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Ontario Mathematics Gazette

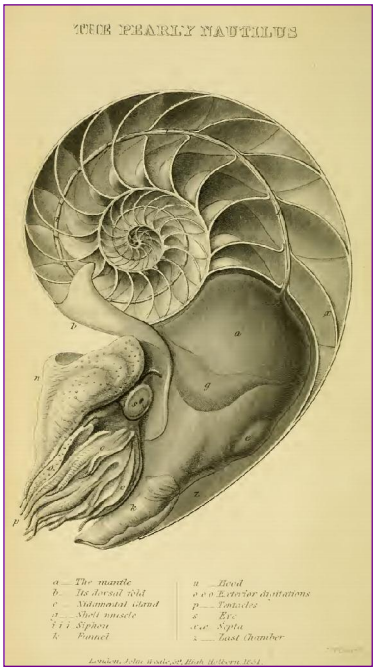
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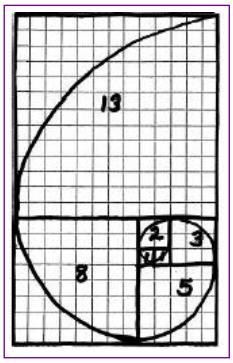
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Vol. 53 #2
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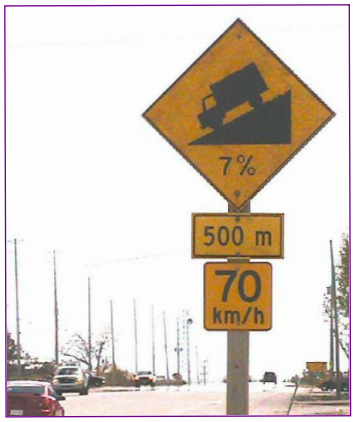
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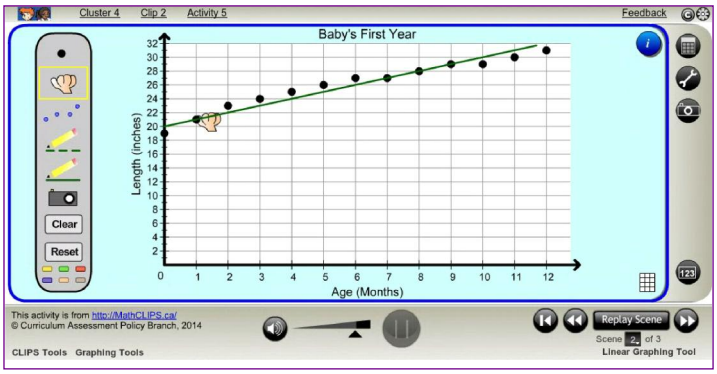




▲ See Hey, It's Elementary – Mathematics and Nature

▲ See Consistency Checking the Math of a Hill

▼ See Provincial Digital Learning Resources— Using CLIPS on a Mobile Device





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- Embed videos

For more information and to set up your FREE OTF Planboard account, visit **otffeo.on.ca**, go to Learning and click on OTF Planboard.

Submission of Articles

The *Ontario Mathematics Gazette* (*OMG*) is looking for news items, articles, and good ideas that are useful to mathematics teachers and mathematics teacher education. We are seeking submissions, preferably from mathematics teachers K–12 and other mathematics education professionals, that describe innovative and creative approaches to mathematics teaching.

Please keep in mind the following criteria when making submissions to the *OMG*:

- The ideas/activities must be of interest to the readership.
- The ideas/activities must be fresh and innovative.
- The mathematics content must be appropriate for the readership.
- The mathematics content must be accurate.
- The article must be well written and easily understood.
- The article and its ideas must be free of sexual, ethnic, racial, or other bias.
- The article must not have been previously published, nor should it be out for review by other publications.
- The article must be original.

Articles must be word-processed in MS Word, double-spaced with wide margins, not exceeding 10 numbered pages of text, and prepared according to the *Publication Manual of the American Psychological Association, Sixth Edition*. Figures and diagrams should be drawn by computer, if possible, or drawn in black ink in camera-ready form. Embedded images must also be submitted separately in jpeg or tif format. Proof of the photographer's permission is required, and for **photos of students** under the age of 18, the written permission of a **parent or guardian is required**.

You must submit **one complete copy** of your article, embedded with any tables, figures, and captioned photographs or graphics, to the Editor, Dan Jarvis, along with **separate files for each of the text, graphics, and/or photographs**. Please e-mail all files to Dan Jarvis at **dan.jarvis@oame.on.ca**.

Your name should not appear anywhere in your article, including websites, so that your article can be sent out for blind review. Your name, full mailing address, and e-mail address must be included on a separate sheet. Upon review, you will be notified as to whether your article has been accepted for publication (as is, or pending minor or major revisions) or rejected.

The Editor reserves the right to edit manuscripts prior to publication. Once an article is published, it becomes the property of OAME.

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Submission of Advertisements

Advertisements for publication in the *Ontario Mathematics Gazette* should be sent to **Robert Sherk** at the above address. Courier is recommended to avoid possible delays. Deadlines for advertisements are January 23 for the March issue, April 1 for the June issue, July 1 for the September issue, and October 1 for the December issue.

Full-page advertisements are to be on 8.5" by 11" paper with a minimum of 0.5" margins and single sided. Each advertisement should be print ready, and colour advertisements should have no bleeds.

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Editor's Message



DAN JARVIS
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Dr. Daniel Jarvis is Professor of Mathematics and Graduate Education in the Schulich School of Education at Nipissing University, North Bay, Ontario. His research interests include instructional technology, integrated curricula, and teacher professional learning.

Greetings! In preparing for this task, I have read back through a number of past editorials, as well as the informative 40th-Anniversary Special Edition (which provided historical background for the organization, chapter activities, and the journals), all of which have been made so conveniently accessible as part of the online digital archive collection. As an OAME member, please be sure to avail yourself of these well-organized and searchable (article/author) archives of both the *Gazette* and the *Abacus* publications. *(Note: Anyone with access to* Abacus *issues March 1977 – March 1986 and/or April – Sept. 1991 is encouraged to contact Tim Sibbald (**tim.sibbald@oame.on.ca**), who can facilitate the archival process.)*

In 1962, Ralph Stanton, the first editor of the *Ontario Mathematics Gazette*, began his editorial with the following words: "It is with some trepidation that I look forward to the task of prefacing each issue of the *Ontario Mathematics Gazette* with an editorial." I share his sense of mild anxiety, knowing how far the publication has come in the interim. He also explained two original goals for the journal: "We are trying to develop a typically Canadian magazine which will differ from corresponding American publications in two respects: the percentage of articles on mathematics, rather than on 'education,' will be higher; the articles on purely American themes will be lacking" (*OMG*, Vol. 1 #1, p. 3). At present, it is fair to say that the *Gazette* remains a predominantly Canadian-focused publication; however, the emphasis on both mathematics and on mathematics education seems to have developed into a more equitable situation. This dual focus had actually become apparent as early as 1972, when the co-editors, J.S. Griffiths and William Eames, promoted the following types of submissions to the journal: "Main Articles: We seek discussive and expository articles reflecting the content or mode of current mathematics or mathematics teaching; and, Mathematical Notes: We seek short papers which give new insights, new proofs of old

theorems, applications (in any sense of the word) of standard material, which are suitable for classroom presentation or for student projects" (*OMG*, Vol. 11N1, pp. 2-3).

Editor Hans Heinig, in 1983, noted the following observations as he exited the role, "The quality of a journal such as the *Gazette* depends to a considerable extent on the quality of the articles submitted for publication. I was most fortunate that many educators in the United States and Canada submitted excellent articles of high-quality standards" (*OMG*, Vol. 21 #3, p. 2). I am confident that such quality will continue to characterize the many articles, letters, reviews, and reports that will be submitted.

As discussed in detail in the anniversary edition (Vol. 50 #4), the *Gazette* has been transformed in a number of small but meaningful ways over the decades. In 1992, for example, editor Jack Weiner reflected on one such calculated executive decision: "We have gone from three issues per volume to four, reducing the size from 64 pages per issue to 48. There are many reasons for this, but the most important, in my view, is immediacy. We can communicate about the many important issues, such as destreaming and standardized testing, as they arise" (*OMG*, Vol. 31 #1, p. 1). While key issues may have changed since '92, the principle of timely discussion focusing on relevant issues in mathematics and mathematics education remains. For example, in 2002, on the heels of major provincial educational reforms in Ontario, editor Louis Lim highlighted, among other pressing issues, the urgent need for teacher mentoring: "Our new teachers are determined men and women, but with the constant 'teacher bashing' in the media, the numerous changes to curriculum, instruction, and assessment, along with a system that can isolate teachers in their own classrooms, we 'teachers in the trenches' need to support and mentor our new colleagues so that they do not become discouraged" (*OMG*, Vol. 40 #3, pp. 2-3).

The role of the teacher [i.e., both teachers of math (non-specialists), and mathematics teachers (specialists)] has become arguably even more complex, in light of the many interim changes in curriculum, assessment, and technology. In 2011, editor Marian Small touched upon one particular area of perceived tension among elementary teachers: "The difficulty for many primary and junior teachers is that they are torn between a curriculum and educational culture that values that students understand the meaning of statements like 3 x 4 = 12… and a public culture that wants things to be like they always were—children should just memorize their facts! … We certainly want students to have good understanding of what they are doing, but there are certain fundamentals that should become second nature for most students for their ultimate success" (*OMG*, Vol. 50 #2, pp. 2-3).

I would of course be remiss if I failed to mention that all of the other previous *Gazette* editors whom I have not heretofore highlighted (J. Hogarth, D. Crawford, T. Davidson, A. Harris, B. Smith, R. Connelly, S. Pravica, B. Onslow, E. Wood, J. LeSage, J. Egsgard, T. Romiens, M. Hurrell, S. Craven) have also made substantial and much appreciated contributions to the association. Immaculate Namukasa deserves a special thank you for taking the time to patiently train me in the secret arts of editorial workflow.

Which brings us to the present day, and the current issue, which includes three articles and ten regular columns, dealing with a rich variety of contemporary issues in mathematics education and research.

In *Consistency Checking the Math of a Hill*, Tim Sibbald challenges teachers and students to carefully observe, test/measure with various methods, and discuss mathematical phenomena occurring and/or described within the urban environment. Pack your required gear (bike, level, camera, inclinometer, mobile Google Maps access), as you scale a local slope on the Trans-Canada highway! Co-authors Meth Devendra and Yixuan Xu explore the *Exponential Function* and the graph of its logarithmic inverse, particularly with base numbers greater than 1, and how this affects graphic display. Researcher Martha Koch, in her article entitled *Approaches to Integrating the Provincial Grade 9 Mathematics Assessment into an Effective Classroom Assessment Strategy*, shares her findings from a qualitative study involving 19 Grade 9 Mathematics teachers, who share insights regarding if/how/why they have incorporated EQAO achievement results within their overall student assessment practices.

Regular columns include the following highlights: OAME President, Paul Alves (President's Message), reminds us of the importance of learning skills as part of formalized classroom assessment; Todd Romiens (OAME/NCTM Report) discusses the American *Common Core State Standards* (CCSS) document, magic squares, and the game of SKUNK; Stewart Craven (Fields Institute Report) updates members on the September/October meetings of the MathEd Forum; Mary Bourassa (Technology Corner) introduces two fascinating websites (Estimation 180, and Robert Kaplinsky); Shawn Godin (What's the Problem?) elaborates on a journey to Grandma's house; Lynda Colgan (Hey, It's Elementary) takes us on a golden tour of mathematics and nature; Mirela Ciobanu (In the Middle) looks at "assessment as learning" techniques, including connections to the *Ontario Numeracy Assessment Package* (ONAP); Ann Kajander (MB4T) explores visual patterns with students; Agnes Grafton, Ross Isenegger, Greg Clarke, and Markus Wolski (Provincial Digital Learning Resources) highlight the use of CLIPS on mobile

devices; and, Assessment Abby (eponymous) answers a submitted question regarding the marking (or not) of student homework. As usual, this issue also features the *Abacus* insert co-edited by Mary Lou Kestell and Kathy Kubota-Zarivnij, and focusing on elementary math teaching resources.

I would also like to bring to your attention two new publications in the area of mathematics education that may be of interest. Co-edited by Alexander Carp and Gert Schubring, the *Handbook on the History of Mathematics Education* (2014), is described by publisher Springer as "the first comprehensive International Handbook on the History of Mathematics Education, covering a wide spectrum of epochs and civilizations, countries and cultures." The second Springer collection, edited by Stephen Lerman, is entitled *The Encyclopedia of Mathematics Education* (2014), and features a number of notable Canadian mathematics education scholars such as Richard Barwell (*Language Background in Math Education*); Olive Chapman (*Math Teacher Educator as Learner*); Brent Davis and Elaine Simmt (*Complexity in Math Education*); Viktor Freiman (*Technology Design in Math Education*); Carolyn Kieran (*Algebra Teaching and Learning*); Peter Liljedahl (*Teacher Beliefs, Attitudes, and Self-Efficacy in Math Education*); Wolff-Michael Roth (*Interdisciplinary Approaches, Zone of Proximal Development, and Activity Theory in Math Education*); and Nathalie Sinclair (*Teaching Practices in Digital Environments*).

Finally, I would also like to take just a moment to pay tribute to a friend and former Nipissing University colleague, Dr. Doug Franks, who recently passed away from brain cancer on October 19, 2014. Some of you will perhaps know Doug from past Canadian Math Education Study Group (CMESG) conferences, and/or through a variety of university/research connections. Doug was an influential mentor for me at a number of life stages (B.Ed. instructor, practicum Faculty Advisor, M.Ed. Thesis Supervisor, co-researcher/co-author). From my perspective, Doug encompassed, in his professional pursuits and friendships with colleagues and students, an example of someone who was at once highly informed and intelligent, yet also refreshingly genuine and compassionate. He was the kind of "human academic" one would wish to emulate—striking a healthy balance between family loyalty and university life commitments. I would like to dedicate this issue of the *Ontario Mathematics Gazette* to my late friend, Doug Franks. ▲



▲ President's Message



PAUL ALVES
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At the time of writing this message, I had just completed progress reports for my classes and had enjoyed a professional learning day with my teacher colleagues in advance of the Thanksgiving long weekend—a natural time for reflection and for considering what lies ahead professionally and personally. Progress reports require us as teachers to provide feedback on student learning, especially with regard to their learning skills. Although learning skills are not factored into the evaluation of a student's understanding of the expectations, they could arguably better predict a student's readiness to learn than many other diagnostic tools. *Growing Success* makes a point of identifying learning skills as a key part of the evaluation process that provides parents, students, and teachers important information. "As students move through the grades, they develop and then consolidate their learning skills and work habits in preparation for postsecondary education and the world of work" (Ontario Ministry of Education, *Growing Success*, p. 12). I like to ask my students to provide input into the progress reports by doing some reflection and self-evaluation. I ask them to fill in the chart pictured below. Note that I use it at other reporting times as well.

The discussion in class usually focuses on what each one of the learning skills means and the impact they have on overall learning. The particular focus in my class this year has been on self-regulation, as I have been concerned with perseverance on tasks and problems—a challenge in math, as the mindset of many students is that if the solution or pathway to a solution doesn't present itself early and easily, then they "don't get it" or can't do it. This is an issue with particular resonance, given that there is a focus on mindsets and how different mindsets can impact student learning. Did I mention that the theme of OAME 2015 is "Building Mathematical Mindsets"?

This year, the discussion in class turned to how I would evaluate myself in each of these learning skills. To be honest, I wasn't prepared for this turn in the discussion. Although my instinctual reaction was to ask the students if they had any homework that needed completing, it did make me pause and think about how

Progress Report		Learning Skill
Teacher Rating	Student Rating	
		Responsibility ✓ Do you complete homework and assignments on time? ✓ Do you pay attention to the quality of work you do? ✓ Do you take responsibility for your learning?
		Organization ✓ Do you create and follow a plan to complete tasks? ✓ Do you establish priorities and manage your time well to complete tasks and achieve goals? ✓ Do you bring required materials to class?
		Independent Work ✓ Do you meet deadlines? ✓ Are you focused and do you remain on task in class?
		Collaboration ✓ Do you work well in a group? ✓ Do you participate in class and share your ideas? ✓ Do you listen passively and actively?
		Initiative ✓ Do you try to solve problems and take care of concerns? ✓ Do you recognize what needs to be done and do it?
		Self-Regulation ✓ Do you set goals and monitor your progress? ✓ Do you seek clarification or assistance when needed? ✓ Do you assess and reflect on your strengths, needs, and interests? ✓ Do you persevere and make an effort when responding to challenges?

these learning skills still play a critical role in my teaching life. After all, aren't we always telling our students that we want them to be lifelong learners? The learning skills aren't limited to the classroom or school halls. Off the top of my head, I think I would evaluate myself with at least a "G" or "E" in the Independent Work learning skill because I did get my progress reports done on time, and this "President's Message" did make it in before the deadline, but my evaluation of the other skills started me thinking and provided a topic for good discussion in the class, in my department, and something to ponder on the drive home from school. As we head into the end of the calendar year, and for many the halfway point of the school year, it may be a good time to think of how you would evaluate yourself in the learning skills that we ask all of our students to demonstrate in their learning.

In my last "President's Message," I asked you as a member to consider how you can become involved in OAME. Have you contacted your local chapter representative? Did you attend one of the many chapter events that were available in the Fall? Did you attend the leadership conference or are you planning on attending the provincial conference in May? As the new year approaches, you may be looking for a resolution. One of the easiest ones to keep may be getting involved in your local chapter.

In closing, I leave you with a couple of notes and reminders. First, welcome to the new editor of the *Gazette*, Dr. Daniel Jarvis. Many of you will recall his many submissions to the *Gazette* (I would highly recommend "Up, Down, Sideways: Difficulties of a Daughter and Dad in Learning Mathematics" [Vol. 44 #1, Sept. 2005], a great read about the involvement of parents in student learning.) Dan will be in charge of putting the issues together, and we are certain that he will carry on the great tradition from previous editors. And finally, I would like to revive a feature of the "President's Message" from the time when Jacqueline Hill was OAME president. Jacqueline would end each column with a question for the members, and the first few who would reply to her would receive a small prize. I still have my "I ♥ Math" button from the time I was the lucky winner. And so here is my question: "Who are the keynote speakers for OAME 2015?" If you are one of the first three members to either reply to me by e-mail or tweet your answer to **@oamecounts**, you will win a small prize for your efforts. On behalf of the OAME executive and board, I wish you all the best for 2015! ▲

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▲ Technology Corner – Great Math Resources Estimation 180 and Robert Kaplinsky

MARY BOURASSA
E-MAIL: mary.bourassa@oame.on.ca



Mary teaches mathematics at West Carleton Secondary School in Ottawa. She is a strong advocate for the appropriate use of technology in the classroom. She has presented workshops internationally, authored mathematics resources, is a past VP of OAME and a Past President of COMA. An award-winning teacher, Mary continually strives to learn new and better ways of helping students learn and love mathematics.

Great Math Resources: Estimation 180 & Robert Kaplinsky

1. Estimation 180

Andrew Stadel (**@mr_stadel**) started this site (**www.estimation180.com/**) in the Fall of 2012 and has been adding to it since. It is a daily estimation challenge (a school year in the United States has 180 instructional days), whose purpose is to increase students' number sense. Students see a picture or video and have to estimate the length/height/weight, etc. of whatever is shown. There is a nice progression of estimations that allows students to build upon their knowledge and learn how to use a frame of reference. The first estimation asks for Mr. Stadel's height. He is standing with a fence behind him that could be used as reference. The next estimation has a picture of Andrew and his wife and asks to estimate his wife's height. This is followed by an estimation of his son's, again with him in the picture, then of a lamppost (aside: this picture has great shadows). There is a huge variety of topics ranging from bacon to song length to number of staples to weight of an Easter bunny. You can enter your estimate on the website to help Andrew compile data. (See Figure 1)

I had my Grade 10 Applied Mathematics (MFM2P) students use small whiteboards for this activity and we usually did several at a time. Some students were very

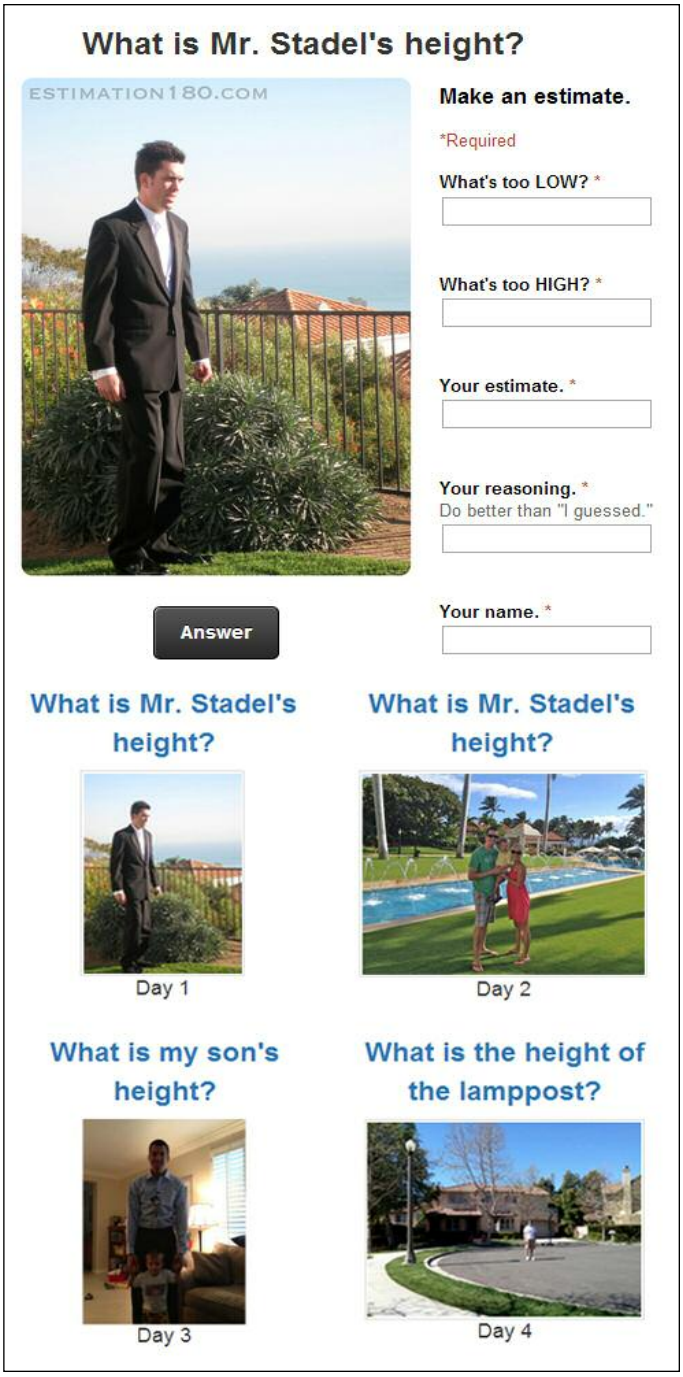


Figure 1

reticent to write a guess initially, but I prompted them with "What would be too low? What would be too high? Choose something in between those two numbers." and they did. This was really important, as estimating is a skill that our Grade 10 students need throughout many of the activities we do and as part of their board-wide summative task. It also helped build students' number sense so that they could better reflect on whether their answers were reasonable. I did Estimation 180 at the beginning of most classes—it is a great warm-up activity

that also allows any students who are late to seamlessly join the class. They truly enjoyed it and I saw a noticeable improvement in their estimations throughout the semester.

Andrew and Michael Fenton (@mjfenton) have also created this handout so students can track their progress. It is available on the website.

ESTIMATION 180 NAME:
Building number sense one day at a time *Period:*

Day #	Description	⬇ Too Low	Too High ⬆	My Estimate	My Reasoning	Answer	Error	|Error| as %
Ex. A	Tyler's age (months)	24	36	30	He looks a little older than my cousin (who is 2)	26	⊕ ⊖ 4	4/26 ≈ 15%
Ex. B	*Bohemian Rhapsody*	4:00	5:00	4:30	10% of song = 30 sec 300 sec total = 5 min	5:56	+ − 86	86/356 ≈ 24%
							+ −	
							+ −	

2. Robert Kaplinsky

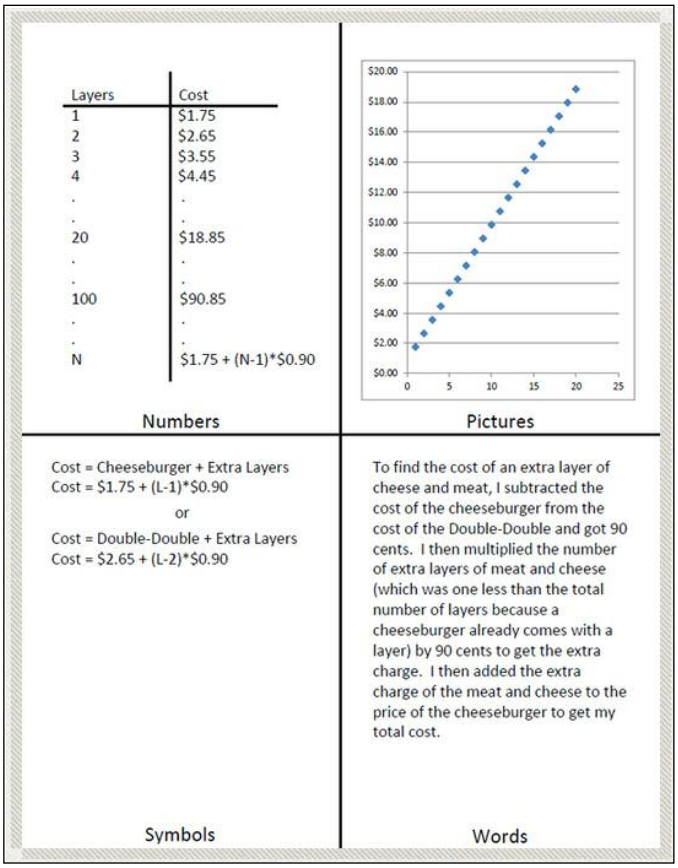
Robert Kaplinsky (**@robertkaplinsky**) has a website (**http://robertkaplinsky.com/**) filled with great lessons for Grades 2–12 math.

Here is an example:



How Much Does A 100x100 In-N-Out Cheeseburger Cost?

In-N-Out, a popular American fast-food chain, allows you to order a cheeseburger, or a double cheeseburger (two beef patties, two slices of cheese) or any number of extra "layers," where a layer would be a beef patty and a slice of cheese. This lesson gets students to figure out how much a 100 x 100 cheeseburger would cost, in terms of money and of calories. Robert does an incredibly good job of setting up the problem, and includes questions to ask and items to consider. He provides different ways of looking at the problem:



He also has a picture of the receipt showing the actual answer. But that is not all! He shows different versions of the equation representing the cost function along with the calorie information needed and the corresponding equation. He clearly states and shows what information you, as the teacher, will need. In this case, pictures of a cheeseburger, a double cheeseburger, a 20 x 20 cheeseburger and a 100 x 100 cheeseburger. And he shows sample student work of low, medium, and high quality.

If you have not checked out what is available from Robert Kaplinsky, do so now! ▲

▲ Call for Manuscripts

The *Ontario Mathematics Gazette* is inviting manuscripts for all grade levels.

Instructions for submission of manuscripts are found on page 1 of any *OMG*.

Contact the Editor for further details.

OAME/AOEM AWARDS FOR EXCELLENCE IN MATHEMATICS EDUCATION



The purpose of the Ontario Association for Mathematics Education's Awards is to formally recognize individuals and educators who demonstrate an outstanding contribution to mathematics education in Ontario. The successful candidate of each award enhances the learning environment, providing opportunity for students to do, see, hear, and touch mathematics in a profound and meaningful way. If you know someone who qualifies for this honour, please nominate her/him/them for one of the following awards.

OAME AWARDS

OAME/AOEM Life Membership Award recognizes an educator who has contributed in a significant way to OAME/AOEM; demonstrates outstanding leadership in mathematics education; and has accumulated ten or more years of membership in OAME/AOEM.

Award for Outstanding Contribution to OAME/AOEM and Mathematics in Ontario recognizes an individual who has made a significant contribution to OAME/AOEM, but who is not necessarily an educator or a member of OAME/AOEM.

Award for Exceptional and Creative Teaching in Elementary Mathematics recognizes an exceptional and creative elementary teacher who demonstrates excellence in mathematics education and contributes to the overall development of students.

Award for Exceptional and Creative Teaching in Secondary Mathematics recognizes an exceptional and creative secondary teacher who demonstrates excellence in mathematics education and contributes to the overall development of students.

Award for Leadership in Mathematics Education recognizes an educator who has demonstrated leadership by contributing in a significant way to the development of mathematics teachers and enhancing mathematics education in Ontario.

Secondary School Department Award for Exceptional and Collaborative Mathematics Teaching recognizes a secondary mathematics department which fosters collegiality, teamwork, and excellent classroom teaching; contributes to the overall development of students; and demonstrates leadership in the mathematics education community.

Elementary School Staff Award for Exceptional and Collaborative Mathematics Teaching recognizes an elementary school (a school accommodating any or all of Grades JK–8), whose staff fosters collaborative and excellent mathematics teaching; contributes to the overall development of student learning of mathematics; and demonstrates leadership in the mathematics community.

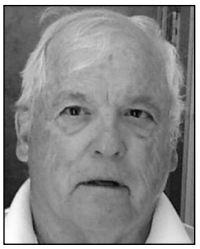
OAME/AOEM Awards Nomination Process

1. Complete the Award Nomination Cover Page appropriate to the specific award, found at **www.oame.on.ca**, or contact the Executive Directors at **EDs@oame.on.ca** for an electronic version. You may include a one-page letter of recommendation.
2. Provide up to three letters of support, of no more than two pages each. These supporting documents may be gathered from a variety of sources to reveal a full picture of the nominee. The support documents will provide precise information about the nominee's contributions as they relate primarily to the specific Award.
3. Include the nominee's Curriculum Vitae of up to five pages. (C.V. not required for Secondary School Department Award and Elementary School Staff award)
4. Send the completed cover page and supporting documents by January 15, 2015 to: OAME Executive Directors, 4751 Mack Avenue, Town of Plympton-Wyoming, ON N0N 1J6, or email: **EDs@oame.on.ca**

DEADLINE: Nominations are due by January 15, 2015 to the Executive Directors (contact info above).
The awards will be presented at the Annual OAME Conference in Toronto, May 8, 2015.

▲ OAME/NCTM Report

TODD ROMIENS
E-MAIL: todd.romiens@oame.on.ca



Todd recently retired from teaching K–12 methodology in the Faculty of Education at the University of Windsor. He is a past president of OAME and a life member, as well as a previous editor of the Gazette*.*

As many of you may know, National Council of Teachers of Mathematics (NCTM) has a new president. Diane J. Briars, a mathematics consultant and former mathematics director of Pittsburg Public Schools for 20 years, has taken over the position. She is an accomplished author and contributor to the recent *Common Core State Standards* (CCSS) document. This document is an updated version of the *Curriculum and Evaluation Standards for School Mathematics* (1989). Although the CCSS has been adopted by more than 40 states, and is designed to assist educators in teaching mathematics more effectively, it is still coming under criticism because teachers deem the federal government to be dictating how math should be taught. In fact, the Governor of Louisiana has launched a lawsuit against the current administration, claiming that it is "meddling in the state educational policy." The basic premise of the lawsuit is that unless states comply with this document, they will risk losing federal funding, which is unconstitutional. I will keep you posted.

The curriculum for Ontario was developed by members of OAME and other stakeholders in consultation with the Province, who recognized mathematics teachers as the specialists most knowledgeable the direction mathematics teaching in Ontario should take.

On another note, I would like to encourage all of you once again to take advantage of the free lessons and activities that are available to you at the NCTM Illuminations website. There is a variety of material for all grade levels that can be accessed at **www.illuminations.nctm.org/**. Just choose a grade level and begin—membership is not a requirement.

In this column, I will briefly describe two lessons that are available for Grades 6 to 8. Each includes objectives, materials that may be required, assessment ideas, extension activities, questions to pursue, related resources, and activity sheets. These are taken from the NCTM publication *Teaching Mathematics in the Middle School*.

Magic Squares

Magic Squares have always been an interest of mine, and in this lesson, the author traces their history back to 2200 BC China. They have been used in various areas of study, including astrology, natural phenomenon, and human behaviour.

A couple of methods for creating Magic Squares are outlined in the lesson. Pheru's method (1315 AD) is an algebraic way of constructing any odd-numbered dimensioned Magic Square (3x3, 5x5, etc.). The method is explained and an activity sheet is included. I wasn't aware of this technique until I read this lesson.

A method I have used with my students was created circa 1687 by French mathematician Antoine de la Loubère. It is a more simplistic way of creating Magic Squares of odd-numbered dimensions. It is not algebraic in nature and much easier to follow.

Magic Squares of even dimensions are not easy to create (don't give students the problem of trying to create one). There are ways of achieving this, but not in this lesson. Works by Martin Gardner may be of interest in this regard.

Magic Squares provide fun ideas to reinforce number skills and to connect to other historical information regarding mathematical development.

The Game of SKUNK

This is a dice game for students to practise decision-making skills, leading to a better understanding of choice versus chance, using mathematical probabilities. The game is easy to play and, in my experience, students enjoy playing it.

Start with a score sheet with the letters S K U N K at the top of the page. The object of the game is to accumulate points by rolling two dice. Each letter of SKUNK represents a different round of the game. Play begins with the "S" column and continues through to "K."

- At the beginning of each round, each player stands. Then, a pair of dice is rolled. (Everyone playing uses that roll of the dice. Players do not roll themselves; rather, the teacher rolls for everyone.)
- A player records the total on the dice in his or her column, unless a 1 comes up.
- If a 1 comes up, play is over for that round, and all points accumulated thus far in that column are lost.
- If a 1 does not occur on either die, the player can choose to continue to stand, to try and accumulate more points, or sit down and take the points accumulated thus far to add to his or her total.
- At the end of the five rounds, the player with the highest total wins.

Probabilities for different rolls should be explored (usually after the game is played), and strategies should be discussed. Consult the lessons for more details. ▲

▲ What's the Problem? Going to Grandma's

SHAWN GODIN
E-MAIL: shawn.godin@ocdsb.ca



Shawn is the head of Mathematics, Business Studies, and Computer Science at Cairine Wilson Secondary School in Orleans, Ontario.

Welcome back, problem solvers. Last time, I left you with the following problem:

> Nadia walks along a straight path that goes directly from her house (N) to her Grandmother's house (G). Some of this path is on flat ground, and some is downhill or uphill. Nadia walks on flat ground at 5 km/h, walks uphill at 4 km/h, and walks downhill at 6 km/h. It takes Nadia 1 hour and 36 minutes to walk from N to G and 1 hour and 39 minutes to walk from G to N. If 2.5 km of the path between N and G is on flat ground, determine the total distance from N to G.

This problem was question #24 from the 2014 Pascal Contest, produced by the Centre for Education in Mathematics and Computing (CEMC) at the University of Waterloo. You can check out past Pascal and other mathematics and computing contests at the CEMC website, **http://cemc.uwaterloo.ca**.

Students struggle with rate problems. A possible strategy for a problem like this is to play around with some situations and see what happens. Back when I was in school, that meant using the latest technology, our personal slate boards. If we wanted to experiment, we had to do many calculations, by hand, and hope to see some patterns that may be useful in solving the problem.

Today, fortunately, we have many other tools that let students experiment. This way, we can keep everybody engaged. I have created a sketch in the Geometer's Sketchpad® that consists of three sliders. Students can change the lengths of each type of path and see what effect this has on the time needed to do the individual parts, as well as the time needed for the whole trip. The sketch is available in my "What's the Problem?" Google drive folder at **https://drive.google.com/folderview?id=0ByDlaUaj8StpanhnUWo2bEV6ZE0&usp=sharing**.

There are some advantages to the brute force method. For one thing, it allows all students to engage in the problem (in the contest, just over 50 percent of students tried the problem, and just under 30 percent of those were successful). While playing with the problem, students may observe some patterns that allow them to solve it in another way. As well, students may detect some other properties of the problem's overall structure that may have been missed, if we just answered the question. Some observations that students may make are as follows:

- If we ignore the condition that 2.5 km of the trip is on flat ground, there are many situations that correspond to the times in the problem. (A few can be found with the sketch; others can be determined if the sliders allowed for additional decimal places.)
- If the uphill and downhill sections are equal, the two trips will be equal.
- The difference in time between the two trip depends only on the difference in distance uphill and downhill (on the original trip).

If a student focuses on the last point, they will see that each time we go 1 km further downhill on the trip to Grandma's, the trip home will be 5 minutes longer. From this, we can tell that, in order to solve our problem, Nadia must travel 0.6 km further downhill than uphill on the trip to Grandma's. Once we know this, we can easily use the sliders to see that she travelled 2.4 km uphill, 2.5 km on flat ground, and 3.0 km downhill on her trip to Grandma's house, for a total of 7.9 km.

This leads us to an algebraic solution to the problem. Since the trip home took longer, then there must have been more downhill distance than uphill distance on the trip to Grandma's (why?). If we let x represent the "extra" downhill distance, in kilometres, on the trip to Grandma's, then the time, in minutes needed to travel that distance on the way to Grandma's is

$$\frac{x\text{ km}}{6\text{ km/h}} \times \frac{60\text{ min}}{1\text{ h}} = 10x\text{ min}$$

Similarly, on the way home, the extra distance (now uphill) would take

$$\frac{x\text{ km}}{4\text{ km/h}} \times \frac{60\text{ min}}{1\text{ h}} = 15x\text{ min}$$

Hence, the difference in time is $5x$ minutes, which must correspond to the 3 extra minutes, yielding x must be 0.6 km. Thus, the extra downward distance will take

6 minutes on the trip to Grandma's. If we also take into account the 2.5 km on flat ground, which corresponds to 30 minutes, then we are left with 1 hour, or 60 minutes, where Nadia was travelling an equal amount uphill and downhill. If we let y represent this common distance, then the relations above for x can give us the times for each part, thus we need

$$10y + 15y = 60,$$

which yields the common distance of 2.4 km. The total trip therefore is

$$2.4 \text{ km} + 2.5 \text{ km} + (2.4 \text{ km} + 0.6 \text{ km}) = 7.9 \text{ km}.$$

We saw earlier that if the distance on flat ground wasn't specified, there would be many solutions that satisfy the original time constraints. Let's generalize the problem and let f,u,d represent the number of kilometres travelled on flat ground, uphill and downhill, respectively, and let t_1, t_2 represent the times Nadia takes on the trip to Grandma's and home again, respectively, in minutes. Then, when we convert the speeds to kilometres per minute, the problem can be represented by the system of equations

$$15y + 12f + 10d = t_1$$
$$10u + 12f + 15d = t_2$$

Some algebra yields

(1) – (2) $\quad 5u - 5d = t_1 - t_2$

(1) – (2) $\quad 25u + 25d = t_1 - t_2 - 24_f$

and

(1) – (2) $\quad 25u + 25d = t_1 - t_2 - 24_f$

5 x (3) + (4) $\quad 50u = 6t_1 - 4t_2 - 24f \quad (5)$

(4) – 5 x (3) $\quad 50d = 6t_2 - 4t_1 - 24f \quad (6)$

which lead to

$$u = 0.12t_1 - 0.08t_2 - 0.48f \quad (7)$$
$$d = 0.12t_2 - 0.08t_{12} - 0.48f \quad (8)$$

and finally

$$u + f + d = 0.04(t_1 + t_2 + f) \quad (9)$$

From the last equation, we can see that, for fixed trip times, the total distance only depends on the distance travelled on flat ground. You can quickly check that substituting $t_1 = 96$, $t_2 = 99$ and $f = 2.5$ yields the solution we had discovered earlier.

If we think further about equation (9), and possibly doing some calculations, we will soon discover that if you travel a certain distance at 4 kilometres per hour, then travel the same distance at 6 kilometres per hour, your average speed is **not** 5 kilometres per hour, but slightly slower. It turns out that when dealing with rates, the most natural "average" is not our usual average, the *arithmetic mean* (AM), but the *harmonic mean* (HM), given by

$$\frac{1}{HM} \frac{\frac{1}{x_1} + \frac{1}{x_2} \ldots + \frac{1}{x_n}}{n}$$

So, for example, if you run a lap around a track at 10 kilometres per hour, then jog a lap at 6 kilometres per hour and finally walk a lap at 4 kilometres per hour, your average speed will be the harmonic mean (HM), given by

$$\frac{1}{HM} = \frac{\frac{1}{10} + \frac{1}{6} + \frac{1}{4}}{3}$$

which yields $HM = \frac{180}{31}$ km/h.

Another interesting "average" that you may not have encountered is the geometric mean (GM), given by

$$GM = \sqrt[n]{x_1 \times x_2 \times \ldots \times x_n}$$

The average that we are used to, the arithmetic mean, is good when we are dealing with additive situations. The geometric mean is useful when dealing with multiplicative situations like rates of growth. For example, if a population of 1000 grows by 25 percent one year, then 8 percent the next year, and finally 28 percent a third year, the final population would be 1728. The arithmetic mean of the three rates is $20\frac{1}{3}\%$, but using that as a growth rate for three consecutive years, population growth from 1000 to about 1742 would be a bit too high. Yet the geometric mean

$$\sqrt[3]{1.25 \times 1.08 \times 1.28} = 1.2$$

or a 20 percent growth rate, brings us from 1000 to 1728, as desired.

The arithmetic mean, geometric mean and harmonic mean, when used with positive numbers, always satisfy

$$HM \le GM \le AM$$

A beautiful geometric demonstration of this fact exists. For two positive numbers a,b, the three means are given by .

$$AM = \frac{a+b}{2}, \quad GM = \sqrt{ab}, \quad HM = \frac{2ab}{a+b}.$$

If we draw a circle with diameter equal to the sum $a + b$, then the radius of the circle is the arithmetic mean of the two numbers. Segments equal in length to the geometric and harmonic means can be constructed as shown in Figure 1. I will leave it to you to verify that the segments have the desired length. A sketchpad sketch of

this is included as the second page of the sketch discussed earlier.

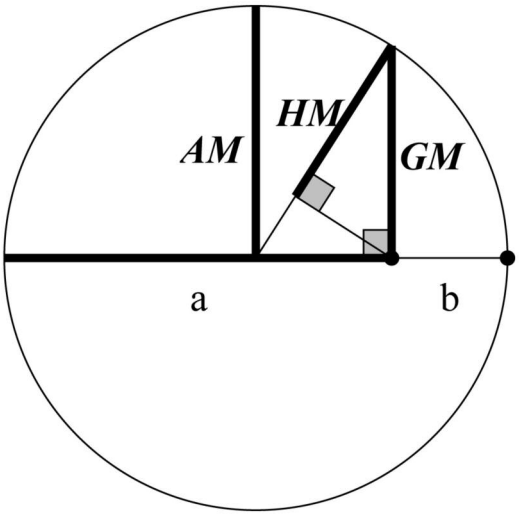


Figure 1

Now for your homework.

The point A(0,0) is the vertex of a square ABCD, and the point M(5, 0) is the midpoint of side BC. Determine the coordinates of vertices B, C, and D.

Until next time, happy problem solving! ▲

Mathematics as an expression of the human mind reflects the active will, the contemplative reason, and the desire for aesthetic perfection. Its basic elements are logic and intuition, analysis and construction, generality and individuality.

Richard Courant

NOMINATIONS FOR OAME/AOEM BOARD OF DIRECTORS 2015–2016

Each year, elections are held for a President-Elect, a Vice President (3-year term), and two Directors (up to a 3-year term) of OAME/AOEM. OAME/AOEM members running for Director will declare their intent to work on either Grades JK–6 or Grades 7–12 issues based on their personal experience, expertise, and interests.

Nominations for these positions are to be submitted by 4:00 p.m., January 15, 2015, to: Sonia Ellison, Past President, 94 Pleasant Avenue, Dundas, ON L9H 3T4 or by e-mail to sonia.ellison@oame.on.ca

Include the nominee's qualifications with the Nominations Form. A nominee must be a member of OAME. By March 2015, the Nomination Committee will publish a slate of candidates on the website, including qualifications, for consideration by the membership. Voting will be on the website, and is open to all OAME/AOEM members. The election results will be released at the Annual General Meeting of the OAME/AOEM in Toronto, May 2015.

NOMINATION FORM

I nominate:_____

for the position(s) of:

☐ President-Elect ☐ Vice-President ☐ Director

If for Director, area of personal experience/expertise, interest:

☐ Grades JK–6 or ☐ Grades 7–12

Nominee's Address: _____

_____

E-mail: _____

Telephone: Home (_____)_____

Work (_____) _____

Nominator's Name : _____

Nominator's Address: _____

_____

E-mail: _____

Telephone: Home (_____)_____

Work (_____) _____

▲ Hey, It's Elementary – Mathematics and Nature

LYNDA COLGAN
E-MAIL: colganl@educ.queensu.ca



Lynda Colgan's career has included roles as a classroom teacher, a university professor, and newspaper columnist. Her contributions to mathematics and its teaching have been recognized through awards such as the Marshall McLuhan Foundation Distinguished Teacher Award. Lynda always exhibits a passion for mathematics and views her professional mission as dispelling the myth that math is the bad guy.

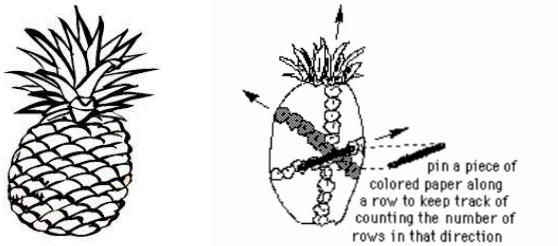
> "Real mathematics is not crunching numbers, but contemplating them—and the mystery of their connections." Charles Krauthammer

It has been my great privilege over the past two years to play a role in the *Parent Forums* organized across the province by the Education, Quality, and Accountability Office. In my workshop, *Parents as Partners in Mathematics Education*, I suggest simple ways that parents can support their children's learning by doing informal mathematics together—at the park, in the kitchen, walking to school, and at the store. To emphasize the point that mathematics pops up in the most unusual places, from Dan Brown's mystery novels to Leonardo da Vinci's enigmatic Mona Lisa, I take parents on a virtual field trip via a pictorial slide show, during which we begin to explore a surprising set of numbers that appears over and over again in nature—whether we are talking about flowers, fruit, vegetables, or animals.

Since most of the audience members have seen a pineapple, pine cone, daisy, sunflower, or snail's shell, my first challenge is to dispel the myth that the natural world is a random mixture of shapes and numbers. The first step, I tell the participants, is to be willing to don a pair of special glasses. Just as the 3D-glasses at the theatre give viewers new perspectives about what they see, the special glasses for the workshop will provide them with a mathematical eye: special lenses that will allow them to dig deeply into the obvious, visible patterns that abound, and discover the mathemagic within.

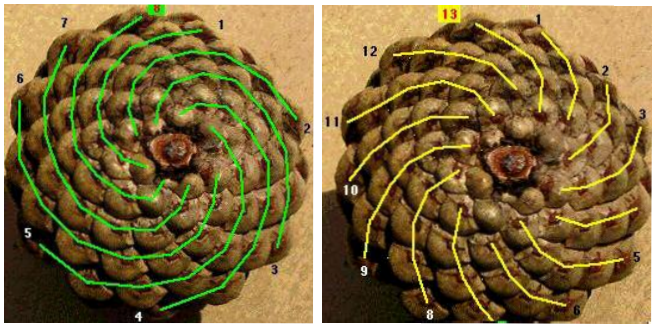
What follows is an annotated summary of the presentation and a related activity for you to try with your students.

The next time you go to the grocery store, look for a healthy, well-rounded pineapple. This will be your experimental specimen. Carefully examine the rows of scales on your pineapple (using the hint provided in the sketch below to help you keep track of the rows). Did you locate three directions of rows (as illustrated in the sketch below)? Now count the number of rows in each direction. You should find that the number of rows in the three spiral sets on the pineapple is 8 to the right, 13 to the left, and 21 vertically. This is called the *Fibonacci Spiral*.



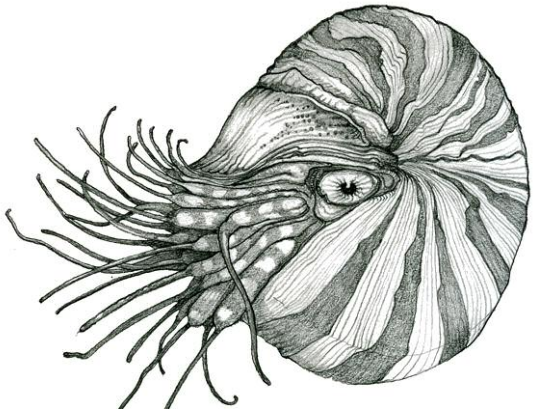
The *Fibonacci Spiral* occurs in pine cones as well. Since pine cones are easy to find in local parks, gather a few and then carefully examine the way in which the scale-like seeds are organized around it. This time, you will notice two sets of spirals—one going counter-clockwise and a second going clockwise. If you mark and count the number of spirals going each way, you will find that most often, the number of clockwise spirals and the number of counter-clockwise spirals are usually 5 and 8, or 8 and 13 (which is the case in the pine cone shown below).

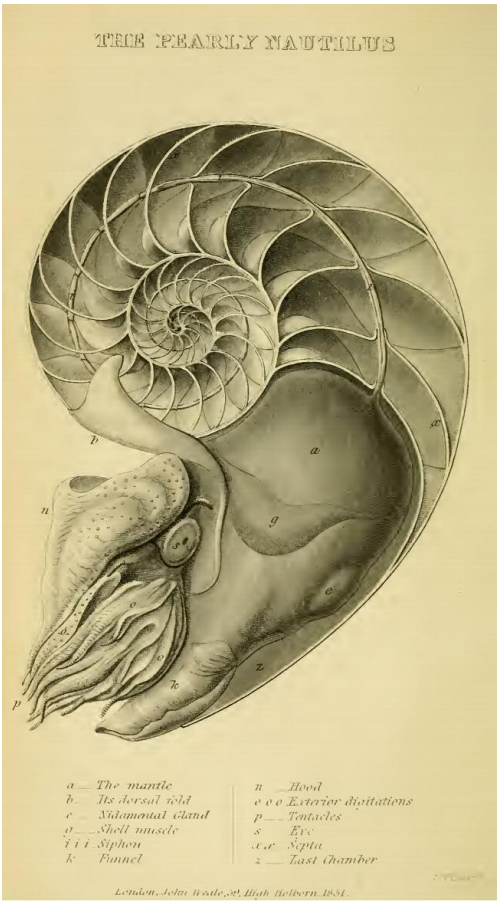




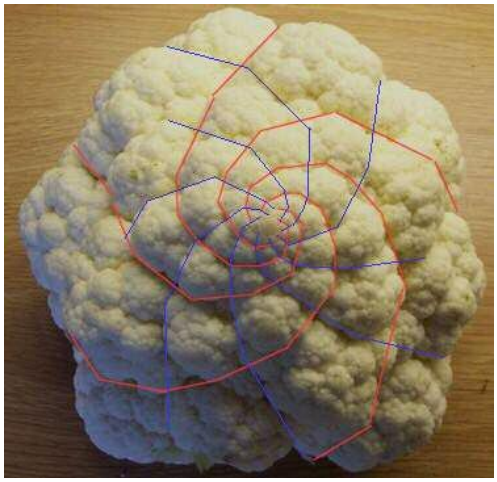
Illustrations of Fibonacci spirals in pine cones from *The Math Marvels* project were completed by Mrs. O'Brien's Sequential III Honors math class at Ward Melville High School in East Setauket, New York. The other projects are available at **www.3villagecsd.k12.ny.us/wmhs/Departments/Math/Obrien/mathmarvels.html**.

The illustration below on the left, drawn by Corey Woessner, shows a chambered Nautilus Pompilius, a mollusk related to the squid, octopus, clam, and snail. The cross-section of another nautilus was drawn by Richard Owen in 1832 in an article entitled *Memoir on the Pearly Nautilus*. In this illustration, it is possible to see that the shell of the nautilus is comprised on many individual chambers, each of which is individually sealed and contains an amount of gas that provides the animal with buoyancy. It is also possible to see how the Fibonacci spiral curve in the internal chambers develops as the nautilus grows.





The intricate *Fibonacci Spiral* pattern also occurs in the arrangement of seeds on the head of a sunflower, in the florets of broccoli and cauliflower, an artichoke, and a snail shell. Look at the picture of the cauliflower head below. Did you notice that it is almost a pentagon in outline? If you examine the cauliflower carefully, you can see a centre point, where the florets are smallest. Look again, and you will see the florets are organized in spirals around this centre in both directions. How many spirals are there in each direction?



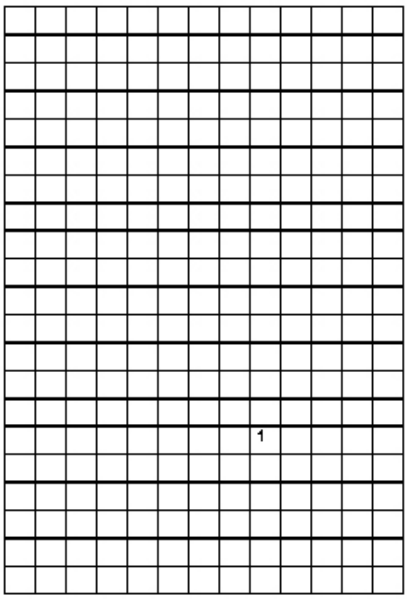
Although it is too late to use a sunflower from your own garden at this time of year, if you do plant a seed and observe the tiny florets on the sunflower head that turn into seeds, you will find that they are arranged in two intersecting families of spirals, one winding clockwise and the other winding counter-clockwise. If you count the number of florets along a spiral, you are likely to find 21, 34, 55, 89, or 144.



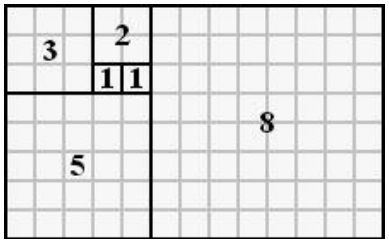
The *Fibonacci Spiral* is based upon the Fibonacci numbers (a sequence named for the thirteenth-century Italian mathematician, Fibonacci, who was born in the

1170s and died in the 1240s. There is now a statue commemorating him, located in Pisa at the Leaning Tower end of the cemetery next to the Cathedral.). Fibonacci's famous numbers are 1, 1, 2, 3, 5, 8, 13,.... As you can see, the pattern in this sequence of numbers is made by adding two numbers to get the next number in the sequence (i.e., **1** + **1** = **2**, 1 + 2 = **3**, 2 + 3 = **5**, 3 + 5 = **8**, 5 + 8 = **13**, and so on. Can you figure out how the sequence continues?

We can use these numbers to create the spiral that is so common in nature. You will need a piece of graph paper like the one shown below.

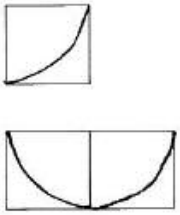


1. Look at the graph paper. The first number in the Fibonacci sequence, 1, has been drawn for you. Outline the square with a dark marker.
2. Go to the square to the right of the 1. Outline that square to represent the next number in the pattern, another 1!
3. Use the line above the two "1" squares to outline a square that is 2 squares long and 2 squares high. This represents the next number in the Fibonacci sequence—2.
4. Now move to the right of the 1 and 2 squares. Use the right side of the 2 square and the right side of the second 1 square to draw a square that is 3 squares high and 3 squares long. 3 is the next number in Fibonacci's pattern!
5. Use the bottom of both 1 squares and the bottom of the 3 square to make the next number in the pattern—a square that is 5 squares long and 5 squares high.
6. Move to the left of the 2 square, the 1 square, and the 5 square. Use their left edges to make the 8 square.

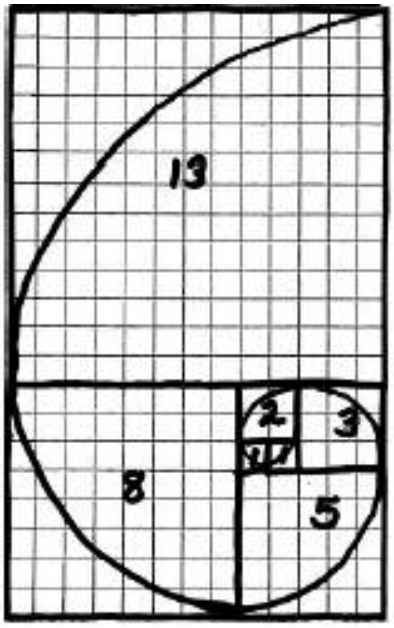


7. Finally use the top of the 8 square along with the top of the 2 and 3 square to make a 13 square.

The final step is to draw the Fibonacci spiral. The best way to do this is to connect one corner of each square (1, 1, 2, 3, 5, 8, 13) to the opposite corner of that square with a smooth curve. Two examples are shown below.



If you followed directions, you will have used the entire graph paper and made a Fibonacci Spiral like the one shown below.



Plants and shells do not know about the Fibonacci sequence or spiral—they just grow in the most efficient ways. Nature's efficiency notwithstanding, the recurring patterns and the invisible mathematical order that seems to inform our physical world have changed how I think

about everything. If you'd like to read more on this topic, I encourage you to read *Exploring the Math and Art Connection: Teaching and Learning Between the Lines* (2012) by Daniel Jarvis and Irene Naestad (Brush Education), *The Self-Made Tapestry: Pattern Formation in Nature* (2003) by Philip Ball (Oxford University Press), *Life's Other Secret: The New Mathematics of the Living World* (1999) by Ian Stewart (Wiley), or *Mathematics in Nature: Modeling Patterns in the Natural World* (2006) by John A. Adam (Princeton University Press).

In closing, I would ask you to think about what Albert Einstein had to say on the topic: *The fairest thing we can experience is the mysterious. It is the fundamental emotion which stands at the cradle of true art and science. He who knows it not and can no longer wonder, no longer feels amazement, is as good as dead, a snuffed-out candle.* ▲

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▲ In The Middle – Whose Learning Is it Anyway? Increasing Students' Engagement through Assessment as Learning Techniques

MIRELA CIOBANU
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Mirela Ciobanu is currently a middle school teacher in Toronto District School Board. Mirela is a keen advocate of classroom inquiry, particularly in the area of formative assessment. Her recent work includes conducting an action research in mathematics as part of the Teacher Learning and Leadership Program for Experienced Teachers developed and funded by the Ontario Ministry of Education in partnership with the Ontario Teachers' Federation.

> *"When students are active, engaged, and critical assessors, they make sense of the information, relate it to prior knowledge, and use it for new learning. This is the regulatory process of metacognition. It occurs when students monitor their own learning and use feedback from this monitoring to make adjustments, adaptations, and even major changes in what they understand."*
> ~ Rethinking Classroom Assessment with Purpose in Mind, Western and Northern Canadian Protocol

How do we communicate achievement to students? How do they picture learning? Can they describe it as something concrete? Are they able to celebrate it? Are they aware when new learning has taken place?

Having math-loving, engaged, motivated, and self-regulating students is the dream of every math educator. This is the result of a systematic set of actions that is continually employed in the classroom, at all levels of instruction, from carefully choosing a task, designing a lesson or a unit, to mapping an entire year of teaching. It requires consistent modelling, a change in the way we approach supporting students, and praising them. It is definitely a change in which we model perseverance and the type of role models we showcase in our mathematics classes.

Assessment as learning is a type of formative assessment that is based on the premise that students are deeply engaged in the learning that takes place in the classroom. This engagement is the result of motivation and the latter cannot be achieved unless a growth mindset is purposefully and systematically cultivated both in the classroom and at home.

Students cannot self-regulate and become independent learners unless they have these habits of mind modelled and reinforced consistently. They need to know that their individual learning and performance is not a well-kept secret by their teacher and that they are empowered to take an active role in it, from being able to describe their strengths, learning styles, to identifying models that work best for them and set new learning targets.

The highly acclaimed research of Stanford University psychologist Dr. Carol Dweck around the development of a "growth mindset" offers educators a different look at the type of conversations we should have with students. It emphasizes the importance of recognizing making mistakes and persevering with challenges as essential to new learning.

Through this article, I describe ways in which teachers can systematically and intentionally work on creating and strengthening an assessment-centred classroom environment during the course of each math unit studied. This type of environment is conducive to making students' thinking visible, and it provides them with opportunities to monitor and track their learning process. The educators who create such environments "encourage students to examine how their ideas changed over the course of a unit of study. Having an opportunity to examine their own ideas and share how and why they have changed is a powerful moment that connects the student to the teaching and learning process" (Keeley & Tobey, 2011).

The steps described below are the result of the action research I conducted during my participation in the 2012–2013 Teacher Learning and Leadership Program for Experienced Teachers, developed by the Ontario Ministry of Education in collaboration with Ontario Teachers' Federation. The student artifacts suggested could be collected in the student's **Mathematics Portfolio** (Figure 1), which contains not only evidence of the successes, but also evidence of the sustained effort and perseverance demonstrated by the students throughout each unit of study to achieve their learning targets. In the end, the students select not only the work samples that best represent them, but also the "mountains" that were harder to climb.

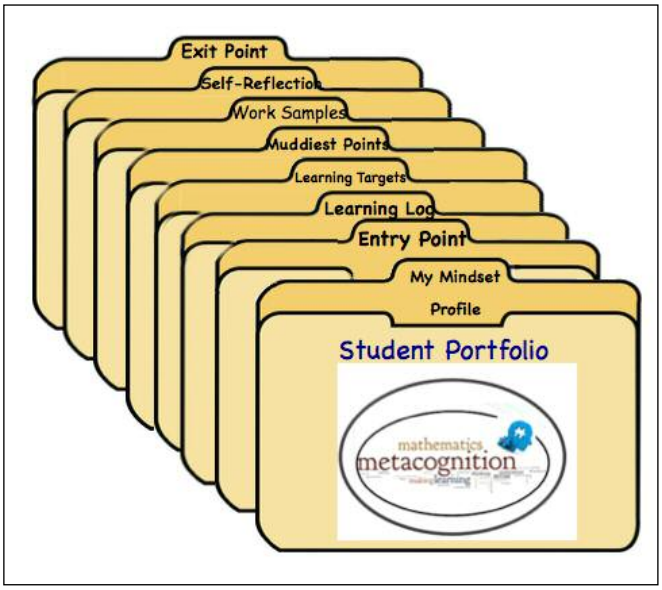


Figure 1: *Suggested student mathematics portfolio with sections focusing on Assessment as Learning*

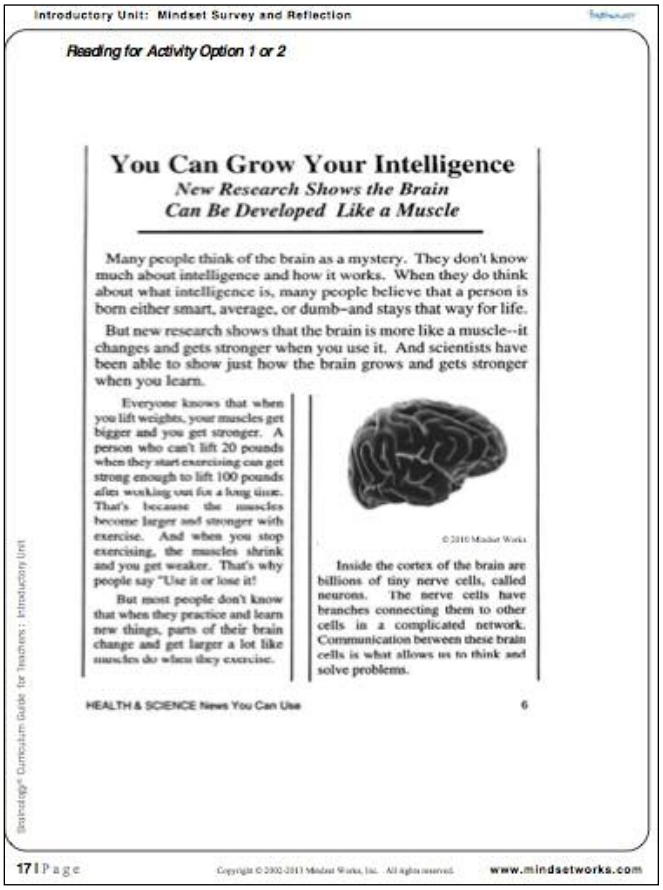
STEP 1: Cultivating a Growth Mindset. Mistakes and challenges, welcome!

The type of conversations students get engaged in starting in September is crucial for the development of their growth mindset and it lays the foundation of the game change that will take place in the classroom. This is supposed to be an environment where mistakes must happen, as they are crucial to new learning. They happen because learning is challenging. If everything were easy, then classes would be boring and learning would not occur.

Every student is expected to be able to track his or her learning, monitor it, reflect on it, abandon strategies and models, adopt new ones, and set new courses of action. It is the continual communication between teacher and student *through* the learning process and about it that allows this to happen.

It is important for students to hear that they belong to a community where all of them will be challenged and will make mistakes in order to learn. Dr. Dweck's research has also been the basis of Brainology®, an online program that aims at developing students' motivation by teaching them about how the brain works and about study skills they need to employ. Knowledge about the brain and effective study habits lead to increased motivation and achievement. The program, which contains ready-to-use teaching and assessment resources, animated modules, electronic journals for

reflections, reference brain guides, and formative tasks, can be found at **www.mindsetworks.com**, but they are not free of cost. However, some resources are offered free of charge. One excellent resource is the article about brain's plasticity and the notion of intelligence growth (Figure 2). This article is especially important because it casts a different light on the terms "smart" and "dumb," and this hits a sensitive chord, especially with those students who have a fixed mindset and who might think intelligence is something innate and only some students are good at math, while others are struggling. This article uses a scientific perspective, which usually works wonders with students in terms of reliability and trust. They learn that intelligence develops as a result of working one's brain, of sustained effort, and persevering with challenging tasks.


Introductory Unit: Mindset Survey and Reflection

Reading for Activity Option 1 or 2

You Can Grow Your Intelligence

New Research Shows the Brain Can Be Developed Like a Muscle

Many people think of the brain as a mystery. They don't know much about intelligence and how it works. When they do think about what intelligence is, many people believe that a person is born either smart, average, or dumb--and stays that way for life.

But new research shows that the brain is more like a muscle--it changes and gets stronger when you use it. And scientists have been able to show just how the brain grows and gets stronger when you learn.

Everyone knows that when you lift weights, your muscles get bigger and you get stronger. A person who can't lift 20 pounds when they start exercising can get strong enough to lift 100 pounds after working out for a long time. That's because the muscles become larger and stronger with exercise. And when you stop exercising, the muscles shrink and you get weaker. That's why people say "Use it or lose it!

But most people don't know that when they practice and learn new things, parts of their brain change and get larger a lot like muscles do when they exercise.

Inside the cortex of the brain are billions of tiny nerve cells, called neurons. The nerve cells have branches connecting them to other cells in a complicated network. Communication between these brain cells is what allows us to think and solve problems.

HEALTH & SCIENCE News You Can Use 6

Brainology® Curriculum Guide for Teachers : Introductory Unit

17 | Page

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www.mindsetworks.com

Figure 2: *Article about brain plasticity that can be used at the beginning of the year. (Free download from* **www.brainology.us/websitemedia/youcangrowyourintelligence.pdf)**

Of particular great use is the student mindset survey, which can also be completed at the beginning of the school year, to help students know themselves as mathematics learners and identify target behaviours they need to change related to the way they regard learning.

During special school events, like Curriculum Night or Family Math Night, parents and guardians should hear a consistent message from educators. It is important for teachers to suggest ways in which family members can support the program, but fundamentally, it is important to shift the way we use praise as feedback and perseverance both at home and in the classroom. Dr. Dweck's website, www.mindsetonline.com, offers various links to her articles, interviews, and tips about praising children. A free handout for parents about praise, motivation, and achievement that I created, based on Dr. Dweck's ideas, can be downloaded for free from **http://mirelaciobanu.edublogs.org/teacher-resources/** (Figure 3). It offers parents practical tips about how to effectively praise their children, and to encourage the development of a growth mindset through a focus on effort, skill, and the perseverance demonstrated in order to solve a task or achieve a goal.

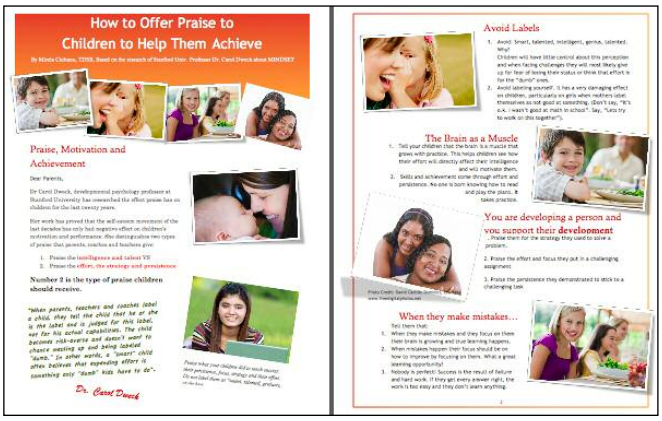


Figure 3: *Parent handout based on Dr. Carol Dweck's research about feedback and praise (Free download from* **http://mirelaciobanu.edublog.org**)

STEP 2: Students' Learning Strengths and Targets: A Different Use of the Ontario Numeracy Assessment Package (ONAP)

Like most teachers in Ontario, I use the ONAP (Ontario Numeracy Assessment Package (Simmons, Bodiam, Hanna, Milot, Tonner, & Small, 2008) as a diagnostic tool at the beginning of each math unit. In parallel to its effective use by the teacher, I use this tool to communicate to students their learning strengths and "targets."

Students are already familiar with the concept of "learning strengths," as it defines concepts and skills they bring with them from previous years. "Learning targets" refers to gaps students have in their learning continuum or "strengths they have not gained yet." The

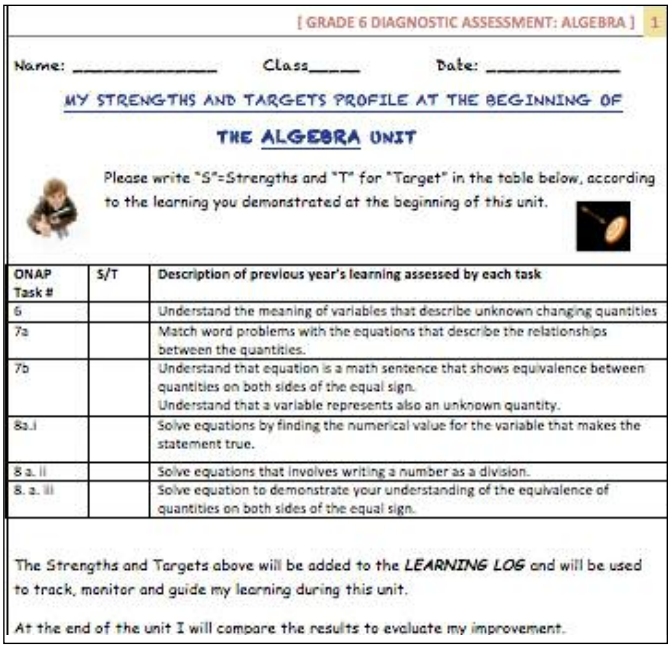
word “targets” assumes that the students are ready to do something to achieve them and does not have a negative connotation. It differs from “weaknesses,” “needs” or “gaps,” and it implies the action of “actively doing something” to get “there,” to reach a goal. The word “yet” has an unexpected effect on students, as Dr. Dweck has shown, and it does not involve the decisiveness of the current situation. Thus, students know that when focused on their learning and working hard, they will get there eventually. They are on their way.

2.1 Using the ONAP

Prior to administering the diagnostic test, teachers administer Part A to the entire class, as suggested by the ONAP, as a warm-up in order to activate students’ prior knowledge. I then select Part B and develop an individual “Strengths and Targets Profile” for each student for that unit or strand, which is based on the performance tasks in the test. Teachers select only the performance tasks that match the unit of study covered, and not the entire test for the strand. If additional tasks are needed, the ONAP test can be supplemented with samples of assessment tasks in Part C or the previous year’s curriculum.

2.2 Individual Students’ Strengths and Targets Profile

Following the test, each student will be able to describe his or her individual performance in terms of strengths (S) and targets (T) for every task completed. This profile is a chart that shows the correlations between the test tasks he or she completed and the specific learning goals unpacked from the specific curriculum expectations for each of them. Grade 6 teachers may download these profiles for various strands, from the author’s blog, at **http://mirelaciobanu.edublogs.org/teacher-resources/** (Figure 4). Thus, the students develop an awareness of what they know and can do, as well as what they need to focus on to improve. All along, they use specific math terminology to support the communication of their knowledge and skills. They will continue to use this vocabulary throughout every unit and the entire year to describe their performance. The strengths describe concepts and skills brought from previous year(s) as they enter the new learning stage of the developmental continuum of the current school year. It is an expectation that I state at the beginning of each year that every student in class needs to know who he or she is as a mathematics learner at all times.



[GRADE 6 DIAGNOSTIC ASSESSMENT: ALGEBRA] 1

Name: _____ Class_____ Date: _____

MY STRENGTHS AND TARGETS PROFILE AT THE BEGINNING OF THE ALGEBRA UNIT

Please write “S”=Strengths and “T” for “Target” in the table below, according to the learning you demonstrated at the beginning of this unit.

ONAP Task #	S/T	Description of previous year’s learning assessed by each task
6		Understand the meaning of variables that describe unknown changing quantities
7a		Match word problems with the equations that describe the relationships between the quantities.
7b		Understand that equation is a math sentence that shows equivalence between quantities on both sides of the equal sign. Understand that a variable represents also an unknown quantity.
8a.i		Solve equations by finding the numerical value for the variable that makes the statement true.
8 a. ii		Solve equations that involves writing a number as a division.
8. a. iii		Solve equation to demonstrate your understanding of the equivalence of quantities on both sides of the equal sign.

The Strengths and Targets above will be added to the *LEARNING LOG* and will be used to track, monitor and guide my learning during this unit.

At the end of the unit I will compare the results to evaluate my improvement.

Figure 4: *Sample of student’s “Strengths and Targets Profile” completed at the beginning of each mathematics unit/strand*

2.3 Communicating the Diagnostic Data to Students

Another significant change is moving from the use of the Class Task Scoring Sheet, which involves the recording of a score for each task and specific expectation, to simply recording an “S” (Strength) or “T” (Target) for each task the student completes. Thus, the teacher still gets a detailed picture of the class with its strengths and areas of needs, as well as the profile of each student, when getting ready to plan instruction. I have used the scoring sheet before and found that communicating a score alone would not support the students as it supported me in terms of informing my instruction. The “Learning Strengths and Targets Profile” that I created can be completed by each student while we take up the test together as a class or after I mark the test, and only score each task with an “S” or “T.” So far, I have not seen any student who did not have any strengths. The impact this strategy has on the struggling students is huge. It increases their confidence; it provides them with descriptive individualized feedback on what they can actually do and for which they should be praised. At the same time, the identified targets make them aware of what they still need to achieve. Moreover, just as it is important for a struggling student to become aware of his or her learning strengths, it is equally important to help high-achieving students to realize that they can still identify areas of improvement in their learning. And finally, for all students, it is vital to see

themselves comfortable with the fact that they all have areas that need further improvement and that the only difference is made by how they will be using their strengths, put in effort, and reach those identified targets.

STEP 3: The Learning Log

After the profile has been completed, each student is asked to record his or her strengths and targets in the Learning Log for that unit (Figure 5). This tool is specifically designed to help students track, monitor, and celebrate their learning after each math class. Students will see how they progress, and what new learning they acquire each day. They have recorded their initial strengths and targets as they "enter" the unit. New learning targets, specific to the current year of study, are added to their previous learning targets at the end of each lesson. These are the learning goals and success criteria that the class identified together at the end of the lesson. If the student feels confident about reaching them, he or she can add them to the "New Strengths Gained" column.

STEP 4: Choosing Individual Learning Targets at the Beginning of a Lesson

Teachers may encourage students to identify an individual learning target from their Learning Log at the beginning of *each* lesson and then comment on his or her success at the end of that instructional unit. This can also be based on the most recent descriptive feedback the student has received and it may constitute a next step. Setting goals and being accountable are two habits that need to be modelled and practised before the students use them independently.

STEP 5: Focusing on Challenges: "Muddiest Point"

Coined by Harvard professor Frederick Mosteller in 1989, the expression "muddiest point" appeared in his article "The 'Muddiest Point in the Lecture' as a Feedback Device." This type of activity can provide the teacher with important information about the way students cope with challenge during problem-solving tasks or difficult concepts during a lesson. Teachers may ask students to stop working on the task to identify and describe in detail the "muddiest point" or the biggest difficulty concerning the solving of that task. The more detailed the description that the student is able to give, the better. The more students are able to zoom in on errors or difficulties in their learning, the bigger their achievement will be. When a student is able to correctly identify, in great detail, the moment learning lost its meaning, he or she demonstrates a high motivation to succeed. Additionally, research has shown that a student's strong ability to use subject-related terminology accurately in a situation like this is related to a high level of achievement. In time, when students feel comfortable asking for support and clarification independently, a variation of this activity can be done at the end of a lesson as an exit slip.

STEP 6: A Focus on Problem-Solving Skills

Teachers may draw students' attention to their own problem-solving habits by recording themselves while working on the problem. Live Scribe Pens, for example, are great for this. Then, have students use the problem-solving checklist, the work sample in front of them and, while listening to their pencast or other peers' pencasts, identify strengths and targets for improvement.

STEP 7: End-of-Unit Reflection on Learning and Exit Point

At the end of the unit, students reflect on their learning, and they should be able to describe themselves as learners during that specific unit. They can be asked to write reflections, using the learning log as a reference resource, identifying targets they still have to reach as they progress through the school year. When completing the final test or the summative task, the students are asked to assess whether they feel that their learning assessed through those tasks represent strengths or are still targets. They can be asked to write an "S" or "T" near each task they complete during an exit test, or a reflection at the end of a summative performance task indicating areas they perceive themselves as strong in, as well as potential targets they need to reach. When this type of dialogue is done consistently in the classroom, teachers can be surprised by how accurate a student's self-evaluation is at the end of a unit.

STEP 8: Parent–Teacher–Student Conference

This portfolio serves as an important discussion piece during parent–teacher–student interviews. Students can take the lead during these conferences and describe their performance in great detail. Additionally, the continuous focus on learning targets makes parents realize that their children's learning process does not have to end just because a unit has ended, and that they are engaged students, who truly show self-regulation in learning. Parents are usually stunned by their children's

ability to use mathematics vocabulary proficiently and also by the focusing on the road ahead when describing what they are still hoping to achieve by identifying next steps and methods or models they can still use to improve their learning.

Overall, a classroom in which students have a lot of opportunities to become aware of how their learning happens is a classroom where student voice truly matters and is heard. The student is the link between assessment, feedback, and new learning.

All the techniques presented above can become components of a Math Portfolio created for *each* unit or strand. At the end of the school year, the work samples, reflections, learning logs, and learning profiles can be used for an end-of-year reflection.

All along, the students should have gained more appreciation for their mistakes and what they learn from them, and should see struggle as announcer of new learning. It is important to realize that they persevered and did not give up, that they remained, in the words of Dan Meyer, patient problem solvers, as life outside the classroom will need them to be.

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All science requires mathematics. The knowledge of mathematical things is almost innate in us. This is the easiest of sciences, a fact which is obvious in that no one's brain rejects it; for laymen and people who are utterly illiterate know how to count and reckon.

Roger Bacon

▲ MB4T (Mathematics by and for Teachers): Visual Patterns – An Opportunity for Growth in Algebraic Understanding

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Ann Kajander is an experienced classroom teacher currently teaching mathematics and mathematics education at Lakehead University. Her research interests relate to teachers' enhanced learning of mathematical concepts. She and her classroom-teacher colleague, Tom Boland, are have written and published a new book for teachers called Math Without Memorization: Big Ideas in Modelling and Reasoning for Elementary Teachers and Other Learners.

By far, the most exciting development in the current curriculum documents, both in Ontario and elsewhere, in my opinion, is the introduction of patterning to develop the underpinnings of algebraic reasoning. This has huge potential on many levels.

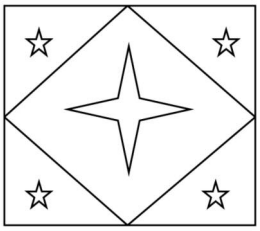
Patterns and Geometry

Many children are attracted to visual patterns… and playing with blocks and tiles forms an early mode of play. Hence, many children already have a familiarity with manipulating visual shapes. It isn't hard to see that moving and exploring geometric shapes, tiles, and so on, aligns with the paradigm of exploration and problem solving in mathematics classrooms. There is also an inherent human desire to create and enjoy designs and patterns. So creating early patterning activities for children can easily exploit such prior experiences.

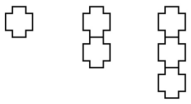
Some important questions to consider when designing tasks might be asking when is it art, and when is it mathematics (and can an exploration be both), and within mathematics, when is it geometry, and when is it patterning, (and again, can and should it be both?). And at what point is the purpose of patterning to develop "standard" algebraic reasoning, or can and should this topic be more than that? The next series of columns will explore a few of these general themes.

Mathematical Patterns

One question I am often asked is whether a geometric design, such as



is a "pattern" as described in the mathematics curriculum. One of the greatest strengths of mathematics is that it gives us a way to predict future events or results based on the use of patterns and generalizations. The types of geometric patterns that are most useful to use in mathematics to develop reasoning are those which involve repetition or change, and hence may need us to make predictions about how they further develop. We can imagine many of these, for example:

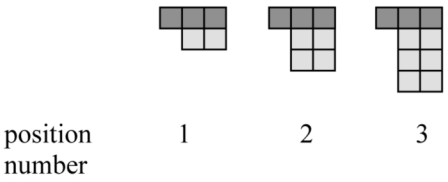
 or 

Thus, the idea of "prediction" is an important one in choosing useful patterns for mathematics learning.

If we number the elements in a sequential pattern series that changes in a predictable way, we have an obvious connection to the early ideas of a variable; a symbol used to represent a certain position or stage of the pattern. This idea of "any" location or value is a key concept. In my own classroom experience, I found that students who were deemed "at risk" in early secondary mathematics classrooms, particularly Applied classes, often struggled with a fundamental understanding of this idea, that is, the notion of a variable.

How Does Patterning Support Algebra?

One possible interpretation of what elementary patterning is *for* is that it is simply to lead us back to a traditional early high school approach to linear algebra, what I refer to as the "all roads leading to $y = mx + b$" or even "all roads leading to calculus" approach. What I mean by this is that visual patterns can be used to directly lead students to the view that linear algebraic pattern rules "should" follow a particular format, namely the one leading eventually to the $y = mx + b$ traditional slope-intercept approach. One way to "lead" students to this view is to purposely offer only patterns for which such a (position number and constant) pattern rule is the obvious one, and even go so far as to colour-code the pattern to emphasize the "multiplier plus constant" solution format (for example, see Beatty, 2014). For example, this pattern



is designed to lead students directly to the traditional "2 × (position number) + 3" format.

Our curriculum, on the other hand, describes patterning in a way which encourages a more flexible approach, stating for example in a Grade 8 Patterning Expectation that patterns should involve "using one or more algebraic expressions" (Ontario Ministry of Education [OME], 2005, p. 109). This expectation is followed up by a description of several different ways a student might see and describe a pattern. A sample student response to describing the "toothpick pattern," shown to follow,



is provided in the curriculum examples as: "(e.g., 'Using toothpicks, I noticed that 1 square needs 4 toothpicks, 2 connected squares need 7 toothpicks, and 3 connected squares need 10 toothpicks. I think that for n connected squares I will need 4 + 3(n – 1) toothpicks because . . .')" (OME, p. 116). This sample response is *not* a pattern rule following the "multiplier plus constant" form of the rule. Changing the student's rule as just provided to a different form (or asking a student to do so) actually obscures the student's thinking and reasoning from view. Furthermore, it does not support our goal of "different ways" (Boaler, 2014) of finding methods.

Students who "see" and visualize a pattern in a certain way may be confused if they are corrected by a teacher to a different way of constructing the rule. On the other hand, encouraging students to share and explain their varying ways of constructing rules enhances algebraic flexibility (see Boaler and Humphreys, 2005, for wonderful examples of this, as well as Kajander and Boland, 2014).

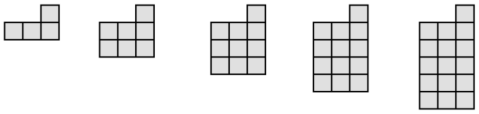
Another Example

An elementary student is asked to find a rule to connect the input number and the output number in this pattern:

Input	Output
2	4
3	7
4	10
7	19
11	31

The student solves the problem by (correctly) recognizing that the output values are all "one more" than multiples of 3. But, he explains, they are the "wrong" multiples, that is, the multiples of one less than the input number. He explains his completed rule as follows: "First you take the input number and subtract one. Then you multiply that by 3. Then you add one." The student's rule is one to share and celebrate. However, "correcting" this rule to align with the "multiplier plus constant" format (as was found in the answers in the back of the book, by the way), does little to support the student's understanding. The student, who actually received this "correction" to the multiplier-plus-constant form of the rule, was left with confusion and frustration as to what was "wrong" with his rule. (Written in algebra, the student's rule could have been recorded as $3 \times (n - 1) + 1$… very cool!).

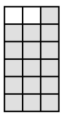
Visual patterns may or may not have clues as to how to "see" the pattern rule. Using colour coding does emphasise a desired way to create the rule, and I would not recommend it. If the input/output chart task as just given was instead rewritten as a visual pattern, it could look like this:



Position number ... 2 3 4 5 6

Such a visual layout makes it clear why the student's thinking, described by "subtracting one from the input number," might be a way to think of the pattern.

Other students might "see" the pattern differently. The visual presentation of this pattern does not underprivilege multiple ways of constructing a rule. For example,



adding in extra squares (which some of our students have called "ghost" squares) for the sixth picture would allow students to find the "3 times position number minus 2" format as an alternative.

Authors such as Boaler and Humphreys (2005) argue that encouraging such *multiple* ways of seeing and interpreting patterns strengthens understanding of the connection between the visual pattern and the pattern rule. In their classroom video, *The Border Problem*, the lesson is focused on finding and describing multiple ways to count the tiles in a square border pattern. In a recent webcast, Boaler reminds us that, as flexible understanding of number leads to later operational strength, flexible ways of seeing patterns leads to algebraic strength (Boaler, 2014). In particular, she recommends questions that allow "space inside [them] to think, learn, and discuss," as opposed to those that simply require "repeating the teacher's methods."

Getting Better at Finding Multiple Solutions

The Ontario Grade 8 curriculum describes two different ways students might think about the "toothpick" pattern shown earlier, each of which would lead to a different formulization of the algebraic rule. Are there more?

For the "toothpick" pattern,



1 2 3 ..... position number

students might be encouraged to take part in a visualization exercise, to inspire them to think of yet other ways to describe the pattern, if they have run out of ideas. For example, students can be asked to imagine one of the pictures with all of the top and bottom rows of toothpicks as separated from the vertically placed toothpicks, like this:



or, to imagine the "perimeter" of each picture in the pattern separately from the toothpicks contained inside it.

Each of these ways of seeing the pattern can result in a differently constructed rule (see Kajander and Boland (2014) for solutions to these ways of imagining the toothpick pattern as just described, as well as for other similarly focused patterning examples).

Pushing to find further ways to characterize patterns supports deep and flexible understanding, as students often have to work to "see" the patterns in a way that was not their first response. (It's a bit like the "figure and

ground" pictures used to enhance visualization in art class.)

The curriculum, as well as researchers such as Boaler (2014) are encouraging the support of students' *multiple* ways of seeing and describing patterns. In addition to providing the potential to deepen understanding, these multiple ways align more fully with "real life" patterns, which do not come conveniently colour-coded into red and yellow regions. The deeply "mathematical" piece, I argue, is the ability to recognize a predictive pattern, and then be able to describe the pattern, using mathematical language. I would prefer to see this emphasized, rather than create pictures that push students to the same old (often superficially understood) formats, which, in the very worst case, may involve "guessing" what two numbers to put in a template.

The emphasis on creating one's own rules based on different ways of visualizing patterns moves problem solving, reasoning, communicating, connecting, and reflecting to the foreground of our classroom discourse. This is very different from filling in the blanks on a prescribed template. It's harder, slower, and more challenging—but more exciting, rewarding, and ultimately, more deeply mathematical.

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▲ Provincial Digital Learning Resources—Using CLIPS on a Mobile Device

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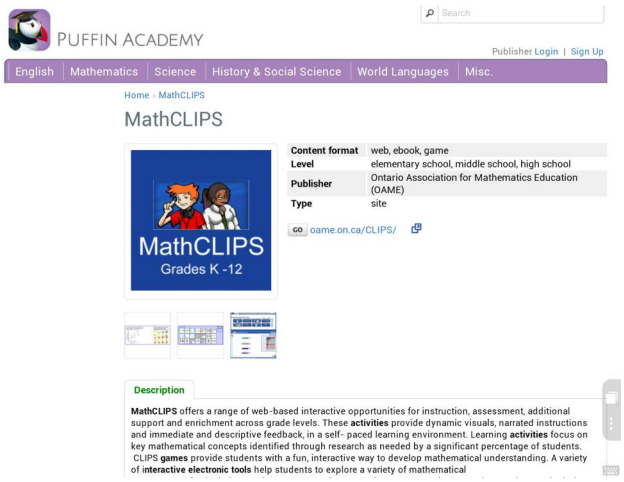
Mobile devices continue to be more popular, more powerful, and more pervasive in schools. You might think that since most of MathCLIPS.ca is written in Adobe Flash, and since Flash has dropped its support of mobile browsers, that a mobile device would be useless in running the activities at MathCLIPS.ca. This is not the case! Most of the activities are quite usable using Flash-enabled browser apps. In addition, now that the 21 Sketchpad files found at MathCLIPS.ca have been upgraded to version 5, Sketchpad Explorer can be used to interact with the Sketchpad activities on an iPad.

Puffin Academy

Puffin Academy is a free app, available for both iOS and Android, which enables the use of Adobe Flash-based websites. All websites are educational and have been vetted and deemed safe. Recently, mathies.ca,

MathCLIPS.ca and **ePractice.ca** were whitelisted as "web apps" for *Puffin Academy*.

Since the activities and learning tools found on these sites were developed to be used with a mouse on a computer, some of the smaller images are sometimes challenging to interact with on a mobile device.



Here are some helpful hints to make the interactions work as smoothly as possible:

1) Use two fingers to pinch and zoom in on smaller buttons and widgets.
2) To drag and drop, press and hold on the target object. When a circle appears on your touch point, you can then move your finger to drag and drop the target object.
3) Use the trackpad found in the *Puffin Academy* toolbar to interact with small objects. To bring up the toolbar, swipe from the left edge toward the centre. Then, choose the "Mouse" icon.
4) To drag and drop using the trackpad, place two fingers on the trackpad and wait for the mouse cursor to change into a hand icon. Drag the object, using two fingers.
5) From the toolbar, select "Theater Mode." Theater mode automatically maximizes the size of the Flash content and allows you to drag objects directly without the press-and-hold action indicated above. However, theater mode also disables the zoom feature.

Sketchpad Explorer

Ontario's publicly funded schools have enjoyed a provincial licence for The Geometer's Sketchpad® 3, 4, and now 5, with student and teacher take-home rights (for more information, visit mathies.ca | Learning Tools).

KCP Technologies has a free iPad app, called *Sketchpad Explorer*, which allows you to view and interact with Sketchpad 5 files.

To see a list of all sketches available on **MathCLIPS.ca**, click the Search icon at the top of the wrapper, select GSP files, and click "Display All."



You are presented with a scrollable list of files. Clicking on an activity title navigates to its Start page, where there is a "Download" button.

Download the file and follow the instructions.



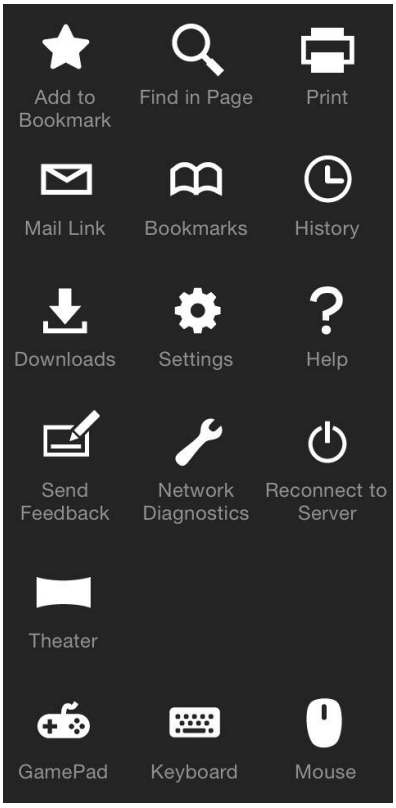
(GSP 675 kB)

If the button is clicked within Puffin Academy, you can specify the download location to be "This Device." You will be notified when the download is complete.

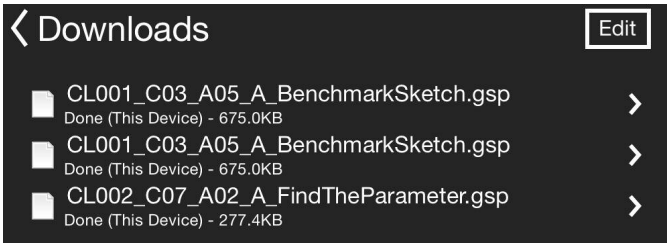
Click the icon with the three dots at the top right of the page (Actions Overflow).



Select "Downloads" from the list of actions.



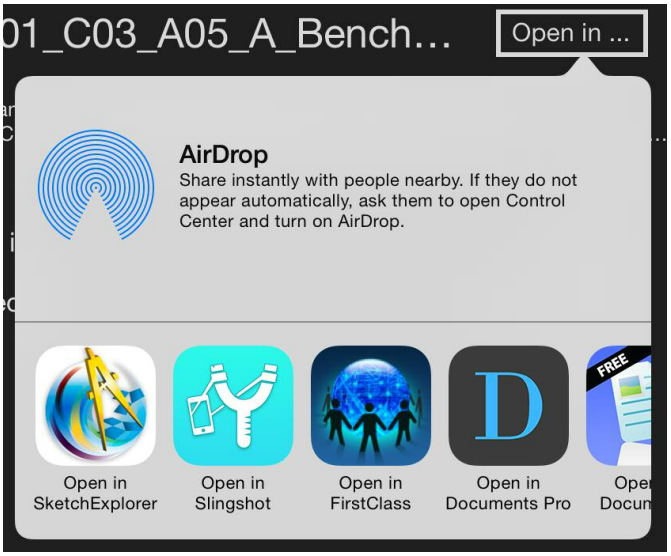
Select the desired sketch.



Select "Open in…."



Select "Open in SketchExplorer."



At the time of writing, clicking "Save" from the MathCLIPS Search results list does not work in *Puffin Academy*. Alternatively, sketches can be saved on a computer and transferred to the iPad, using iTunes File Sharing (see **www.dynamicgeometry.com/ General_Resources/ Sketchpad_Explorer_for_ iPad.html)** or using the Dropbox app.

Once the sketch is open in *Sketchpad Explorer*, you will be able to drag objects and select buttons just like you would in Sketchpad. In addition, there are three buttons at the bottom right of the screen.



They allow you to obtain information about the app, temporarily magnify a region of the sketch and access saved and sample sketches, respectively. This is in addition to the panning and zooming you expect when two fingers are used. The web page, **www.dynamicgeometry.com/ General_Resources /Sketchpad_Explorer_for_iPad.html**, outlines the various features of the app.

Among the cooler features is the ability to

- flick a parameter value up or down
- replace an image in a sketch with one of your own
- shake the iPad to erase traces
- change the magnified region and to interact with it with a second finger
- display touch indicators for demonstration purpose

Exploring MathCLIPS activities on mobile devices is a nice extension for those who have access to the devices and are willing to put up with the screen-management issues and the idiosyncratic user interfaces of the apps used.

Why not visit *Puffin Academy* today? Download one of the GSP files or try out the recently revised Linear Graphing tool. Revisions include allowing both solid and dotted lines and a wider grid option.



▲

Pure mathematics is, in its way, the poetry of logical ideas.

Albert Einstein

Consistency Checking the Math of a Hill

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Among the challenges faced by math educators is that many problems in practical situations are handled so readily that students do not recognize that there was a problem. Perhaps the most pervasive is the cashier who claims not to do any math. While I am not sure about the usage of the word "do", as we don't hear people saying "I do English"; I am confident that many cashiers perform math without recognizing it. On some occasions, it is clear that cashiers are circumspect and cognizant of what they are doing; anyone who has received erroneous change and had instant recognition of their claim by a cashier knows that some form of math process is alive and well.



Figure 1: *Thibeault Hill, North Bay.*

Another common issue, and the one that gave rise to this article, came from looking at consistency between a few sources of information about a particular section of the Trans-Canada Highway. Thibeault Hill (Figure 1) is the section of the Trans-Canada that climbs the escarpment located at the north side of North Bay. It is sufficiently steep that a special lane was added years ago that would forcibly stop any truck that should run into brake difficulties (see YouTube "North Bay runaway truck ramp test").



Figure 2

Students in Grade 9 learn about slope, and road signs present a practical interpretation of that concept. For example, a sign on Thibeault Hill indicates a 7% slope (see Figure 2), meaning that for every 100 m moved horizontally, it rises or falls by 7 m. Using right angle trigonometry, an inverse tangent calculation leads to an angle of 4° for the hill.

If students turn to Google Maps they can use the geometric scale to measure the length of the hill. However, this is where the complacency about mathematics becomes an issue. The map from Google is flat (see Figure 3), and it has only one scale that applies to the map, whether you are measuring a flat section of road (such as McKeowan Avenue) or a sloped section like Thibeault Hill. This raises the question of whether or not Google uses a flat Earth model, and what errors would arise if this were the case. In remote sensing, where satellite images of the Earth are used to measure the Earth, a known issue is that distances can be underestimated or "foreshortened" because of the third dimension.

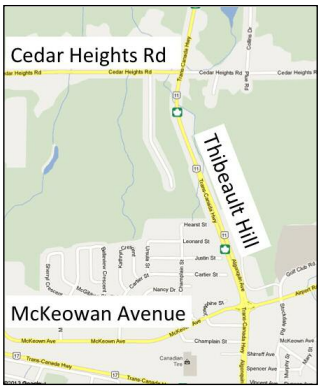


Figure 3: *Map of a Thibeault Hill from Google Maps.*

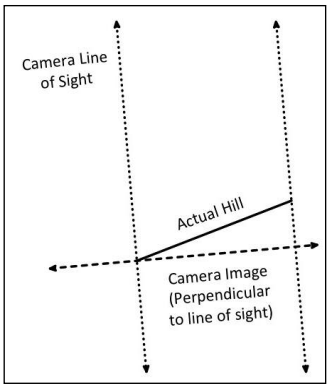


Figure 4: *Foreshortening geometry.*

Foreshortening arises because photographs are essentially taken perpendicular to the line of sight of the camera. In Figure 4, the solid line represents the actual hill. The photographic lines of sight for the two ends of the hill are shown using parallel dotted lines. The dashed line, which is perpendicular to the dotted lines, is what the photograph will show. The hill will appear as having

the length of the camera image (after scaling the photograph) unless compensation for the third dimension is determined. This detail raises a doubt about how Google Maps represents the third dimension. Does it provide a scale based on assuming the area being mapped is flat or does the map get drawn in a manner that compensates for the foreshortening?

This particular problem is well suited to hands-on exploration for anyone with a hill in the vicinity of his or her school. In my case, I often travel up and down Thibeault Hill because Nipissing University is located on the escarpment. This has afforded me different opportunities to think about measuring the hill. On one occasion I counted bicycle wheel revolutions while plummeting down the hill. Another time, while a video camera was running, I drove up the hill, which provided the distance on the odometer as well as speed and time measurements that could be shown using a projector or SMART Board. It should be noted that I used the satellite view in Google Maps to select easily identified start and end points for distance measurements. My measurements, regardless of method, were consistent giving a length of 650 m for the length of the hill. This is an important message for students: Making multiple measurements and checking for consistency is superior to making one measurement.



Figure 5: *Using an inclinometer.*

The slope of the hill is easily measured in two ways: using a level, or an inclinometer. Using a level and measuring a 90 cm length with a 7.5 cm drop gives a 1/12 slope. The short 90 cm length could be measured in many places on the hill allowing one to ask whether the section measured is representative of the entire hill—an excellent communication question, and one that ties to teaching about representative sampling in statistics. Using an inclinometer provided an angle measurement of 5°. However, the measurements should raise concerns about accuracy, which is why problems like this one are beneficial for our students. With the level, this was done on the tarmac at the side of the road and had the perception of differing from the road surface because of the slope of the road for proper drainage of water from the surface. The angle measurement (see Figure 5) was made by looking along the surface of a concrete wall, behind the inclinometer in the figure, which runs up the side of the road. Traffic at the time appeared to slow more than usual and there was the unusual distraction of wondering how much the occasion appeared like a speed trap (Note, for cross curricular physics, without friction, one could use the energy potential $E = mgh$ conversion to kinetic energy $E = 0.5mv^2$ to show that brakes are necessary to avoid speeding!)

With the differing measurements, how does one reconcile them? This is a great question for students! The 7% grade, from the sign, led to a 4° slope, using an inverse tangent, that is not far off the measured 5°, from the inclinometer. The 1/12 slope can be compared in various ways: In terms of equivalent fractions multiplying by eight and one third shows 1/12 = 8.3/100, or an approximate 8% slope. Alternately, an inverse tangent, $\tan^{-1}(1/12)$, gives an angle of 4.8° which is consistent with the measured 5° angle and, again, not far off from the 4° calculated from the road sign. There is also the practical consideration that I was measuring the steepest portion as opposed to an average slope, which could be a potential tie to calculus. The 7% figure, from the road sign, is consistent with average slope to a stoplight that is near the bottom of the hill.

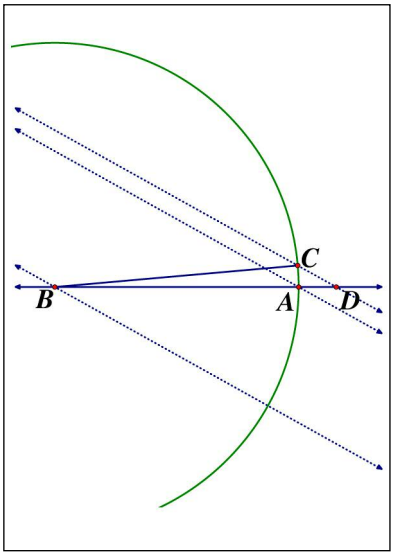


Figure 6: *Geometry of forelengthening.*

What about the length of the hill? I measured 650 m while the result using Google's scale was 750 m. Regardless of the method, one thing that is clear is that the Google Map is not foreshortened! In fact it is forelengthened. But how can the distance be longer? Consider Figure 6 where the dotted lines represent the line of sight of a camera that is some distance

away. BC is the true length of the hill and is 5° from the actual horizontal length of the hill, BA. When the image is corrected for the angle of the camera, the length BD will be the calculated length because the correction determines the distance on the horizontal. So, while the hill has length BC, the map will render it as having length BD after the correction for the camera angle is applied. In this illustration the ratio BD to BC is exactly the ratio of 750 m to 650 m and would explain the discrepancy. However, your students may want to debate the likelihood of a satellite image being taken at that angle, and since I am not going to say at what angle this hypothetical camera is positioned, your students may want to use the Sine Law to determine it.

How much does all this matter? Your students may simply want to be pragmatic and use Google Maps because it is convenient. However, there are practical considerations for civil engineers who have to run town services up the escarpment to service Nipissing University, Canadore College, and various houses and businesses that are atop the escarpment. This can be used as a what-if math challenge. What range of possible hill heights can be determined from the 4-5 degree range of angles and the 650-750 m slant height of the hill? A calculation gives 650 Sin(4) = 45 m as the least possible height and 750 Sin(5) = 65 m as the greatest. To convey to students the importance of the 20 m difference, one could suggest that 20 m matters on the school football field, and is similarly important in maintaining city services?

The opportunity afforded by performing a consistency check of information using a local hill is considerable. Students can measure in different ways, using a level, inclinometer, or map scale, that are connected to different conceptual ideas or different ways of articulating the same idea. The focus of this article has been on a case of multiple approaches to one measurement without assuming that the conveniently supplied Internet answer is the full story. For teachers who would like a statistical component, one could have students compare different measurements made by different students. ▲

Mathematics is a game played according to certain simple rules with meaningless marks on paper.

David Hilbert

▲ Ask Assessment Abby: Should Homework Be Marked?

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Ask Assessment Abby A^3 is a regular column in the OAME *Gazette*, where teachers can share concerns and best practices about assessment, evaluation, and reporting of mathematics. Please send your questions to Ask Abby at **assessmentabby@oame.on.ca**.

One colleague suggests I mark homework and another says that I shouldn't. My question is simple: Do I mark homework?

Help me with my homework
Confused in Classroom

Well, the answer is yes… and no. It depends on the homework. If homework can be used as evidence of students' ongoing and evolving learning, and teachers provide continuous feedback and opportunities to revise and re-demonstrate their understanding, homework can be assessed and support evaluation. In this case, homework is a rich learning opportunity. *Growing Success* (2010, p. 39) states that "assignments for evaluation must not include ongoing homework that students do in order to consolidate their knowledge and skills to prepare for next class." Thus, asking students to complete a number of questions from the textbook is homework that should not be evaluated.

Keep Math Rich with Students in Mind,
Assessment Abby

If you have any questions or comments for Assessment Abby: **assessmentabby@oame.on.ca**

Growing Success: **www.edu.gov.on.ca/eng/policyfunding/growSuccess.pdf** ▲

▲ Approaches to Integrating the Provincial Grade 9 Mathematics Assessment into an Effective Classroom Assessment Strategy

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Martha Koch is a mathematics education researcher with an interest in assessment practice, teacher professional development, and innovative approaches to curriculum design and implementation. After conducting research in Ontario for a number of years, she has recently become an Assistant Professor in the Faculty of Education at the University of Manitoba. She teaches courses in mathematics education and educational research methods and is an executive member of the Canadian Assessment for Learning Network (CAfLN).

Large-scale testing and classroom assessment are often viewed as fundamentally different activities in terms of their purpose, administration, and technical qualities. In Ontario, Grade 9 mathematics teachers have a unique opportunity to bring these two activities together. These teachers have the option of scoring some or all of the items on the Education Quality and Accountability Office (EQAO) Grade 9 Assessment of Mathematics before returning the test booklets for official scoring, and they can include the scores from the assessment as part of their students' grades (EQAO, 2013; Ontario Ministry of Education [OME], 2010). Teachers who decide to use the assessment for this purpose choose the items they want to score, create their own scoring guides, and determine how much the assessment will contribute to each student's grade. Teachers must also decide how the EQAO items they have chosen to score align with the Achievement Chart categories in the mathematics curriculum (OME, 2005). In some schools, individual teachers make these decisions based on the needs of the students in their classroom. In other schools, the decisions are made collaboratively, with all teachers in the department following the same approach (Koch, 2010). While many teachers choose to include the assessment within the 30% summative part of students' grades, EQAO and Ministry policies do not preclude using the assessment in the 70% term grade (EQAO, 2013; OME, 2010).

Various aspects of teachers' use of the EQAO Grade 9 Assessment as part of students' grades have been investigated by EQAO (Kozlow, 2011, 2012; Pang, Kozlow, & Rogers, 2012) and by mathematics education researchers (Kitto, 2006; Koch, 2010). To further explore this practice, in this article, I draw on interviews I conducted with 17 Grade 9 mathematics teachers, who describe how they use the EQAO assessment in their students' grades. More specifically, I share examples of innovative practices these teachers have developed. These practices are aligned with *Growing Success* (OME, 2010), Ontario's assessment policy, and they also reflect established research on effective assessment practice (Assessment Reform Group, 2002). The practices that are described relate to three principles from *Growing Success*: assessment should be fair and equitable for all students; teachers should provide multiple opportunities for students to demonstrate their learning; and students should have opportunities to develop their self-assessment skills. As will be shown, the teachers in this study strive to ensure that the way they include the EQAO Grade 9 Assessment in students' grades is consistent with their overall classroom assessment strategy and helps to improve students' mathematics learning.

Data Gathering

Before describing the ways these teachers use the EQAO assessment, I provide some background information about the participating teachers and the data-gathering process. The 17 teachers interviewed (11 female, 6 male) were from four school boards (two small rural boards, one mid-sized urban board; one large urban and suburban board) and had a range of teaching experience from two to 25 years. Six of them had been mathematics department heads at some time in their careers. Two of the teachers were using the EQAO Grade 9 Assessment as part of their students' grades for the first time, while the others had included the assessment in students' grades on numerous occasions. Most had used the EQAO assessment for grades in both the Academic and Applied Grade 9 mathematics courses. In addition, many had experience teaching in more than one school and had seen a variety of approaches for including the assessment in grades.

Individual interviews were conducted with each teacher. Teachers were asked to describe how they use the EQAO Grade 9 Assessment in their assessment practice, their reasons for doing so, and any concerns they have with this use of the assessment. The interviews were an average of 33 minutes in duration and were audio-recorded and transcribed for analysis. The analysis of the interview transcripts provides many valuable insights into how and why teachers use the EQAO assessment in their students' grades, as well as their concerns about certain aspects of the assessment, such as the high proportion of multiple-choice items and the limited value of the assessment for students taking the Applied course. In this article, I focus on several practices these teachers engage in that instantiate the three *Growing Success* assessment principles mentioned earlier.

In reading this article, Grade 9 mathematics teachers and mathematics department heads are invited to reflect on their use of the EQAO assessment, discuss the implications of other possible approaches, and consider the merits of incorporating one or more of the approaches described in the article in their assessment practice. A few of the practices that are described will surprise some readers and may foster a renewed debate about the ways Ontario's assessment policy can be interpreted and enacted.

Observations

Fair and Equitable for All Students

Growing Success indicates that teachers should make assessment decisions for individual students and for groups of students that are fair and equitable. Many of the practices the teachers in this study have developed help to ensure that their use of the EQAO assessment meets this principle. In this section, I focus on three dimensions of these teachers' use of the assessment where fairness and equity are particularly evident: the selection of items to score, decisions with regard to the amount the assessment contributes to grades, and how teachers develop scoring guides for the open-response items.

Although I present a number of individual teacher's practices, I want to point out that the extent to which individual teachers make decisions regarding how the EQAO assessment is used in grades varied considerably among the teachers in this study. Some teachers indicated that these decisions are made at the departmental level to ensure that all teachers score the same items in the same way, and weight the EQAO assessment for the same amount in students' grades. In these cases, the mathematics department head felt that consistency across teachers is an essential element of fair and equitable assessment practice. In other cases, teachers indicated that some decisions are left to individual teachers within the department. For instance, individual teachers choose which items to score and establish their own scoring guide, but all teachers in the department weight the assessment for the same amount. However, there were also cases where each teacher within the department was encouraged to use his or her judgment to make decisions about item selection, scoring, and weighting based on the characteristics of the students within their classrooms. In these cases, the mathematics department head ascribed to the view that fair and equitable treatment of students does not necessarily mean using the same approach for every student.

The teachers interviewed used a variety of approaches to select which EQAO items to score. Among the 17 teachers in the study, 4 indicated they score only the multiple-choice items, 12 score a selection of both the multiple-choice and open-response items, and 1 teacher prefers to score only the open-response items. Though individual teachers may choose to score different item types, everyone interviewed indicated that they do not score items they feel they have not adequately covered during the course, or items that are worded in ways their students might find confusing.[1] For example, one teacher states, "What I plan on doing is taking the EQAO test after they're done writing it and selecting the questions that I feel best represent the course" (Shannon interview, p. 1).[2] Similarly, another teacher indicates, "One thing I did this year that I hadn't done in the past is I noticed there was a pattern; there was one question that I chose to mark and none of my Applied kids got it, so I just took it off" (Whitney interview, p. 3). And a third teacher states, "Because I'm selecting which questions, if a topic hasn't been covered or… if it's different from how I would teach it in class or I don't think it's relevant to what I did in my classroom or aligned with what we did, then I like to be able to remove that" (Leslie

[1] *At the same time, almost every teacher interviewed commented on the high quality of EQAO items, and many indicated that they use some of the released EQAO items as learning activities in their classrooms.*

[2] *All participating teachers have been given pseudonyms to help ensure anonymity.*

interview, p. 5). Several of these teachers went on to explain that they had not intentionally omitted or overlooked aspects of the curriculum, but rather, that unexpected events (illness, snow days, school closures, etc.) sometimes resulted in a concept being covered in less depth than they would have liked. In essence, these teachers avoid using EQAO items that cover material their students have not had an adequate opportunity to learn.

Fair and equitable treatment of all students is also a key factor for many of these teachers when deciding on the amount the assessment contributes to their students' grades. For instance, one teacher counts the assessment as 5 percent of most students' final grades, but does not include the assessment in the grades for some students in his class. As he explains:

> There are a couple of girls who do very well in assignments so far and haven't done very well on tests, whether it's anxiety or other things, so what I am planning on doing with them is… having an alternate assignment that basically would be based on what you would learn from the EQAO, and that way, they can demonstrate [their learning]. (William interview, p. 2)

In making different decisions about how the assessment contributes to grades based on his individual students' needs, William is demonstrating that equitable assessment practice does not mean using exactly the same approach for all students, and that assessment can be used to support each students' mathematics learning.

Similarly, another teacher reported that she counts the assessment as the equivalent of two quizzes within the term grade for students in her Grade 9 Applied mathematics class, rather than as part of the summative grade.[3] She explains:

> I know there has been talk at some point about counting it as part of their culminating or doing something else, but because I don't find EQAO is geared to the way I teach my students, the questions aren't worded the way that I word my questions to my students, I don't believe in taking that and putting it in the culminating activity mark, which weighs more heavily… EQAO is not designed for Applied students. The whole thing with Applied students is that they're not test writers, most of them. I find just the whole idea of this big two-day test for them is a bit overwhelming sometimes, and so I'm definitely more lenient with theirs. (Margot interview, pp. 1–2)

Margot went on to explain that she counts the assessment as a greater portion of grades in the Academic course because she feels students in that course have stronger test-writing skills. Other teachers indicated that they are careful to balance the amount the assessment contributes to grades so that it motivates students as they complete the items, but does not create undue stress or overwhelm them.

A third aspect of these teachers' practices that focuses on fair and equitable treatment of students is the way that teachers develop scoring guides for the open-response items. Each of the teachers who mark the open-response items explained that they develop scoring guides that are consistent with the way they score similar items on the unit tests they develop. For instance, one teacher states, "For the long-answer questions, I would assess them based on how I've been assessing on all my tests" (Shannon interview, p. 2); and another states, "I just go through and I [mark] based on if I were to set the test, what would I be looking for in their answer" (Margot interview, p. 2). These teachers, as well as several others who mark the open-response items, went on to explain that they feel that using a scoring approach their students are familiar with increases the fairness of including the EQAO items in students' grades.

In all of these examples, we see that teachers are focused on using the EQAO Grade 9 assessment in ways they consider to be fair and equitable for their students. In some cases, this means that different approaches are taken for individual students within a class or for students taking different mathematics courses (i.e., Applied versus Academic). In addition, these teachers carefully consider the teaching, learning, and assessment they have used throughout the course as they make decisions about their use of the EQAO assessment. Thus, while the EQAO assessment is a standardized test when it is administered and scored as part of Ontario's accountability system, these teachers use their judgment to make context-specific decisions that meet the needs of the students in their classroom.

Providing Multiple Opportunities to Demonstrate Learning

According to *Growing Success*, teachers should use assessment practices that "are ongoing, varied in nature,

[3] *Four of the 17 teachers (from two different participating districts) include the EQAO assessment in the term part of the grade, rather than in the summative grade.*

and administered over a period of time to provide multiple opportunities for students to demonstrate the full range of their learning" (OME, 2010, p. 6). It may seem counter-intuitive that this principle could be addressed through the use of student responses to items on a large-scale test. However, three of the teachers I interviewed explained that they treat the EQAO Grade 9 Assessment as an additional opportunity for students to demonstrate their learning, rather than deciding in advance on a certain percentage that the assessment will contribute to all students' grades.

One teacher I interviewed described how most of the teachers in his department use the responses to the EQAO items as additional evidence to help them come to a judgment of each student's understanding for each overall learning expectation. To begin this process, the teachers get together to decide which overall learning expectation from the Ontario curriculum is most closely aligned with each item on the EQAO assessment. Then, they review the assessment information they have gathered for each student during the semester. They identify those overall learning expectations where they feel they need additional evidence to help them determine the most consistent level of understanding for a given student, and then examine that student's responses to the related EQAO assessment item(s). As Philip explains:

> What I use the EQAO most for, or where it's most influential is for students who have not shown me evidence that is passing the expectations. Then that's usually where I'm depending more on the EQAO 'cause it is sort of their last chance to show me that they understand something. And so if they're able to show me something good there, I'm able to say that's at least a pass for that overall expectation. (Philip interview, p. 4)

Thus, rather than deciding on a fixed percentage that the EQAO assessment will contribute to every students' grade, and using the same selected items for every student, the EQAO assessment items are used to help the teachers at this school judge each student's level of understanding of the overall learning expectations.

A teacher from another board described a related practice where she scores all the EQAO items for a particular mathematics topic where she has not had a chance to gather as much assessment evidence during the semester. For instance, one year, she had not had time to gather enough assessment evidence for geometry, so she scored all of the EQAO multiple-choice and open-response items related to geometry and included the results as part of the term grade for that unit. In another example, a teacher explained that she decides which EQAO items to score on a student-by-student basis. For instance, if a particular student was absent for a significant time period during the semester and had limited opportunities to demonstrate his or her understanding of a particular curriculum area, she can look at the student's EQAO responses to gather additional insights about his or her understanding of that mathematics content area.

In each of these practices, we see that rather than using a one-size-fits-all approach, these teachers have developed strategies where the EQAO assessment items become another of the multiple opportunities they give students to demonstrate their mathematics learning.

Developing Self-Assessment Skills

Both *Growing Success* (OME, 2010) and many research-based descriptions of effective assessment practice (Assessment Reform Group, 2002) emphasize the positive impact of self-assessment on students' learning. While a provincial assessment may not seem like an opportunity for developing self-assessment skills, several of the teachers I interviewed have found ways to do this. In some cases, students engage in self-assessment before taking the test, and in other cases, the self-assessment activities take place after the test is written.

One teacher has students in his Grade 9 Academic mathematics class develop study guides for one another as a way of preparing for the EQAO assessment. Working in teams, his students are asked to reflect on what they know best and what they are less confident with from the course material. They then choose one area of the curriculum to become experts in and develop review questions for one another to complete. Emphasis is placed on students becoming more aware of their own level of understanding for each content area and focusing their review efforts on their areas of need. Another teacher described how immediately after writing the EQAO assessment, she has students in her Grade 9 Applied mathematics course reflect on what they did and did not know as they completed the EQAO items, to help them determine the areas they need to focus on to prepare for their in-school final examination. She states:

> I'm really big on metacognition. [I tell the students] "So now when you look at the kinds of things you couldn't do on the EQAO, now you know where to

> study for your exam" … they know which ones they did well on and which ones they didn't … It helps them to know what they know … I get kids to commit to the unit or strand that they want to work on, and then I can have stations set up … they can work together … to think about the kinds of problems that they're having the most trouble with. So, you can tailor their review to what they actually need to work on, and they're more willing to work when you do that. (Diane interview, p. 29)

Another teacher has a class discussion after her students complete the EQAO test to enable the students to decide which items she should include in their grades. Similarly, another teacher explained that she allows individual students to indicate which open-response items they would like her to score by identifying the items they think they completed successfully.

Each of these practices shows how the EQAO assessment can be integrated within a teacher's classroom-assessment strategy to ensure that students are actively engaged in the assessment process and have an additional opportunity to reflect on what they know and where they need to continue working. In these instances, the EQAO test, which is often seen only as a summative assessment, is also serving as a formative assessment tool that helps to improve students' mathematics learning. In fact, these approaches demonstrate how the EQAO assessment can be used to support students' learning, even though teachers are not permitted to provide students with detailed feedback on their responses to the EQAO items.

Closing Remarks

The teachers who participated in this study describe how they use their professional judgment to develop a coherent classroom-assessment strategy that includes the EQAO Grade 9 Assessment. They have developed practices that align with current assessment research and with the principles of Ontario's assessment policy. Many of these teachers have combined assessment *of*, *as*, and *for* learning in interesting ways as they make decisions to support their students' learning. For some, the EQAO assessment is another opportunity for students to demonstrate their understanding of the curriculum expectations. The EQAO assessment can be used in flexible ways to meet the needs of individual students or groups of students, and provide feedback to guide students' ongoing learning at the same time that it is used to help teachers determine report card grades.

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Marcus du Sautoy

In mathematics the art of proposing a question must be held of higher value than solving it.

Georg Cantor

Exponential Function: Graph of $y = b^x$

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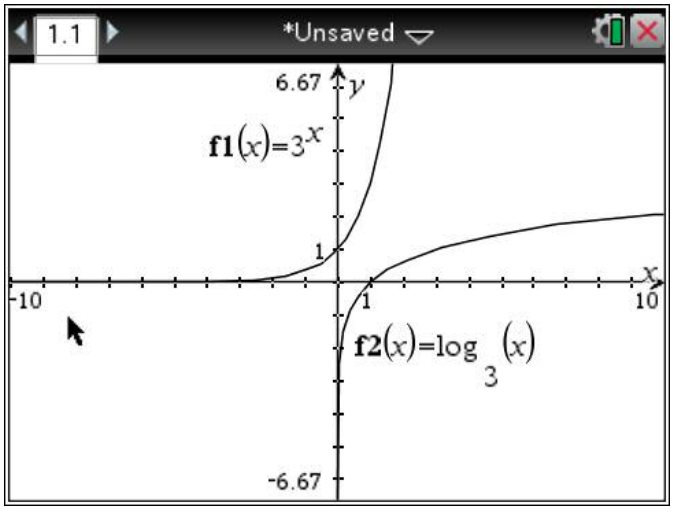


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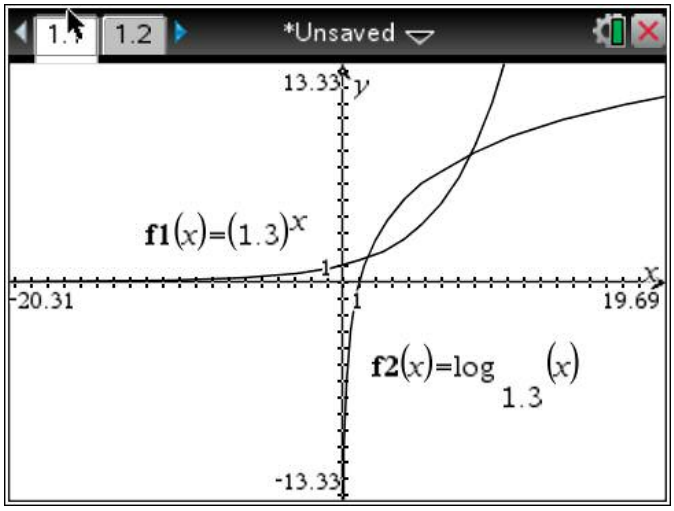
Yixuan Xu graduated from Boren Sino Canadian School and is now reading for her Bachelor of Science Degree in Finance & Accounting at Purdue University, Indiana, USA.

When I was teaching Ontario Curriculum MHF4U (Advanced Functions) math class to my Grade 12 students, we came up with an interesting problem. We were discussing the graph of exponential function and its inverse graph, the logarithmic. I sketched the graph of $y = 3^x$ and its inverse graph, $x = 3^y$ on the board, and asked the class to pick a base number greater than 1, and sketch their own.



$y = 3^x$, and $y = \log_3 x$

After a while, one student came up and reported that his calculator was not working properly and that his inverse graph looked different. He chose base number $b = 1.3$. I asked him to validate his graph, using a table of values. His results were correct and his calculator was also correct, but the shape of his graph was different.



$y = 1.3^x$, and $y = \log_{1.3} x$

This prompted a class discussion about why the graphs look different when we have base numbers greater than 1. The students already knew that it would be different if the base number was b,$0 < b < 1$. In trying to explain this scenario, we stumbled upon the fact that for some base numbers the exponential and inverse graph intersect, and for others they do not. To explain this phenomenon, we considered the fact that the inverse graph is a reflection on the $y = x$ line. This led to the statement that, if $b^x > x$, then there is no intersection; otherwise, the exponential and its inverse graph will intersect.

At that point, we finished the class, and I left the students with the problem of solving $b^x = x$. I emphasized that they need to solve for both the variable x and parameter b. I suggested that since we were looking for equality, we can seek the intersection of $y = x$ and $y = b^x$. The next day, Yixuan Xu came to me with the solution to the problem. This is her solution, using calculus.

Differentiating $b^x = x$ we get $b^x \ lnb = 1$

Hence $lnb = \frac{1}{b^x} = \frac{1}{x}$

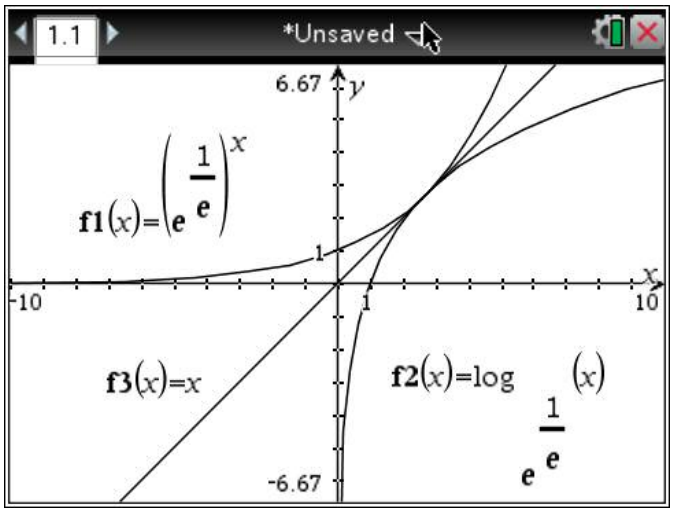
Then $b = e^{\frac{1}{x}}$

Using $x = b^x$ we now have $x = \dfrac{1}{\left(e^{\frac{1}{x}}\right)^x} = e$ and, so $b = e^{\frac{-1}{e}}$.

Then equation $b^x = x$ has solution $x = e$, and $b = e^{\frac{-1}{e}}$.

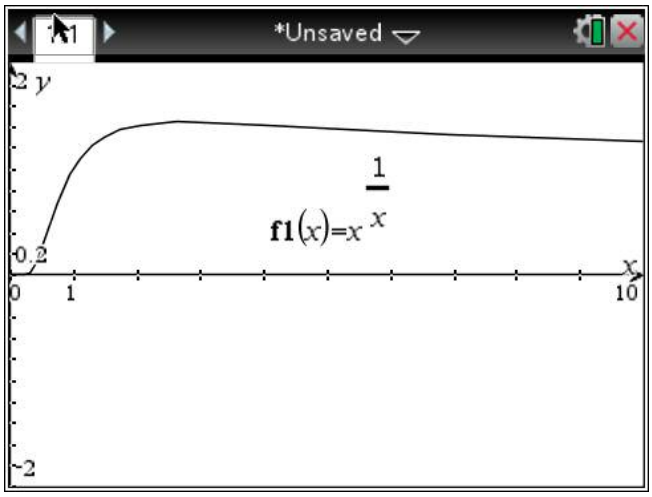
We can also restate the above statement as follows. The solution to $y = b^x$, $y = x$ is given by (e,e), when $b = e^{\frac{-1}{e}}$ At this point, $y = x$ is a common tangent to the graphs of functions $f(x) = b^x$ and its inverse function $f(x) = \log_b x$. We claim that the line $y = x$ is a common tangent, since Yixuan's proof is based on equating the derivatives.

Now we can sketch our exponential and logarithmic graphs again: $y = \left(e^{\frac{1}{e}}\right)^x$, and $y = \log_{e^{\frac{1}{e}}} x$.

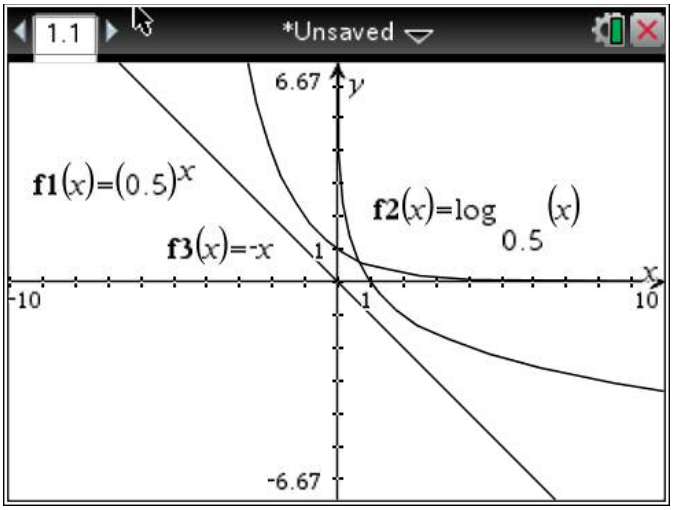


Then the graph of the function $f(x) = b^x$, when $b > e^{\frac{1}{e}} \approx 1.4446$ lies above the line $y = x$ and does not intersect with the graph of its inverse function $f(x) = \log_b x$, but when $1 < b \le e^{\frac{1}{e}}$, these graphs intersect.

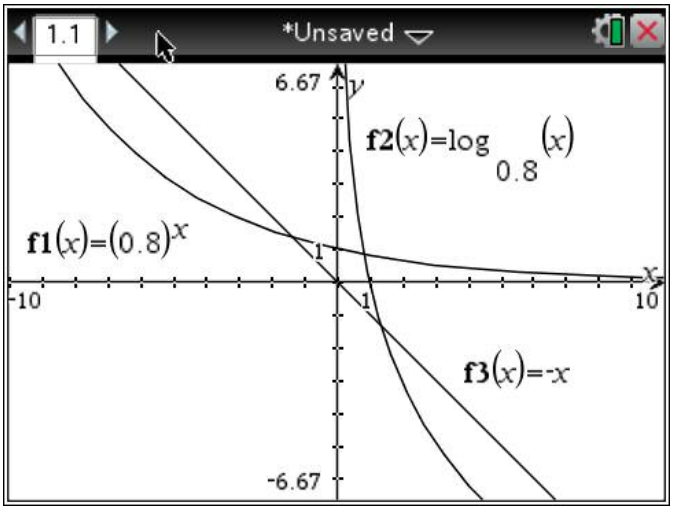
Interesting connections may be made to the famous Steiner's problem, where one seeks the maximum of the function $f(x) = x$. Steiner's problem has maximum at $\left(e, e^{\frac{1}{e}}\right)$.



We can develop a similar result when the base number is b, $0 < b < 1$:



$y = 0.5^x$ and $y = \log_{0.5} x$ (These graphs lie above the line $y = -x$.)



$y = 0.8^x$ and $y = \log_{0.8} x$ (These graphs intersect the line $y = -x$.)

When $0 < b < 1$ we can solve the equation $b^x = -x$ using a similar method as above.

Differentiating $b^x = -x$ we get $b^x \ln b = -1$

Hence, $\ln b = \frac{-1}{b^x} = \frac{1}{x}$

Then $b = e^{\frac{1}{x}}$

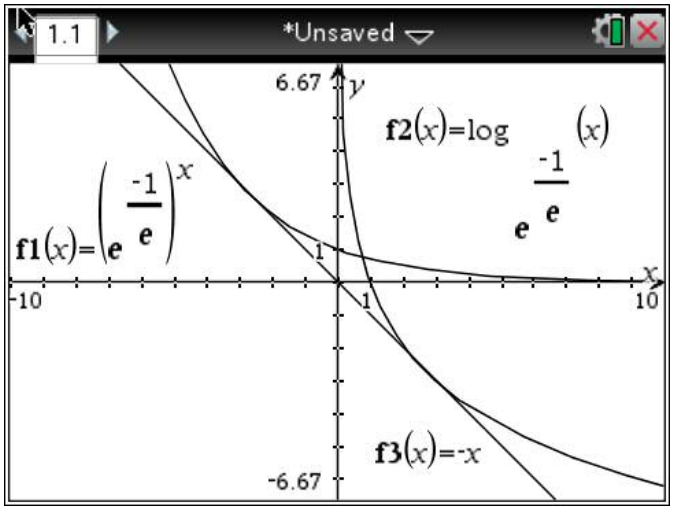
Using $b^x = -x$ we now have $\left(e^{\frac{1}{x}}\right)^x = -x$. This gives $x = -e$

Also, $b = e^{-\frac{1}{e}}$.

Then equation $b^x = -x$, $0 < b < 1$ has solution $x = -e$, and $b = e^{-\frac{1}{e}} = \left(\frac{1}{e}\right)^{\frac{1}{e}}$.

We can also restate the above statement as follows. The solution to $y = b^x$, $y = -x$ is given by $(-e,e)$ when $b = e^{-\frac{1}{e}}$. Then $y = -x$ is a common tangent to $f(x) = b^x$ and its inverse function $f(x) = \log_b x$. We claim again that $y = x$ is a common tangent, since the proof is based on equating the derivatives. Note that $y = -x$ is a tangent to $y = \log_b x$ at $(e,-e)$.

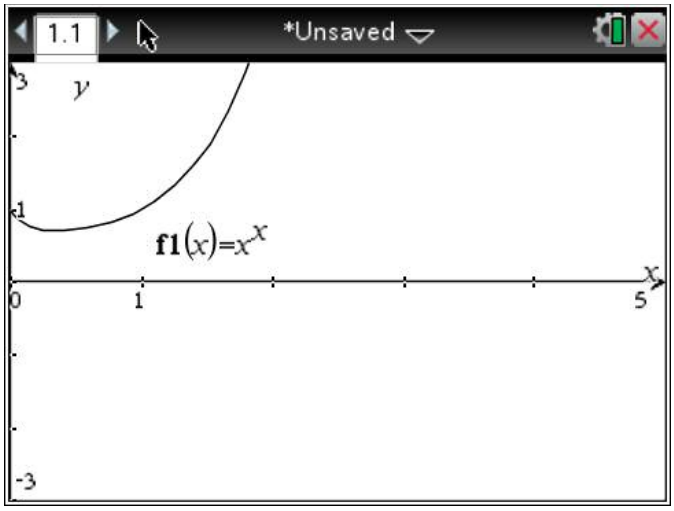
$y = \left(e^{\frac{-1}{e}}\right)^x$, and $y = \log_{e^{\frac{-1}{e}}} x$ (These graphs share a tangent line $y = -x$.)



Now we have the result for the base number b, $0 < b < 1$:

When $0 < b < e^{-\frac{1}{e}} \approx 0.6922$, the graphs of $y = b^x$, and its inverse graph $y = \log_b x$ lie above the line $y = -x$, but when $e^{-\frac{1}{e}} \le b < 1$, the graphs of $y = b^x$ and its inverse intersect the line $y = -x$.

Interesting connections may be made to a problem similar to Steiner's, where we seek the minimum of $f(x) = x^x$. This problem has minimum at $\left(\frac{1}{e}, e^{-\frac{1}{e}}\right)$.



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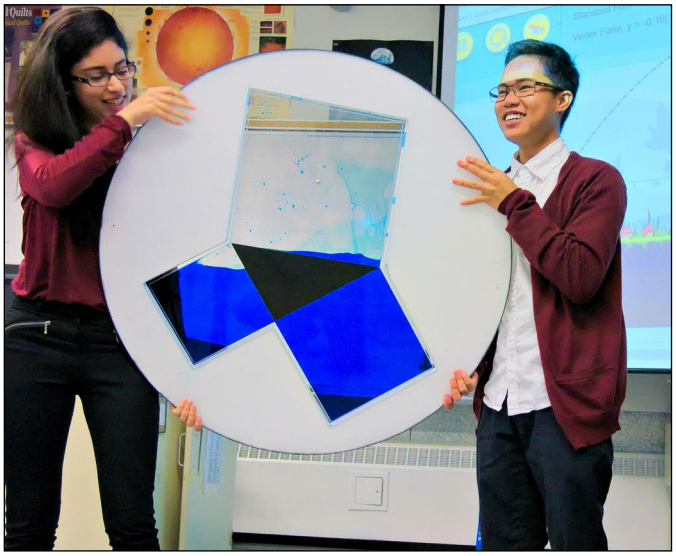
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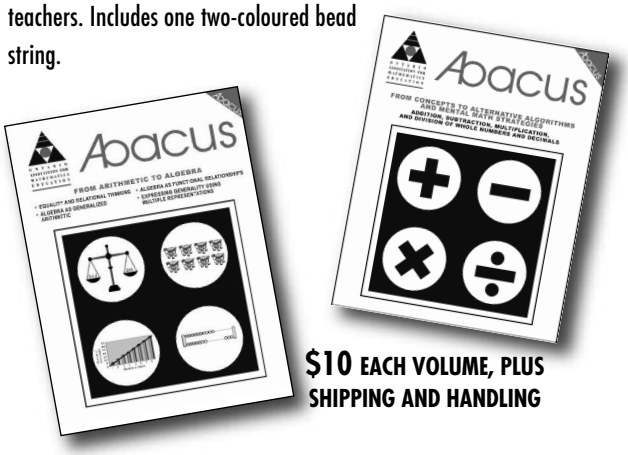
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▲ Fields Institute MathEd Forum Report

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Stewart Craven presently teaches courses for the Continuing Education Department and the Transition Years Program at York University. He is an active participant in the Fields Mathematics Education Forum.

As of this writing, the MathEd Forum has met twice since my last column.

The September theme was "Mathematics and Equity" and featured four speakers: **Terrence Richard Blackman** (Morgridge College of Education, University of Denver): Mathematics, mathematicians, mathematics education and equity; **Allan Pitman** (Faculty of Education, Western University): The changing definitions and contexts of school mathematics; **Steven Khan** (Faculty of Education, Brock University): A conversation that requires more of us: From principles to action in achieving excellence in mathematics education (in Ontario); and **Ana Isabel Sacristán** (Department of Mathematics Education, Cinvestav–IPN, Mexico): *On the integration of digital technologies in mathematics classrooms: Support and innovation for learning?* This meeting was very well attended and, as always, the discussion was thought provoking.

The October theme was "Mathematical Literacies: Problem Solving in Evolving Digital Environments" and featured five speakers: **Yves Bourgeois** (University of New Brunswick, NB): *Why are we (apparently) so bad [at] getting our youth skilled for the labour market?*; **Michael Perklin** (Bitcoin): *Bitcoin, blockchains, and the coming crypto-revolution*; **Charles Anifowose** (Vretta): *Immediacy: How 21st century practices are supercharging math intuition among today's learners*; **Tracy Solomon** (Hospital for Sick Children, Toronto): *Towards an evidence base to inform mathematics instruction in Canada*; and **Viktor Freiman** (Moncton University, NB): *Building new opportunities for (interdisciplinary and multi-sectoral) collaboration and partnership*.

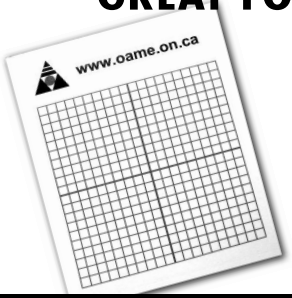
The Steering Committee of the MathEd Forum is responsible for selecting themes and then organizing speakers to fill out each program. The following is a sample of some the ideas that have been put forward for future meetings:

a. Discussing the significance of PISA results (international testing)
b. What mathematics do our students need to thrive in the twenty-first century?
c. Who are the customers of education? Businesses, parents, politicians, institutes of higher learning? Who is not a customer of education?
d. Revisit the existing high school math curriculum to critically evaluate it. There have been two major changes since 2000: What is working, what is not working?
e. How does research inform mathematics education?
f. How can teaching the history of mathematics be integrated into the curriculum?
g. How should we go about teaching statistics? How do we build understanding about difficult concepts such as uncertainty? Should we use innovative approaches such as computer game technology in the teaching of statistics?

First, I would like to ask you to consider coming to our meetings. (Agendas and information can be found at: **www.fields.utoronto.ca/programs/mathed/forum/**) Second, I would like to invite you to make suggestions about what themes you think should be addressed by the Fields MathEd Forum. (Please send me an email at: **numeratecitizen@mac.com**) ▲

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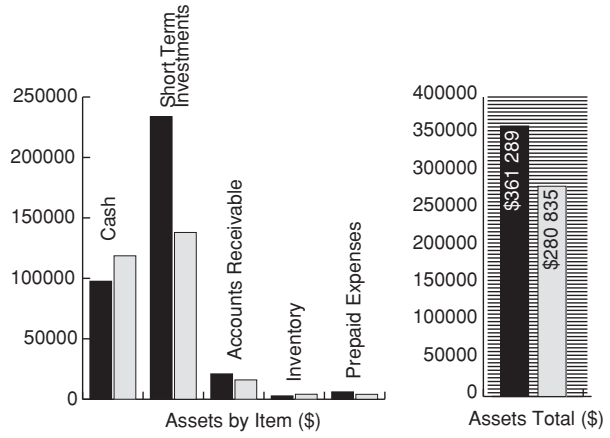
▲ OAME Financial Statement 2013-14

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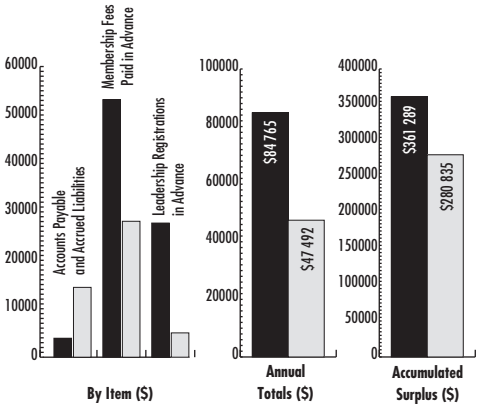
On the basis of information provided by management, I have compiled the statement of financial position of Ontario Association for Mathematics Education (Association Ontarienne pour L'Enseignement des Mathématiques) as at August 31, 2014, and the statement of operations, changes in net assets, and cash flow for the year then ended. We have not performed an audit or a review engagement in respect of these financial statements and, accordingly, we express no assurance thereon. Readers are cautioned that these financial statements may not be appropriate for their purposes.

London, Ontario,Septmeber 24, 2014, SKRYPNYK GROUP,Professional Corporation, Authorized to pratise public accounting by CPAofO

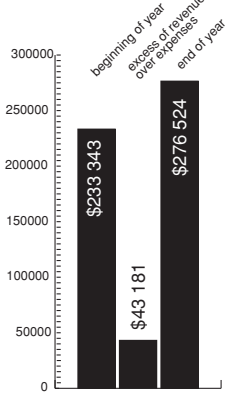
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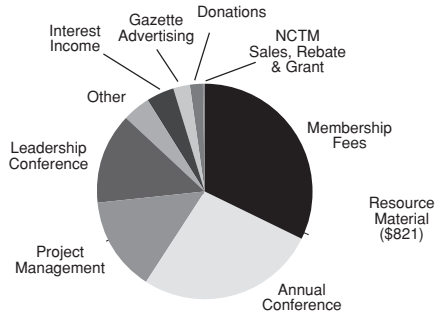
Liabilities and Fund Balance



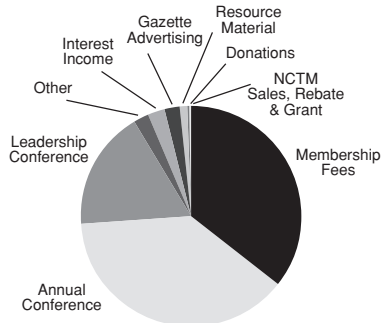
Operating Fund Balance



Revenue



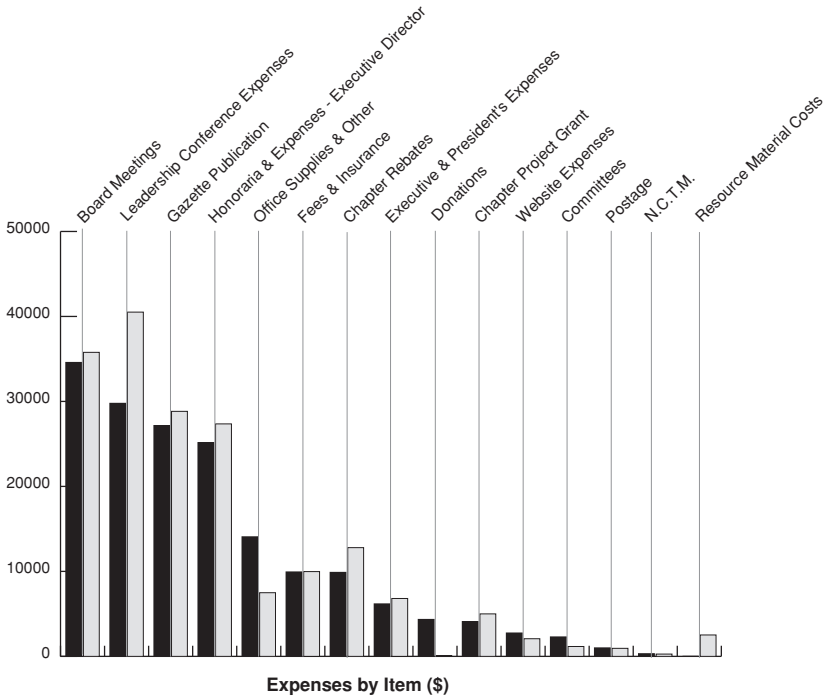
2014 ($214 762)



2013 ($235 514)

2014/2013

Expenditures



8 PAGE INSERT ELEMENTARY MATH

ONTARIO ASSOCIATION FOR MATHEMATICS EDUCATION

Abacus

Volume 53 • Number 2 • December 2014

SPATIAL REASONING WITHIN NUMBER SENSE AND NUMERATION CONTEXTS

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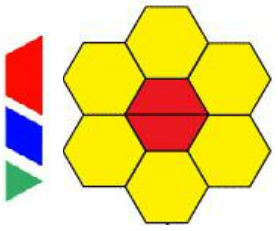
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KATHY KUBOTA-ZARIVNIJ currently focuses on K to 6 mathematics professional learning at the provincial level. She is a long-time OAME board member and OAME past president. In relation to her work in mathematics education, she uses complexity thinking to interpret mathematics teaching and learning for students and teachers. **kathkubo@rogers.com**

Number sense in the early grades is more than matching a numeral to a quantity. It is about having a "feel" for how numbers compare to one another. It is the basis for how we differentiate quantities that are reasonable from those that are unreasonable. Such number reasonableness or number sense is derived from a spatial understanding of number.

What type and number of pattern blocks could cover this pattern block flower?



Determining the area relationships between the hexagon, trapezoid, rhombus and triangles is achieved through spatial reasoning; that is how the area of one pattern block relates to another pattern block.

For example:

- 6 triangles, 3 rhombus or 2 trapezoids cover 1 hexagon
- 1 trapezoid, 1 rhombus and 1 triangle cover 1 hexagon
- 3 trapezoids, 3 rhombus and 3 triangles cover 3 hexagons

Solutions	hexagon	trapezoid	rhombus	triangle
1	7	0	0	0
2	0	14	0	0
3	0	0	21	0
4	0	0	0	42
5	1	12	0	0
6	?	?	?	?

For this problem, spatial reasoning (and proportional reasoning) is used to discern these number and measurement relationships:

- the larger the pattern block, the fewer number of pattern blocks
- the smaller the pattern block, the greater the number of pattern blocks

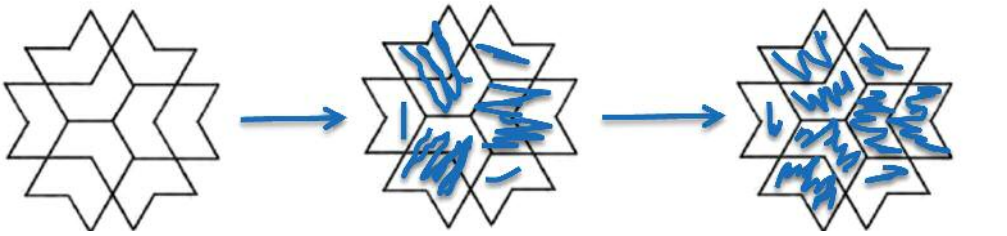
In this issue, the Research Summary and problems (with multiple solutions) focus on the use of spatial reasoning within number sense contexts. The problems we provide are designed for use with a three-part problem-solving lesson structure.

RESEARCH SUMMARY – SPATIAL REASONING WITHIN NUMBER SENSE AND NUMERATION CONTEXTS

Children use imagery in their thinking and the properties of images will have consequences on children's concepts and reasoning (Piaget & Inhelder, 1971). For example, students who have a well-developed spatial sense tend to solve tasks in more meaningful ways, not just as rote learners (Reynolds & Wheatley, 1997). Wheatley (1998) reports that students develop the capacity to solve problems in multiple ways, as their network of mental imagery for mathematical concepts and relationships develops. Consider this problem that relates spatial organization and quantity.

How many same and different size hexagons comprise this shape

Challenges like this, called Quick Draw (Wheatley, 2007) provide learners with experiences in making direct connections between spatial reasoning and number concepts. In this example, students select a particular size shape in the drawing that is repeated.



Original shape (3 large + 3 small) hexagons 3 x 2 + 3 = 9 small hexagons

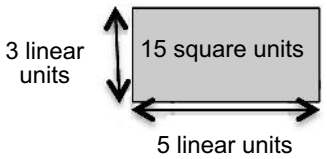
Though number reasoning tasks or problems are predominately viewed as numerical in nature, evidence is emerging that spatial attention plays an important role in preschool and kindergarten children's ability to successfully complete number naming, numeration and symbolic magnitude comparison tasks. For example, Ansari et al. (2003) reported visual spatial abilities account for children's response to cardinality tasks and to questions like, "How many?".

Some studies examined mental representations or imagery in arithmetic and revealed differences between high and low achievers (Pitta & Gray, 1996; Gray, Pitta & Tall, 1997). Low achievers had a tendency to highlight surface details and emphasized the concrete qualities within situations (focused to the descriptive qualities of numbers). Mathematically high achievers focused their attention on the relationships and abstract qualities of numbers (Pitta & Gray, 1996). According to Gray, Pitta and Tall (1997), different perceptions of these objects are at the heart of different cognitive styles that lead to success and failure in elementary arithmetic.

Spatial reasoning for numbers enable the capturing of a mathematical concept or relationship as some form of representation, such as diagrams, graphical displays and symbolic expressions. Representations or models illuminate particular aspects of a mathematical concept or relationship. For example, Mowat and Davis (2010) explain that understanding of number as a "position in space" is the spatial component of understanding number as movement along a continuous measurement scale. Numbers 4, 9 and 15 are located as points or positions on the number line. These numbers also have a linear unit value; that is, number 4 has a length of 4 linear units.

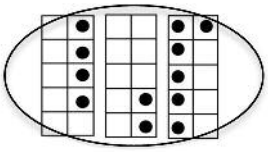
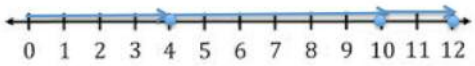
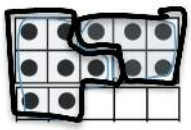
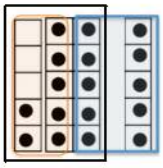
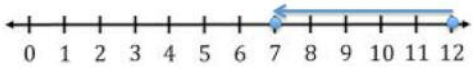
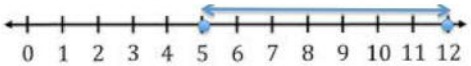
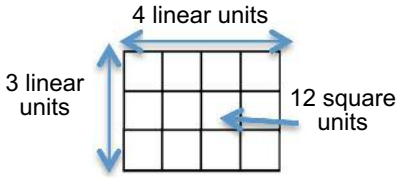
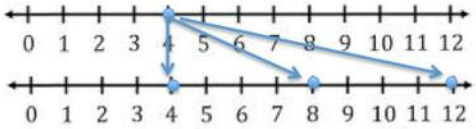
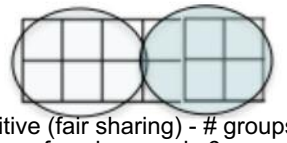
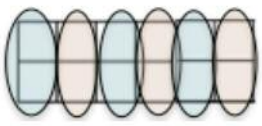
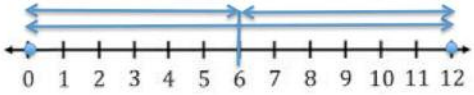
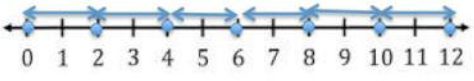
Newcombe (2010) explains that "space provides a concrete grounding for number ideas, as when we use a number line, use base-10 blocks, or represent multiplication as area. (p. 34).

3 linear units x 5 linear units = 15 square units



Representations can help students organize their mathematical thinking and make mathematical ideas more concrete and available for reflection. As students' develop their mathematical knowledge, they develop an increasingly large repertoire of mathematical representations, as well as knowledge of how to use them productively. Such knowledge includes choosing and moving between representations and learning to ask such questions as, "Would a grid or number line model provide greater clarity than a set of symbolic calculations to solve this problem?"

LINKS TO MANIPULATIVES: USING GRIDS AND NUMBER LINES

Number Operation	Grid	Number Line
Addition Joining or Combining of 2 or more addends; Sum = addend + addend + addend 12 = 4 + 6 + 2	 Join 3 quantities, 4, 2 and 6 to make 1 quantity, 12	 start at 4, add on 6, then 2, end up at 12
Subtraction *Separation* (1 whole separated into 2 parts) OR *Comparison* (comparing 2 wholes for same and difference; comparing subtrahend to minuend (referent) Minuend – subtrahend = difference 12 – 5 = 7	 Decomposing 1 whole, 12, into two parts, 5 and 7 minuend subtrahend  Comparing 2 wholes, 12 and 5; same is 5, difference on minuend is 7	 start as 12 and separate 5, the result or difference is 7  mark 12, 5 show difference of 7
Multiplication *Area of rectangle* *Scalar* (1 to many correspondence) Multiplicand x multiplier = product 4x1=4; 4x2=8, 4x3=12	 4 linear units 3 linear units 12 square units Area of a rectangle with known side lengths and unknown area (product)	 4 is the multiplicand; multiplier is 1, 2, or 3 which maps on to the products of 4, 8 or 12
Division *Fair Sharing* (Partitive – number of groups known, size of each group unknown) or *Grouping* (Quotative – size of 1 group known, number of groups unknown) Quotient – result of division Dividend ÷ divisor = quotient 12 ÷ 2 = 6	 Partitive (fair sharing) - # groups is 2, size of each group is 6  Quotative (grouping) – size of each group is 2 and # of groups is 6	 Partitive (Fair Sharing) - 12 divided into 2 groups, size of each group is 6  Quotative (Grouping) – size of each group is 2, so # of groups is 6

LESSON LEARNING GOALS

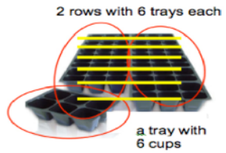
Identify the curriculum expectations (MOE, 2005) grade-before and grade-after the grade specific learning goal, in order to:
- see the development of mathematical knowledge, skills, and strategies across grade levels
- determine the mathematical focus of the problems chosen for Before (activation), During (solving problem), and After (Practice) parts of the lesson

Grade 4	Grade 5	Grade 6
-multiply two-digit whole numbers by one-digit whole numbers using a variety of tools, student-generated algorithms and standard algorithms	-multiply two-digit whole numbers by two-digit whole numbers, using estimation, student generated algorithms and standard algorithms	-solve problems involving the multiplication ... of whole numbers (four-digit by two-digit) using a variety of tools and strategies

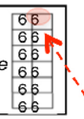
BEFORE (Getting Started) – 5 to 10 minutes

- Activating students' mathematical knowledge and experience using a prompt or problem, related to the math in the lesson problem.
- Includes student responses to highlight ideas and strategies.

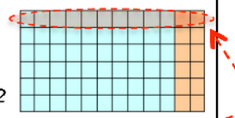
Planting Trays Problem:
Garden plants are sold in flats. Two trays with 6 cups each are attached side by side and 6 rows of them are joined to form a flat.



Solution 1
1 tray with 6 cups, 6 times, twice
6 x 6 x 2 = 36 x 2 = 72



Solution 2
12 rows x 6
(10 + 2) x 6
= 60 + 12 = 72



Solution 3
6 cups and 12 trays in a flat

Tray Number	1	2	3	4	5	6	7	8	9	10	11	12
Number of cups	6	12	18	24	30	36	42	48	54	60	66	72

***Picture** or **Visualize** equal size groups in rows and columns:*
- *12 equal size groups of 6*
- *6 equal size groups of 10 and 2 (or 12)*

DURING (Working on It) – 15 to 20 minutes

- Understanding the problem – Teacher asks, "What information from the problem are we using to solve it? Explain." Teacher records in a list below the problem, the information that students identify.
- Students solve the problem on chart paper (landscape) with markers (visible for class discussion) in pairs or small groups.
- Teacher circulates to record different student solutions, in addition to the ones the teacher anticipated.

Flowers Problem:
The social committee needs about 250 to 300 flowers for decoration. Boris orders 12 groups of 24 flowers. How many should be delivered?

What information are we going to use to solve the problem?
- *Ordered 12 groups of 24 flowers*
- *Needs 250 to 300 flowers*
- *How many flowers should be delivered?*

AFTER (Consolidation) – 20 to 25 minutes

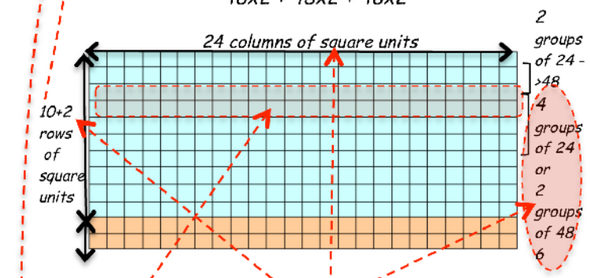
- Teacher selects 2 or more solutions for class analysis and discussion in a sequence (1st, 2nd, 3rd, etc.) based on mathematical relationships between the solutions and the lesson learning goal.
- Students (authors) explain and discuss their solutions with their classmates; co-construction of success criteria towards the lesson learning goal
- During whole class discussion, solutions are organized (often re-organized) to show mathematical elaboration from one solution to the next and towards lesson learning goal.
- Teacher mathematically annotates (math terms, math symbols, labelled diagrams, concise explanations) on and around solutions to make mathematical ideas, strategies, and models of representation explicit.

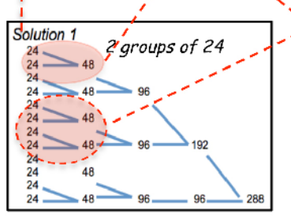
288 = 24 x 12

product *result of multiplication number of square units* — ***multiplicand*** *size of the group # of columns* — ***multiplier*** *# of groups # of rows*

distributive property *(decomposing factors into 10s, 1s)*
→ 24x12 = 24 x (10+2) = 24x10 + 24x2

288 = 24 x 12
24x2 + 24x2 + 24x2 + 24x2 + 24x2 + 24x2
48x2 + 48x2 + 48x2





Solution 2
24 x 10 = 240
24 x 2 = 48
24 x 12 = 288
They will have enough flowers at 288.

Adding Doubles Or Pairs of Same Size Numbers

Distributive Property
Decompose Multiplier into 10s, 1s

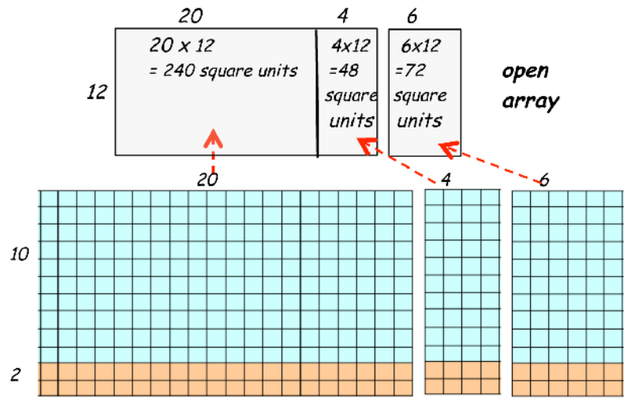
How does the table of values relate to multiplication?

Number of Groups	Number of Flowers in 1 Group	*Multiplicand size of group* *multiplier number of groups*	Number of Flowers
1	24	24 x 1 = 24	24
2	24	24 x 2 = 48	48
5	24	24 x 5 = 120 (24x2+ 24x2 + 24x1 = 48+48+24)	120
10	24	24 x 10 = 240 (24x5+24x5=120+120)	240
12 = 2+10	24	24 x 10 + 24 x 2 = 240+48 = 288	288

- *number of groups -> term number*
- *number of flowers -> term value*
- *number of flowers in group X # groups = # flowers*
- ***multiplicand x term number (multiplier) = term value***

Array → # rows x # columns = multiplicand X multiplier

12x20 < 12x24 < 12x30



- *Under calculated to 20 groups, needed to add 4 more groups*
  12x24 = 12x20 + 12x4 = 12x2x10 + 48 = 240 + 48 = 288
- *Over calculated to 30 groups, needed to subtract 6 groups*
  12x24 = 12x30 - 12x6 = 12x3x10 - 72 = 360 - 72
  = 360 - 60 - 12 = 300 - 12 = 288

Solution 3
12 x 20 < 12 x 24 < 12 x 30
12 x 2 = 24; 12 x 3 = 36
12 x 4 = 48; 12 x 6 = 72
12x20 =12 x 2 x10 = 24 x 10
= 24 tens = 240
need 4 more groups of 12
12 x 4 = 48

Compensation (Area Model)

Solution 4
number of flowers = number in 1 group x number of groups
24 flowers in 1 group
number of flowers = 24 x number of groups.

Number of groups	Number of flowers
1	24
2	48
5	120
10	240

48 + 240 = 288

distributive property 24x12 = 24 x (10+2)

Size of Group, # of Groups (Table of Values)

AFTER (Highlights/Summary) – 5 minutes

- Teacher and students revisit the student solutions for key ideas, strategies, and models of representation that are related to the lesson learning goal (anticipated success criteria)
- Teacher lists key ideas, strategies, and models of representation so the students can see how the mathematical details from their solutions relate explicitly to the lesson learning goal; summary of co-constructed

Highlights/Summary:

288 = 24 x 12 = 24x10 + 24x2

product result of X # square units = # of columns X # of rows — *multiplicand group size* — *multiplier # of groups* — *distributive property decompose factors to 10s, 1s* — *array (# rows x # columns)*

Represent the multiplication of 2-digit by 2-digit whole numbers by:
- *Distributive Property – decomposing one or more of the factors into 10s and 1s*
  (24x12 = 24x(10+2) = (24x10) + (24x2) = 240+48 = 288
- *Compensation – under or over calculating the product to closest 10s then adjusting by adding or subtracting*
  24x12 = 20x12 + 4x12 = 240 + 48 = 288
  24x12 = 30x12 - 6x12 = 360 - 72 = 360-60-12 = 288
- *Table of Values –*
  multiplicand x term # (or multplier) = term value

AFTER (Practice) – 5 to 10 minutes

- Teacher chooses 2 or 3 problems, similar to the lesson problem for student to solve in pairs as a scaffold and individually.
- Problems could vary by number (choice, size), problem contexts, or what is unknown or needs to be solved
- Students are asked to solve these problems using a strategy different from the one they used for the lesson problem.

Trash Problem :
62 people picked up at least 15 pieces of trash at the BBQ. At least how many pieces of trash were picked up?

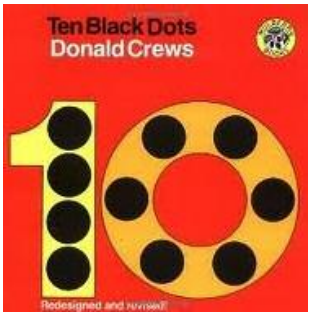
Solution 1
62 groups of 15 pcs trash
15 pcs trash x 62 people
= 15 x 20 = 300
+ 15 x 20 = 300
+ 15 x 20 = 300
+ 15 x 2 = 30
15 x 62 = 930
15 pcs trash x 132 people
= 930 pcs trash

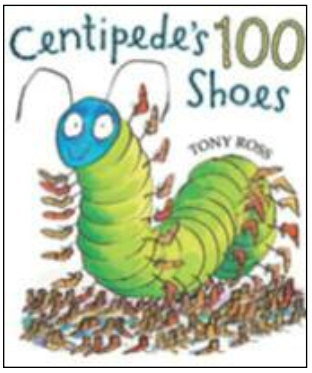
Solution 2
Size of group is 15 (trash pc)
# of groups is 62 (people)

Number of people	Pieces of trash
1	15
2	30
10	150
20	300
60	900
62	900+30

Note: Mathematical annotations include mathematical vocabulary, symbols, elaborations of mathematical details from solutions, labels describing the method/strategy, and questions to further thinking. All annotations are records of students' mathematical discussion.

Note: A classroom board is longer proportionally than these 2 pages. Due to the space constraints on these pages, the mathematical annotations are recorded above the solutions with arrows, rather than on and around the solutions.





LINKS TO LITERATURE:

Ten Black Dots by Donald Crews provides a spatial representation of numbers in different ways (e.g., rows and columns of black dots). It illustrates how numbers can be decomposed and re-composed. "five buttons can make buttons on a coat…" shows five dots as a column of black dots, while the next page, "…or the portholes on a boat" Five is decomposed into horizontal rows of two and three dots.

- Read Ten Block Dots as an introduction to using quick images with dot cards, ten frames.

Centipede's 100 Shoes by Tony Ross is about a centipede (with 42 legs) that buys 100 pairs of shoes and realizes the next morning that he bought too many shoes. So, he sells them along with 42 socks.

- Represent the shoes and socks that were sold to the beetles, wood lice, grasshopper, spiders and worms. Use ten frames, 10x10 square grid or array of dots.

LET'S DO MATH – APPLE PROBLEMS

Before (Getting Started)

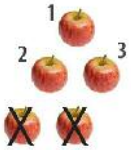


Apple Problem A:

Johnny Appleseed saw 5 apples in the bowl on the kitchen counter. He took 2 apples. How many apples were left in the bowl?

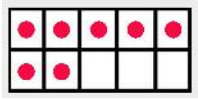
Solution 1

I pictured the apples and I took 2 away. I counted those left and there were 3.



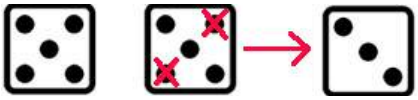
Solution 2

I showed 5 dots in to top of a 10-frame. Then I showed 2 dots in the bottom. When I compared the numbers the difference was 3.



Solution 3

I thought of the 5 apples and saw the 5 on the die. Taking 2 dots off makes the number 3.



During (Working On It)

Apple Problem B:

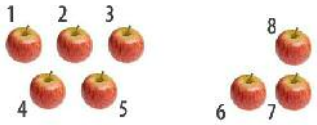
Jani Appleseed noticed 5 apples in a bowl on her kitchen counter. How many apples would there be once she added 3 apples to the bowl?



After (Consolidation)

Anticipating Student Responses:

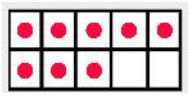
Solution 1



I saw the apples in my head and I counted them 1, 2, …5, 6, 7, and 8.

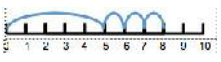
Solution 2

Put each apple in 10-frame and saw 2 less than 10 or 8.



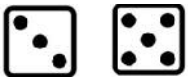
Solution 3

Start at 5 and jump 3 more to 8



Solution 4

Each dot is an apple
5 apples + 3 apples = 8 apples.



Coordinating Discussion for Student Learning:

Why might solution 1 be chosen first for student discussion followed by solution 2, 4, and 3?

- *Solution 1* – counting each object using one-to-one correspondence, stable order and cardinality
- *Solutions 2 and 4* – using an organized space (i.e., 10-frame, dice faces) to represent quantity and join them to determine the sum
- *Solution 3* – using number line

LET'S DO MATH – PROBLEMS



Before (Getting Started)

Ticket Problem 1:

Six people give out tickets for door prizes. 100 tickets are shared equally among 6 people. How many tickets did each person get?

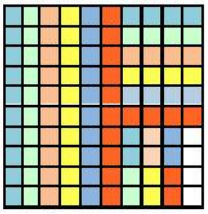
Solution 1

I imagined the 6 hands and the 100 tickets. Then I started sharing.
10 x 6 = 60 too little
15 x 6 = 90 too little
16 x 6 = 96 and 96 + 4 = 100
So, each worker gets 16 tickets to give out and there were 4 left over.



Solution 2

6) 100 — 10
60
40 — 5
30
10 — 1
6 — 16
4



So 100 = 16 x 6 + 4
Each of 6 people get 16 tickets each with 4 left over.

During (Working On It)

Flowers Problem 2:

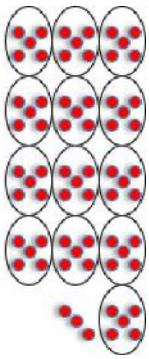
The garden centre donated 68 flowers to the community centre. The flowers are bundled in groups of 5 to decorate tables. How many tables could be decorated?



After (Consolidation)

Anticipating Student Responses:

Solution 1



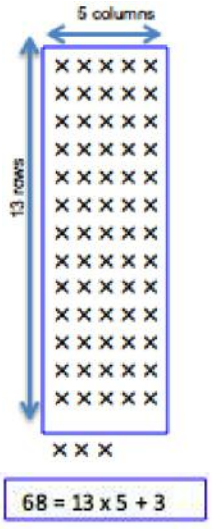
I put 68 counters in groups of 5 and got 13 groups with 3 left over. So we can decorate 13 tables with 5 flowers each and have 3 extra flowers.

Solution 2

Number of Groups	Number of Flowers
0	68
1	63
2	58
3	53
4	48
5	43
6	38
7	33
8	28
9	23
10	18
11	13
12	8
13	3

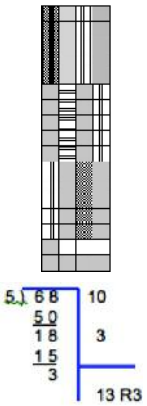
So 68 ÷ 5 = 13,
3 flowers left over so cannot make another group, so 3 left over.

Solution 3



68 = 13 x 5 + 3

Solution 4



68 flowers can make 13 groups of 5 and there will be 3 left over.

Coordinating Discussion for Student Learning:

Problem 1 is a division by sharing problem and 2 is a division by grouping problem.

Why might solution 1 (for Problem 2) be chosen first for student discussion, followed by solution 3, 2 and 4?

- *Solution 1* – counters represent 68 flowers, grouped 13 times with 5 in each group and 3 left over.
- *Solution 3* – the dimensions for 13 groups of 5 flowers in an array (i.e., 5 flowers in each row; 13 in each column; 65 of the flowers are accounted for and there are 3 left over)
- *Solution 2* – Shows skip counting starting with the 68 flowers and subtracting 5 flowers at a time as each table is decorated. When there are 13 groups, there are 3 flowers left so that cannot make another group and there will be 3 left over.
- *Solution 4* – model repeated subtraction; know that 10 groups of 5 means 50 of the flowers will decorate 10 tables; the remaining 18 flowers make 3 groups of 5 with 3 flowers left over.

NEXT STEPS FOR YOUR PROFESSIONAL LEARNING

Application to Your Classroom

- Consider how these number sense problems include visual-spatial reasoning in relation to proportional reasoning.
- To suit your students' learning needs, begin designing number reasoning tasks or problems that are relative to the grade specific curriculum expectations in the Number Sense and Numeration strand, being mindful of the models of representation (e.g., area, set, linear, graphic, number) of different types of numbers and number operations.
- Provide problems that have students represent number relationships and operations using models of representation, such as the area model (e.g., square unit grids), set model (e.g., groups of same and different objects organized in rows and columns, equal size groups, 2x10 and 10x10 rekenreks, five and ten frames), table of values and linear model (e.g., 1x100 rekenrek, metre stick, open or unitized number lines).
- Incorporate spatial reasoning daily for practice, such as Quick Images. Quick images materials include dot patterns, ten frames, dominoes, coloured squares that the learner views for a few seconds to decide on the quantity presented.
- Practise noticing the different strategies and the breadth of the mathematics your students use to solve number reasoning tasks or problems, such as, decomposing or composing numbers using proportional shapes, like pattern blocks, square tiles, base ten blocks.
- Think ahead of the kinds of mathematical annotations you will record on and around the student solutions to make explicit spatial reasoning strategies for number representations and concepts like addition as combining, multiplication as equal size groups in an array or division as fair sharing or equal size grouping.

SUGGESTED READINGS

Some readings used in the Research Summary and in the development of the problems and solutions are listed below

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CLOSING

Our next issue will focus on spatial reasoning within measurement contexts. As well, the next issue will outline the ways that teachers need to know these notions for teaching, as well as, the ways that teachers can use these mathematical ideas and pedagogical strategies within learning and teaching mathematics within a three-part problem solving lesson framework.

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Send your teaching strategy ideas, problems and student solutions to the *Abacus* Co-Editors via email

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