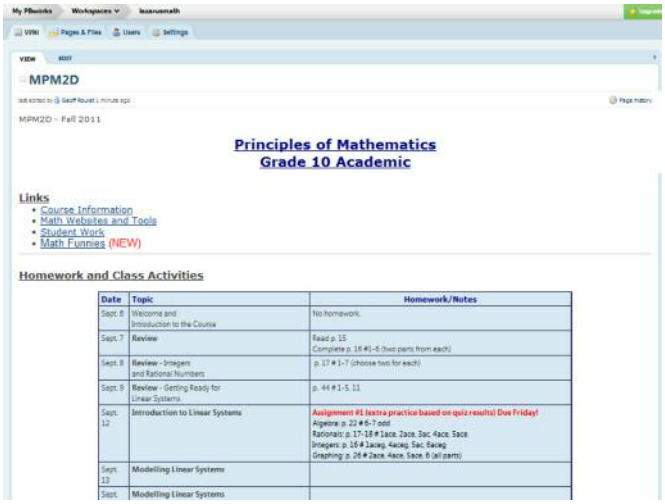


Figure 4: Wiki home page

An important first step in this approach to blended learning was to introduce students gradually to the wiki, the dynamic mathematics software, and to collaborating in a math-talk learning community. In preparing students to participate in a blended collaborative learning culture, a math-talk learning community was introduced in the classroom during the first week of school. The classroom was set up so that students sat in groups of four. They also contributed to co-constructing criteria for communicating in group and whole-class activities. Some of the criteria they identified in this activity included listening, everyone participating, and "sharing is caring." Math-talk activities were supported throughout the entire course.

As the community developed, students were introduced to the wiki and to GeoGebra. Initially, the wiki was a place where students or their parents could access course information, including details about daily work and homework (See Figure 4). This allowed for a gradual introduction to the wiki, which is important because initially, the various features of a wiki may be confusing to students (Lavin, Beaufait, & Tomei, 2008). In the same way, GeoGebra was also introduced in stages. This tool was used by the teacher to facilitate whole-class discussions and by students as they explored mathematical ideas in class.

Online safety was an important consideration in this classroom. On the wiki, students were identified by self-created pseudonym names. Access to wiki pages was also controlled by the teacher. We used a wiki that allowed control of access to each page. We had control over which pages were open for individual, group, class, or public access for viewing/editing. Pages that contained course information that would be of interest to parents, were open to the public for viewing, but not for editing. Pages that students contributed to were password protected. Each student had a username and password and had access to view and/or edit the pages that we approved. For example, a page for individual student work could be open only to that individual or to the entire class. A page for group work could be closed to the group or open to the entire class for viewing and/or editing. Page access depended on the purpose of the task.

Stage 2: Posting Mathematics Text

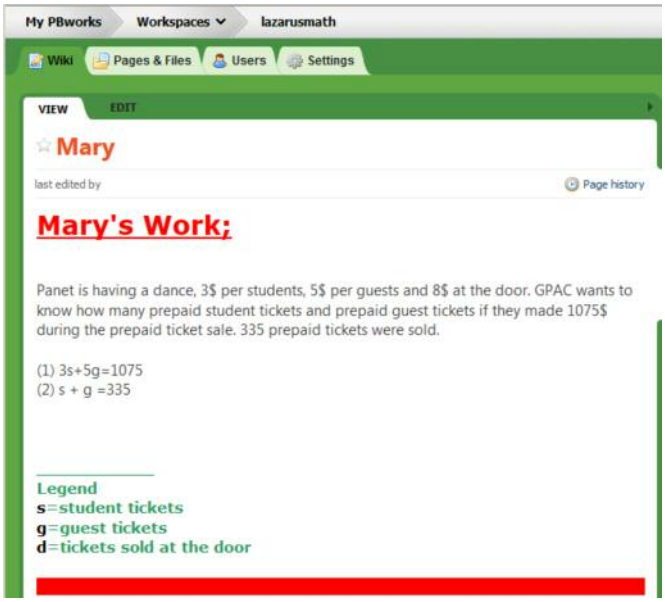


Figure 5: Individual student wiki page

For their first online experience in this course, students posted text to the wiki. This was done in class so that any issues could be addressed. Given the potential for delays, such as computer loading time, we found that it is important to have a task for students to work on during wait time.

Students worked individually for this activity, posting to their own wiki pages (see Figure 5). Each student created and posted two unique linear systems word problems along with the corresponding algebraic representations. One problem was required to have a unique solution and the second either infinite or no solutions. Prior to this posting, students had opportunities in class to create different types of problems and to get feedback from the teacher and peers. Their wiki postings can be used in a variety of ways. For example, students can visit each other's pages and comment on the constructions or contribute other representations. We found that such interaction requires classroom initiation and modelling, and for this, a few examples were shared and discussed in class.

Stage 3: Embedding GeoGebra Applets

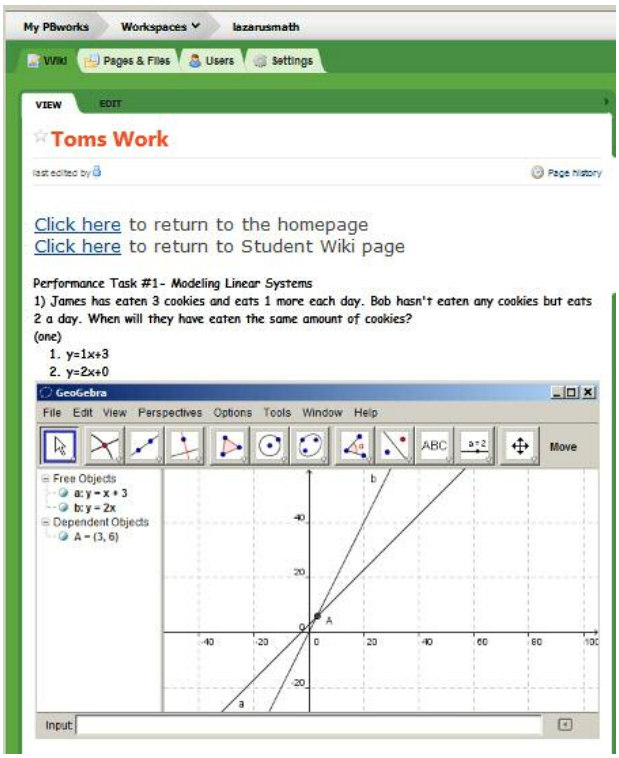


Figure 6: Embedding GeoGebra applets

Once students were comfortable with accessing and posting text to the wiki, they were introduced to embedding GeoGebra applets to wiki pages. This activity was also done in class to allow for troubleshooting any problems. Students embedded graphical models of the problems that they had created (see Figure 6).

Stage 4: Embedding Jing Videos



Figure 7: Season's Greeting Jing video

Jing, screencast software, was introduced next. Jing allowed students to record their screens and voices. This software was introduced in a fun way, just before the holiday season, when students worked in groups to create and share a season's greeting that they developed using GeoGebra (see Figure 7). For more information on how to embed GeoGebra applets and Jing videos on wiki pages in PBworks, visit the Collabmath wiki (**http://collabmath.pbworks.com**).

Stage 5: Commenting on Each Other's Work

Once they gained experience with participating in a math-talk learning community and with using the technologies, students started to comment on each other's work (see Figure 8). For this online collaboration, they worked together to develop a strategy for using similar triangles to determine the height of an inaccessible object. This collaboration was part of the "MPM2D Performance Task" (see below) that was initiated in the classroom, but students also contributed to the wiki between classes.

Students were encouraged to post text, GeoGebra models, and videos to explain their approaches. Prior to participating in this activity, the class contributed to a second conversation about communication in mathematics, this time with respect to what it means to contribute in an online collaboration. Similar to the approach to co-constructing criteria for participating in a math-talk community, we developed criteria for collaborating on a wiki. These criteria, which emphasized expanding on their peers contributions rather than merely agreeing or disagreeing, were included in a communication rubric for evaluating participation in the online discussions.

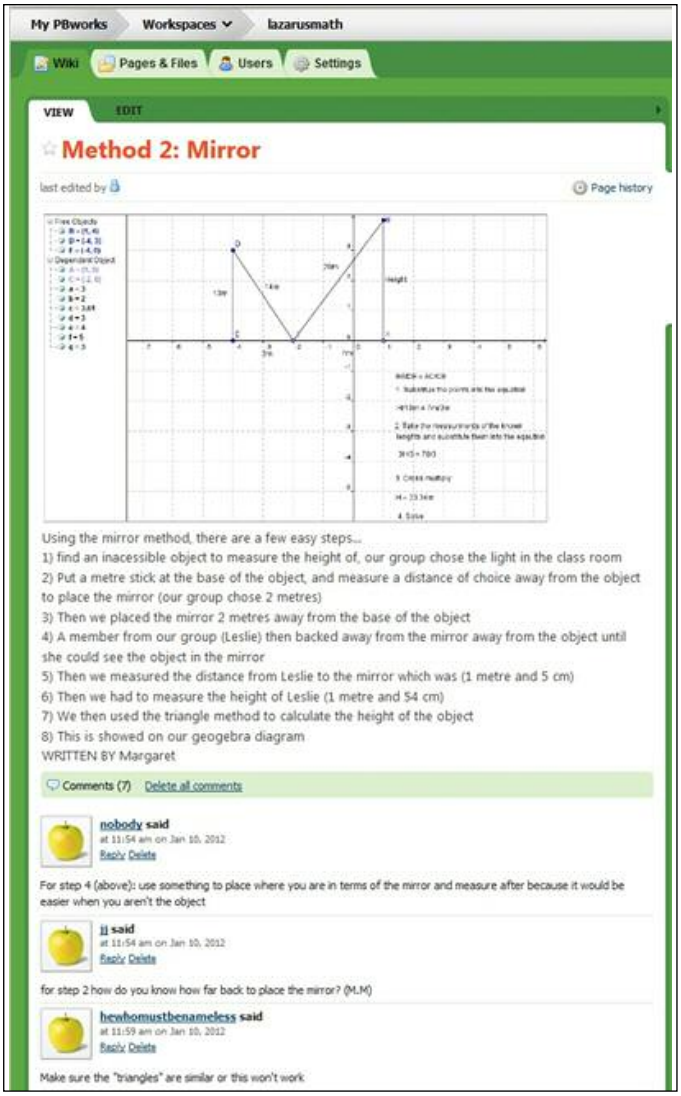


Figure 8: Collaborating on the wiki

Stage 6: Combining Skills

Students combined their technological and mathematical skills in an individual task. Each student selected a different inaccessible object and applied the group-generated strategy to determine the height of that object. For this individual component of the performance task, students were encouraged to submit their work on a wiki page (Figure 9) or in a form that they felt was most appropriate (e.g., paper and pencil).

MPM2D **Name:** _____

Performance Task
Solving Problems Related to Similarity

Curriculum Expectation
Use your knowledge of ratio and proportion to investigate similar triangles and solve problems related to similarity.

Task:

i. Choose an inaccessible object, the height of which you will measure.

ii. Select an appropriate method for measuring the height of this object. Choose one from the back of this page or develop a new one, if you wish.

iii. Think of how you can use the tools and a diagram similar to the one shown to measure the height of the object.

Part I: Online Collaboration and Jing Video for Developing a General Strategy

iv. Visit **lazarusmath.pbworks.com** and go to the *Student Work* page. *Click on Height of an Inaccessible Object*. Select a link to the method that you chose and post your strategy.

v. You must respond to the strategies posted by other members of this group. You can use text, a GeoGebra model, and/or a Jing video to share your thinking.

vi. Then, with a partner or in a group of up to four people, you must create a Jing video that summarizes the strategy. Use GeoGebra to model your work for this video. Refer to the ideas that are already posted to the web page.

vii. You will post this video to a new page on the website. Miss Lazarus will help with this. Your contributions will be graded using the rubric generated by our class for *Thinking* and *Communication*.

Part I is due by _____.

Part II: Measuring the Height of Your Object

viii. Apply the strategy that your group came up with to measure the height of the object that you selected in (i).

ix. Your solution may be submitted in writing or posted to your web page, using a Jing video. This is your choice. For this final solution, you must summarize all steps and calculations for measuring the height of your object. Refer to the strategy that your group generated. You should include at least one diagram in your own solution. Explain how accurate your diagram is and consider any possible sources of error. This individual report will also be graded using a rubric for *Thinking* and *Communication*.

Part II is due by _____.

Findings and Considerations

Blended collaboration provides new ways for students to interact with each other and with mathematical ideas. Combining a wiki, dynamic mathematics software, and screencast videos influenced how students questioned, reacted to, and elaborated on ideas. When a GeoGebra applet was shared on the wiki, for example, students could elaborate on the construction or explore other possibilities by manipulating the object and then re-posting to the wiki page. Screencast videos were also valuable, as they allowed students to post more detailed explanations as they recorded their screens and voices while interacting with objects.

Since the wiki and GeoGebra exist on the Web, access was available beyond class time, and students were encouraged to use these tools on their home computers or on the computers available in the school library. Taking into account that in some cases computer access was limited, when students collaborated online, they were generally given class time, and the activities were extended for longer periods so that they had sufficient time to visit the library, if they needed to. Initially, student commitment to using the online tools varied. There was a wide range of experience, skills, and motivation. As students gained computer access, experience, and comfort, however, they also tended to gain enthusiasm. They demonstrated this on the wiki, where in addition to sharing mathematical thinking, they were also posting jokes, season's greetings messages during the holiday, and thank-you messages.

Introducing tools in stages helped students develop the required skills and reduced the potential for frustration, but still some had considerable difficulty mastering all of the online activities. This range in skills raised issues related to assessment and evaluation. In terms of formative assessment, the online collaborative component provided a lens into student thinking that was valuable for supporting student learning. The range in technological skills, however, presented a problem for summative assessment that we addressed by giving more in-class support and providing students with choice in how they submitted their work. In adapting to meeting the technological needs of students in this class, we question the implications for assessment and evaluation in a technology-rich learning environment.

Our experience with this project showed that a wiki, GeoGebra, and Jing can be combined to provide an online collaborative environment that complements in-class activity. Looking back over the project, we see

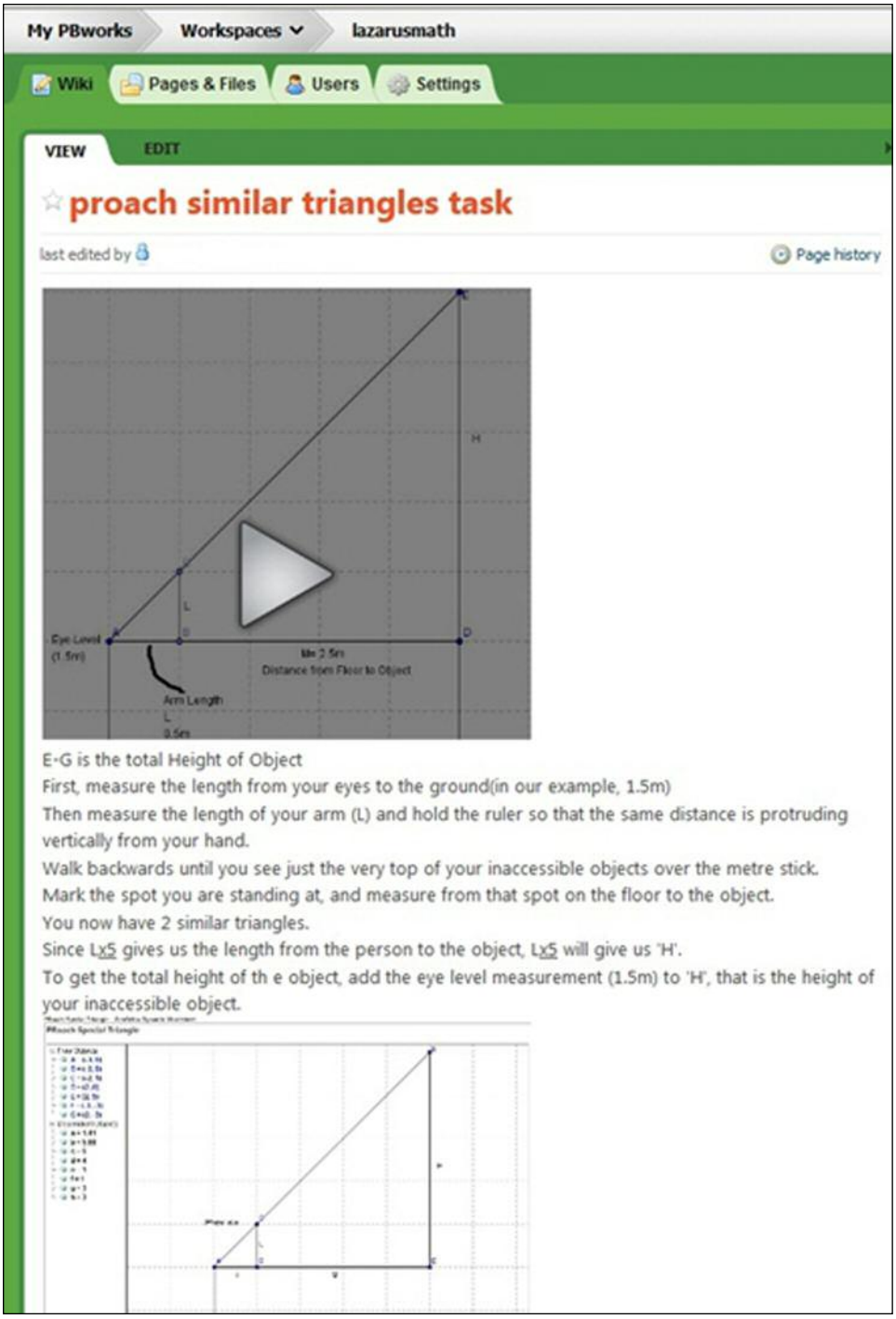


Figure 9: Combining the wiki, GeoGebra, and Jing

many ways in which student opportunities to share online could be increased or enhanced. Even though we may not have explored all of the possibilities, we found that students did benefit from sharing their work and exploring contributions on the wiki.

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8 PAGE INSERT ELEMENTARY MATH

ONTARIO ASSOCIATION FOR MATHEMATICS EDUCATION

Abacus

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REPRESENTING AND INTERPRETING PROBABILITY

TABLE OF CONTENTS

Abacus Editor Greetings 1
RESEARCH SUMMARY 2
Representing and Interpreting Probability
LINKS TO LITERATURE 3
- *Jumanji*
LET'S DO MATH 4-5
(Bansho (Board-Writing) Plan)
- One Die, Two Dice
LET'S DO MATH 6
- Two Choices plus Dishwashing
LINKS TO MANIPULATIVES . . . 7
- Colour Tiles and Spinners
LET'S DO MATH 7
- Predicting Likelihood
NEXT STEPS FOR YOUR PROFESSIONAL LEARNING. 8
- Suggested Readings

A Publication of the OAME/AOEM

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Is there anyone who hasn't wished that they could win a provincial lottery? It is often said that we have the same chance of winning the lottery as the chance of being struck by lightening. If only a few people are hit by lightening, then only a few people win the multi-million dollar lottery prize. If that is the case, why do we buy lottery tickets? These two examples of probability are two examples of very unlikely events; that is, an event that probably won't happen. Weather forecasts are another way we use probability every day. What does it mean when the probability of precipitation (POP) is 50%? It may mean that there is 0% chance of rain in one part of the city and 100% chance of rain in another part of the city. A 50% chance of a rainstorm covering the entire city would also lead to a POP of 50%. Probability influences our lives every day. In fact, anyone who thinks and plans for their future uses probability in their planning process (e.g., deciding what the chances are of a specific event) In this issue, the Research Summary and problems (with multiple solutions) focus on strategies for representing probability situations theoretically and through experiments and simulations. The problems we provide are designed for use within a three-part, problem-solving lesson structure.

LINKS TO LITERATURE AND MANIPULATIVES

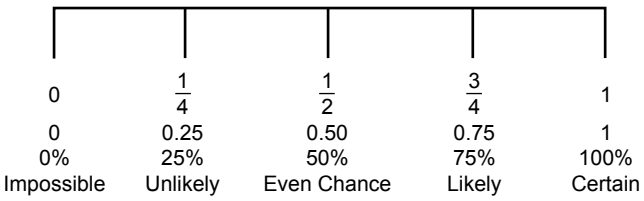


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As another school year comes to a close, it is likely that many of you are making plans for the summer based on the probability that the weather will be sunny and hot. Students also have experiences with probability. Watch a child opening presents and you might notice that they are making predictions about the contents. "This gift probably has new clothes in it." Also, probability provides an opportunity to explore the concept of fairness in a variety of social justice issues. In this issue I will take yet another look at colour square tiles in the context of probability, using a classic picture book as a context for solving a problem involving probability.

RESEARCH SUMMARY – REPRESENTING AND INTERPRETING PROBABILITY

Probabilities are associated with situations where the outcome is not known in advance or cannot be predicted. It is the study of uncertainty. Further, "probability is the study of measures of likelihood for various events or situations" (Small, 2009, p. 197). For younger students, probability language includes descriptions of situations, such as certain, impossible, and/or somewhere in between (like or unlikely). Also, young students could describe events using probability language like, always happens, likely to happen, unlikely to happen, never happens. Older students relate such probability descriptions to quantities (decimals, fractions, percentages) across a scale from 0 to 1, inclusive. An event that cannot occur has a probability (of happening) equal to 0 and the probability of an event that is certain to occur has a probability equal to 1.



Sample Space

In order to quantify probabilities, we need to define the sample space of an experiment and the events that may be associated with that experiment. The sample space is the set of all possible outcomes in an experiment. For example:

- If a die is rolled, the sample space S is given by $S = \{1,2,3,4,5,6\}$
- If two coins are tossed, the sample space S is given by $S = \{HH, HT, TH, TT\}$, where H = head and T = tail.
- If two dice are rolled, the sample space S is given by
  $S = \{(1,1),(1,2),(1,3),(1,4),(1,5),(1,6);\ (2,1),(2,2),(2,3),(2,4),(2,5),(2,6)\ ;$
  $(3,1),(3,2),(3,3),(3,4),(3,5),(3,6)\ ;\ (4,1),(4,2),(4,3),(4,4),(4,5),(4,6)\ ;$
  $(5,1),(5,2),(5,3),(5,4),(5,5),(5,6)\ ;\ (6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\}$

An event is a subset of the sample space; a specific outcome of an experiment. For example:

- A die is rolled. Event, E is the set of possible outcomes where the number on the face of the die is even. $E = \{2,4,6\}$
- Two coins are tossed. Event, E is the set of possible outcomes where there is exactly one Head. $E = \{(HT),(TH)\}$
- Two dice are rolled. Event, E is the set of possible outcomes where the sum of the numbers on the faces of the two dice is equal to four. $E = \{(1,3),(2,2),(3,1)\}$

Theoretical Probability

Theoretical probability is based on the fact that all outcomes are equally likely. For example:

$$P(A)=\frac{n(A)}{n}$$

$P(A)$: probability of event A

$n(A)$: number of occurrences of A (number of favourable outcomes)

n: Total number of possible outcomes (sample space)

- What is the probability of getting a 3, when a die is rolled? The event is "getting a 3" is $E = \{3\}$. Sample space, $S = \{1,2,3,4,5,6\}$.
  The number of possible outcomes in E is 1 and the number of possible outcomes in S is 6. Probability of getting a 3 is $P(3) = 1/6$.
- What is the probability of getting an even number when a die is rolled? The event "getting an even number" is $E = \{2,4,6\}$. Sample space is $S = \{1,2,3,4,5,6\}$.
  The number of possible outcomes in E is 3 and the number of possible outcomes in S is 6.
  The probability of getting an even number, P(even number) = $\frac{3}{6}=\frac{1}{2}$.

Tree diagrams, area models, and tables are used to organize and determine theoretical probability.

Experimental Probability

Unlike theoretical probability, experimental probability uses real data from situations to determine how likely outcomes will occur in the future. For example, here is some data about people's colour preferences. If a person is selected randomly from the group of 20 people what is the probability that this person likes green? So, Event, *E* is "likes green". The probability of choosing a person who prefers the colour, green is represented as a fraction or ratio of the number of green outcomes and the total number of outcomes.

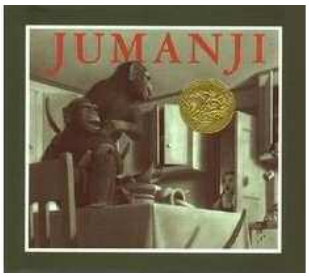
Color	Outcomes
red	6
blue	6
green	8
Total	20 (sample size)

P(*E*) or *P*(likes green) = number of green/total number of outcomes

P(*E*) or P(likes green) = $\frac{8}{20} = \frac{2}{5}$

According to Small (2009) "experimental probability is determined by the results of an experiment that has already occurred. In contrast, theoretical probability involves analyzing possible outcomes in advance and using logic and reason to predict what is likely to happen" (p. 203). The results of experimental probability approaches a theoretical probability when enough random samples are used; that is, with large enough sample spaces.

LINKS TO LITERATURE: *JUMANJI*



In the book, *Jumanji*, written by Chris Van Allsburg, in 1981, Peter and Judy find a game under a tree. They play the game and with each roll of the dice, the events described by the game board begin to materialize around them.

You may stop the story at various points to investigate the probability of rolling the number identified in the story. For example, read up to the point where Peter says, "If you roll a 12, you can get out of the jungle." Challenge the students to plan an investigation that determine the possible outcomes of rolling a sum of 12.

Strategy #1 – Cross Classification

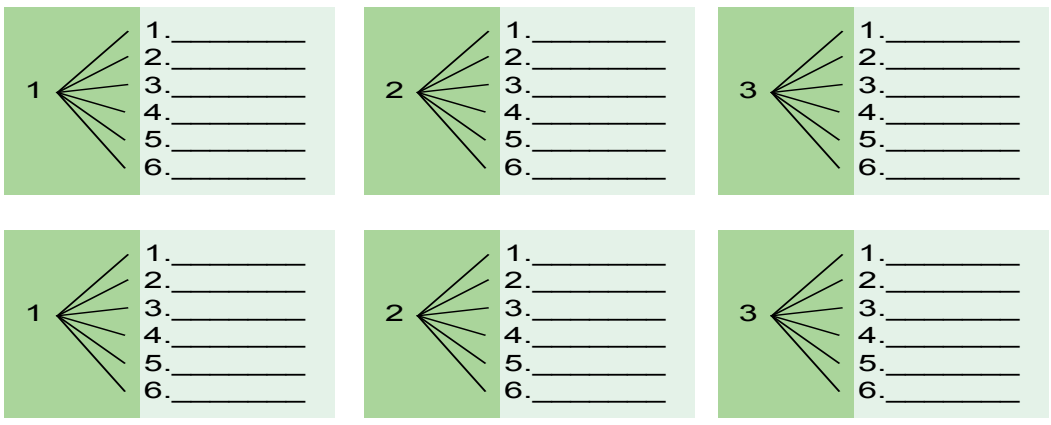
+	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Strategy #2 – Concrete Graph

2	3	4	5	6	7	8	9	10	11	12
1+1	1+2	1+3	1+4	1+5	1+6	2+6	3+6	4+6	5+6	6+6
	2+1	3+1	4+1	5+1	6+1	6+2	6+3	6+4	6+5	
		2+2	2+3	2+4	2+5	3+5	4+5	5+5		
			3+2	4+2	5+2	5+3	5+4			
				3+3	3+4	4+4				
					4+3					

Possible sums from two dice

Strategy #3 – a tree diagram



The book, *Jumanji*, was made into a movie in 1995. Watch the trailer on YouTube **http://www.youtube.com/watch?v=OJKHQLM8AbM** and investigate the probability in the following line from the trailer, "In the jungle you must wait until the dice say five or eight."

LESSON LEARNING GOALS

Identify the curriculum expectations (MOE, 2005) grade-before and grade-after the grade specific learning goal:
- to see the development of mathematical knowledge, skills, and strategies across grade levels
- to determine the mathematical focus of the problems chosen for Before (activation), During (lesson), and After (Practice) parts of the lesson

BEFORE (Getting Started) – 5 to 10 minutes

- Activating students' mathematical knowledge and experience using a prompt or problem, related to the math in the lesson problem.
- Includes student responses to highlight ideas and strategies.

One Die, Two Dice Problem:

Consider the faces of a die. How many times should you expect a 5 to show, when you roll one die 30 times?



Solution 1

Rolled the one die and predict four 5s.

1	2	3	4	5	6
𝍸	𝍸	𝍸	𝍸	𝍸	𝍸

Solution 2

- *Equally likely chance of getting any number 1, 2, 3, 4, 5, or 6.*
- *30 rolls, get five 1s, five 2s, five 3s, five 4s, five 5s, and five 6s.*

***Probability Experiment of 30 Rolls** – close but not exactly the same*

***Equally Likely** - each face has equal chance of coming up on each die roll - 30 rolls ÷ 6 faces = 5 times for each face*

DURING (Working on It) – 15 to 20 minutes

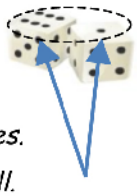
- Understanding the problem – Teacher asks, "What information from the problem are we using to make a plan to solve it? Explain." Teacher records in a list below the problem, the information that students identify.
- Students solve the problem on chart paper (landscape) with markers (visible for class discussion) in pairs or small groups.
- Teacher circulates to record different student solutions, in addition to the ones the teacher anticipated.

Predicting with Dice Problem:

If you roll two dice at least 10 times, how many times would you expect a 5 to appear?

a) Make a prediction. Explain.

b) Keep track and record your results.



What information are we going to use to solve the problem?

- *Roll 2 dice at the same time, at least 10 times.*
- *Keep track of dice combinations for each roll.*
- *Sum of dice (e.g., 1 die and 6 die -> sum of 2 dice is 7)*
- *How many times will a 5 will come up?*

AFTER (Consolidation) – 20 to 25 minutes

- Teacher selects 2 or more solutions for class analysis and discussion in a sequence (1st, 2nd, 3rd, etc.) based on mathematical relationships between the solutions and the lesson learning goal.
- Students (authors) explain and discuss their solutions with their classmates; co-construction of success criteria
- During whole class discussion, solutions are organized (often re-organized) to show mathematical elaboration from one solution to the next and towards lesson learning goal.
- Teacher mathematically annotates (math terms, math symbols, labelled diagrams, concise explanations) on and around solutions to make mathematical ideas, strategies, and models of representation explicit to students.

Experimental Probability

- ***Collect Data** - Record the number (frequency) of each dice sum for over 36 dice rolls (total number outcomes)*
- *Use data from experiment to determine "how likely a dice sum of 8" will come up after 36 dice rolls*
- ***Sample Space**- total possible outcomes in experiment - number of dice rolls is 36*
- ***Probability** of dice sum of 8 - using data collected*
- ***Event, E** is "dice sum of 8" (favourable event) using d*

P(E) or P(sum of 8) = number of sum 8/total number of outcomes = 5/36

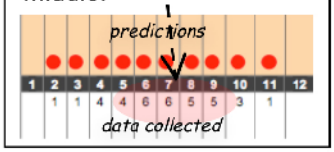
Predicting Outcomes

- *all numbers on the face of a die have equal chances*
- *different combinations 2 dice that sum to 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 and 12*
- *more combinations for sums of 6, 7, 8 shown in both the data experiment OR by listing all possible sums*

Solution 1

Our prediction was really off. We expected all sums to be equally likely but it does not look like this is correct.

The outcomes are bunched up around the middle.



Solution 2

Sums	How	# times got it	Sums	How	# times got it
1	never	0	7	1+6, 2+5, 3+4, 4+3, 5+2, 6+1	5
2	1+1	2	8	4+4, 3+5, 2+6, 6+2, 5+3	5
3	1+2 2+1	2	9	5+4, 4+5, 3+6, 6+3	4
4	2+2, 1+3, 3+1	3	10	5+5, 4+6, 6+4	1
5	1+4, 4+1, 2+3, 3+2	5	11	5+6, 6+5	1
6	3+3, 2+4, 4+2, 1+5, 5+1	7	12	6+6	1

Predicted All Sums Equally Likely Compared to Collected Data

Predicted All Sums Not Equally Likely - Listed All Possible Sum Combinations

Note: Mathematical annotations include mathematical vocabulary, symbols, elaborations of mathematical details from solutions, labels describing the method/strategy, and questions to further thinking. All annotations are records of students' mathematical discussion.

Grade 5	Grade 6	Grade 7
- determine and represent all the possible outcomes in a simple probability experiment - represent using a common fraction, the probability that an event will occur in simple games and probability experiments	- express theoretical probability as a ratio of the number of favourable outcomes to the total number of possible outcomes, where all outcomes are equally likely - predict frequency of outcome of simple probability experiment or game calculating and using theoretical probability of that outcome	- make predictions about a population given a probability - represent in a variety of ways all possible outcomes of a probability experiment involving 2 independent events

Theoretical Probability

- all outcomes for numbers (1 to 6) on dice are equally likely, BUT sums of dice rolls are not equally likely - some sums have more combinations of dice

- list all possible outcomes in a table or on a bar graph

- represent theoretical probability of an event (rolling dice sum 7) using fraction → 6/36

$$P(A) = \frac{n(A)}{n}$$

number of occurrences of A (number of favourable outcomes) → 6

probability of event A

total number of possible outcomes (sample space)

- all possible combinations of sums from 1 to 12, using 2 dice → 36

36 = 1+2+3+4+5+6+5+4+3+2+1

= 6+5+1+5+1+4+2+4+2+3+3

= 6+6+6+6+6+6 = 6x6 = 36

Probability of rolling dice sum of 7 = 6/36

Solution 3

Our prediction looked like group 1. But, we could not get 1 but we could get 12. So we started looking at what dice gave what sums. 5 comes up 4 times.

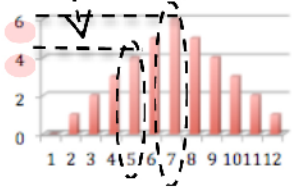
Sums	Total ways to get it	Probability
1	Impossible	0
2	1	1/36
3	2	2/36
4	3	3/36
5	4	4/36
6	5	5/36
7	6	6/36
8	5	5/36
9	4	4/36
10	3	3/36
11	2	2/36
12	1	1/36

Comparing Predictions to All Possible Combinations of Sums

Solution 4

We tallied and graphed it. Getting 1 has a probability of zero as it can never occur with 2 dice.

The probabilities go up from impossible to most likely at sum of 7 and then back down to unlikely at 2, 3, 11, and 12. 5 comes up 4 times.



Graphing All Possible Combinations of Sums (Mountain)

AFTER (Highlights/Summary) – 5 minutes

- Teacher and students revisit the student solutions for key ideas, strategies, and models of representation that are related to the lesson learning goal (anticipated success criteria)
- Teacher lists key ideas, strategies, and models of representation so the students can see how the mathematical details from their solutions relate explicitly to the lesson learning goal; summary of co-constructed success criteria

Highlights/Summary:

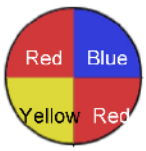
- ***Probability*** *describes how likely an event will occur.*
- ***Experimental probability*** *is data collected as the frequency of events occurring.*
  - *→ If rolling 2 dice 36 times (sample space), how often does the sum of 7 come up? Table shows it came up 6 out of 36 dice rolls → Experimental Probability (sum of 7)=6/36*
- ***Theoretical probability*** *is a ratio of the number of possible occurrences of a specific event in relation to total number of possible outcomes (sample space).*
  - *-> For a combination of sums using 2 dice, there are 36 different possibilities (sample space): for sum of 7 - there are 6 possibilities → Theoretical Probability(sum of 7)=6/36*
- ***Is the experimental probability always the same as the theoretical probability?***

AFTER (Practice) – 5 to 10 minutes

- Teacher chooses 2 or 3 problems, similar to the lesson problem for student to solve in pairs as a scaffold and individually.
- Problems could vary by number (choice, size), problem contexts, or what is unknown or needs to be solved
- Students are asked to solve these problems using a strategy different from the one they used for the lesson problem.

Coin Flipping Problem

What is the probability of spinning a red and flipping a head?





Solution 1

P(red) is $\frac{1}{4}$ and P(H) = $\frac{1}{2}$

So the probability of Red followed by heads is 1/8

Solution 2

Probability (RH) = $\frac{2}{8} = \frac{1}{4}$

Colour	Coin		Colour	Coin	
R	H	RH	B	H	BH
	T	RT		T	BT
R	H	RH	Y	H	YH
	T	RT		T	YT

Before (Getting Started)

Two Choices Problem:



Rosa just got married and is starting to think about having a baby. Her dream is to have one boy first and then a girl. What is the probability that she will have the children in the order she hopes for?

Solution 1

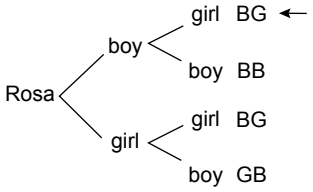
The likelihood that she will have a boy is half.

The likelihood that she will have a girl is half.

So the probability that Rosa will have a boy and a girl is one-half.

Solution 2

The probability that Rosa will have a boy followed by a girl is ¼ or one/fourth.



During (Working On It)

Dishwashing Problem:

Janine has three children two girls, Anise and Lada, and one boy, Ephrim. After breakfast, lunch and dinner, one child is chosen at random to do the dishes.

What is the probability that Anise will have to do the dishes in 2 of the 3 meals in a day?

After Consolidation

Solution 1

We could have AAA, AAL, AAE, ALA, AEA.

So the probability of Anise at two meals is 5 favourable outcomes.

Solution 2

Probability (A in 2 meals) = $\frac{7}{27}$.

B	L	D	DAY
		A	AAA
A	L	L	AAL
		E	AAE
		A	ALA
A	L	L	ALL
		E	ALE
		A	AEA
E	L	L	AEL
		E	AEE

B	L	D	DAY
		A	LAA
A	L	L	LAL
		E	LAE
		A	LLA
L	L	L	LLL
		E	LLE
		A	LEA
E	L	L	LEL
		E	LEE

B	L	D	DAY
		A	EAA
A	L	L	EAL
		E	EAE
		A	ELA
E	L	L	ELL
		E	ELE
		A	EEA
E	L	L	EEL
		E	EEE

Solution 3

We could have AAA, AAL, AAE, ALA, AEA, EAA, and LAA. So the probability of Anise at two meals is 7 favourable events out of 27 possibilities (3 x 3 x 3) or P(A in 2 meals) = $\frac{7}{27}$.

Coordinating Discussion for Student Learning:

Why might solution 1 be chosen first for student discussion followed by solutions 2 and 3?

- *Solution 1* – Students identified some but not all of the favourable outcomes
- *Solution 2* – Students use the table to list all the possible outcomes – shows total favourable and total sample space.
- *Solution 3* – Students list favourable events and show how to calculate the total number of outcomes in the sample space.

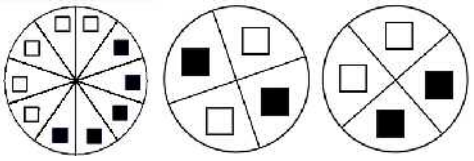
LINKS TO MANIPULATIVES: COLOUR TILES AND SPINNERS

"If you take a white tile out of the bag, you win a prize. Which bag will you choose? Why?"

Students who think additively may choose Bag A because it has twice as many white tiles as Bag B. Students who think multiplicatively may see that both bags are half white and half black.



Have students take and replace the tiles in each bag and keep track of the colours that they take. Then have students use the square tiles and fraction circles to create spinners.If the spinner points to a white square, you win a prize. Compare the results of spinning a spinner and taking tiles out of a bag. How do the results of using two of the following spinners compare?



LET'S DO MATH

Before (Getting Started)

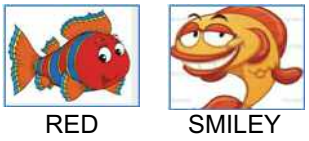
Predicting Likelihood Problem:

Which of the events in the chart are *impossible, unlikely, equally likely, likely* or *certain*? Identify at least one event for each category and then add two events of your own under the appropriate title.

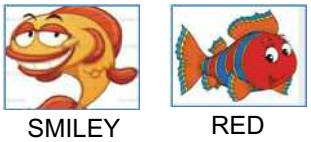
During (Working On It)

Predicting Fishy Events Problem:

The picture cards to the right represent the fish in a small aquarium. They are put in a bag.







1. What is the likelihood of each event?
   - picking ANGEL with all cards in the bag
   - picking SMILEY with all cards in the bag
   - picking RED with all cards in the bag
2. Which 3 cards are in a bag so that each probability statement is true?
   a) you are certain to pick RED
   b) it is likely that you will pick RED
   c) it is likely that you will pick SMILEY
   d) it is impossible to pick ANGEL

After (Consolidation)

Anticipating Student Response

Solution 1

Impossible	Unlikely	Equally likely	Likely	Certain
13	4	15	1,3	2

EVENTS

1 It will rain in April in Ontario.
2 My dog will sleep in my bed tonight.
3 I will grow to be taller than my mother.
4 I will play baseball this week.
5 I will play soccer this year.
6 I will read 10 books.
7 I will find an elephant in my closet tonight.
8 I will brush my teeth once today.
9 I will drink 4 full glasses of water.
10 I will have beans for dinner.
11 I will have some fruit today.
12 The sun will shine today.
13 The sun will shine tonight over my house.
14 When I roll a die, I will get a 4.
15 When I flip a coin, I will get heads.

Solution 1

1. a) Probability (Angel) = unlikely (Angel is 1 out of 6 cards)
b) P(SMILEY) = unlikely (2 out of 6 is still less than half)
c) P(RED) = equally likely – it is just as likely that you will pick RED as pick any other fish (3 RED and 3 other fish)

2. a) Put 3 RED cards in the bag only
b) Put 2 RED and 1 SMILEY in the bag
c) Put 2 SMILEY and 1 Angel (or RED) in the bag
d) put 3 other cards in bag – omit ANGEL

Solution 3

1 a) P(ANGEL) = $\frac{1}{6}$

b) P(SMILEY) = $\frac{2}{6} = \frac{1}{3}$

c) P(RED) = $\frac{1}{2}$

2 a) 3 RED cards

b) 2 RED 1 other

c) 2 SMILEY 1 other

d) 3 cards no ANGEL

Solution 2

Impossiblie	Unlikely	Equally Likely	Likely	Certain
P(ANGEL) = impossible with 0 ANGEL card out of 3 cards to pick	P(ANGEL) P(SMILEY)	P(RED)	P(RED) = likely with 2 RED out of 3 cards to pick P(SMILEY) = likely with 2 SMILEY out of 3 cards to pick	P(RED) = certain with 3 RED out of 3 RED to pick

Coordinating Discussion for Student Learning:

Why might solution 2 be chosen first for student discussion followed by solutions 1 and 3?

- *Solution 2* – This solution shows the probabilities on the likelihood line from impossible to certain. So it is good to start with these understandings and using words to describe events.
- *Solution 1* – Students go through and support each argument with an explanation of why the likelihoods are where they are.
- *Solution 3* – Students here represent the probabilities with fractions.

$$\frac{\textit{number of favourable events}}{\textit{number of possible events}}$$

NEXT STEPS FOR YOUR PROFESSIONAL LEARNING

Application to Your Classroom

- Try these probability problems yourself first and then offer them to your students.
- To suit your students' learning needs, vary the ways that you have students engage with probability concepts. Use different materials, such as counters, square tiles in bags, dice, spinners, and attribute blocks.
- Consider the purposes for representing and interpreting probability in local classroom and school community contexts. Some interesting contexts for probability include predictions of the weather and predicting outcomes of dice, card and board games and game playing strategies.
- Practise noticing the different strategies and the breadth of the mathematics your students use to describe, show and make sense of probability outcomes in relation to experimental and theoretical situations. Take note of the kinds of probability language students' use in the earlier years and the ways that students make sense of using fraction, decimal and percentage representations of probability. As well, provide ample opportunities for students to create theoretical models and compare them to experimental results, using tree diagrams, tables and cross-classification charts.
- Think ahead of the kinds of mathematical annotations you will record on and around the student solutions to make explicit key mathematical ideas, strategies for reading data as the students discuss and analyze each other's solutions.

SUGGESTED READINGS

Some readings used in the Research Summary and in the development of the problems and solutions are listed below:

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CLOSING

Our next issue will focus on proportional reasoning across the five strands (i.e., number sense and numeration, measurement, geometry and spatial sense, patterning and algebra, data management and probability). As well, the next issue will outline the ways that teachers need to know these notions for teaching, as well as, the ways that teachers can use these mathematical ideas and pedagogical strategies within a problem solving-based teaching and learning lesson framework.

CALL FOR *ABACUS* SUBMISSIONS

Send your teaching strategy ideas, problems and student solutions to the *Abacus* Co-Editors via email kathkubo@rogers.com or mkestell@sympatico.ca

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