8-PAGE INSERT ELEMENTARY MATH
ABACUS

Ontario Mathematics Gazette

OAME – ONTARIO ASSOCIATION FOR MATHEMATICS EDUCATION

AOEM – ASSOCIATION ONTARIENNE POUR L'ENSEIGNEMENT DES MATHÉMATIQUES

Vol. 53 #1
September 2014

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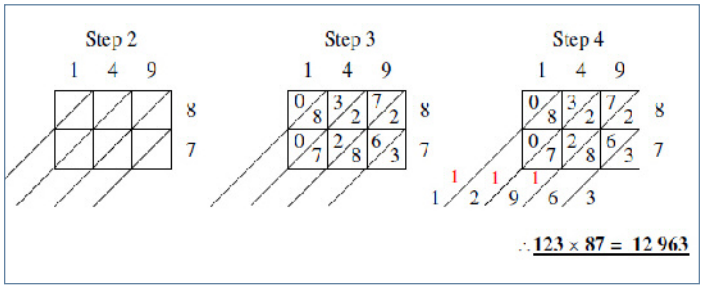
At the beginning of the winter, there were at least 66 students registered in a ski class. After the class started, 11 boys transferred into this class and 13 girls transferred out. The ratio of boys to girls in the class was then 1 : 1. Which of the following is not a possible ratio of boys to girls before the transfers?

(A) 4 : 7 (B) 1 : 2 (C) 9 : 13
(D) 5 : 11 (E) 3 : 5

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See Hey, It's Elementary – The Power and Responsibility of "Why"



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Please keep in mind the following criteria when making submissions to the *OMG*:

- The ideas/activities must be of interest to the readership.
- The ideas/activities must be fresh and innovative.
- The mathematics content must be appropriate for the readership.
- The mathematics content must be accurate.
- The article must be well written and easily understood.
- The article and its ideas must be free of sexual, ethnic, racial, or other bias.
- The article must not have been previously published, nor should it be out for review by other publications.
- The article must be original.

Articles must be word-processed in MS Word, double-spaced with wide margins, not exceeding 10 numbered pages of text, and prepared according to the *Publication Manual of the American Psychological Association, Sixth Edition*. Figures and diagrams should be drawn by computer, if possible, or drawn in black ink in camera-ready form. Embedded images must also be submitted separately in jpeg or tif format. Proof of the photographer's permission is required, and for **photos of students** under the age of 18, the written permission of a **parent or guardian is required**.

You must submit **one complete copy** of your article, embedded with any tables, figures, and captioned photographs or graphics, to the Editor, Immaculate K. Namukasa, along with **separate files for each of the text, graphics, and/or photographs**. Please e-mail all files to Immaculate K. Namukasa at **immaculate.namukasa @oame.on.ca**.

Your name should not appear anywhere in your article, including websites, so that your article can be sent out for blind review. Your name, full mailing address, and e-mail address must be included on a separate sheet. Upon review, you will be notified as to whether your article has been accepted for publication (as is, or pending minor or major revisions) or rejected.

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The *Ontario Mathematics Gazette*—ISSN 0030-3011—is indexed in the Canadian Education Index and is published four times per year. Its Canadian Publication Mail Product Sales Agreement Number is 40051074.

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Advertisements for publication in the *Ontario Mathematics Gazette* should be sent to **Robert Sherk** at the above address. Courier is recommended to avoid possible delays. Deadlines for advertisements are January 23 for the March issue, April 1 for the June issue, July 1 for the September issue, and October 1 for the December issue.

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▲ Editor's Message

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Immaculate Kizito Namukasa is an associate professor at the University of Western Ontario. Her research projects include "Mathematical thinking and activity of elementary and middle school learners," "New teachers learning to teach mathematics," "Expert teachers implementing innovative mathematics methods and environments in their classrooms," "Parents doing mathematics with their children in community settings," and "Complexity research inquiry and critical mathematics education in Canadian and international contexts."

At two sites this summer, I had the opportunity to give further thought to the role of manipulatives in teaching mathematics—both elementary and secondary—at a working session at the PME/PME-NA Annual Conference, an international mathematics education conference, and at a six-day math workshop for parents with their elementary school children at the University of Western Ontario. With the increase in design and use of tablet-based apps for learning mathematics, inquiry on manipulatives is at the forefront of forums about the use of learning tools. At the conference session, experts with manupulatives shared and reflected on their work on virtual manipulatives. Beyond making math easier and fun for many students, they reflected on aspects such as the role of concrete, virtual, and tablet-based manipulatives in learning mathematics. Manipulatives are in a broader network of learning tools. Also, earlier definitions of manipulatives need to be elaborated on in light of computer-based and computer-enabled manipulatives.

In a parent–child project in May at Western, parent–child diads (and triads) engaged in mathematics activities that utilized several learning tools. Children were then invited to draft interfaces for digital games based on the math activities. A researcher from Information Science, Lu Xiao, and I organized this workshop. Graduate students also comparatively analyzed close to a hundred iOS apps for teaching integers to middle grade students. This work is already revealing interesting findings, including a taxonomy of

apps based on what is visualized or, even better, gamified in an app. In some apps that replicate workbooks, it is only drill or quizzes, lists of unrelated practice exercises that are gamified; yet in others, it is math manipulatives, concept building, problem solving, or consolidation tasks that are gamified. In addition, we found that apps differ by the understanding of the designer of what is involved in learning. Some are overly prescriptive, whereas others include room for adaptation by the learner via user choices and inputs, and for emergence of novel learning. In the workbook-like apps, we discovered that it was the parents who had to externally motivate or push their children to use these apps. Computer activities or apps that are well designed to go beyond providing mere random practice motivate learners to use them. They build on and extend classroom mathematics. Only a handful of children in the parent–child workshops at Western, including those who have regular access to or own a tablet themselves, were able to provide an example of such apps. The great news is that workshop attendees indicated that their teachers had recommended these games. A larger number of them were, unfortunately, using workbook-like apps or games. And some simply did not play any math games on computers, tablets, or phones, despite the numerous other games they played on these devices.

If you are wondering what apps have to do with concrete manipulatives, wonder no more. Type in a search engine for apps, such as the apps store, a name of a specific manipulatives, say, geoboards or base-ten blocks, and watch how much work app designers have already done to replicate concrete manipulatives or, even better, games that are based on concrete materials. The complementary relationships between concrete and virtual manipulatives is key. Students who have used concrete geoboards understand better when they are working with an app based on geoboards. Many virtual manipulatives have been designed to build on and to extend progress made with concrete manipulatives. Several teachers who extensively utilize manipulatives in their class would tell you that they use those materials that promise to pay off in terms of being reusable for several topics and in terms of having pictorial or symbolic counterparts.

But what is your reason for utilizing manipulatives—concrete, virtual, and touch-tab manipulatives. With Stanley and Tuchtie (Namukasa, Stanley, & Tuchtie, 2009), we talked about rationales for using manipulatives that go beyond using these tools because they make math fun and easy for many students. We outline rationales offered in mathematics education for manipulative materials in use as early as the seventeenth century, to more recent ones that draw from complexity science. Manipulatives bridge gaps for learners, including those between topics such as number sense and patterning; between elementary and secondary math topics; and between concrete situations and abstract mathematics language. Manipulatives also provide mathematics representations and structures needed to learn abstract concepts. They lay the foundation for mathematics concepts by providing multiple representations and embodiments of concepts, as well as by engaging various and integrated senses/responses beyond visualization or touch in learning. Furthermore, the actions, operations, and activity afforded by manipulatives help in mediation of mathematics activity and in negotiation of math meaning. For bodily knowing, in addition to providing the structures for abstract knowing, is legitimate knowing in itself. So the relationship between the materials and tools, and the mathematical concepts associated with them, is a close one. Gadamer (1992) asserts that concepts are constantly in the process of being formed, or, more generally, that the "thing in itself" (in this case, mathematical concepts) is nothing but the continuity with which the various perceptual perspectives shade one another. "[E]very 'shading' of the object of perception is exclusively distinct from every other, and each helps co-constitute the thing in itself as the continuum of these nuances" (Gadamer, 1992, p. 447). Complexity research supports the view of some concepts to be dynamic and dependent on tools used. Mathematical concepts such as number concepts, patterning, and symmetry appear to arise from many human activities and could be usefully construed as emergent concepts. They are likely to arise from activity and interactions with various learning tools.

This issue introduces a new column, *Ask Assessment Abby*. Among the firsts is also the first message from the new OAME president, Paul Alves. It is also a issue where several OAME announcements are made: OAME 2015 conference; OAME Awards 2015; OMO competition proceedings. As well, proceedings of the OAME June Board meeting are shared in the Directors' Dialogue. Two articles are published—the first, by Ami Mamolo and Kevin Thomas, shares a project with a social-justice focus; and the second, by Ian Brodie and George Gadanidis, shares the focus of Math Performance Festivals run by Western. This issue also includes a book review of the *The ANIE: A Math*

Assessment Tool That Reveals Learning and Informs Teaching a chapter sample of Mathematical Models for Teaching by Ontario authors Ann Kajander and Tom Boland; and an article announcing the launch of a Canada-wide math competition, COMC. The regular columnists have once again written inspiring pieces on such topics as ratio problem solving (*What's the Problem?*); the power and responsibility of "why" questions (*Hey, It's Elementary*); NCTM activities and resources for new teachers (*OAME/NCTM Report*); and provincial digital learning resources (*What's New with Clips?*).

Let me conclude my message with this quotation by David Reid in his National Presentation at the PME/PME-NA conference in Vancouver about Ontario mathematics education:

> Mathematics education research in Ontario benefits from a strong network among researchers and with a variety of stakeholders through interactions with the Ontario Association of Mathematics Education, the Ontario Mathematics Coordinators Association, and the Ontario Ministry of Education. Monthly meetings of the Fields Institute Mathematics Education Forum helps to support this network. Researchers in Eastern and Western Canada do not benefit from the same level of integration, organization, and support, but nonetheless have built strong research communities...

This has been an amazing two-year experience—editing nine issues. This issue is the last issue of my term as *Gazette* editor. Thank you, *Gazette* readers. Thank you, *Gazette* writers. I have had the good fortune of working in the company of many positive and supportive OAME leaders, associate editors, reviewers, and columnists. For those readers who would like to keep in touch, you may find my math blog at **www.namukasa.ca/**. Have a rewarding new school year!!!

References

Gadamer, H.-G. (1992). *Truth and method.* (2nd rev. ed). New York: Crossroad.

Namukasa, I.K., Stanley, D., & Tutchie, M. (2009). Virtual manipulative materials in secondary mathematics: A theoretical discussion. *Journal of Computers in Mathematics and Science Teaching, 28*(3), 277–307.

Reid, D. (2014, July). National Presentation at the PME/PME/NA Annual Conference. Retrieved from **www.pme38.com/program/** ▲

▲ President's Message



PAUL ALVES
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Welcome to another school year and to the first issue of volume 53 of the *Gazette*! I am writing this message on a warm August afternoon, but thoughts of the classroom are not far off as I anticipate the start of another school year. My goal with this first President's Message is to tell you a bit about me and, perhaps more importantly, to use that story as an invitation.

I will be starting my eighteenth year of teaching (I had to double-check that number), and in the Fall, I will be working with a great group of educators at Fletcher's Meadow Secondary School in Brampton. Going back over those 18 years, my teaching career has seen many detours, and every one of them has enriched me as an educator and person.

I started teaching in Dufferin-Peel at Cardinal Leger Catholic Secondary School. It was at Cardinal Leger where I first learned about OAME and our local chapter, CHAMP. I can clearly recall a whiteboard, with the CHAMP logo emblazoned on it, that sat in a central location in our math office. Usually taped to its surface was the latest CHAMP newsletter and various flyers advertising events for CHAMP and OAME. My first annual conference in 1997 was as a member of the Leger math department, and I still have some of the materials from the workshops I attended at that conference. I subsequently moved from Leger to Orangeville District Secondary School (ODSS) in the Upper Grand District School Board. Although it was a short time at ODSS, I benefited greatly from my time teaching with a very dynamic and creative group of teachers. They taught me a great deal and encouraged me by their example to be involved in our professional organizations.

From ODSS, I moved to Bramalea Secondary School in the Peel District School Board. It was while I was at Bramalea that I first became involved with CHAMP. I answered a call for volunteers to help prepare math contests that we administered in our chapter for Grades 7–11 students. I couldn't have imagined that answering a simple request to help create questions for a math contest would have brought me such a rewarding professional experience. I have met so many talented

and devoted math teachers through my association with CHAMP and OAME. One lasting impression that has formed is the idea that the best professional development is provided by teachers for teachers. It is from involvement at the grassroots level that some of the best learning can occur. When groups of teachers come together to discuss and learn from a shared experience, we all become better teachers. I think the highlight for me and my involvement with CHAMP came at OAME 2014. As exhausting as it was, it was so rewarding to be a part of a group of dedicated volunteers who were doing something purely for the good of mathematics education in our province. And so now, although my road has had a few twists along the way since I was hired at Bramalea, the one constant has been my professional association with CHAMP and OAME. It has reminded me that to teach is to learn.

As this school year begins, I would invite all of our members to consider becoming involved as well. Check the Chapters section of the website and find out who your chapter representative is and reach out to him or her. Find out how you can become involved in your local chapter. OAME exists because of the vitality of its local chapters, and the chapters exist because of the involvement of volunteers. It could be the beginning of a very rewarding professional experience.

It will be a busy year for OAME. In addition to the many events organized by the chapters, the Leadership Conference is happening November 6–8, 2014 in Toronto. We are looking forward to a great conference with Amy Lin, Dr. Cathy Bruce, Shelley Yearley, and Dr. Jo Boaler. Registration information can be found by visiting **http://oame.on.ca/main/files/conference/L2014.pdf**. The provincial conference will take place May 7–9, 2015 at Humber College. The keynote speakers—Marian Small, Dan Meyer, and Nora Newcombe—will be headlining a great program with other noted mathematics educators. If you would like to submit a proposal, please visit the Session Proposals page at **www.oame2015.ca**. Another highlight is the provincial OMO playdowns happening June 5–6, 2015. OMO will be hosted by COMA and will be taking place in Ottawa. As you can see, this will be a busy year for OAME, and we hope that you can be a part of the many events being hosted by chapters and the organization.

In closing, I would like to pass on my thanks to a couple of people. First, a big thank you goes to outgoing president, Sonia Ellison. Sonia is just one of the many educators I have been lucky enough to learn from through my involvement in CHAMP and OAME. OAME has gone through some structural changes in recent years, and it is through the leadership of past presidents like Cathy Hall and Sonia that we have emerged as a healthy and vital organization. I will rely on Sonia for her wise counsel as I navigate my way through this year as president. Secondly, our Editor, Immaculate Namukasa, will be moving on to other professional pursuits this year. Being an editor of any publication is a task that requires a great deal of patience, organization, and knowledge of the subject matter. We were so fortunate to have Immaculate at the helm because she was the perfect blend of each of those traits. We wish her well as she moves on in her professional pursuits.

I hope you enjoy this issue of the *Gazette* and that you look for ways to become professionally involved. If you have any ideas or suggestions as to how OAME can serve its membership, please feel free to send me an e-mail. And don't forget to follow us on Twitter **@oamecounts**. Have a wonderful start to the school year. ▲

▲ Call for Manuscripts

The *Ontario Mathematics Gazette* is inviting manuscripts for all grade levels.

Instructions for submission of manuscripts are found on page 1 of any *OMG*.

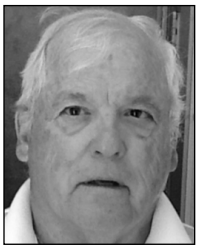
Contact the Editor for further details.

Erratum in:

Book Review of *Mathematical Models for Teaching: Reasoning Without Memorization* 2014 June; 52(4): 35. Boland, T., & Kajander, A. [corrected to Kajander, A., & Boland, T.].

▲ OAME/NCTM Report

TODD ROMIENS
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Todd recently retired from teaching K–12 methodology in the Faculty of Education at the University of Windsor. He is a past president of OAME and a life member, as well as a previous editor of the Gazette*.*

Welcome to another school year that I hope will be exciting for your students and you. If you haven't already done so, it is a good time to join the National Council of Teachers of Mathematics. This is especially true if you are just starting out in your teaching career.

NCTM was founded in 1920. It is now the world's largest organization concerned with mathematics education, with more than 80 000 members worldwide. It presents itself as "the public voice of mathematics education, supporting teachers to ensure equitable mathematics learning of the highest quality for all students, through vision, leadership, professional development, and research."

NCTM holds annual and regional conferences for teachers, publishes four print journals, and has been highly influential in the direction of mathematics education in the United States and Canada.

Teaching Children Mathematics is a journal that supports improvement of pre-K–6 mathematics education by serving as a resource for teachers so as to provide more and better mathematics for all students in these grades.

Mathematics Teaching in the Middle School helps teachers in Grades 5–9 by acting as a resource for them, as well as for supervisors and teacher educators.

The *Mathematics Teacher* is a journal devoted to improving mathematics education in Grades 8–12.

NCTM does not conduct research in mathematics education, but it does publish the *Journal for Research in Mathematics Education*. It is the most influential periodical in mathematics education research worldwide.

All of these journals are teacher resources that serve as forums for the exchange of mathematical ideas, activities, and pedagogical strategies and/or sharing and interpreting research. They provide a medium for sharing activities to deepen the understanding of mathematical ideas and linking mathematical education research to practice.

You can go to **www.nctm.org** to obtain membership, which comes with one of the journals listed above. Memberships with electronic versions of the journals, which are somewhat less expensive, are also available. Additional publications for all interests and grade levels can be found at **www.nctm.org/catalogue**, where they are listed with prices.

If you have the means to attend a conference, please note the following regional conferences:

- Indianapolis, Indiana – October 29–31, 2014
- Richmond, Virginia – November 12–14, 2014
- Houston, Texas – November 19–21, 2014

The annual meeting will be in Boston, Massachusetts from April 15–18, 2015. Each conference will have over 200 presentations, many hands-on workshops and activities, collaboration with like-minded educators, and a large exhibit hall.

NCTM offers electronic resources to facilitate the learning process. You can explore these to see how exciting mathematics teaching can be.

The Illuminations website (**www.illuminations.nctm**) contains classroom-ready lessons for pre-K–12 teachers. The site also contains interactive tools that help you to bring math concepts to life in class. Reviews of web resources round out the collection.

The *Figure This* website (**www.figurethis.nctm**) provides 80 math challenges for middle school students. Each challenge explains how math is used in the real world, and provides a hint to get started, complete solutions, questions to ponder, and resources for further exploration. The website's design helps families do math at home.

The *Reflections* website (**www.reflections.nctm**) is designed to enhance your ability to critically reflect on your teaching methods by watching videos of other teachers. Reflections helps you to look closely at your math teaching through examination of lessons and sample student work. It includes lessons on algebra in Grades 3–8.

A new online resource entitled *Real World Math* was launched by NCTM in August 2012. It is intended for use with middle school or high school students, and contains over 150 articles, lesson plans, and activities, selected because they connect math to the real world and the

classroom. *Real World Math* can be accessed through any internet device, and if you purchase it, you will have ongoing continuous access without ever having to renew the subscription. Recently, 40 new activity sheets, 20 lesson plans, 27 URLs, and 2 applets have been added to the material. Each article includes lesson plans, activity sheets, web links, and applets to provide the tools necessary to meet learning objectives.

Because of the vast amount of material available, I encourage all to join up and reap the many benefits of being a member. ▲

▲ Thank You

The OAME would like to thank the following members for reviewing articles for the *Gazette* during the last year. Peer review is an important element in ensuring the ongoing quality of this publication.

Melissa Baker
Cathy Chaput
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Members who are interested in reviewing articles can email Tim Sibbald (**tim.sibbald@oame.on.ca**) to be added to the list. Please indicate which divisions of math education you feel comfortable reviewing. Note that no one is ever obliged to review an article; it is always "by request," to respect varying workloads.

▲ What's the Problem? Ski School

SHAWN GODIN
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Shawn is the head of Mathematics, Business Studies, and Computer Science at Cairine Wilson Secondary School in Orleans, Ontario. He is a past member of the OAME Board of Directors and is currently Editor-in-Chief of the Canadian Mathematical Society's problem-solving journal, Crux Mathematicorum.

Welcome back, problem solvers. I hope you had a great restful summer. Last time, I left you with the following task:

> At the beginning of the winter, there were at least 66 students registered in a ski class. After the class started, 11 boys transferred into this class and 13 girls transferred out. The ratio of boys to girls in the class was then 1 : 1. Which of the following is not a possible ratio of boys to girls before the transfers?
>
> (A) 4 : 7 (B) 1 : 2 (C) 9 : 13 (D) 5 : 11 (E) 3 : 5

This problem was question #25 from the 2013 Grade 8 Gauss Contest, produced by the Centre for Education in Mathematics and Computing at the University of Waterloo. You can check out past Gauss and other mathematics and computing contests at the CEMC website, **http://cemc.uwaterloo.ca**.

Looking at the problem, we see that after the transfer, the ratio was 1 : 1; that is, there were equal numbers of boys and girls. To make the numbers equal, 11 boys had to transfer in and 13 girls had to transfer out, so at the start, there must have been 24 more girls than boys.

When one does not know what to do, there is no shame in trial and error. In many cases, the overall structure of the problem will reveal itself, or at least a hint of a pattern or strategy may come to mind.

Solution 1: Suppose we look at combinations that are permitted, and the ratios that are created. Figure 1 shows a spreadsheet set-up that can be used to solve the problem. We will choose a value for the number of "girls before," from which we can calculate the number of

girls after 13 left. The number of "girls after" must be the same as the number of "boys after," so since 11 boys came in, there were 11 fewer at the start. Notice in column E, the common greatest factor of "girls before" and "boys before" is determined so the ratio could be put in lowest terms (columns F and G).

	A	B	C	D	E	F	G
1	Girls before	Girls after	Boys before	Total before	Common factor	Ratio boys	Ratio Girls
2	40	=A2-13	=B2-11	=A2+C2	=GCD(A2,C2)	=C2/E2	=A2/E2
3							

Figure 1

Given a reasonable starting point of 40 girls, and copying the formulas down, we can see what happens when the number of girls is increased by one. Figure 2 shows the possibilities for 40 through 50 girls. Notice that options (B) 1 : 2 and (D) 5 : 11 appear in rows 6 and 10, respectively. Thus, our problem is solved, since the ratio 5 : 11 appears only if there are 64 students in total.

	A	B	C	D	E	F	G
1	Girls before	Girls after	Boys before	Total before	Common factor	Ratio boys	Ratio Girls
2	40	27	16	56	8	2	5
3	41	28	17	58	1	17	41
4	42	29	18	60	6	3	7
5	43	30	19	62	1	19	43
6	**44**	**31**	**20**	**64**	**4**	**5**	**11**
7	45	32	21	66	3	7	15
8	46	33	22	68	2	11	23
9	47	34	23	70	1	23	47
10	**48**	**35**	**24**	**72**	**24**	**1**	**2**
11	49	36	25	74	1	25	49
12	50	37	26	76	2	13	25

Figure 2

You may not be convinced that this is the only place that 5 : 11 occurs, but if you convert each of the ratios to a decimal, you will see that the ratio of boys to girls is increasing as the number of girls increases, so any ratio that appears will only appear once. I invite you to add a column that calculates the ratio as a decimal and to extend the table to find the other three choices to verify that (D) is the only situation that doesn't satisfy the conditions of the problem.

Next, let's use trial and error in another way. Since this is a multiple-choice question, we will examine the choices one at a time.

Solution 2: If the ratio was 4 : 7, then for every 4 boys, there were 7 girls. The possibilities for the number of boys and girls would then be: 4 and 7, 8 and 14, 12 and 21, etc. To satisfy the conditions of the problem, we need to have 24 more girls than boys, and for there to be at least 66 students in total. The first ten cases are examined in Figure 3.

Number of Boys	Number of Girls	Difference	Total
4	7	3	11
8	14	6	22
12	21	9	33
16	28	12	44
20	35	15	55
24	42	18	66
28	49	21	77
32	**56**	**24**	**88**
36	63	27	99
40	70	30	110

Figure 3

We see from our table that the ratio of 4 : 7 is possible. If we started with 32 boys and 56 girls, there would be 88 students, and the ratio would be 32 : 56 = 4X8 : 7X8 = 4 : 7. If 11 boys transferred in, there would now be 43 boys, and if 13 girls transferred out, there would be 43 girls, so the ratio would be 1 : 1 and the problem is satisfied. More importantly, a pattern has emerged. Notice that the numbers in the difference column were all multiples of 3, and the numbers in the total column were all multiples of 11. If this pattern carries over to the other choices, it will make our solution easier.

In Figure 4, we explore the case of 16 boys and 28 girls. Using two colours of tiles or linking cubes, we can represent the possibilities by using a number of groups of 4 for the boys (white squares) and the same number of groups of 7 for the girls (shaded squares). It is easy to see that in each case, the total number of students has to be a multiple of 11 (a group of boys coupled with a group of girls). Similarly, we can see that the difference between the number of boys and the number of girls must be a multiple of 3 (the lighter-shaded squares).

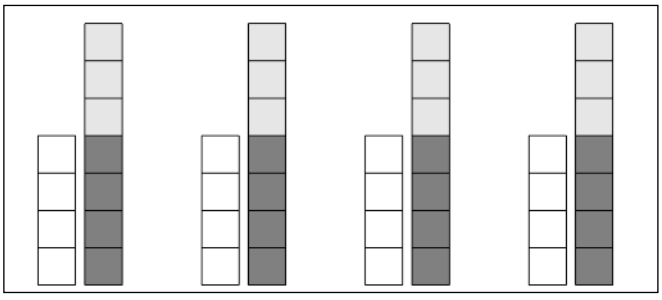


Figure 4

Similarly, in each case, the total will be a multiple of the sum of the terms of the ratio, while the difference will be a multiple of the difference of the terms of the ratio. Thus, for

example, for choice (C) 9 : 13, the total number of students will be a multiple of 22, while the difference is a multiple of 4. We can see these patterns emerge in the first three cases of the remaining choices in Figure 5 (B – number of boys, G – number of girls, D – difference between number of boys and girls, T – total number of students).

(B) 1 : 2			
B	**G**	**D**	**T**
1	2	1	3
2	4	2	6
3	6	3	9

(C) 9 : 13			
B	**G**	**D**	**T**
9	13	4	22
18	26	8	44
27	39	12	66

(D) 5 : 11			
B	**G**	**D**	**T**
5	11	6	16
10	22	12	32
15	33	18	48

(E) 3 : 5			
B	**G**	**D**	**T**
3	5	2	8
6	10	4	16
9	15	6	24

Figure 5

So how does this help? We know that we start with a difference of 24 between the number of boys and girls. We know that the difference is a multiple of the difference between the terms of the ratio, so we can determine which case provides this difference (possibly one of the choices will not give a difference of 24, in which case, we would be done) and see if it gives rise to the correct number of students in total.

We will use this method for the first case, (A) 4 : 7, that we have tabulated in Figure 3. Since 7 – 4 = 3, the differences are a multiple of 3. To have a difference of 24, we must look at case 24 ÷ 3 = 8. Thus, the total number of students present, which is a multiple of 11, would be 8 x 11 = 88. Since 88 is at least 66, the conditions of the problem are satisfied and this ratio of students would be possible. A summary of all possibilities are examined in Figure 6.

Option	Difference (multiple of)	Total (multiple of)	Case number	Total
(A) 4 : 7	3	11	24 ÷ 3 =8	8 x 11 = 88
(B) 1 : 2	1	3	24 ÷ 1 = 24	24 x 3 = 72
(C) 9 : 13	4	22	24 ÷ 4 = 6	6 x 22 = 132
(D) 5 : 11	6	16	24 ÷ 6 = 4	4 x 16 = 64
(E) 3 : 5	2	8	24 ÷ 2 = 12	12 x 8 = 96

Figure 6

So we see from our table that option (D) is impossible because there would have to be 64 students at the start for the action to work, but the problem specified that there were at least 66 students.

Solution 3: Finally, let's look at the problem algebraically. If we let g and b represent the numbers of girls and boys at the start, then we have two conditions:

$$b + 11 = g - 13 \quad (1)$$

$$\text{and } b + g \geq 66. \quad (2)$$

The first equation rearranges to $b = g - 24$, and when we insert this into the second equation, we get $2g - 24 \geq 66$ or $g \geq 45$, which makes sense looking at Figure 2. From here, there are a couple of ways that we can proceed.

Solution 3a: Now if we continue as in Solution 2, we can consider the options one at a time. If we start with (A) 4 : 7, this means that $b = 4k$ and $g = 7k$ for some positive integer k. Thus, since $b = g - 24$, we have $4k = 7k - 24$, which yields $k = 8$, so there would be 32 boys and 56 girls. Since 56 ≥ 45 (and indeed 88 ≥ 66), the conditions of the problem are satisfied and this option is valid. Using a similar method, we can check the remaining four options. I encourage you to try it out and see that it gives (D) as the culprit.

Solution 3b: From our earlier work, we deduce that the ratio of boys to girls will be $(g - 24) : g$. We usually write ratios in lowest terms, so we need to know when we can reduce our ratio. Since both terms contain a $g - 24$, any common factor of $g - 24$ and g will be a common factor of g and 24.

Suppose the greatest common factor of g and 24 is 8. We can then write $g = 8G$ for some positive integer G. Thus the ratio of boys to girls would be $(8G - 24) : 8G$, which reduces to $(G - 3) : G$. From this, we see that the terms of the ratio are three apart, just like option (A) 4 : 7. This leads to $g = 8 \times 7 = 56 \geq 45$, so this case satisfies the conditions of the problem. Figure 7 looks at all possible greatest common factors of g and 24, from which we can see that (D) is the only one that doesn't satisfy the condition $g \geq 45$.

Greatest Common Factor	Reduced Ratio Boys to Girls	Option	Number of Girls
1	$(G-24) : G$	None	
2	$(G-12) : G$	None	
3	$(G-8) : G$	None	
4	$(G-6) : G$	(D) 5 : 11	44
6	$(G-4) : G$	(C) 9 : 13	78
8	$(G-3) : G$	(A) 4 : 7	56
12	$(G-2) : G$	(E) 3 : 5	60
24	$(G-1) : G$	(B) 1 : 2	48

Figure 7

From Figure 7, we can deduce that, if we drop the condition on the original size of the class, then we can see all possible ratios. For example, the ratio $(G - 1) : G$ that corresponds to a greatest common factor of 24 tells us that all ratios where the first term is one less than the second term will appear, that is 1 : 2, 2 : 3, 3 : 4, and so

on (corresponding to 24 boys and 48 girls; 48 boys and 72 girls; 72 boys and 96 girls).

Similarly, the ratio ($G - 2$) : G tells us that all ratios where the first term is 2 less than the second term will appear. These ratios will be reduced only if G and 2 have no common factors, that is, G must be odd. Thus, 1 : 3, 2 : 4, 3 : 5, and 4 : 6 are all possible, but 2 : 4 = 1 : 2 and 4 : 6 = 2 : 3, so the only new ratios would be 1 : 3, 3 : 5, 5 : 7, and so on. So we can go beyond what the problem has asked us and deduce the nature of the ratio of boys to girls under the condition that if 11 boys transfer in and 13 girls transfer out, the ratio would be 1 : 1. It just goes to show that a math problem is never done.

Now for your homework.

Nadia walks along a straight path that goes directly from her house (N) to her Grandmother's house (G). Some of this path is on flat ground, and some is downhill or uphill. Nadia walks on flat ground at 5 km/h, walks uphill at 4 km/h, and walks downhill at 6 km/h. It takes Nadia 1 hour and 36 minutes to walk from N to G and 1 hour and 39 minutes to walk from G to N. If 2.5 km of the path between N and G is on flat ground, determine the total distance from N to G.

Until next time, happy problem solving! ▲

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▲ Hey, It's Elementary – The Power and Responsibility of "Why"

LYNDA COLGAN
E-MAIL: colganl@educ.queensu.ca



Lynda Colgan's career has included roles as a classroom teacher, a university professor, and newspaper columnist. Her contributions to mathematics and its teaching have been recognized through awards such as the Marshall McLuhan Foundation Distinguished Teacher Award. Lynda always exhibits a passion for mathematics and views her professional mission as dispelling the myth that math is the bad

Ask any parent, grandparent, or caregiver to single out the most challenging stage of child-rearing and he or she is likely to respond, "The ***Why***? phase." A parenting magazine uses the following example to make the point: "Sweetheart, you need to take a bath." "***Why***?" "Because you got dirty at the park." "***Why***?" "Because you were having so much fun playing." "***Why***?" The probability of an infinite loop under such circumstances is approximately 0.99.

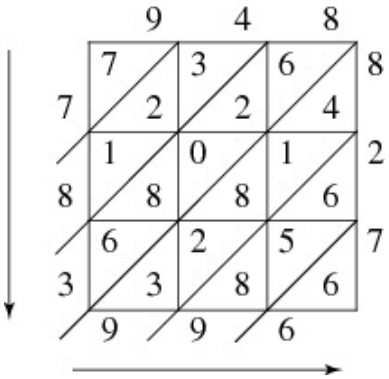
Although the inexhaustible "whys" from a pre-Kindergartner may test the patience of even the most imperturbable adult, early childhood educators warn that ignoring youngsters' questions may have a negative impact: stifling curiosity and the desire to learn about the things they see, hear, and do. Simply stated, when a child asks a question, he or she is taking an active role in his or her own learning. Our role is to foster and encourage such important growth and development.

Developmental experts explain that the "why" stage is an important milestone in terms of communication—an important step mastering the art of conversational give-and-take. More importantly, they point out, is that your child is seeking out the most knowledgeable person that he or she knows and requesting information that he or she needs or wants. This act, experts agree, demonstrates the importance of *strong relationships* in helping children learn and grow: children turn to the people they love and trust to help them learn.

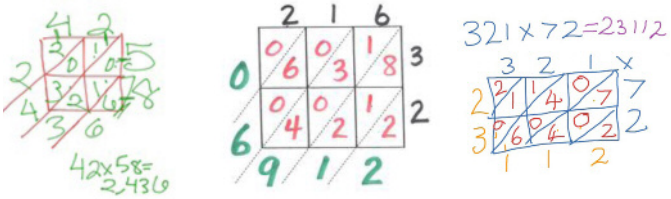
For all the same reasons that "whys" are important at home, they are important at school, regardless of grade.

The fact that "why" questions may seem simple on the surface, however, are anything but, arose recently in one of my pre-service education classes during the last academic year, when one of my teacher candidates opted to teach *Gelosia* multiplication during a microteaching activity. The budding teacher enthusiastically explained that lattice multiplication, also known as sieve multiplication or the jalousia (gelosia) method, dated back to tenth-century India and was introduced into Europe by Fibonacci in the fourteenth century. She added that the name "gelosia" was actually given to this method in Italy, since the grid resembled the lattices or gratings that covered the windows of buildings. The gratings, known as "gelosia" in Italian, were put on the windows of buildings in order to protect high-class Italian women from the public view—in the old Byzantine custom of Italy, wives and daughters of Venetian nobles were kept sequestered in their homes.

She then explained that this form of lattice multiplication was an alternative, and much simpler algorithm for the standard multiplication procedure with which all of her classmates would be familiar. She illustrated the steps of the process by employing a grid to calculate 827 x 948. When she announced that the product was 783 996, and challenged her amazed classmates to check her work, using a calculator, the energy in the room was palpable.



The fascination with the method continued, as other teacher candidates quickly computed products to self-generated questions, using the grids.



Just as the teacher candidate was preparing to thank her peers for their enthusiastic participation and congratulate them on their computational success, she was stopped in her tracks by a small voice from the back of the room that asked, "***Why*** does this work?"

She was dumbfounded by the question. She had selected the topic because it represented a simple, but fascinating, technique for multi-digit multiplication. She had tried numerous examples herself to make sure that "it" worked and that she could troubleshoot an arithmetic error that occurred due to a misapplication of the procedure. But she had not bothered to research "why" the method worked in all cases, including decimal numbers: her presentation had abridged a robust multiplication method with a rich history to a "novelty."

The ensuing conversation in class was fascinating and is the basis for this column. I opened the discussion by saying that having a deep understanding of multi-digit multiplication is a major student expectation in the Junior Division, according to *The Ontario Curriculum: Mathematics*. I added that according to mathematics educators, this understanding goes beyond procedural fluency (where students merely know [or have memorized] a series of steps to reach a solution). I went on to say that influential research, such as that found in *Adding It Up*, suggests that when students see a multiplication problem, such as 26 x 48, they should be able to solve it using multiple approaches; write accompanying word problems to illustrate its meaning; and engage in dialogue to demonstrate the ways in which they reasoned through the strategies they employed to find a solution. I ended my input by positing that although the Gelosia model can be challenging visually and spatially for some students, it may also have benefit because students are working with single-digit multiplication—the building block for more complex problems.

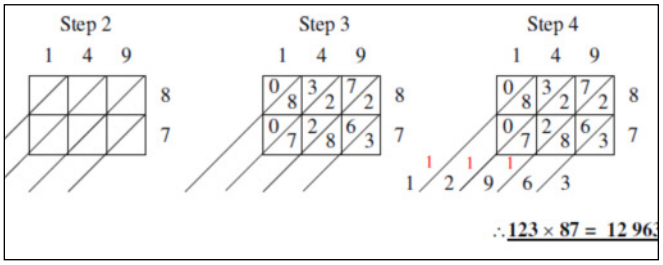
The rebuttals to my opening remarks were fast and furious:

- Gelosia is fun and it works. Who cares "***why***?"
- I thought you said we should make math engaging. Gelosia does that. Now you are saying that is wrong. ***Why***?
- The kids won't care "***why***" Gelosia works, as long as they get the right answer.

And then, the same young teacher who posed the original question, asked, "What happens if a child in your class or a parent asks you to explain '*why*?' How do you respond?"

There was silence. The teacher candidate's microteaching lesson had accomplished two goals (one unanticipated). Her peers learned the Gelosia lattice method of multi-digit multiplication: a centuries- old, reliable procedure. They had also learned that "math tricks" without substantiation are much like junk food: lacking nutrition (in this case, for the brain), substance, and mathematical sustenance.

So, the last part of the Three-Part Lesson on Gelosia multiplication fell to me. What follows are my examples for the steps to find the product of 149 x 87.



"***Why***" does the Gelosia/lattice method work? First, we turn to the distributive property to multiply 469 x 37 and show the mathematical development of the "magic" of the lattice method.

469 x 37 = (400 + 60 + 9) x (30 + 7)

= (400 + 60 + 9) x 30 + (400 + 60 + 9) x 7

= 400 x 30 + 60 x 30 + 9 x 30 + 400 x 7 + 60 x 7 + 9 x 7

= 12 000 + 1800 + 270 + 2800 + 420 + 63

= 17 353

A helpful strategy is to draw a chart to organize the expanded multiplication process.

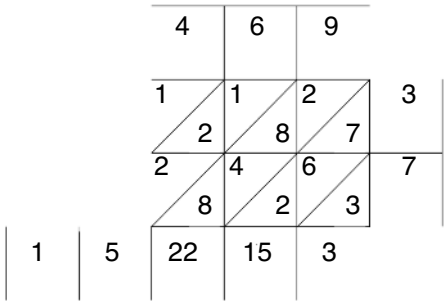
		400	60	9	
		12 000	1800	270	30
		2800	420	63	7
12 000	**4600**	**690**	**63**		

Adding the diagonals, we get 12 000 + 4600 + 690 + 63 = 17 353

This action gives us a clue. If we "drop" the zeroes, we get 12, 46, 69, and 63. To get the product using our traditional algorithm for multi-digit multiplication, we would write these numbers (without their zero placeholders) in a new place value chart as shown, and find the sum.

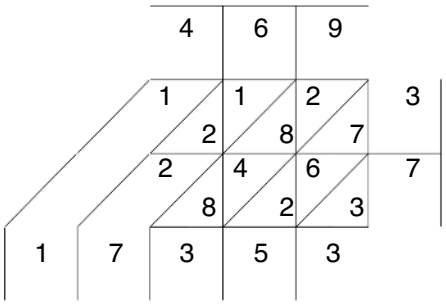
1	2			
	4	6		
		6	9	
			6	3
1	7	3	5	3

The next step to explain the left-to-right "digits of the product" trick emerges when we make a simple change to the first chart we drew: the addition of diagonal lines to split each product into two digits. Now, when we sum the diagonals, and use a place value chart, we get



1				
	5			
	2	2		
		1	5	
				3
1	7	3	5	3

If we regroup as we sum the diagonals from right to left, and then read the digits from left to right, the product magically appears: 17 353.



As I finished the spontaneous demonstration to show "why" Gelosia multiplication worked, I asked my students to suggest adjectives describing what they believed to be the most important characteristics of an elementary math teacher. Their responses were consistent: Kindness. Patience. Compassion. Sense of humour.

I asked them to dig deeper and to reflect upon the lessons that we had all just experienced. Slowly, the tenor of the conversation changed. One teacher candidate said, "If Justine had not asked 'why' Gelosia multiplication worked," and Lynda hadn't explained it to us, we would have left the class just knowing a new 'trick,' and now I think that's not going to be good enough."

The student who prepared the lesson replied, "I thought that I was doing the right thing by putting all my energy into researching the history and finding cool pictures for my PowerPoint presentation. I was so happy to be able to do all the steps in my head and get the right answer every time. I thought children would feel the same way. They'd think it was fun, not math. I was so proud of myself for making multiplication interesting. It never even occurred to me to delve further. To think about "***why***."

Her colleagues were supportive, agreeing that preparing for the ***why*** instance was not part of their typical lesson planning either—the assumption being that the "math behind the trick" would be too advanced for most students anyway. The few teacher candidates who worried aloud that the "math behind the trick" might be too complex for them to understand were reminded by their peers that since they would never let their students give up on themselves, it was clear that the same principle applied to teachers as learners.

The original goal of the lesson about Gelosia multiplication was to introduce a topic with the potential to stimulate elementary students' curiosity about a novel multi-digit multiplication process. The young teachers in class that day learned much more: a testament to the power of why. ▲

NOMINATIONS FOR OAME/AOEM BOARD OF DIRECTORS 2015–2016

Each year, elections are held for a President-Elect, a Vice President (3-year term), and two Directors (up to a 3-year term) of OAME/AOEM. OAME/AOEM members running for Director will declare their intent to work on either Grades JK–6 or Grades 7–12 issues based on their personal experience, expertise, and interests.

Nominations for these positions are to be submitted by 4:00 p.m., January 15, 2015, to:
Sonia Ellison, Past President,
94 Pleasant Avenue, Dundas, ON L9H 3T4
or by e-mail to sonia.ellison@oame.on.ca

Include the nominee's qualifications with the Nominations Form. A nominee must be a member of OAME. By March 2015, the Nomination Committee will publish a slate of candidates on the website, including qualifications, for consideration by the membership. Voting will be on the website, and is open to all OAME/AOEM members. The election results will be released at the Annual General Meeting of the OAME/AOEM in Toronto, May 2015.

NOMINATION FORM

I nominate:_____

for the position(s) of:

☐ President-Elect ☐ Vice-President ☐ Director

If for Director, area of personal experience/expertise, interest:

☐ Grades JK–6 or ☐ Grades 7–12

Nominee's Address: _____

_____

E-mail: _____

Telephone: Home (_____)_____

Work (_____) _____

Nominator's Name : _____

Nominator's Address: _____

_____

E-mail: _____

Telephone: Home (_____)_____

Work (_____) _____

▲ Directors' Dialogue

ANN MICHELE STENNING
E-MAIL: a.stenning@tvdsb.on.ca

Holiday Inn Hotel, Toronto Pearson International Airport, 970 Dixon Rd., Toronto on June 6–7, 2014

Friday, June 6, 2014

Sonia Ellison welcomed board members and thanked those who had supported her during this past year as OAME's president. In 2014–2015, Sonia will take on the role as Past President, while Lynda and Fred Ferneyhough will continue as Executive Directors. OAME's President will be Paul Alves, with Tim Sibbald as President-Elect and Bill Otto and Judy Mendaglio as Vice-Presidents. Directors with one more year in their mandate include Kyla Kadlec, Pat Kehoe, Sandra Jean Price, Frank DiPietro, William Lundy, and Jane Silva. Directors with two-year terms are Jill Lazarus and Lori Schuyler, while Jung Choi-Perkins and Anne Yeager will serve as directors for three more years.

Goodbyes and appreciation were expressed to departing Chapter Representatives including Jen Gravel (ISOMA), Sheridawn Maloney (NOMA), and Gina Micomonaco (Y^{4}MA). Other volunteer positions coming to an end include *Gazette* Editor Immaculate Namukasa and board members Cathy Chaput, Cheryl Geoghegan, and Suhana Kadoura.

Those returning in 2014–2015 as Chapter Representatives include Bart Vanslack (TEAMS), Melissa Black (COMA), Lori Yee (PRMA), Freda Liu (SAME), Christine Sasso (SWOAME), Jelena Loncar (GVMA), Claudio Attanasio (CHAMP), Pamela Chun (GOLDEN), Paulene Washington (MAC^2), Tom Boland (NWOAME), and Ann Michele Stenning (WOMA). New Chapter Representatives Charlotte Aust (ISOMA), Kathy Pilon (QSLMA), and Chi Hang (Y^{4}MA) were welcomed to the board meeting.

Paul Alves and Dwight Stead presented highlights from the annual OAME conference held at Humber College this past May. The conference organizers reported a profit along with positive feedback from the delegates. Sonia Ellison thanked Paul, Dwight, and members of their organizing committees for their effort.

The remaining part of the night was spent with board members working to organize OAME 2015.

Saturday, June 7, 2014

On Saturday morning, board members continued working on OAME's annual conference for 2015. Just before lunch, the general meeting of the board of directors began. After the agenda and minutes from February's board meeting were approved, Lynda Ferneyhough reported on OAME's finances. Lynda noted that increased memberships and income from a Ministry project will protect OAME's investments.

After lunch, Paul Alves announced that the Leadership Conference will be held November 6–8, 2014 at the Holiday Inn. Amy Lin is to speak on Thursday night, Jo Boaler will speak for the entire day on Friday, while Cathy Bruce and Shelley Yearly will speak on Saturday.

Greg Clarke reports that there were 26 teams at the Lakehead University Orillia campus for the 2014 Ontario Mathematics Olympics. Board members viewed a short highlight video, which included the chants that teams were required to create. Melissa Black announced that COMA is hosting OMO 2015 at Carleton University on June 5–6, 2015.

Claudio Attanasio announced that the OAME 2015 website is up and running and a twitter feed is operating. Board members were subsequently allowed to ask questions pertaining to the afternoon announcements.

CHAPTER REPORTS

CHAMP ran a conference at Humber College for their members.

COMA held a Welcome Back Social at the Centurion in the Fall. Dr. Lynda Colgan from Queen's University and "Prime Radicals" were featured at the event. Door prizes consisted of Prime Radicals DVDs that were donated by Dr. Colgan. The annual mini-conference was held at the University of Ottawa SITE building. Marian Small was the keynote speaker, while Jennifer Capitani, Robin McAteer, and Anne Holness held breakout sessions. The event also included publisher displays and a wrap-up luncheon at Draft Pub. The Carleton-Ottawa Math Olympics ran this year and for the first time, the event featured entire teams of junior students. COMA will host OMO 2015, co-chaired by Melissa Black and Linda LoFaro. Venues have been secured, while sponsorship and marketing campaigns are underway. COMA also sent some of its members to OAME's annual conference in their efforts to "pay it forward." Future activities offered by COMA include their Welcome Back Social with Marian Small in the Fall; the Annual Ron Bender Mini-

Conference at Ottawa University in November; the Carleton-Ottawa Math Olympics next April; and the Ontario Math Olympics, hosted by COMA, in June.

GOLDEN offered a mini-conference in April and sent two teams to OMO this past Spring. Math contests for both elementary and secondary students are also planned for next year. In the Fall, GOLDEN will offer a Fall social.

GVMA continues to plan their annual Fall conference for Thursday, November 6.

ISOMA held a mini-conference in January and Math Olympics in April. Thank you to members from the Holy Trinity School Math Department for organizing and hosting this Math Olympics. There were 144 students that made up 36 teams from 20 ISOMA schools. All mathletes participated in four events. Based on the results from this event, two teams from University of Toronto Schools were selected to represent ISOMA at OMO this past Spring. ISOMA was also able to host Dan Meyer for a full-day PD session. Held at Holy Trinity School, 45 teachers participated in the event. Thank you to John Doma and his colleagues for organizing and hosting this event.

MAC² hosted OMO 2014. There were 26 teams of Grades 7 and 8 boys and girls. Students searched for mystery digits hidden around Lakehead University, played Sudoku, and participated in the individual, pairs, and team competitions. There was also an OMO numbers game and a team relay, with points that were earned from each event.

NOMA broke with tradition and hosted a mini-conference outside of Sudbury this Spring. Dr. Marian Small presented to members from the Kindergarden to Grade 6 group, and Amy Lin presented to the Grades 7 to 12 group. The sessions were held at Nipissing University and teacher candidates from the school of education attended. A remote connection allowed a group of educators to meet in New Liskeard and join Amy Lin's session. Since many school boards financially support participants who attended the annual OAME conference in Toronto, NOMA's membership has grown tremendously.

QSLMA has fielded two teams for OMO and has supported a local team of mathletes who are attending the American Regional Math League Competition in Pennsylvania. The Fall mini-conference will feature Dr. Egan Chernoff, who will be speaking about "math wars." An interactive question-and-answer session will be included along with a dinner and social hour, and the interaction will continue with a dinner and informal social hour in the evening.

SWOAME has just completed a one-day workshop featuring Dan Meyer. There were 60 local teachers in attendance. We have also finished a series of monthly evenings on math with local teachers. The evenings were informal and discussions focused on ways to be creative with curriculum delivery. Many resources and ideas were shared. The feedback was positive and therefore we will be looking in the Fall to continue these sessions.

TEAMS held the 20th Annual Mini-Conference in October 2013. Over 250 delegates attended to hear Marian Small's talk on Proportional Reasoning. For the second session, delegates were able to choose from over ten breakout sessions. In April 2014, TEAMS held the Toronto Math Olympics (TMO). Over 80 students participated in the TMO, and TEAMS will be send three all-star teams to the OMO in Orillia. TEAMS directors continue to have monthly meetings and are busy planning the next Annual Mini-Conference for the end of October 2014. The mini-conference will be held at John Polanyi Collegiate Institute in October. TEAMS is again planning to have a featured keynote speaker, with breakout sessions to follow. Registration will again be on MCIS.

WOMA hosted their Spring mini-conference in April at White Oaks Public School in London. Upon arrival, the attendees were offered snacks and refreshments. After two professional development sessions, participants were served a pizza supper, drinks, and dessert. To finish the event, a band introduced the audience to songs written using mathematical ideas and concepts.

Y⁴MA held a successful IMO event in April, with schools from three school boards participating. Three teams represented the chapter at the OMO provincial competition in May, and the annual general meeting for Y⁴MA was held in June. ▲

▲ What's New with CLIPS? A New Column Name!

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Greg is from the Simcoe Muskoka Catholic District School Board and Ross is from the Near North District School Board. Both are currently on assignment with the Ontario Ministry of Education as Provincial Math Leads, working on the CLIPS project, in conjunction with their colleague, Agnes Grafton, from the Brant Haldimand Norfolk Catholic District School Board.

Welcome back to a new school year! As often happens, a new school year brings changes, and we are certain this year will be no different. From our point of view, the name for this column is changing to more accurately reflect the work we do and the projects and resources we intend to highlight throughout the year.

Developing the CLIPS website (**www.mathclips.ca**) was one of the Ministry's first extensive digital resource projects originally designed for Grades 7–12. During the past year or two, this work has expanded to include not only CLIPS, but also the launch of the mathies.ca website, as well as the development of games for primary and junior students; various virtual learning tools; and "how to" video supports. We are particularly excited to be embarking on the development of various apps for release on both iOS and Android devices. So, in order to reflect this new reality, we have decided to rename this column "Provincial Digital Learning Resources – What's New?"

So, what is new in the world of Provincial Digital Learning Resources? Looking into our crystal ball, and crossing our fingers, we are hoping to have at least a couple of items ready for the Fall, or at least be well into the review and approval process. Our work continues during the summer months, when we are fortunate to have a number of teachers from across the province assist with programming. Here is a list of some of the things that this team is working on:

- A virtual number line tool – original release will work with whole numbers and integers, will be available for computer download and as an app for iOS devices and Android tablets
- A relational rods app (iOS and Android)
- Diamond Drop Sign of a Sum – a new integer addition game
- A grid tool with annotation features

As always, new releases will be featured on **mathies.ca**, in the "What's New" link inside **MathCLIPS.ca** and announced via the EduGAINS RSS feed.

Have a great year! ▲

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A Mathematical Exploration of the Nutrition North Canada Program

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Helping students develop awareness of issues of social justice is increasingly recognized as an important educational objective and one with value beyond social studies classrooms. In this article, we present a task for intermediate or secondary school mathematics, which was designed to develop mathematical skill and understanding while raising students' awareness of an important social issue in Canada. Specifically, the task deals with some of the mathematics related to controversies and inequities with the Nutrition North Canada (NNC) Program (e.g., CBC, 2012)—a subsidy program aiming to improve access to perishable healthy foods, as well as country or traditional foods, in isolated northern communities across Canada.

The complete task is presented in Appendix A in handout form suitable for classroom use. The task centres around data management and data relations, and was designed to allow students the freedom to explore the problem in their own way and draw their own conclusions about the social and political issues involved. The task is suitable for use in many Ontario mathematics courses including Grades 7 and 8 Mathematics, Grades 11 and 12 Foundations for College Mathematics, and Grade 12 Mathematics of Data Management. In Appendix B, we identify the general expectations that can be addressed through this task for these five courses. A list of specific expectations is too lengthy to include here, but may be found in the Ontario curriculum documents. Because each group of learners is unique, we leave the details of how these expectations might be addressed at different grade levels and in different courses to the teacher. Of course, younger learners are likely to require more focused instruction, simplification of resources and, perhaps, more intervention. For older learners, the teacher may allow the students more freedom in how they approach the task and intervene only to point out complexities students may be overlooking or to direct students in ways that allow for the exploration of specific learning outcomes.

Though we focus on data concepts, there are several other meaningful connections that can be made to the Ontario curriculum (e.g., to geometry and spatial sense, distance and measurement, ratios, percentages, and fractions), and we invite others interested in these connections to join our exploration. In Appendix C, we offer a list of online resources that have background and reference material that may be useful as students investigate different possibilities within the task. We wrap up the paper with ideas that could help other teachers develop similar tasks around different issues, as well as with questions related to the NNC program that could be used for further exploration and discussion.

Developing a Mathematical Understanding of the NNC Program

The handout we provided in Appendix A is structured into two parts. Part A gives the context and includes a brief introduction to the NNC Program, with an online reference for further research. It focuses on NNC's services to Nunavut for two main reasons: Nunavut has the lowest (current) population density of provinces and territories, and its far north communities have a strategic purpose (arctic sovereignty). Part B draws attention to some of the controversy surrounding the program and attempts to point out, in a way that makes sense to students, the resource inequity and disparate opportunities between remote Canadian communities accessible only by air and water and the rest of Canada. Some statistical facts set the stage, though the provided information was deliberately kept minimal—enough to highlight key ideas that could then be researched

independently by the students during work periods or at home. The task can be supplemented by several web-based resources (see Appendix C), which the teacher can choose to strategically reveal to help scaffold students' investigations.

Question 1: Exploring the Weekly Costs of Groceries

> *What is the weekly cost of groceries for a family of four in your neighbourhood? Please explain how you determined this. Then, compare your result with data about Nunavut from the Aboriginal Affairs and Northern Development Canada website, which has documented costs of food in remote northern communities (see* **www.aadnc-aandc.gc.ca/eng/1100100035986/1100100035987**).

While this question might at first glance appear straightforward, it requires the consideration of appropriate definitions for a number of terms, as well important choices about the overall approach to the problem. Depending on the intended learning goals of the teacher, each of these decisions can be determined in advance or left open to students.

In their responses, students can explore different ways of obtaining data. They may choose to collect their own data (primary data), or to research publicly available data (secondary data). Each approach addresses different expectations and affords different challenges. We chose to leave the decision open to help foster problem solving, inquiry, and debate about when, why, and how to make use of these different data sources and their respective limitations. However, even if using only secondary data, there is still ample opportunity for inquiry and investigation.

For example, students may recognize that different families have different food- purchasing habits and that these habits can have a large impact on the cost of groceries. Either on their own or through prompts from the teacher, students may address factors such as quantity of food, choice of foods and food brands, and choice of retailer. Exploring how to measure the cost of groceries offers the possibility to learn about the National Nutritious Food Basket (NNFB). Developed by the Government of Canada, the NNFB is a "survey tool that is a measure of the cost of basic healthy eating that represents current nutrition recommendations and average food purchasing patterns" (*Nutritious Food Basket Guidance Document*, 2010, p. 7), as indexed by the average (across retailers) minimum cost of 67 foods. Much of the variation involved in the cost of groceries can be controlled for using this as a standard.

There is an opportunity here for students, on their own or as a class, to critically analyze the standards and assumptions of the NNFB. Some possible questions to prompt students are: "Do the foods and quantities in the NNFB approximate their own and their family's eating habits?" and "Does the NNFB use the average or minimum cost of foods across brands and retailers, and why?" Of particular importance for answering the question in this task is how to define a family of four. The NNFB Guidance Document defines it as "a man and woman, each aged 31–50 years; a boy, 14–18 years of age; and a girl, 4–8 years old" (*Nutritious Food Basket Guidance Document,* 2010, p. 7). The teacher may wish to prompt students to reflect on why this definition is used and to question its appropriateness for this task. In the end, students may choose to accept the NNFB definition for a family of four or create their own.

In addition, the phrase "in your neighbourhood" has different interpretations that can influence how the task is addressed. For example, if interpreted literally, then students may need to collect primary data from their neighbours. If "in your neighbourhood" is interpreted more liberally as referring to their municipality, then online public data can be used. The second part of this question instructs students to use secondary data about Nunavut from the Aboriginal Affairs and Northern Development Canada website; this resource also includes data for many other municipalities, making a comparison more immediate. The link provided directs students to the Revised Northern Food Basket (RNFB)—a tool similar to the NNFB and used "to monitor trends in the cost of healthy eating in isolated northern communities eligible for the Nutrition North Canada (NNC) program" (**www.nutritionnorthcanada.gc.ca/eng/1369314079798/1369314090524**).

Considerations when using primary data. If students decide to collect their own data, the task may be both more challenging and considerably more time consuming. Some key decisions that students will be faced with include how to define their neighbourhood and what sort of data to collect. For instance, neighbourhoods are not typically defined by fixed boundaries within a municipality. Instead, they are usually loosely defined within a community and can change over time. As such, students may be invited to research their own community and make their own decisions about how to define their neighbourhood.

Students may decide to survey neighbours about their food-purchasing habits. This is a rich opportunity that will truly immerse students in all the complexities involved in measuring grocery costs. Challenges in creating their own survey may lead to important questions such as: "Which purchases should be included in the cost of groceries?" and "How will respondents determine how much they spend per month?" Alternatively, students may design a survey that parallels that of the NNFB's, which is available in the *Nutritious Food Basket Guidance Document*. Both avenues offer students the opportunity to discover the rationale behind a tool like the NNFB, to experience first-hand the challenges of conducting such a survey, and to recognize the need for such a tool.

Considerations when using secondary data. Using the NNFB and a definition of a family of four, it is possible to find data for "the weekly cost of the groceries" for most Canadian municipalities. As students explore the RNFB data for Nunavut communities and make comparisons with NNFB data in other Canadian municipalities, there are a number of challenges that they may recognize. Some examples are: (1) the RNFB uses average food costs across brands, while the NNFB uses minimum food costs; (2) the RNFB uses a different reference family of four, defined as "a man and a woman aged between 31 and 50, and a boy and a girl aged between 9 and 13" (**www.nutritionnorthcanada.gc.ca/eng/1369314079798/1369314090524**); and (3) available data may be from different years. In addition, there is a large variation in the weekly cost of RNFB across the many Nunavut communities. For example, data from 2013 shows a minimum weekly cost of $400.22 in Rankin Inlet and a maximum weekly cost of $478.83 in Taloyoak. In order to make comparisons, students will need to decide how to deal with this variation. The teacher may wish to discuss the merits of using a maximum value, a minimum value, a mean value, or some form of weighted mean value.

Question 2: Exploring the Cost of Living, Income, and Representation

> *Determine the cost of living of an average family of four in your neighbourhood and compare it with the household income of an average family of four in your neighbourhood. Represent the data in two different forms. What do the results tell you? Do the different representations give different insights into the data? Please explain.*

Many of the issues discussed for question one also emerge when considering cost of living and household income, and rather than revisit these, we choose to focus on possible ways that students may choose to represent the data.

As students engage with the concept of cost of living, they can consider the different contributing expenses. They may also encounter the Canadian Consumer Price Index (CPI), which "is a measure of the rate of price change for goods and services bought by Canadian consumers" (*Your Guide to the CPI*, p. 7). The CPI is not a measure of the cost of living, but it is closely related. Whether or not a connection to the CPI is made, students will likely recognize the need to categorize the different costs that contribute to the cost of living. Some of the categories used by the CPI are food, shelter, clothing and footwear, transportation, health and personal care, education, alcoholic beverages and tobacco products. These costs, together with household income, constitute the data that can be represented.

There are many ways of representing this type of data; some of the most common are table, bar graph, and pie chart. This question affords students an opportunity to explore the relative merits of each representation (or any others they come up with). They may initially be inclined to use a pie chart because of the categorical (nominal) data. However, question 3 may lead them to a comparison with Nunavut cost-of-living data, providing an opportunity for discussion about the effectiveness of pie charts versus bar graphs for this type of comparison.

Question 3: A New Look at the NNC Program

> *A local newspaper has asked you to write an opinion piece about the Northern Nutrition Canada's service to Nunavut. Analyze the goals of the NNC program in relation to the needs of Nunavut's residents. Your analysis should include, but need not be limited to, a critique of the subsidy package, its cost effectiveness, and its affordability.*

This question is the consolidation piece—an opportunity for students to analyze their investigations, make inferences and arguments, and use these to articulate their understanding of the program. The invitation to write an opinion piece, and the assurance that the validity of the arguments, rather than position of the opinions, would be the focus of any assessment, allows students to consider and explain issues based on

their own perspectives. The question directs attention toward aspects of the NNC program that connect explicitly to the two previous investigations, without indicating how those investigations should be used. This allows room for a teacher to address how data may be (re)presented for the purpose of meeting a particular agenda—an issue relevant to mathematics, as well as literacy and social science.

Students may pose, focus on, and answer their own questions about the situation through this investigation. For instance, students with an interest in history may consider critiquing the program with respect to its predecessor, the Food Mail Program (FMP), a similar initiative that emerged in the 1960s to support food transportation to communities accessible only by air transport. The FMP offered a transport subsidy that was eventually deemed cost ineffective, and in April of 2011, it was replaced with NNC, which offers a retail subsidy. This historical look immediately invites scrutiny of the differences between types of subsidies, as well as the rationale behind the change. While intended to improve efficiency and decrease transportation costs, families are still faced with very high costs for basic necessities. Further, certain staples such as flour are not included in the subsidy package, though it is difficult to imagine how a country food such as bannock could be made without it.

Investigating Further

This task was designed as a Rich Learning Task (RLT). For teachers interested in creating their own tasks that address topical social issues in a mathematical way, Flewelling and Higginson's handbook on RLTs (2002) has good tips for structuring learning activities. RLTs are intended to give students opportunities to choose from large sets of mathematical and non-mathematical skills, and use them in integrated, creative, and purposeful ways to solve problems. In our case, thinking about how the mathematical investigations from questions 1 and 2 could help students reflect on their opinion pieces in question 3 is one of these opportunities. RLTs are "typically structured to give the learner the opportunity to engage in an inquiry, investigation, experiment, or problem-solving episode" (Flewelling & Higginson, 2002, p. 40). Tasks that have a real-world or socially relevant context also aim to help students independently address things like "so that's why I need it [the mathematics], so that's how it's used, so that's what it leads to, so those are its limitations, so that's what it tells me about this kind of situation" (Stocker, 2008, p. 20). RLTs tend to be time consuming, so taking an interdisciplinary approach may be worthwhile. One of the benefits of RLTs, in our view, is that they provide opportunities for both students and teachers to engage collaboratively as problem posers and solvers, asking, "How will we figure this out together?" (Stocker, 2008, p. 15).

This is exactly what happened for us with this task, and it led to new questions that we might not have thought of otherwise. Taking this approach with our students allowed us to see different connections to the curriculum and gave us an opportunity to discuss with students questions that they brought forward that were both relevant to the social issue as well as mathematically important. We conclude with a list of questions that invite different avenues of investigation:

- How is food transported to these remote locations?
- How are the subsidies being passed on to the consumer?
- How is the subsidy distributed across the communities served by the program? (e.g., Is the distribution of the $53.9 million per-year subsidy proportional to population?)
- Does the RNFB reflect healthy food-purchasing habits of the Nunavut population?
- What are some of the reasons for maintaining populations in these far northern regions?

Acknowledgements

This task was developed as part of a research study funded by the Social Sciences and Humanities Research Council. Early stages of this research helped shaped the current work, and we would like to acknowledge those contributions from G. Gunthrope, L. Martin, and R. Pali.

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Appendix A – The Task

PART A: Introduction – What is Nutrition North Canada (NNC)?

Nutrition North Canada is a subsidy program that seeks to improve access to perishable healthy food, as well as country or traditional foods, in isolated northern communities. It was launched on April 1, 2011 and is based on a market-driven model. The subsidy is transferred to retailers and suppliers that apply and are selected to register with the program, which then must pass on the savings to consumers. There are some conditions for communities to be eligible for the program, such as the lack of year-round surface transportation (i.e. no permanent road, rail or marine access). In some places, like Nunavut, residents are upset that non-perishable basics like flour are not subsidized under the new program. They must cope with a cost of living that can be significantly higher than many other places in Canada. You can find more facts on this program at: **http://www.nutritionnorth canada.ca/**

PART B: The Project – Exploring NNC in Nunavut

Nunavut has approximately 30 000 residents, with about 56 percent of residents under the age of 25. Statistics Canada reports that the median family income in Nunavut in 2010 was $ 62 680 per year, which is among the lowest in Canada. Despite this, cost of living can be quite high. Due to the challenges of vast distances, a small but growing population, the high cost of materials and labour, and extreme climate, it is difficult to maintain Canada's high standards of living in Nunavut. This is one of the reasons that the federal government has included Nunavut in the Nutrition North Canada program.

In addition to subsidizing food costs, one of the goals of NNC is to provide culturally appropriate nutrition education and health initiatives, working in conjunction with the Canadian government. However, recent media attention to the NNC has questioned the effectiveness of the program, suggesting that prices have increased since its implementation, and they question why basic necessities such as flour are not being subsidized (e.g., see: **www.cbc.ca/news/canada/north/story/2012/06/09/north-nunavut-food-price.html**). It is unclear if the needs of Nunavut's residents are being met, and whose responsibility it is to provide for these needs.

1. What is the weekly cost of groceries for a family of four in your neighbourhood? Please explain how you determined this. Then, compare your result with data about Nunavut from the Aboriginal Affairs and Northern Development Canada website, which has documented costs of food in remote northern communities (see: **http://www.aadnc-aandc.gc.ca/eng/1100100035986/1100100035987**).
2. Determine the *cost of living* of an average family of four in your neighbourhood and compare it with the *household income* of an average family of four in your neighbourhood. Represent the data in two different forms. What do the results tell you? Do the different representations give different insights into the data? Please explain.

   Note: Statistics Canada website offers helpful information for this question. Visit: **http://www12.statcan.ca/census-recensement/2006/dp-pd/fsfi/ index.cfm?LANG=ENG&VIEW=D&format=flash&PRCODE=01&TOPIC_ID=7**
3. A local newspaper has asked you to write an opinion piece about the Northern Nutrition Canada's service to Nunavut. Analyse the goals of the NNC program in relation to the needs of Nunavut's residents. Your analysis should include, but need not be limited to, a critique of the subsidy package, its cost effectiveness, and its affordability.

Appendix B – Connections to the Ontario Curriculum

General Categories of Expectations	Course
Primary data collection (surveys)	7, 8, CM11, DM12
Secondary data collection	7, 8, CM11, DM12
Data display	7, 8, CM11, DM12
Populations and sampling	7, 8, CM11, DM12
Sampling bias	7, 8, CM12, DM12
Use of data to make inferences and convincing arguments	7, 8, DM12
Critical analysis of data in the media	CM12, DM12
Indices (e.g., CPI)	CM12, DM12

Ontario Mathematics Courses Legend

7-Grade 7 Mathematics; 8-Grade 8 Mathematics; CM11-Foundations for College Mathematics, Grade 11; CM12-Foundations for College Mathematics, Grade 12; DM12-Mathematics of Data Management, Grade 12

Appendix C – Web-Based Resources

Consumer Price Index **http://www.statcan.gc.ca/tables-tableaux/sum-som/l01/cst01/cpis01a-eng.htm**

Cost of the Nutritious Food Basket – Toronto 2013 **http://www.toronto.ca/legdocs/mmis/2013/hl/bgrd/backgroundfile-61625.pdf**

Cost of the Revised Northern Food Basket in 2012–2013 **http://www.nutritionnorthcanada.gc.ca/eng/1369313792863/1369313809684**

Eating Well with Canada's Food Guide – First Nations, Inuit, and Métis **http://www.hc-sc.gc.ca/fn-an/food-guide-aliment/fnim-pnim/index-eng.php-a1**

Fact Sheet: Revised Northern Food Basket **http://www.nutritionnorthcanada.gc.ca/eng/1369314079798/1369314090524**

Fact Sheet: The Nutrition North Canada Program **http://www.nutritionnorthcanada.gc.ca/eng/1367932314461/1367932387670**

Food Mail Program **http://publications.gc.ca/collections/Collection/R2-221-2002E.pdf**

Food Secure Canada **http://foodsecurecanada.org/community-networks/northern-remote-food**

A National Nutritious Food Basket **www.hc-sc.gc.ca/fn-an/surveill/basket-panier/index-eng.php**

Nutrition North Canada **www.nutritionnorthcanada.gc.ca/eng/1351088285438/1351088295799**

Nutritious Food Basket – Guidance Document **www.mhp.gov.on.ca/en/healthycommunities/public-health/guidancedocs/NutritiousFoodBasket.PDF**

The Revised Northern Food Basket **http://publications.gc.ca/collections/collection_2008/inac-ainc/R3-56-2007E.pdf** ▲

▲ Success by Numbers: Canada Math Competitions Help Prepare Students for Challenges Ahead

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The vital importance of mathematics, while generally acknowledged, may not necessarily be appreciated. Math is often misunderstood and maligned, especially by students. Math is dull and boring. Math is way too hard. Math is anything but cool. At the same time, math can be fun and rewarding. Math can be beautiful and creative. And perhaps more importantly, math can nurture strong analytical and problem-solving skills, key ingredients to career success. One of the tools in the arsenal to bridge the discord between general perceptions and critical reality is the math competition.

Since 1969, the Canadian Mathematical Society (CMS) has been staging national math competitions to encourage students to explore, discover, and learn more about mathematics and problem solving. Along the way, thousands of students have become a bit more comfortable with math and a bit more confident in what they can achieve.

Why Math Competitions?

Math competitions are a primary extracurricular activity that can both stimulate and cultivate student interests. Like sport, whether for recreation or competition, math

competitions require some basic knowledge and understanding. Coupled with discipline and practice, math exercise will lead to improved self-confidence, enjoyment, and personal success. And math competitions are much more than just an intellectual pursuit for gifted students. A good math competition provides opportunities for "Hey, I can do that!" self-discovery and nurtures a bit of "Now, maybe I can try that!"

Teachers play a critical role in math education, and competitions represent much more than another opportunity to organize! Most math competitions provide a plethora of supplementary resource materials, from sample and past problem sets to current weekly or monthly problems, as well as other resources. In the case of the CMS, those resources range from access to the *A Taste of Mathematics* (ATOM) series of books, to math camps, to other competitions, and even to *CRUX*, the international problem-solving journal. All these resources can be valuable and rich tools in their own right, and the selection, adaption, and combination of these resources allows tailoring to individual classroom or student interests.

Teachers can choose from many math contests and competitions. Some contests are singularly university centric, such as the CEMC contests at Waterloo, whereas some competitions are designed for a regional or provincial audience, like the OAME Ontario Mathematics Olympiad. The CMS series offers a national problem-solving competition built on a partnership with universities across Canada.

The Sun Life Financial Canadian Open Mathematics Challenge (COMC) Competition

This flagship national competition is open to any student (and indeed anyone) in any location at any grade level. It attracts thousands of participants from across Canada and internationally each year. Although the competition focuses on upper-level high school students, performance awards are available at multiple grade levels. Furthermore, every student in Canada who participates is equally eligible for prizes. Top-performing students receive plaques and certificates, and their school receives a plaque as well. At the same time, graduating students who perform well may be considered for scholarships, and other students may be considered for an invitation to a CMS regional, speciality, or national math camp.

The 2.5-hour competition, normally staged at the school in early November, consists of three parts. Part A has four basic questions worth four marks each that should be easily answered by the average student. Part B has four intermediate questions worth six marks each and will provide stimulation for students with an interest in math. Part C has four more difficult questions worth ten marks each to provide challenges for students with advanced interests.

While the competition is nationally focused, performance is recognized at "best in Canada" and "best in grade in Canada" levels, as well as "best in province" and "best in grade in province" levels. For Ontario, given the high number of participants, there are also regional levels of recognition. In addition to awards, plaques, certificates, and prizes, top-performing students are also automatically invited to participate in the advanced CMS competition. Some students who score very close to qualifying for an automatic invitation may be invited to a second-round CMS competition and another chance to advance.

One distinguishing element of the competition is that each year, based upon feedback from schools, the CMS adjusts the structure to better reflect overall curriculum content and increase the value to both teacher and student. The piloted Problem of the Week exercises, introduced in 2013, received favourable feedback and will continue in 2014. Feedback on the presentation of the 2013 questions and their level of difficulty is being used to develop better and more meaningful questions for 2014.

For students with advanced interests, this is the only competition that can eventually lead to a student being chosen by the CMS for Math Team Canada and competing in the International Mathematical Olympiad.

The Sun Life Financial Repêchage Competition

Students who come very close to qualifying for an invitation to the advanced CMS competition are invited to participate in the take-home Repêchage competition in early February. About 75 students are given one week to complete eight questions. The best-performing students will be invited to the advanced competition, and each student receives one of the *A Taste of Mathematics* books.

The Sun Life Financial Canadian Mathematical Olympiad (CMO) Competition

This three-hour advanced competition is usually written in school in late March and typically consists of

five challenging math problems. Approximately 80 students are invited to participate. The top-scoring students receive awards and prizes, and the top score is awarded the Sun Life Financial Cup. All students receive a one-year online subscription to CRUX. Top-performing students may also be invited to a national math camp or to be part of Math Team Canada.

Math Team Canada (MTC) and the International Mathematical Olympiad (IMO)

Canadian students have consistently performed very well on the international stage at the IMO competition. The IMO is an intense world-class three-day competition. Each day, students have 4.5 hours to solve three questions. At the July 2013 IMO in Colombia, MTC took two gold, two silver, and two bronze medals and ranked 11th out of 97 countries with 550 participants. Once six students are selected by the CMS to be on the team, just like in any sport, CMS assembles training and coaching staff to provide an intense preparation program. Training takes place at the Banff International Research Station (BIRS), and then the team flies off to the IMO venue. In 2014, the IMO venue is South Africa, and in 2015, it will be Thailand.

Other Competitions

In addition to staging national competitions, the CMS is the supporting organization for Canada's participation in the Asian Pacific Mathematics Olympiad (APMO). Domestically, the CMS also supports national, provincial, and regional math competitions across Canada. For example, in Ontario, the CMS has supported the Caribou Mathematics Competition staged by Brock University and the Math@Mac Online Competition staged by McMaster University. Across Canada there is a good supply of contests and competitions to explore and try as a student works toward the CMS Canada math competitions and Math Team Canada!

Thank You, Teachers

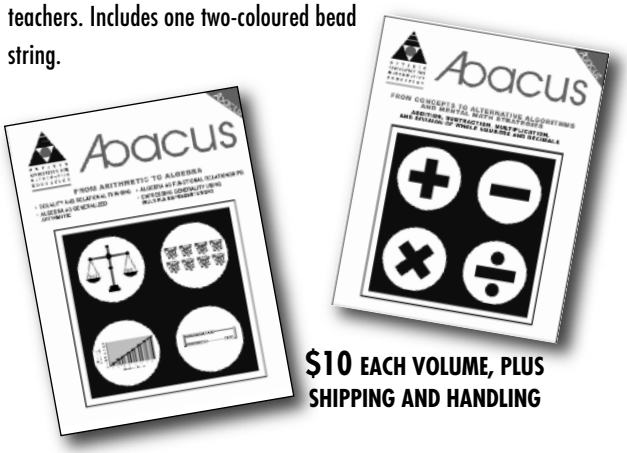
None of the math contests and competitions could be staged without the support and efforts of the teachers who organize events at their schools. The CMS is especially grateful to those teachers who support the Canada math competitions. For 2014, the CMS will be introducing a teacher thank-you program, including the distribution of sponsored education materials, a small token of appreciation for making a big difference.

Success by Numbers

If web stats are to be believed, the CMS Canada math competitions attract an awful lot of students and teachers who seem to particularly enjoy the breadth of materials available. Meanwhile, direct participation in the CMS national competitions grows each year as more and more students and teachers become aware of the opportunities. And while it is always reassuring to note success by numbers, perhaps the more rewarding aspect of the CMS math competitions is the knowledge that each student can participate and compete to his or her own level of interest. The collateral benefit should be students who are better prepared for career competitions ahead, where success with numbers can really mean success by numbers. ▲

ABACUS: A COMPILATION

Volume 46 – This resource for teachers includes the concepts of addition, subtraction, multiplication, and division of whole numbers and decimals, alternative algorithms, mental math strategies, manipulatives, and links to literature. It will be useful for exploring these aspects of teaching and learning personally or with a team of teachers. Includes one two-coloured bead string.



$10 EACH VOLUME, PLUS SHIPPING AND HANDLING

Volume 47 – This resource for teachers includes the concepts of equality and relational thinking, algebra as generalized arithmetic, algebra as functional relationships and expressing generality using multiple representations. With links to manipulatives and literature, it will be useful for exploring these concepts personally or with a team of teachers.

Order by mail, fax, or online:

OAME, 4751 Mack Ave., Town of Plympton-Wyoming, ON N0N 1J6
Phone/Fax: (519) 899-4203, or order online at www.oame.on.ca
Do not send cash or cheque. You will be invoiced, with shipping and handling costs added.

▲ OAME 2015: Building Mathematical Mindsets

WAYNE ERDMAN
E-MAIL: wayne.erdman@gmail.com

Wayne Erdman of Erdman Educational Consulting Ltd. is a mathematics author for McGraw-Hill Ryerson, and is Chair of OAME 2015: Building Mathematical Mindsets. He is also a Practicum Supervisor at OISE.



The 2015 OAME annual conference returns to Humber College North after a successful event in 2014. Rather than being organized by a local chapter, the OAME board of directors is taking on the task and is excited about the chosen theme, "Building Mathematical Mindsets," or motivating life-long learning through supporting a belief in the power of determination and effort. This follows the current research into growth mindsets, in which brains and talent are the starting point, but dedication and hard work create a love of learning and motivation to grow and be productive in business, education, and sports.

To support us in our goal of building mathematical thinking from any age, our keynote speakers will be Marian Small from One, Two... Infinity, Dan Meyer from Stanford University, and Nora Newcombe from Temple University. Marian is an expert on making math meaningful to our students. Dan is well known for his TED talks on exploring the way we teach teachers to teach children. Nora is an authority on spatial development and cognition and spatial thinking in pre-K–12 education. All are very inspiring speakers and we look forward to these insightful presentations.


Dan Meyer


Marian Small


Nora Newcombe

We will also have a strong lineup of featured speakers, including Ruth Beatty from Lakehead University, Damian Cooper of Plan-Teach-Assess, Amy Lin from the Halton District School Board, Chris Suurtamm from the University of Ottawa, and Ron Lancaster and Indigo Esmonde from OISE. We are also introducing three threads (a conference within a conference), where delegates can be embedded in three days of in-depth sessions focusing on a dedicated topic. A Concept Study will be led by Brent Davis of the University of Calgary and Kathy Kubota-Zarivnij of the Toronto Catholic District School Board. "Manipulating Mathematical Thinking with Technology" will be led by Caroline Rosenbloom of OISE and Mirela Ciobanu of the Toronto District School Board. "Supporting Students with Learning Disabilities" will be led by Connie Quadrini of the York Catholic District School Board and Connie Gray of the Simcoe Catholic District School Board.

We will also be holding a lesson study at a local Toronto school, planning numerous field trips in the Toronto area to explore math in the community, and are also bringing back the highly popular OAME Ignite, where our highlighted speakers are invited to share their personal and professional passions, using 20 slides that auto-advance every 15 seconds for a total of just five minutes. There will also be inspirational talks by many of your favourite speakers.

We invite those of you who are thinking about presenting a session at OAME 2015 to contact us by October 31 through the MCIS website. We are always looking for interesting workshops on one of our strands—Understanding Student Mathematical Mindsets; Assessment – Building Positive Mindsets; Mathematics with Manipulatives and Technology; Supporting Students with Special Needs; and Building Mathematics Knowledge for Teaching.

Visit **www.oame2015.ca** for further information about the conference. Place the OAME conference on your calendar—May 7 to 9, 2015—and prepare to build mathematical mindsets with your students. ▲

OAME/AOEM AWARDS FOR EXCELLENCE IN MATHEMATICS EDUCATION



The purpose of the Ontario Association for Mathematics Education's Awards is to formally recognize individuals and educators who demonstrate an outstanding contribution to mathematics education in Ontario. The successful candidate of each award enhances the learning environment, providing opportunity for students to do, see, hear, and touch mathematics in a profound and meaningful way. If you know someone who qualifies for this honour, please nominate her/him/them for one of the following awards.

OAME AWARDS

OAME/AOEM Life Membership Award recognizes an educator who has contributed in a significant way to OAME/AOEM; demonstrates outstanding leadership in mathematics education; and has accumulated ten or more years of membership in OAME/AOEM.

Award for Outstanding Contribution to OAME/AOEM and Mathematics in Ontario recognizes an individual who has made a significant contribution to OAME/AOEM, but who is not necessarily an educator or a member of OAME/AOEM.

Award for Exceptional and Creative Teaching in Elementary Mathematics recognizes an exceptional and creative elementary teacher who demonstrates excellence in mathematics education and contributes to the overall development of students.

Award for Exceptional and Creative Teaching in Secondary Mathematics recognizes an exceptional and creative secondary teacher who demonstrates excellence in mathematics education and contributes to the overall development of students.

Award for Leadership in Mathematics Education recognizes an educator who has demonstrated leadership by contributing in a significant way to the development of mathematics teachers and enhancing mathematics education in Ontario.

Secondary School Department Award for Exceptional and Collaborative Mathematics Teaching recognizes a secondary mathematics department which fosters collegiality, teamwork, and excellent classroom teaching; contributes to the overall development of students; and demonstrates leadership in the mathematics education community.

Elementary School Staff Award for Exceptional and Collaborative Mathematics Teaching recognizes an elementary school (a school accommodating any or all of Grades JK–8), whose staff fosters collaborative and excellent mathematics teaching; contributes to the overall development of student learning of mathematics; and demonstrates leadership in the mathematics community.

OAME/AOEM Awards Nomination Process

1. Complete the Award Nomination Cover Page appropriate to the specific award, found at **www.oame.on.ca**, or contact the Executive Directors at **EDs@oame.on.ca** for an electronic version. You may include a one-page letter of recommendation.
2. Provide up to three letters of support, of no more than two pages each. These supporting documents may be gathered from a variety of sources to reveal a full picture of the nominee. The support documents will provide precise information about the nominee's contributions as they relate primarily to the specific Award.
3. Include the nominee's Curriculum Vitae of up to five pages. (C.V. not required for Secondary School Department Award and Elementary School Staff award)
4. Send the completed cover page and supporting documents by January 15, 2015 to: OAME Executive Directors, 4751 Mack Avenue, Town of Plympton-Wyoming, ON N0N 1J6, or email: **EDs@oame.on.ca**

DEADLINE: Nominations are due by January 15, 2015 to the Executive Directors (contact info above).
The awards will be presented at the Annual OAME Conference in May 8, 2015.

▲ OAME Awards 2014

Life Membership Award: Wayne Erdman

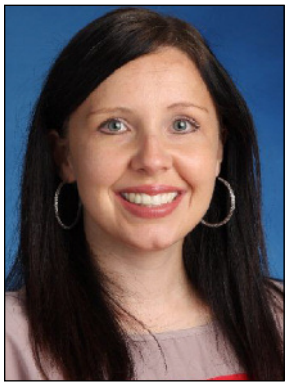


To be the recipient of the OAME Life Membership Award is to shine within the criteria of demonstrating outstanding leadership in mathematics education, as well as making significant contributions to OAME over many years. Wayne Erdman indeed shines. As his nominee stated, "How can one person do so much? How can an educator excel in so many areas?," and concluded it was owed to a sustained passion for mathematics education in our province.

Wayne's passion has cut a broad swath as his career has moved forward. He has led a mathematics department to award-winning successes as the department leader. He has contributed articles to the *Gazette*, as well as having contributed to mathematics textbooks and teachers' resources approved by the Ontario Ministry of Education. And, not least, he has been a driving force in OAME—through his chapter leadership, his provincial conference leadership, and his leadership of the provincial Math Olympiad—over the years since his membership began in 1995.

Wayne is to be congratulated for the passion, drive, and commitment that have earned him this award. However, he certainly has not finished yet!

Award for Exceptional and Creative Teaching in Elementary Mathematics: Stefani Linse



It is a credit to Stefani's expertise in the classroom to be nominated and supported for this award by parents and educators alike. Recognition by this variety of partners in education reflects the appreciation for her qualities as a teacher. Stefani places importance on the strengths of each child in her Grade 3 class, and reinforces them with frequent feedback as she guides them to next steps in their progress. She is a frequent home communicator, a fact which has been noted with appreciation by parents whose children are navigating school with a set of learning challenges.

Stefani also brings to her classroom the skills to work with current methodologies and technology. She is proficient with the use of manipulatives and other technologies to engage students in their mathematics learning, but also understands the value of Accountable Math Talk and student collaboration. Outside of her classroom, Stefani has been part of her board's Collaborative Inquiry in Mathematics projects, and lately is involved in a leadership program with Halton Catholic District School Board.

OAME is proud to add Stefani's name to the roster of outstanding mathematics educators who have earned this award.

Award for Exceptional and Creative Teaching in Secondary Mathematics: Judy Mendaglio



Teaching has not been Judy's first career, yet in this profession, she has obviously found a successful niche. Her classroom is a place of learning where students embrace technology of various types—interactive whiteboards, software, models—to support their engagement and learning. Her effervescent personality can only serve to enhance the activities she brings to her teaching. Testimonials from former students acknowledge the positive impact she has made on her students.

Her commitment to providing authentic and engaging learning experiences for her students has not been limited to the classroom. Judy has been part of the creative team that developed Financial Literacy resources that have been released through the Ontario Ministry of Education. Her involvement in OAME has enriched the organization for the passion about mathematics that she brings with her. We congratulate Judy as a deserving recipient of OAME's award, for her creativity in teaching has truly made a difference. ▲

▲ Turning Problem Solving Upside Down – Book Review of: *The ANIE*

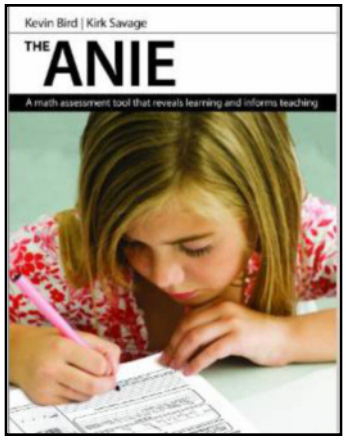
(Bird, K., & Savage, K. (2014). *The ANIE: A math assessment tool that reveals learning and informs teaching.* Markham, ON: Pembroke Publishers.)

TIM SIBBALD
E-MAIL: timothys@nipissingu.ca



Tim is an assistant professor at the Schulich School of Education at Nipissing University and a vice-president of OAME.

ANIE is an acronym for Assessment of Numeracy In Education. It is essentially a scaffolded approach to problem solving that has many tangential benefits. Among those benefits is increased insight into student thinking, a structured approach to formative assessment, opportunity for mathematical creativity, and cross-curricular support for language arts.



The book itself is a straightforward read and is not overly long (about 80 pages). The authors are school administrators from British Columbia, who clearly appreciate the reality of teachers being busy. It is a practical book that could be readily implemented using the supplied blackline masters. The focus is on elementary grades, but extensions to higher grades are clearly feasible.

The overall structure of the assessment is the provision of a knowledge-based question that is followed by a scaffold sequence of tasks related to the question. Students have to estimate the answer, calculate, sketch, explain the sketch, provide a real-life example that could lead to the question, and finally, reflect on their thinking. While there are several interesting features to this approach, a key element is that the students are being led toward making their own word problems through the real-life example. The authors aptly point out that this has the benefit of allowing students to generate the context, rather than have a context imposed on them. They also point out that students are not trying to decipher someone else's text; rather, they are using their own words to construct the scenario.

The blackline masters are provided with an area for assessment information. This has not been designed specifically for Ontario, but the generic version could be adapted easily. An explanation of the use of the assessment area is provided in detail. The only component of the blackline masters that is a cause for concern is that the problem has to fit into one or two text lines. Modification would be necessary for geometry questions, for example.

While I think the approach presented in this book is worthy of exploration, there is potential that routine use could cause monotony that might hinder some students. There is no doubt that the scaffold will help many students. The question is whether it should be used as is for the students who do not need as much scaffold. That said, the structure of the scaffold provides a clear way for teachers to see student reasoning and a good format for formative feedback. Through its use, a teacher would recognize the students who need less scaffold and be able to provide them with a less structured version.

The evaluation of the ANIE approach that is provided in the book is limited. Two examples are provided, where schools had increased scores on a provincial assessment. However, the evidence does not demonstrate whether ANIE improved student comprehension or simply taught students how to work with a structured format. In the same way that completing multiple IQ tests increases one's IQ score, the evidence does not rule out the possibility that ANIE is having an impact for unanticipated reasons. The second example provided as evidence introduced ANIE at the same time as JUMP math and a literacy program. While the evidence shows a positive impact, it is unclear how much was due specifically to the use of ANIE.

Overall, this book is practical, food for thought, and worthwhile for the price tag of approximately $25. It has a practical base and immediate utility. While, as a researcher, I take issue with the evidence, I confess to a professional sense that the approach will have benefits. ▲

An Audience for Math – The Math Performance Festival

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What if…?

If someone asked you, "What did you do in math today?," could you share a math story that would:

- capture the imagination?
- offer a cool perspective on a math idea?
- surprise the person mathematically?
- give that individual the pleasure of mathematical insight?

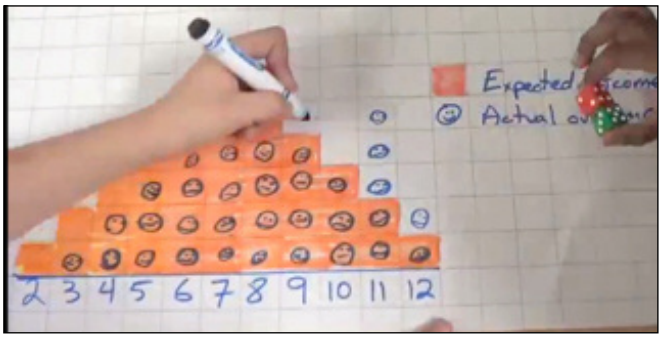
Most students and adults would have a difficult time doing this. Typically, most math is learned in classrooms for the classrooms, and not for "just plain folks" (Brown, Collins, & Duguid, 1989).

What if mathematics education became an occasion for students to *perform* their math learning for the wider community, to help them and others better understand mathematics concepts, and to engage their attention with beautiful math ideas? (Gadanidis & Hughes, 2011).

Since 2008, Western University has been running a Math Performance Festival, supported by the Imperial Oil Foundation, the Fields Institute, and the Canadian Mathematical Society, where students have shared their math performances for a wider audience. In this paper, we share some of the teacher resources available on the Festival website (**www.MathFest.ca**) for creating math performances that capture the imagination and offer the pleasure of mathematical surprise and insight: (1) case stories of classroom practice, (2) interviews with teachers whose students participate in the Festival, and (3) resources and guides to creating a math performance.

A case in point – two primary classes share their math story

Probability is a wonderful topic in the grade 2 and 3 curriculum: learning how to communicate likelihoods, experimenting with random, and not so random events, or using probability to predict outcomes. We wanted to create a math performance to celebrate our learning. Looking at the mathfest.ca website, we were inspired by past submissions, and decided to make a song to summarize our learning. From there, we looked around the resources section of the website and found ideas for writing a math poem and adding music to it.



*The idea that we tried for generating lyrics was to send home a math surprise with the students for parents. The students played a "rigged" dice game, with their parents rolling two dice—this can be seen in our video (**www.youtube.com/watch?v=pChIVgc5QgY**) at the 0:42 mark. Parents were asked to share what they had learned and what mathematics concepts their child had shared with them. Picking and choosing from this feedback, we created the lyrics for our song that outlined what we had all learned about probability. These lyrics (**www.mathfest.ca/mpf2013/TwistandShoutaboutProbability.doc**) were the collected understandings of the parents and students in our classes.*

Excerpt from *Twist and Shout about Probability*

We're talking probability
Exploring odds is fun (Odds are fun)
What are the chances in a game we played
It was so easy to be done (To be done)

Well shake it, shake it, shake the dice now,
See what result comes up (What comes up?)
You never know the answer
It's probability, not luck. (Not just luck)

Let's trick our parents and our teachers,
We'll use an educated guess, (We can predict)
Now that we get how the odds work,
Oh probability's the best. (It passed the test!)

The students wanted to show how they had done the math, so we videotaped them performing probability experiments. As these were quite long, we got the idea to speed up the video to fit our song. In the end, we also wanted to show how we had arrived at our learning, so we created a mini-documentary at the end that was narrated by the students.

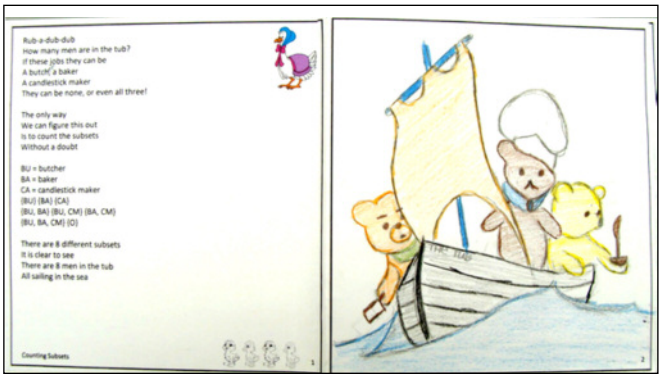
In the end, this was a really fun process. Knowing that we were going to create a math performance added excitement to the fun math that we learned.

Seeing is believing – teacher interviews

There is no substitute for understanding how you can change the way you approach mathematics teaching like seeing and hearing from those who do it. In the resources section of the Festival website (**www.edu.uwo.ca/mpc/resources.html**), there are interviews with teachers talking about how they use the Festival as a springboard to a richer learning experience for their students. There are also multiple links to the resources these teachers used to help create these experiences.

These interviews run the gamut of what, how, and why these performances are great learning opportunities, as well as being a lot of fun for students and teachers alike. Performing creates new audiences for mathematics learning, reaching people who might never have previously seen what we learn in class.

One particularly fine, illustrative example is an interview with Shirley Dalrymple. Shirley shares with us how she assigned a project every year that had her students revealing their knowledge of probability, permutations, and combinations through rewriting fairy tales. These Grade 12 MDM4U students have also shared their work on the website so that others can use them as examples to make their own mathematical fairy tales. "The story really helps them demonstrate to me that they understand the concepts because they have to apply them in a way that isn't reciting a formula, isn't everyone doing the same question, but they have to think of a way to use a concept in a story," Shirley said.



While looking at the stories these students have created to illustrate mathematics through storytelling, we can see creativity being applied to explain mathematics concepts. Having an audience to tell a story to has allowed these students to put mathematics into a different light than what they might have experienced previously.

Documenting a different approach – case stories

The case stories on the Festival website outline a breadth of what is possible in mathematics education. From Kindergarten to Grade 8, from number sense through data management, there is something there for everyone. Each of the case stories outlines how it is possible to take everyday, ordinary math concepts from one of the grade level curricula and teach them through complex, puzzling, and most importantly, interesting and important mathematics, and then share the learning through a performance of some kind. These mathematics activities use what we call a "low floor/high ceiling" approach. Each activity starts with a puzzle, riddle, or story embedded in the context of big ideas, rich with complex and puzzling mathematics. The initial engagement begins with mathematics that everyone can understand, and unfolds in a way that exposes all of the students to the rich, complex mathematics concepts they will encounter later in their education—like learning about fractions in the context of infinity and limit (**http://mathfest.ca/mpf2014/infinity-story.html**). Each case story outlines not only the mathematics that is possible, but is also accompanied by a performance such as a song, a piece of visual artwork, a dance, a comic strip, even complete stories that summarize the mathematics learned.

One example is the case story of "Kindergarten Patterns." Challenged to find a way to get kindergartners to think about and create patterns beyond simple ABAB

patterns, this story shows how even very young children can create complex repeating patterns. The kindergartners began by creating their own simple songs, using Orff instruments. Playing and singing, they then created dances with repeating steps, "wrote down" their music, using coloured bingo dabbers, and created art out of these strips.



Another example was when we heard, "I wish we had more homework like this!" This was the feedback we received from a parent after her child had taught her how much fun you can have making ten from two numbers. Starting with the simple equation ___ + ___ = 10, we led the students through equalities, inequalities, and graphing with coordinates, using the two unknowns from the equation above. Through this, the students produced a series of points from the coordinates, which formed straight lines. When challenged to change the line by changing the equation, the students found lines with different slopes, different *y*-intercepts, curves like hyperbolas, and in the case of the intermediate students, circles. All this came from the simple idea of making ten.



The students' perspectives on mathematics had been changed. From the idea that there are fixed answers to math questions, the students now understood that even simple mathematics like ___ + ___ = 10 is linked to complex, rich, interesting, and important mathematics. Algebra is connected to pattern, which is connected to graphical representations, which is connected to geometry, and so on. Summarizing their learning and the comments of their parents in a song, these children really did wish they had more homework like this.

These case stories demonstrate how it is possible, and desirable, to capture the imaginations of students. Starting with a low floor and having a high ceiling, the activities narrated here led the students into ever more complex and rich mathematics. Their minds were opened as they actively sought more. Even after the students had stopped for the day mathematically, the suggestion that there is always something more to be sought is now in their heads. Learning mathematics has been changed into a quest for knowledge. It is now a journey of understanding with surprise and insight into the beauty and interconnectedness of mathematical concepts.

An invitation

We hope that you and your students will consider participating in the Festival. The Festival is more of a celebration than a contest, and we endeavour to give out as many prizes and medals as our funding allows.

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Gadanidis, G., & Hughes, J. (2011). Performing big math ideas across the grades. *Teaching Children Mathematics, 17*(8), 486–496 .▲

Ask Assessment Abby: Assessing Students' Mindsets

ASSESSMENT ABBY
EMAIL: assessmentabby@oame.on.ca



Ask Assessment Abby A^3 is a regular column in the OAME *Gazette*, where teachers can share concerns and best practices about assessment, evaluation, and reporting of mathematics. Please send your questions to Ask Abby at **assessmentabby@oame.on.ca**.

Ask Assessment Abby,

My school board is very interested in working on student mindset, particularly in mathematics. We also have a focus on effective implementation of *Growing Success*. My question is, "What is 'mindset,' and what are the connections between assessment and evaluation in math and student mindset?"

Signed,

Hopeful and willing to learn

Dear Hopeful,

Great question!

"Mindset" is a word that has been around for a very long time, but was rarely heard until recent years. It now seems to be everywhere. Businesses may look for a "global" mindset for an executive. They may interview to screen out individuals who have an "entrepreneurial" mindset. In education, the importance of both student and teacher mindset in directing student success, particularly in mathematics, has been the focus of a lot of discussion. Much of what we now know about mindset comes from the research of Carol Dweck, a psychologist from Stanford University, who has been studying mindset for 30 years. She uses the adjectives "fixed" and "growth" to describe mindsets. People who believe that their intelligence is not set at birth, and that their skills and understanding will improve with effort and experimentation, are described by Dweck as having a growth mindset. One difference between an individual who has a growth mindset and one having a fixed mindset is how he or she responds to disappointment. The growth mindset will be challenged to improve, with a belief that improvement is possible. The other will become discouraged and shut down. Mawi Asgedom describes a growth mindset simply as "I can do it." A similar, but slightly different, treatment of mindset that may be helpful can be found in the work of Marilee Adams. She refers to the "learner" versus the "judger" mindsets. Teachers in particular may find her concepts useful to inform their practice around assessment and evaluation.

EQAO data tells us that many of our students seem to have fixed mindsets when it comes to math. This is one of the reasons that many school boards are looking at student mindset in mathematics as an area of concern. We probably all recognize the oft-heard phrases, "I just don't get math" and "I was never good at math." These phrases are indicators of a fixed mindset. They serve to release the individual from responsibility to improve. More importantly, when statements like these are spoken in front of a child, particularly if said by a person of influence in that child's life (a parent, a teacher), they provide that student with a reason to walk away from learning math. The fixed mindset is created. Now, while we cannot change the attitudes of the adults in a child's life, we can learn something by trying to understand why this attitude is so prevalent in our adult populations.

You asked about mindset and its connection to assessment and evaluation. Before answering your question directly, allow me to share a true story. Last night, I was playing a game on my tablet. I intended to play for ten minutes while I was waiting for a television program to start. It didn't go according to plan. I played the level, but did not achieve the goal in the number of moves permitted. The ten minutes was over, but I did not even notice. Rather, I elected "Try again." This time, I got closer to achieving the goal, but I still missed the mark. Try again. Same result. After the fourth try, I not only achieved the goal, I did it with ten moves to spare. So what is this story about? While I would welcome your kudos for my accomplishment, I don't really need them because I already feel good about what I did. I persevered. I figured out how to use the tools I was given more purposefully. I created better strategies to solve the problem that was presented to me by the game. How did this all happen? Truthfully, it all happened quite naturally. When I did not achieve the goal that was set for me by the game, the game's goal became my goal. I did not succeed at first, but I knew I would be provided with the opportunity to try again, either immediately or at a later time. I made the decision to try again, even though that meant that I would miss the start of a show that I had been waiting to see. While playing, I consciously decided to be more deliberate in my moves.

The end result was that I was successful.

For a moment, take this story and relate it to how math is often assessed in the classroom. If your experience is like mine, a student is taught a concept and a set of related mathematical procedures, is quizzed after a small set has been taught, is taught more procedures or has to answer "word problems" using that procedure, then is tested. Testing is somehow understood to be different from quizzing. At this point, the unit is done. Except in grades where students are required to write EQAO tests, or in high school, those concepts and procedures may never reappear and be evaluated again. However, neither EQAO tests nor final exams are likely to change their mark on their previous tests. They are "set in stone." It is not surprising, then, if a student does not believe that he or she can still grow mathematically.

Contrast theses two scenarios. Most students do not see math tests as challenges that they take on willingly. On the contrary, they fear tests. The test has a finality attached to it that does not foster a desire to continue to try, to work to "get it," to continue to learn. If the student has not done well on the test, he or she is quite likely to greet a "bad mark" with a shrugging of the shoulders or throwing up of the hands. Students may walk away, resigned to "just not being that good at math." They may develop an aversion to even learning math because learning it seems to inevitably lead to the dreaded test. So what can be done?

Growing Success, the Ontario Ministry of Education's policy document on assessment, evaluation, and reporting, provides a key. It challenges all teachers, including teachers of mathematics, to provide students with formative assessment opportunities. These assessments should include descriptive and timely feedback about what students need to do to improve. Finally, it states that students should be given a variety of vehicles to demonstrate their learning and their achievement of expectations. Each of these offers opportunities for creative thinking around building growth mindsets through assessment and evaluation in math.

If you believe, as I do, that the best time to learn is now, then quizzes that are collected and marked at a later date are not what you are looking for to assess student attainment of skills and procedures. Remember watching the FIFA World Cup? These players were once children who learned about the rules of the game, the skills needed, and the drills needed as part of a team instruction. But they became better players because the coach watched them play and identified areas for growth and improvement. The coach recognizes that all students need a foundation in the rules and regulations and some of the basic moves, but to develop a championship team, each individual player needs individual coaching during practice. Feedback given during the practice allows players, and students, to be redirected when required so that they are able to do their best when it counts. So, one thing you could try in your classroom is to walk around while students are practising and stop to give descriptive feedback about how each student is progressing. At that time, offer one thing that the individual student can do to improve. If there is a common problem, address it with the whole class. (This is a great way for the teacher to build the learner mindset and to move away from the judger mindset.) Now, don't get me wrong. Quizzes are excellent opportunities to provide comments about structure, form, written use of terminology, etc., but even these areas of communication skill can be assessed as students work collaboratively in class. In fact, students themselves will often provide each other with this feedback immediately as they work together on rich tasks. For the student with challenges, the path to improvement is not overwhelming. It has detail and direction. The feedback is positive in its intent as contrasted to the dreaded red "X" (judger mindset). Specific and individualized feedback may provide the student with the growth mindset necessary to continue.

One more thing: I have had great success with projects such as films, journals, model building, presentations, and other alternate forms of evaluation. These offer opportunities for students to check in at various points in the project to obtain feedback from the teacher about how they are doing and what they could do to improve. When students bring their partially completed projects into class and they share what they have done with each other, they often receive additional valuable ideas and feedback. Other students often provide the best motivation for a student to do his or her best work.

I hope this has been helpful, Hopeful, and that you have success building growth mindsets through your assessment and evaluation practices.

Abby

If you have used assessment and evaluation strategies to promote growth mindsets in your students, and would like to share with your OAME colleagues, please send your ideas by email to the OAME *Gazette* at **assessmentabby@oame.ca**, and put "Ask Abby" in the subject line. We will select our favourite responses for publication in future columns. .▲

▲ OMO2014 – The Number Games

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Greg Clarke is from the Simcoe Muskoka Catholic District School Board and is currently on assignment with the Ontario Ministry of Education as a Provincial Math Lead, working on the CLIPS project.



Jodi Caverzan Wells teaches at St. Bernadette Elementary School in the Simcoe Muskoka Catholic District School Board. She has a passion for mathematics and has thoroughly enjoyed teaching students in the intermediate division for 16 years. As well, she held the role of Student Work Study Teacher for one year. She has been a storyboarder for CLIPS and is a member of MAC2.



Stephanie Rogers is a curriculum consultant for the Simcoe Muskoka Catholic District School Board, with a focus on mathematics. She has hosted a local OMO event for the last three years, and very much enjoyed being part of the team that put together the contest for this year's event.

The Ontario Mathematics Olympics, Provincial Finals for 2014, were titled "The Number Games," and they were indeed the best games ever! The host for the event was the MAC2 chapter (Mathematics Association for Cottage Country) at the Lakehead University's campus in Orillia, Ontario. This was the first OAME event held at the Orillia campus; the staff at Lakehead made us all very welcome, and fed us well too!

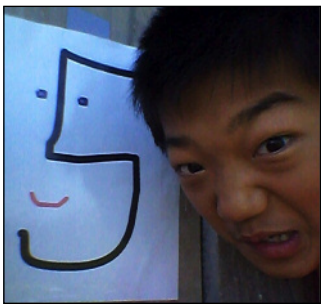
With 26 teams from around the province—from Thunder Bay to Ottawa, Kingston, Toronto, Sarnia/Windsor, and all points in between—the majority of OAME chapters were well represented at this year's competition—and they were impressive! An OMO team consists of four students: two Grade 7's + two Grade 8's (two boys and two girls), in some combination thereof. Each team had to pick a unique number to identify itself, and use that as a basis for the team chant, the team poster, and team spirit! There were some very creative results.

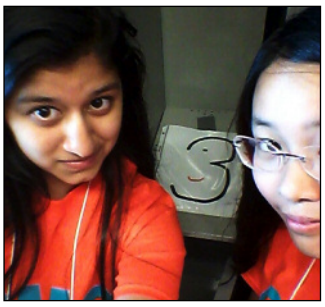
The weather was amazing for the entire weekend, which was great, since the two activities on Friday involved a lot of outdoor hunting for clues and solving mysteries. When the teams arrived, they checked in, and promptly did their team cheer, which was recorded and then compiled into a video montage by Dr. Ruth Beatty, a professor in the Faculty of Education at Lakehead University. Before dinner, a Sudoku challenge had mathletes (and many of the adults!) searching the grounds for clues for missing numbers.

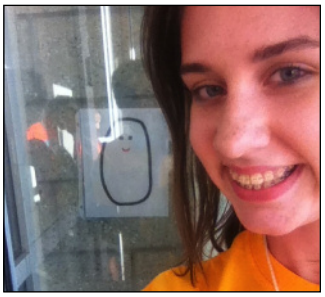


Dr. Ruth Beatty

After dinner, the group engaged in the Dilemma of the Disappearing Digits. Mathletes had to find where the digits had gone from "The Living Math" book, a rare book stored in the Lakehead vaults. Once again, they were traversing the campus, finding clues in QR codes, and searching the grounds for the naughty numbers. When a team found a digit, the students took a selfie with the digit to prove it to the organizers, then continued on their quest to find the next one. Great fun (and some really interesting selfies!) resulted… and the students didn't want to stop… so the time limit was extended. A total of seven teams completed the challenge (some hunting down the organizers relentlessly to prove their triumph).









The contest itself, held on the Saturday, went very smoothly. For the individual competition—congratulations to Daniel from SAME team 42 for top marks for boys; Evelyn from Y4MA team 1618 and Yolanda from TEAMS team 5656 tied for top marks for girls. For the pairs competition—congratulations to Bill and Cassie from Y4MA team 2048 for top marks. For the group competition—congratulations to TEAMS 20-5-1-13-19 and NWOAME -1/12, who tied for top marks.



Pairs Contest

After lunch, the final event was the Team Relay. Each team was given the same starting question, and took up a position around the edge of a large circle on a soccer field outside the main building on campus. The centre of the circle held the scorekeepers and answer-checkers. When a team had an answer, one of the mathletes would run to the centre, check the answer, and if it was correct, he or she would run back to the team with a new question. Otherwise, the team member would run back and try the question again (after a third failed attempt, the team could pass and move on to the next question). As is often the case, this final event was the most fun to watch, and the most fun to participate in—full of frenzy, frantic fractions, and furious figuring.



Relay

When it was all said and done, contests marked, and sums totalled, the final results were as follows:

- GOLD Medals were earned by Team #65536 from ISOMA (coach Sandra Della Maestra)
- SILVER Medals were earned by Team #2048 from Y4MA (coach Christopher Wu)
- BRONZE Medals were earned by Team #20-5-1-13-19 from TEAMS (coach Earl Totten)



It was amazing to see how much fun that the students were having during the contest, and to listen in on the amazing math conversations that took place. A reporter from the local Orillia newspaper dropped in on Saturday and interviewed some of the organizers and mathletes. The article (see link at end of this column) showed different perspectives and gave a nice overview of the event. One quotation from the article highlights the value of events such as these for some of our students: "This is one of the only few places in the world where I actually feel like I'm part of a group." ~ Mathlete

The entire event seemed to go by in a flash. It ran very smoothly, due in large part to the amazing organizing team. Many thanks go out to the MAC2 members who worked tirelessly this past year to organize the contest questions, the hospitality, and the special events. A very special thanks to all of the Lakehead Teacher Candidates—our future math teachers—who volunteered during the event and helped make everything run so smoothly, either by escorting teams around, helping those who were lost, or running and scoring the events themselves.

Good luck to the Ottawa crew from COMA, who will be running next year's provincial competition.

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Link to newspaper article on OMO2014: **www.orilliapacket.com/2014/05/31/mathletes-take-up-the-challenge**

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▲ Chapter Sample – *Mathematical Models for Teaching: Reasoning Without Memorization*

ANN KAJANDER,
TOM BOLAND

This is chapter one from the newly published book by Ann Kajander and Tom Boland.

Chapter 1 ~ Introduction to Mathematics for Teaching

What Is Mathematics?

Teachers' beliefs about what mathematics is and how it should be learned are formed during their own schooling, and often form the basis of their own teaching practice. We have met many elementary teachers who tell us that teaching mathematics is not their strength. While they may be comfortable designing creative, interesting, student-based lessons in other subject areas, they say that they often feel less comfortable with their ability to do this within mathematics. Some other teachers feel that as long as they know "the rules," mathematics is not that difficult. In fact, many teachers were previously taught that mathematics is a rule-based subject, and that there is only one correct answer to any given problem and one method of deriving that answer. As a result, when individuals begin teacher preparation programs, they may believe that mathematics is about following procedures memorized by rote (Ball, 1988a, 1988b; Handal, 2003; Holm and Kajander, 2011, 2012; Thompson, 1992). Nothing is farther from the truth about mathematics! Mathematics is a rich and creative problem-solving-based subject. For descriptions of problem- and inquiry-based mathematics learning, see Ambrose, 2004; Askey, 1999; Ball and Bass, 2000; Boaler and Humphreys, 2005; Kajander et al., 2010; and Weiss and Moore-Russo, 2012.

What Students Need

During the past 25 years, research on the ways students understand and use mathematics (NCTM, 2000; National Mathematics Advisory Panel [NMAP], 2008) suggests that what they need to know about fundamental mathematics has changed and developed greatly. More than ever, students need to be good problem-solvers rather than "mini-calculators." They must be able to effectively represent a mathematics

problem, often taken from a real-world context, to reason it out. They must be able to make connections among mathematical ideas in order to devise appropriate problem-solving strategies. They must be able to interpret and ultimately communicate their solutions. Such mathematical work not only requires students to master new and different types of mathematical understandings and approaches, but teachers must also do the same. This places significant demand on teachers, as described by Sowder (2007). Helping teachers to develop new ways of understanding based on inquiry and sense-making is the purpose of this book.

How Mathematics Is Learned

The way that practitioners use mathematics does not reflect the way it is initially developed and understood by learners (Skemp, 1986), regardless of whether the learners are schoolchildren or mathematicians (Weiss and Moore-Russo, 2012). Skemp illustrates this principle using the following example: We don't learn the concept of "colour" by reading a definition of light wavelengths; rather, we first need to experience and compare things that are red, yellow, blue, and so on before we can construct—and understand—the relatively abstract concept of colour.

The problem for teachers is that once we have learned a certain generalization or "rule," it seems simpler to use (and, therefore, teach) that rule than to provide a lot of problems for students to explore. Problem-based learning can feel messy and, at times, incomplete (Zack and Reid, 2003; Bay-Williams and Meyer, 2005). This may increase the appeal of teaching via procedures, because we, as teachers, might think this approach is easier; however, the critical point is that generalizations are only easier to understand and use after we have gone through the process of constructing their meaning for ourselves. Our minds are uniquely wired to make mathematical generalizations or abstractions, and to use them once we have done so—but this is not how these ideas are learned in the first place! Unpacking the higher-order ideas we have about mathematics to develop lessons that allow students to establish the conceptual building blocks themselves remains the central challenge of good mathematics teaching.

Teachers' Knowledge

Research suggests that teachers' knowledge must be broad in scope and include an understanding of how ideas are initially developed (Lesh and Doerr, 2003; Ma, 1999; Weiss and Moore-Russo, 2012). In our work, we have formed various ideas of what such knowledge should be, a central theme of which is deep and connected understanding, based on models and reasoning and applicable in the classroom. Our own ongoing research about best practices in supporting teacher mathematics development (Kajander, 2010a, 2010b; Hart, Swars, Oesterle, and Kajander, 2012; Kajander, Fredrickson, Casasola, and Boland, 2013; Kajander and Holm, 2013) illustrates the challenges faced by teachers, and provides grounding for this book. In particular, teachers need to know a multiplicity of models that are typically constructed by students, as described by Doerr and Lesh (2003), and how these apply to both mathematics and learning. Knowledge of such models is an aspect of the "distinct body of knowledge associated with mathematics teaching" (Davis and Simmt, 2006, p. 294). Kahan, Cooper, and Bethea (2003) explain how this new field of "mathematics for teaching" described by Ball, Thames, and Phelps (2008) differs from the knowledge typically taught in more general university mathematics classes. Hill, Sleep, Lewis, and Ball (2007) describe this new approach as including knowledge of the various ways students might understand a concept. Such a distinct body of knowledge forms the basis of this book, which is designed to support elementary teachers' deep and conceptual mathematical learning as needed for effective classroom teaching.

Supporting Research

This book is based on two major premises derived from substantial mathematics education research during the last several decades, one of which is about the nature of children's mathematics learning, and the second of which is related to the demands that such learning environments place on teachers' knowledge. The first body of research is about effective classroom learning. The U.S.-based National Council of Teachers of Mathematics (NCTM) produced the first major publication related to mathematics reform in 1989, *Curriculum and Evaluation Standards for School Mathematics*, as well as the more recent *Principles and Standards for School Mathematics* in 2000. These documents were the culmination of several decades of research about children's learning across North America, and describe classroom learning that is far richer, more conceptual, more problem- and context-based, and that makes a greater number of connections between and among concepts than in the past. Further research, described by Lesh and Doerr (2003), supports the effectiveness of such classroom learning in the modern era, particularly for learners that have traditionally been less successful in mathematics. For example, students

who were allowed to learn using inquiry-based methods with models and manipulatives were able to construct meaning for the ideas more effectively than when they were told rules. Curriculum documents all over North America have been revised and developed over the last several decades based on this research. These curriculum documents, which include a much broader and more conceptual approach to mathematics, as well as explicit emphasis on problem solving and *mathematical* models and processes, place significant demands on teachers in terms of the need for richer and deeper mathematics understanding. Kajander and Holm (2013) and Sowder (2007) illustrate the challenges teachers may experience in this regard, particularly if they were taught mathematics using only a rule-based approach.

In a reform-based learning environment, teachers are asked to help students develop mathematical ideas by means of a conceptually based problem-solving paradigm. While providing detailed information on planning and facilitating actual classroom mathematics lessons is not the purpose of this book (the further reading section at the end of this chapter contains books that provide more information about lesson construction), the mathematical understanding needed by teachers to construct and facilitate such classroom lessons is explored in depth.

The second premise of this book is that there is a body of specialized mathematical knowledge that teachers require to facilitate effective classroom teaching. In their research, Ball, Thames, and Phelps (2008), Davis and Simmt (2006), McNeal and Simon (2000), and Silverman and Thompson (2008) describe the development of the field of "mathematics for teaching." Ball, Thames, and Phelps define various types of knowledge for teachers, including a specialized body of content knowledge unique to teaching, which "is mathematical knowledge not typically needed for purposes other than teaching" (2008, p. 400). They further state that teachers' mathematical work "involves a kind of unpacking of mathematics that is not needed—or even desirable—in settings other than teaching" (2008, p. 400). Hence, while the work of a research mathematician might involve developing increasingly powerful and abstract generalizations, the work of teachers involves "prying apart constructs, [and] making sense of the analogies, metaphors, images, and logical constructs that give shape to a mathematical construct" (Davis and Simmt, 2006, p. 301). This knowledge is indeed mathematical; Ball, Thames, and Phelps (2008) claim that "deciding whether a method or procedure would work in general[,] … determining the validity of a mathematical argument, or selecting a mathematically appropriate representation… requires mathematical knowledge and skill, not knowledge of students or teaching" (p. 398). Such specialized mathematical knowledge helps teachers construct good classroom problems while anticipating students' various potential interpretations, and recognizing what students must already understand in order to take on these problems. Teachers also draw on their content knowledge to effectively help students model and reason through problems, recognize their errors, and provide prompts, questions, or further or different problems that will allow them to recognize and clarify their misconceptions. Teachers support effective classroom discussions about possible outcomes and solution methods of the problems, and ultimately assess student progress in order to plan appropriate new problems. Research is becoming more and more conclusive, gathering evidence that suggests that such deep and particular teacher mathematics knowledge supports measurably stronger student achievement in this subject (Baumert et al., 2010; NMAP, 2008; Sowder, 2007).

Determining precisely what content to include in this book has been the focus of a number of our recent research projects. For the last ten years, we have been tracking future elementary teachers' understanding of mathematics as these prospective teachers enter teacher education programs. We have found, over and over, that many teacher candidates initially struggle to explain the concepts in Ontario's elementary curriculum, and similar results are found elsewhere, as illustrated by the research of Ma (1999) and Weller, Arnon, and Dubinsky (2009). We have worked in parallel with elementary classroom teachers who we judged to be highly effective teachers, chronicling what they know, and how they use this knowledge when teaching mathematics in their day-to-day classroom work with students. This work, along with recommendations from documents such as *Foundations for Success*, the report of the National Mathematics Advisory Panel (2008), helped us to decide what to include in this book. Supporting the development of a specialized mathematical understanding, conducive to effective classroom learning, is the primary focus of this book.

Concepts and Procedures

Ball and Wilson (2012) have explained that in mathematics education, possibly more than in any other subject, there is a significant gap between what we know from research about best practices and what current

classroom practice generally looks like. Research in mathematics education over the past three decades indicates that most students learn best (and retain ideas longer) if they are able to construct deep understanding of the concepts themselves, rather than simply memorizing methods and rules. When students learn conceptually, they actually develop the procedural skills along the way. Procedural fluency has been shown to develop alongside conceptual understanding even when it is not the focus of instruction (Stein, Remillard, & Smith, 2007). Students retain computational knowledge and skills developed in this manner longer, and can apply the procedures appropriately and with greater accuracy when solving new problems or those that are not routine problems.

Developing a deep understanding of the fundamental concepts may at first seem like a daunting challenge for some teachers, especially those with "math anxiety," as teachers who are anxious about their mathematical capacity may have survived their own mathematical education largely by memorizing. Also, it may at first seem somewhat counterintuitive to go "backwards" (Ball & Wilson, 2012)—backing up and unlearning or letting go of a previously memorized method—when trying to figure something out that we thought we already understood. Teachers may initially experience an internal resistance when working to construct deep understanding of previously memorized mathematical ideas (Holm & Kajander, 2011, 2012); Eisenhart and colleagues (1993) describe how it can be challenging for teachers to focus their own instruction on these ideas. For example, it seems simpler to accept the rule that states that "the product of two negative integers is positive" than to attempt to develop a model to explain the reasons behind this rule; however, the mathematical power gained from deeply understanding a concept soon makes the effort worthwhile.

Mathematics Anxiety and Mathematical Development for Teachers

Mathematics anxiety may stem from an awareness of inadequate personal understanding of mathematics, and the best way to address such anxiety is by developing enhanced content knowledge (Sowder, 2007). According to Stipek, Givvin, Salmon, and MacGyvers (2001), understanding how previously memorized rules work contributes to the development of more effective teaching practices. In our experience, nothing addresses the "math anxiety" experienced by some teachers more effectively than developing, often for the first time, a conceptually deep, connected, and flexible understanding of previously memorized rules. See Holm and Kajander (2012) for a detailed description of pre-service teachers' experiences as they navigated such a paradigm shift, and how emotionally rewarding it was for them.

Teachers have the responsibility to understand mathematics in a way that will allow them to connect the mathematical ideas to students' levels of development, as well as support the conceptual development of ideas. The process of backing up and redeveloping the models, concepts, and connections in mathematics is not trivial. In some ways, the more sophisticated one's procedural mathematical fluency is, the harder and more counterintuitive this unlearning process may initially seem. Moreira and David (2008) suggest that a strong procedural knowledge base may actually inhibit the desire to unpack and redevelop the underlying ideas, and further argue that strong procedural skill may even block a prospective teacher's willingness to engage with the mathematics more conceptually. However, it must be remembered that the construction of deep and connected mathematical knowledge, as described by Ma (1999), by students and teachers alike, is extremely powerful with respect to both teaching and learning. For students, making sense of mathematics in this way (instead of blindly memorizing rules) supports learning that is rewarding and well retained, rather than frustrating and easily forgotten. We can only assure the reader that engaging in this process will increase both personal understanding of mathematics and the capacity to provide better mathematical learning opportunities for students.

Reasoning and Rigor

One criticism of problem-based learning as the new paradigm for learning mathematics more conceptually is a perceived lack of mathematical rigor and the ability to produce complete and syntactically correct arguments according to generally accepted standards. Traditionalists may argue that rigor is lacking in problem-based learning environments. Historically, higher-level mathematics was taught by exposing students to relatively sophisticated mathematical arguments, such as proofs; this often consisted mainly of examining those proofs already in existence (Beaugris, 2013). This method was thought to teach mathematical reasoning, and instill in students a sense of what achieving mathematical rigor entailed.

A flaw in this traditional approach is a lack of understanding of human mathematical development.

While examining a sophisticated mathematical argument or proof presented in formal language might be interesting and illustrative, it does little to help students develop the ability to construct valid mathematical reasoning and rigorous arguments (Skemp, 1986; Reid, 2011). To use a metaphor, while budding painters might benefit from viewing famous works of art at a gallery, such viewing is hardly a complete educational experience for an artist. The same is true in mathematics. Children need to have the very experiences mathematicians themselves enjoyed when they constructed well-known and commonly used mathematics ideas, methods, and arguments. They need to experiment, conjecture, model, test, look for exceptions, find patterns, construct and extend arguments, and generalize—to name but a few mathematical processes highlighted by the NCTM in their 2000 publication. Ultimately, such skills are the very ones that will allow students to construct mathematical arguments of their own. Such mathematical arguments, based on connected understanding as well as sound reasoning, are the very underpinnings of mathematical reasoning and proof.

Student-generated arguments can become more and more rigorous as students' mathematical sophistication increases (Zack & Reid, 2004), just as, historically, mathematics became more rigorous as the field progressed. Research suggests that such a progression is an effective strategy for learning about the process of mathematical proof (Stylianides & Ball, 2008; Reid, 2011).

In this book, we often employ the stepping stone of a *generic example* (Reid, 2011) to argue an idea using a specific instance, while simultaneously thinking about how the idea might apply more generally. A generic example is a specific example constructed in such a way that it is representative of many other examples, such as a triangle with no special properties (e.g., that is not equilateral or right-angled). MATHEMATICAL TERM A conjecture is an idea or theory proposed as a mathematical truth.

Summary

To construct classroom learning environments that support deep and connected learning for students, teachers must have enhanced mathematical understanding. In their daily work, teachers need to be able to design useful contexts and examples, provide models and strategies, support alternate student methods and solutions, pose good questions, and help students identify misconceptions. Such work requires a deep, rich, and specialized understanding of mathematics. Teacher knowledge of mathematical concepts, models, representations, strategies, and connections directly supports student learning, and is therefore critically important to student success. Providing teachers with the means to develop this enhanced knowledge of mathematics is the purpose of this book. This book will also help all learners of mathematics think about elementary mathematics in a more meaningful way—reliance on a good memory is not required!

Further Reading

Ball, D.L., Hill, H.C., & Bass, H. (2005). Knowing mathematics for teaching: Who knows mathematics well enough to teach third grade, and how can we decide? *American Educator, 29*(3), 14–46. **www.aft.org/pdfs/americaneducator/fall2005/BallF05.pdf**

This article describes and provides examples of what is particular to teachers' specialized knowledge of mathematics for teaching.

Holm, J., & Kajander, A. (2012). Interconnections of knowledge and beliefs in teaching mathematics. *Canadian Journal of Science, Mathematics, and Technology Education, 12*(1), 7–21.

This article describes the journeys of selected pre-service teachers as they strengthen their content knowledge base during teacher preparation.

Small, M. (2012). *Making math meaningful to Canadian students, K–8.* (2nd ed.). Toronto: Nelson Education.

This comprehensive textbook would be useful as a companion resource to *Mathematical Models for Teaching*. It provides information about classroom mathematics teaching strategies. ▲

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SPATIAL REASONING WITHIN GEOMETRY CONTEXTS

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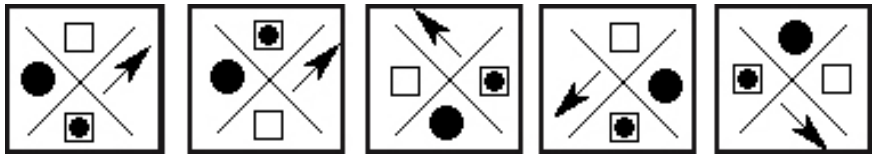
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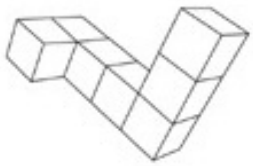
- Did you choose one shape and compare the shape to the right and left of it for each diagram?
- Did you imagine spinning each diagram about its centre or the intersection of the 2 line segments to see which diagrams look the same?
- Did you imagine, in your mind, a rotation of 1 or 2 turns counter-clockwise or clockwise (i.e., 90˚ or 180˚ CW or CCW) to match it with a diagram?
- Did you move your body around the diagram?

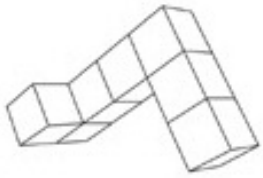
Any of the strategies listed above constitute the use of spatial reasoning, such as: moving one's body in space, perspective-taking, comparing object attributes, imagining moving objects in space. More specifically in the geometry and spatial sense curriculum, spatial reasoning includes: analyzing attributes, properties and geometric relationships for two- and three-dimensional geometric shapes and geometric relationships; describing locations and spatial relationships using coordinate geometry; recognizing and applying transformations (i.e., translations, rotations, reflections, dilatations); recognizing and creating shapes containing line and rotational symmetry; visualize mental images of geometric shapes; and recognize and represent two- and three-dimensional shapes from different perspectives.

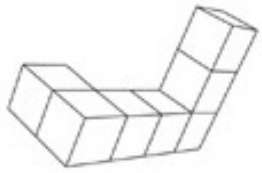
In this issue, the Research Summary and problems (with multiple solutions) focus on the use of spatial reasoning within geometry contexts. The problems we provide are designed for use with a three-part problem-solving lesson structure.

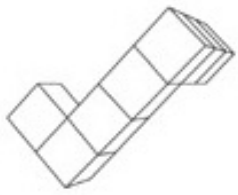
RESEARCH SUMMARY – SPATIAL REASONING WITHIN GEOMETRY CONTEXTS

Bahr and de Garcia (2010) describe spatial reasoning as thinking and reasoning by comparing, manipulating and transforming mental pictures through spatial visualization (i.e., comparing shapes that have changes in position on the plane or in space) and spatial orientation (i.e., the capacity to look at a fixed figure from several different points of view).









According to Linn and Peterson (1985), spatial thinking is not a unitary construct; it is a collection of specific skills, such as representing, transforming, generating and recalling symbolic, nonlinguistic information. More specifically, spatial thinking consists of: (1) mental rotation of two and three-dimensional figures; (2) spatial perception which includes determining spatial relationships with respect to the orientation of one's own body; (3) spatial visualization which includes multiple manipulations of spatially presented information derived from the analysis of the relationship between different spatial representations.

Further, Hegarty and Waller (2004) identified a dissociation of abilities involved with spatial orientation from those involved with spatial visualization, specifically mental rotation. They suggest that the dissociation between tests of perspective taking and mental rotation reflects a distinction between ability to make egocentric spatial transformations (i.e., to imagine the results of changing one's egocentric frame of reference with respect to the environment) and making object-based transformations (i.e., to imagine the results of changing the positions of objects in the environment, while maintaining one's current orientation in the environment).

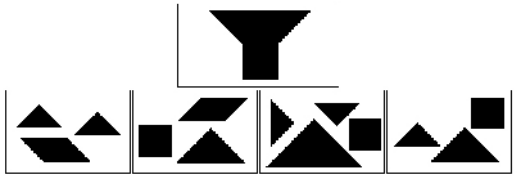
The National Research Council (2006) states that "spatial thinking is a collection of cognitive skills … it is a constructive amalgam of three elements: concepts of space, tools of representation, and processes of reasoning … It is the concept of space that makes spatial thinking a distinctive form of thinking" (p. 12).

1) *Space* – Properties of space include dimensionality, continuity, proximity and separation. In a geometry context, the nature of spaces includes the number of dimensions (two-or three-dimensional) and the kind of coordinate system, such as the Cartesian plane.
2) *Representation* – We use different spatial structures (e.g., maps, multidimensional scaling models (e.g., house floor plan, elevation house plan, orthogonal or perspective maps), dynamic geometry software like Geometer's Sketchpad, and computer-assisted design (CAD)) to help us to perceive, analyse static (two-or-three-dimensional) and dynamic (transformational) properties of objects and the relationship between objects. "These structures help us to remember, understand, reason and communicate about the properties and relations between objects represented in space, whether or not those objects themselves are inherently spatial." (p. 27)
3) *Reasoning* – Reasoning processes include: (a) extracting spatial structure (i.e., boundary, pattern, cluster, centre, path, surface, network, sequence, hierarchy, region); (b) performing spatial transformations (e.g., perspective, rotation, 3D, 2D, scale, interpolation, generalization); (c) drawing functional inferences (e.g., spatial correlation, spatial dependence heterogeneity, extrapolation, prediction, causation).

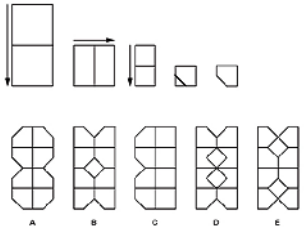
Examples of Spatial Reasoning Tasks or Problems with a Geometry Context

Wright, Thompson, Ganis, Newcombe and Kosslyn (2008) state that the performance of spatial tasks can be improved through practice and training to maximize learning.

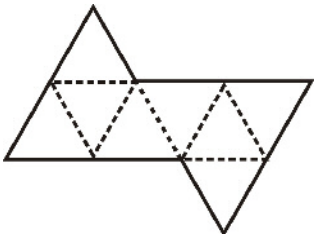
Two Dimensional and Three-Dimensional Figures



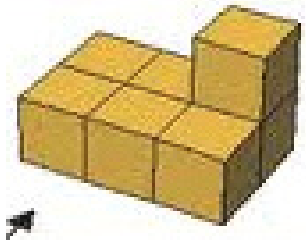
Which set of shapes can make the hexagon?



Which shape is a result of the cut-out sections of the 4 layer square?



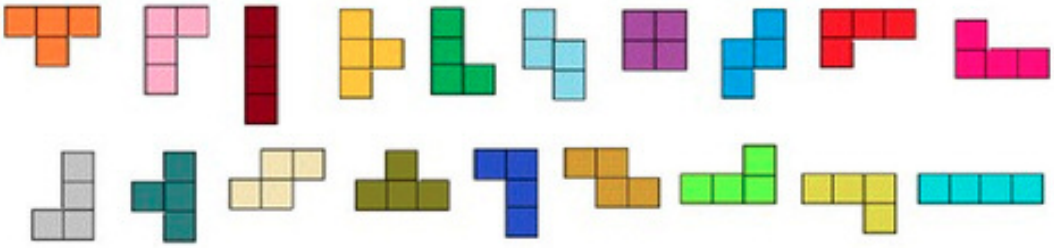
When this net is folded up, what does the 3D object look like?



Sketch 3 different views of this cube structure.

Transformations

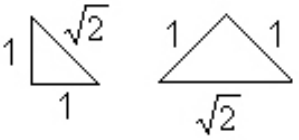
Which shapes are different? Which shapes are the same? Use transformations to explain your choices.



Position in Space

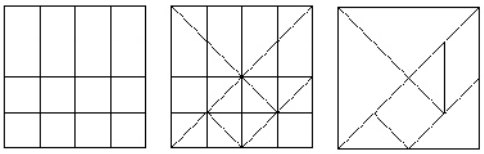
B is on the right of A. C is on the left of B. D is in front of C. E is in front of B. What is the relationship between D and E?

LINKS TO MANIPULATIVES: TANGRAMS



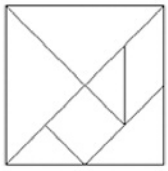
The tangram (Chinese: 七巧板; or *Qi Qiao Ban*) literally means "seven boards of skill or cunning" is a dissection puzzle consisting of 7 flat (2-D) shapes, called *tans*. All tans are made using squares and isosceles triangles (made from half a square). The 7 tangram pieces (or tans) comprise a dissection puzzle of 5 triangles, 1 square and 1 parallelogram that can be put together to form various shapes given only an outline or silhouette. So, 7 tangram pieces are composed of 32 half squares from 16 whole squares.

Tangram pieces (tans) are made by decomposing a 4x4 square into pieces. Some possible spatial reasoning tasks include:



- Explain how to make a tangram from a 4x4 square, by cutting paper or by paper folding.
- If paper folding, what are the least number of folds needed to make the 7 tans?

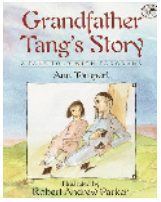
LINKS TO LITERATURE: TANGRAM STORIES

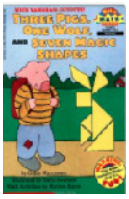


There are several kinds of stories that include the use of tangrams to recognize and apply transformations which require spatial reasoning (e.g., spatial memory and spatial visualization skills) and to become more precise in their use of vocabulary about space. For any of these stories, students can create tangram shapes (e.g., objects, animals, people) and explain the transformations they used

to move the tansfrom being a square to the new tangram shape.

- *Logicville* has tangrams stories with narratives that include different tangram illustrations, such as *The Story of a Crown and Bottle, The Boy Who Cried Wolf, The First Christmas, Weighing an Elephant, Twelve Days of Christmas* (**http://www.logicville.com/puzTS-A1.htm**)
- *Legend of the Tangram* (**http://www.jumpintoabook.com/2013/01/grandfather-tangs-story-storytelling-with-tangrams/**)
- *Grandfather Tang's Story* by Ann Tompert is about a grandfather who is telling a story using tangrams as illustrations.
- *Three Pigs, One Wolf, Seven Magic Shapes* by Grace Maccaroneis about 3 pigs, which use the 7 magic shapes to form solutions to problems they encounter with Big Brad Wolf.





LESSON LEARNING GOALS

Identify the curriculum expectations (MOE, 2005) grade-before and grade-after the grade specific learning goal, in order to:
- see the development of mathematical knowledge, skills, and strategies across grade levels
- determine the mathematical focus of the problems chosen for Before (activation), During (solving problem), and After (Practice) parts of the lesson

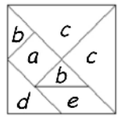
Grade 5	Grade 6	Grade 7
-create and analyse designs by translating and/or reflecting a shape or shapes using a variety of tools)	-create and analyse designs made by reflecting, translating, and/or rotating a shape or shapes by 90^0 or 180^0	-create and analyse designs involving translations, reflections, dilatations and/or simple rotations of two-dimensional shapes using a variety of shapes

BEFORE (Getting Started) – 5 to 10 minutes

- Activating students' mathematical knowledge and experience using a prompt or problem, related to the math in the lesson problem.
- Includes student responses to highlight ideas and strategies.

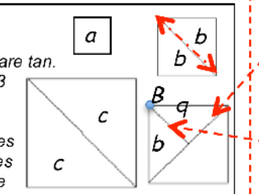
Tangram Shape Problem A:

What different squares can be made using different number of tans.
Explain your strategies.



Solution 1

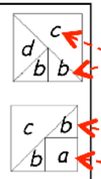
Start with 1 square tan. Choose 2 then 3 tans to make a square
- *2 small triangles*
- *2 large triangles*
- *1 small triangle and 1 large triangle.*



Visualize:
-cut a square in half along the diagonal.
-rotate 1 small triangle b about point, B, 90^0 CW to make 1 square

Solution 2

-2 same size triangles make a larger triangle; rearrange 4 tans to make groups of triangles;
-2 right triangles make a square.
-choose triangle and square tans only to make squares



Imagine how:
-4 small triangles make 1 large triangle
-2 small triangles make 1 square

DURING (Working on It) – 15 to 20 minutes

- Understanding the problem – Teacher asks, "What information from the problem are we using to make a plan to solve it? Explain." Teacher records in a list below the problem, the information that students identify.
- Students solve the problem on chart paper (landscape) with markers (visible for class discussion) in pairs or small groups.
- Teacher circulates to record different student solutions, in addition to the ones the teacher anticipated.

Tangram Shape Problem B:

What different rectangles (not including squares) can be made using different number and combination of tans. Explain your strategies.

What information are we going to use to solve the problem?
- *rectangles have opposite same side lengths*
- *rectangle is an area enclosed by 4 straight sides with 90^0 corner angles*

AFTER (Consolidation) – 20 to 25 minutes

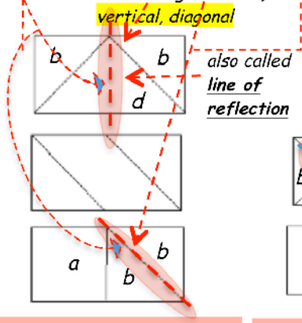
- Teacher selects 2 or more solutions for class analysis and discussion in a sequence (1^{st}, 2^{nd}, 3^{rd}, etc.) based on mathematical relationships between the solutions and the lesson learning goal.
- Students (authors) explain and discuss their solutions with their classmates; co-construction of success criteria towards the lesson learning goal
- During whole class discussion, solutions are organized (often re-organized) to show mathematical elaboration from one solution to the next and towards lesson learning goal.
- Teacher mathematically annotates (math terms, math symbols, labelled diagrams, concise explanations) on and around solutions to make mathematical ideas, strategies, and models of representation explicit.

Visualize (form a mental image)

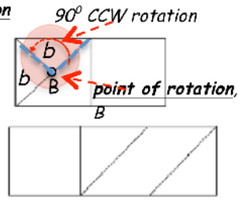
then check by modelling with tangrams.

Reflection *- **flip** a 2-D shape over a line of symmetry and the 2-D shape remains the same size and facing the opposite direction like a mirror image (or the orientation of image is reversed from the pre-image), like a mirror image*

Line of Symmetry
(divides a 2-D shape into two mirror-image halves):



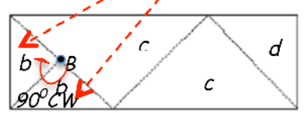
Rotation *– is a circular movement of 2-D shape around a centre (or point) of rotation. Track a rotation (CW or CCW) by measure of **turn** or angle measure by a side length.*



3 tans
4 small triangular units

4 tans
6 small triangular units

Transformations *include reflections (flip), rotations (turn) and translation (slide) of 2-D shapes, where the **pre-image** (shape prior) and **image** (shape after) of the 2-D shape is congruent (i.e., same size, area, angles and polygon slide lengths).*

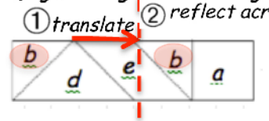


*270^0 CCW or 90^0 CW **rotation** about point, B*



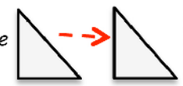
Diagonal Reflection *about line of reflection*

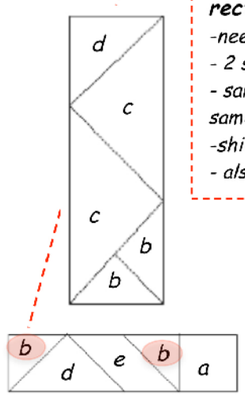
Composition of Transformations *– result of two or more transformations combined, forming a new transformation:*
- → *1st transformation produces an image upon which the second transformation is then performed.*
- → ***Glide Reflection*** *- composition of a translation and reflection (e.g., triangle b to triangle b)*

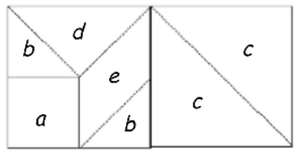


Or reflect then translate

Why do none of the tangram rectangles show a translation?
-need 2 same size 2-D shapes
- 2 sets of same size triangle tans
- same size triangles need to be in the same orientation
-shift or slide a shape
*- also called a **glide***







5 tans
8 to 12 small triangular units

7 tans
16 small triangular units

AFTER (Highlights/Summary) – 5 minutes

- Teacher and students revisit the student solutions for key ideas, strategies, and models of representation that are related to the lesson learning goal (anticipated success criteria)
- Teacher lists key ideas, strategies, and models of representation so the students can see how the mathematical details from their solutions relate explicitly to the lesson learning goal; summary of co-constructed

Highlights/Summary:

Translations are slides, rotations are turns and reflections are flips of two-dimensional (2-D) shapes, which are congruent.

Analyse tangram designs using transformations:
- ***Reflections*** *– flip across a line of reflection a 2-D shape where its image is reversed from its pre-image*
- ***Rotations*** *– turn of 2-D shape around a centre of rotation*
- ***Translations*** *– slide a 2-D shape horizontally, vertically or diagonally*

AFTER (Practice) – 5 to 10 minutes

- Teacher chooses 2 or 3 problems, similar to the lesson problem for student to solve in pairs as a scaffold and individually.
- Problems could vary by number (choice, size), problem contexts, or what is unknown or needs to be solved
- Students are asked to solve these problems using a strategy different from the one they used for the lesson problem.

Tangram Shapes Problem C:

What ways can a trapezoid be made using 7 tans only?

Solution 1

Keep the 2 large triangles on the outer edges of the trapezoid.



Solution 2

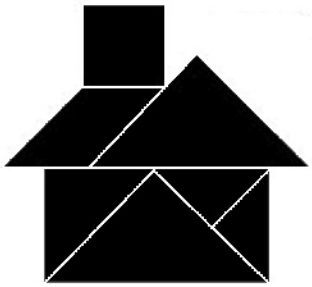
Think of a rectangle and right triangle.



Note: Mathematical annotations include mathematical vocabulary, symbols, elaborations of mathematical details from solutions, labels describing the method/strategy, and questions to further thinking. All annotations are records of students' mathematical discussion.

Note: A classroom board is longer proportionally than these 2 pages. Due to the space constraints on these pages, the mathematical annotations are recorded above the solutions with arrows, rather than on and around the solutions.

LET'S DO MATH – PUZZLE COMPARING PROBLEMS



Before (Getting Started)

Puzzle Comparing Problem 1:

What shapes do you see? How do their dimensions compare?

Solution 1

I see:

- 2 big triangles
- 1 medium triangle
- 2 small triangles
- a rhombus
- a small square

Solution 2

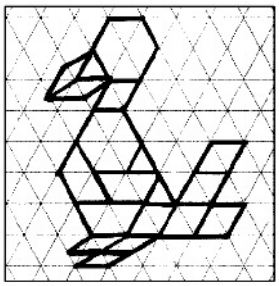
I see:

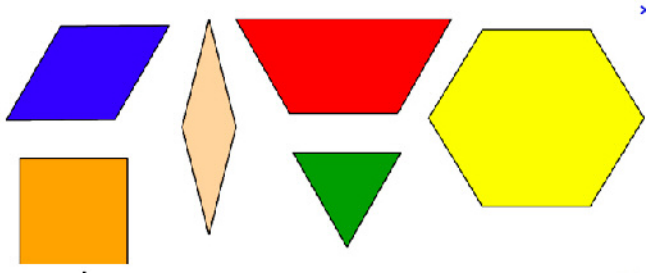
- 2 congruent big triangles
- one side of the large triangle is equal length of one side of the medium triangle
- one side of each of the small triangles is half the length of one side of the large triangle
- the square's side length is equal to the side length of the rhombus.

During (Working On It)

Puzzle Comparing Problem 2:

Pick a section of the pelican and match pattern blocks to it. Explain your thinking.



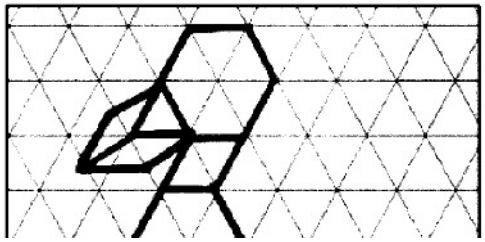


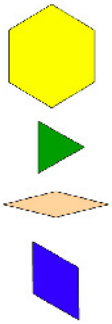
After (Consolidation)

Anticipating Student Responses:

Coordinating Discussion for Student Learning:

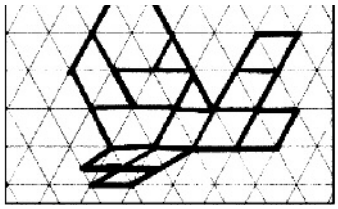
Solution 1

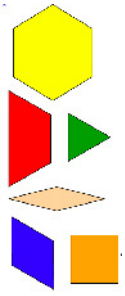




- A hexagon with 6 sides makes the head.
- The bigger rhombus with 4 sides and bigger acute angles makes the neck.
- The beak is made from 1 triangle (3 sides) and two small, light rhombus (4 sides) with the smaller acute angles.

Solution 1



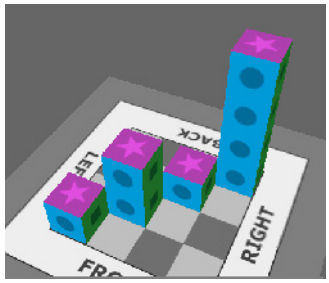


- I see three trapezoids (red). These are 2 units on bottom, 1 unit on top and two sides.
- I see three small rhombus (browny) with opposite sides equal and parallel. All sides are equal to 1 unit.
- I see one larger rhombus (blue) with bigger acute angle and smaller obtuse angle than the small rhombus. All sides are 1 unit.
- I see two little equilateral triangles 1 unit by 1 unit by 1 unit.

Why might solution 1 be chosen first for student discussion followed by solution 2?

- *solution 1* – shows comparing by direct matching of side lengths and angle measure, listing pieces that fit into the shape. The student reasoned using the number of sides and angles.
- *solution 2* – shows a more complicated part of the bird and names the properties that make pattern blocks match the puzzle pieces. Students manipulated and transformed the shapes to make them fit the puzzle pieces.

LET'S DO MATH – DIFFERENT PERSPECTIVES PROBLEMS

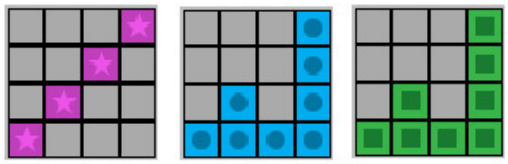


The images in this problem are taken from an applet from Explore Learning – Gizmos
http://www.explorelearning.com/index.cfm?method=cResource.dspView&ResourceID=170

Before (Getting Started)

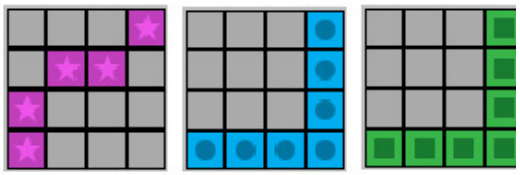
What are the top, front, and right views of this structure?

Solution 1



Top Front Right

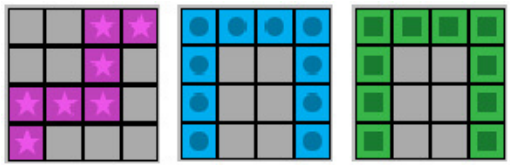
Solution 2



Top Front Right

During (Working On It)

Build a structure on the 4x4 grid so that these are the top, front, and right views of the structure.

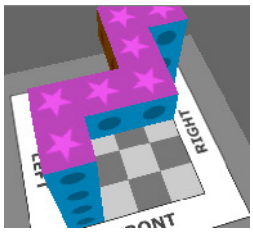


Top Front Right

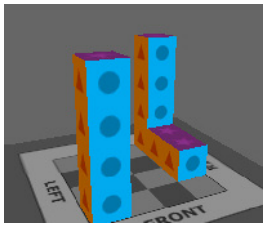
After (Consolidation)

Anticipating Student Responses:

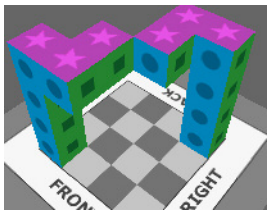
Solution 1



Solution 2



Solution 3



Coordinating Discussion for Student Learning:

Why might solution 2 be chosen first for student discussion, followed by solution 1 and 3?

- *solution 2* – this solution invite the discussion about the continuity or the lack of continuity from the front view. There are no cubes in column 2 or 3 (counting from left). The view from the Right is difficult to see and because on paper you cannot move around and "see" the structure from the Right. You may need to build the model and compare the right view.
- *solution 1* – this solution looks like it will work from the Front and the Right side. The 2 outside edges are 4 cubes high and there is a band across the top from left to right. Imagine viewing this from above and you see that the legs that extend across the structure are only 2 cubes in length rather than three.
- *solution 3* – this is an accurate solution. This perspective shows the Top and Front views clearly and you have to imagine the Right view.

NEXT STEPS FOR YOUR PROFESSIONAL LEARNING

Application to Your Classroom

- Try these spatial reasoning tasks or problems yourself first and then offer them to your students.
- To suit your students' learning needs, begin designing spatial reasoning tasks or problems that are relative to the grade specific curriculum expectations in the Geometry and Spatial Sense strand. However, grade specific curriculum should not limit the kinds of spatial reasoning experiences we make available to students. Results from the Math For Young Children (M4YC) project remind us that early primary students have the capacity to engage in spatial reasoning tasks that are thought to be well beyond their years.
- Provide problems that have students work with 2-D plans, sketches and designs; read and draw maps of different scales, complete and create visual puzzles, draw and model three-dimensional objects given different views (side, top, front) and angles.
- Incorporate spatial reasoning daily, as Quick Images for practise and exploration.
- Concrete or virtual learning materials to use include tessellations, pentominoes, nets, isometric dot paper, blocks for building, tangrams, pattern blocks, PolydronsTM, Geometer's SketchpadTM and everyday classroom objects.
- Practise noticing the different strategies and the breadth of the mathematics your students use to solve spatial reasoning tasks or problems, such as, decomposing or composing shapes to make different shapes, moving one's body in space, perspective-taking, shifting dimensions between three-dimensional figures and two-dimensional shapes, comparing object attributes, imagining moving objects in space.
- Think ahead of the kinds of mathematical annotations you will record on and around the student solutions to make explicit spatial reasoning strategies and concepts as the students discuss and analyze each other's solutions to co-construct success criteria for the lesson learning goal.

SUGGESTED READINGS

Some readings used in the Research Summary and in the development of the problems and solutions are listed below:

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CLOSING

Our next issue will focus on spatial reasoning within number sense contexts. As well, the next issue will outline the ways that teachers need to know these notions for teaching, as well as, the ways that teachers can use these mathematical ideas and pedagogical strategies within learning and teaching mathematics within a three-part problem solving lesson framework.

CALL FOR *ABACUS* SUBMISSIONS

Send your teaching strategy ideas, problems and student solutions to the *Abacus* Co-Editors via email

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