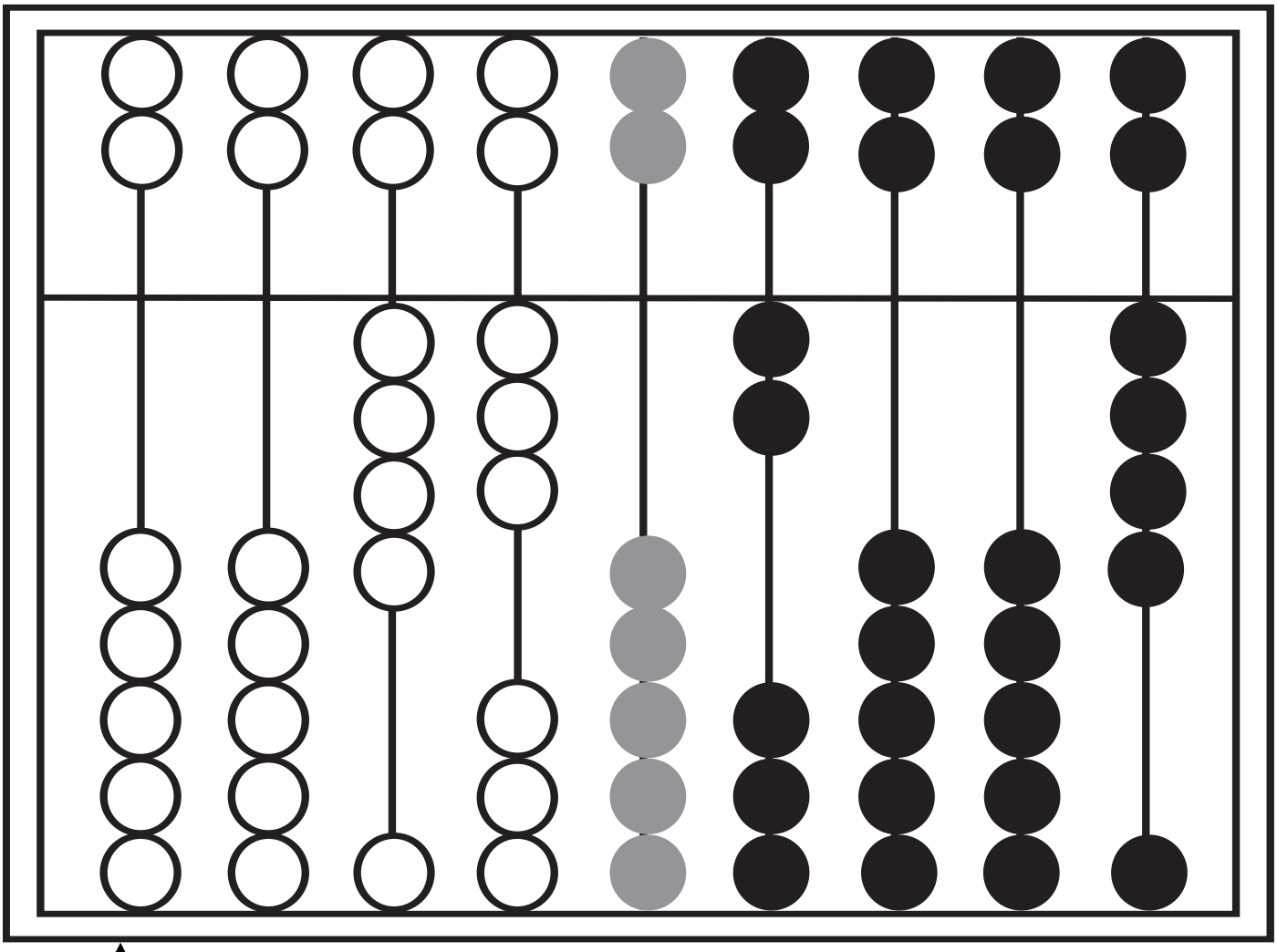
Abacus

Volume 43 2005



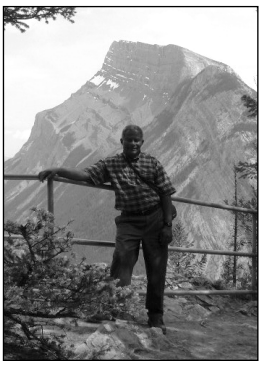
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ONTARIO ASSOCIATION FOR MATHEMATICS EDUCATORS

▲ EDITOR'S CORNER

Trevor Brown



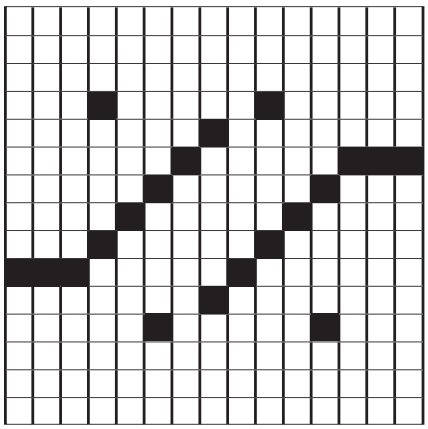
You will notice two new columns in this issue of The Abacus. The Mathematical Games section is an extension of the "What if…?" section which we introduced in the previous issue. Here we use the popular Nim game as the starting point and then proceed to show how by asking a series of "What if?" questions we can generate many related activities with some subtle variations.

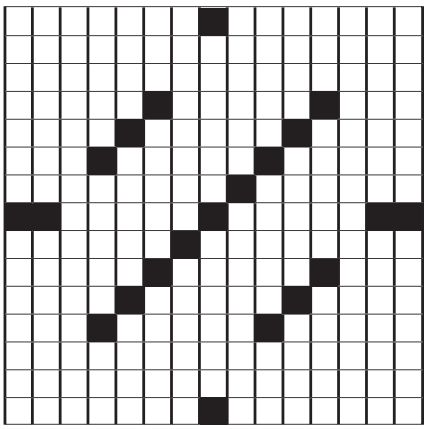
The other new column is the one looking at technology, and how it can be used to deepen and enrich the students' understanding of mathematical ideas and concepts. It is a truism that the proper use of technology allows students to make conjectures. In the two activities in this section, students are encouraged to form conjectures and then proceed to state why they formulated their conjectures.

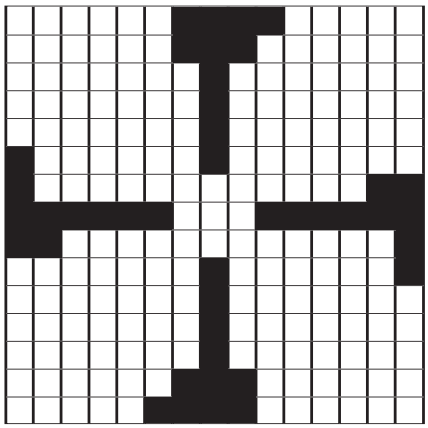
Please send us any ideas and or activities that you have used to enrich and deepen your students' understanding of mathematics.

The Mathematical Eye

For this task you will need to investigate a crossword puzzle. According to *The Million word Crossword Dictionary* by Stanley Newman and Daniel Stark, a crossword puzzle should satisfy the following conditions.







1. The pattern of black and white squares must be symmetrical. Generally this rule means that if you turn the grid upside-down the pattern will look the same as it does right side up.
2. Do not use too many black squares. In the old days of puzzles, black squares were not allowed to occupy more than 16% of a grid.
3. In addition, there is a tendency for crosswords to have odd-valued sides, with 15" x 15" (not 14" x 16") being the predominate size for daily crosswords.

▲ MATHEMATICAL GAMES CORNER

Strategy games are particularly suitable as points of departure for investigations because players invariably try to discover a winning strategy and usually the best way to do this is to analyze the outcomes of the series of moves.

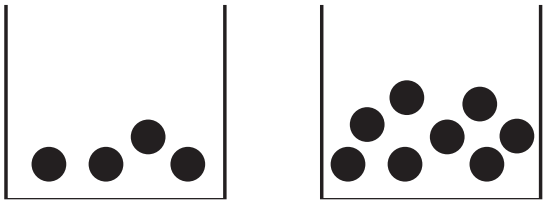
Here we shall look at a range of strategy games that can be investigated by the students. You will want to have them write a report at the end.

Last Counter

This is a game for two players

You will need twelve counters.

Put four counters into one container and eight in the other container.



Each player can take counters in one of two ways:

1. By taking any number they like from just one jar or
2. By taking the same amount from both jars.

The winner is the person who takes the last counter. Devise a winning strategy for the game.

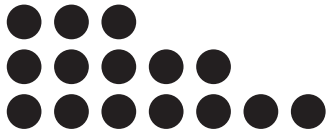
What if:

a) The winner is the person who does not take the last counter?

b) What if you were to change the number of counters, for example, 1 5 counters.

NIM #1

Arrange 1 5 counters into 3 groups of 3, 5 and 7.



Players take turns at taking out counters from these groups.

A player may take as many counters (but at least one) as he/she selects, but all the counters taken must be from the same group.

The winner is the player who takes the last counter (or counters).

What if the winner is the player who does not select the last counter (or counters)?

NIM #2

Make a pile of seven counters.



Two players each take turns to remove either one or two counters from the pile. The pile left with last counter is the loser.

Play the games several times and search for a winning strategy. Tasks,

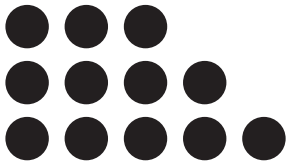
1. Who has the advantage, the first or second player?
2. What if you were to change one of the rules to read, “The player with the last counter is the winner?”
3. What if you were to use six counters, eight counters, nine counters, ten counters, ...? Investigate

Now is the time to ask the “What jf?” questions.

▲ MATHEMATICAL GAMES CORNER

Variations

Make a row of 3 counters, a row of 4 and a row of 5.



Two players each take turns to remove any number of counters from a particular row.

The player left with the last counter is the loser or, the player with the last counter is the winner.

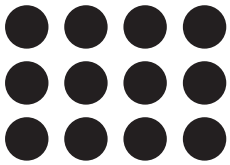
Investigate both situations.

Try changing the number of counters in each row.

What if there were four rows, five rows,...?

BRACHET'S GAME

Put several counters down to make a single group.

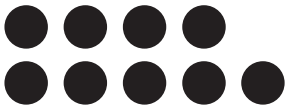


Players alternately take 1, 2 or 3 counters from the group.

The winner is the player who takes the last counter (or counters) or, the winner is the player who does not select the last counter (or counters).

TSYANSHIDZI

Put several counters down to form two groups of different sizes (say a 4 and a 5).



Each player in turn may make one of the following moves:

1. Take any number of counters from one group, or
2. Take the same number of counters from both groups. The winner is the player who takes the last counter (or counters).

SKITTLES

Arrange several counters in a straight line with each one touching (or assumed to be touching) counters on either side.



In their turns, players may take out 1 or 2 counters from anywhere in the line, but 2 may only be taken provided they are 'touching' each other.

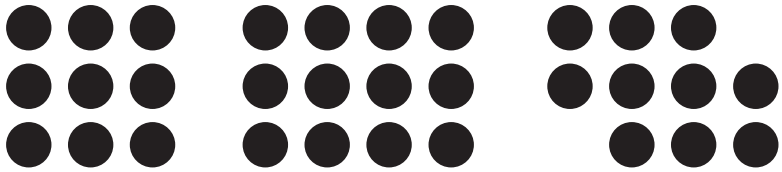
The winner is the player who takes the last counter (or counters).

▲ MATHEMATICAL GAMES CORNER

GRUNDY'S GAME

Arrange several counters into groups.

There can be any number of groups (from 1 upwards) and any number of counters (greater than 3) in each group, but 3 groups with between 9 and 1 3 counters in each group make a reasonable starting position.

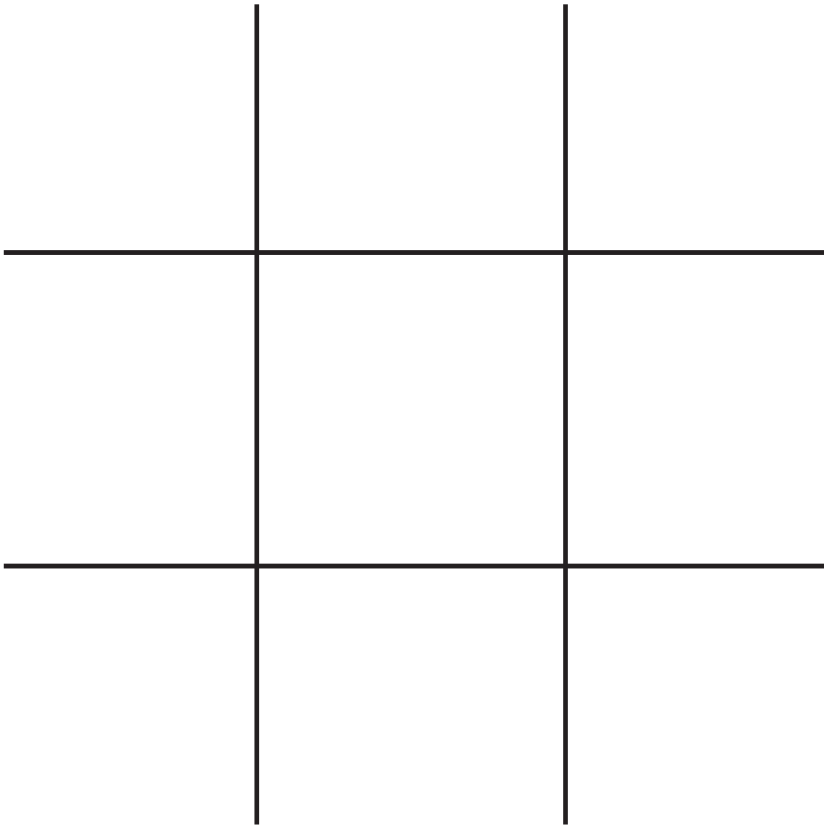


Each player in turn has to divide any one group into 2 groups of different sizes (so a group of 1 or 2 counters cannot be divided).

The winner is the last player who is able to make an allowed move.

QUOX

Played on a 3 by 3 grid, with one player putting in 0's and the other player putting in X's in turns.



In any one turn, players may put their marks in as many cells as they like, provided that all the cells used are in the same straight line vertically or horizontally but not diagonally.

The cells do no have to be 'next to' each other.

The player who makes the last mark (or marks) wins the game.

▲ LITERATURE CORNER

Literature is a wonderful way to introduce mathematics topics to all ages of learners. There are many books that are specifically written to accompany mathematical units.

A Domino Set

A set of dominoes usually consists of 28 rectangular dominoes, each having two squares with 0, 1, 2, 3, 4, 5, or 6 spots.

Every combination is represented. The value of the domino is the sum of the values of the two squares.

Domino Puzzles

by David Fielker

Trevor, please provide image file for the book cover P

'Domino Puzzles" is bursting with rich investigations each requiring no more than a box of ordinary dominoes. Although this book is aimed at age 1 1 upwards, this book has potential for all our students.

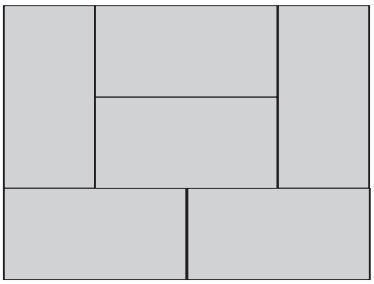
It is written in an informal style, it is perfect for students to work through on their own. Each activity is introduced very clearly and the diagrams are very helpful. The questions are posed in stages, leading students through the mathematical thinking.

Early algebraic concepts and problem solving strategies can readily be developed from the activities.

To order a copy of either book, or for additional information, please call the BEAM orderline on 020 7684 3330.

▲ RECREATIONAL MATHEMATICS

1. Use dominoes to make rectangles. Make a rectangle with 2, 3, 4, 5, 6,...dominoes. What do you notice?



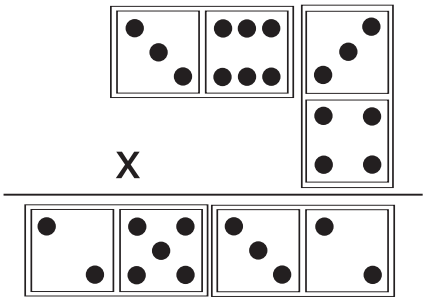
2. John has a box containing domino pieces but he does not think it is a complete set.

   He has 24 dominoes in his box and there are 125 spots on them altogether. Which of his domino pieces are missing?

   Is the solution unique?

3. Here you have four dominoes laid out in the pattern of a multiplication.

   Can you make multiplication products like this using all 28 dominoes?

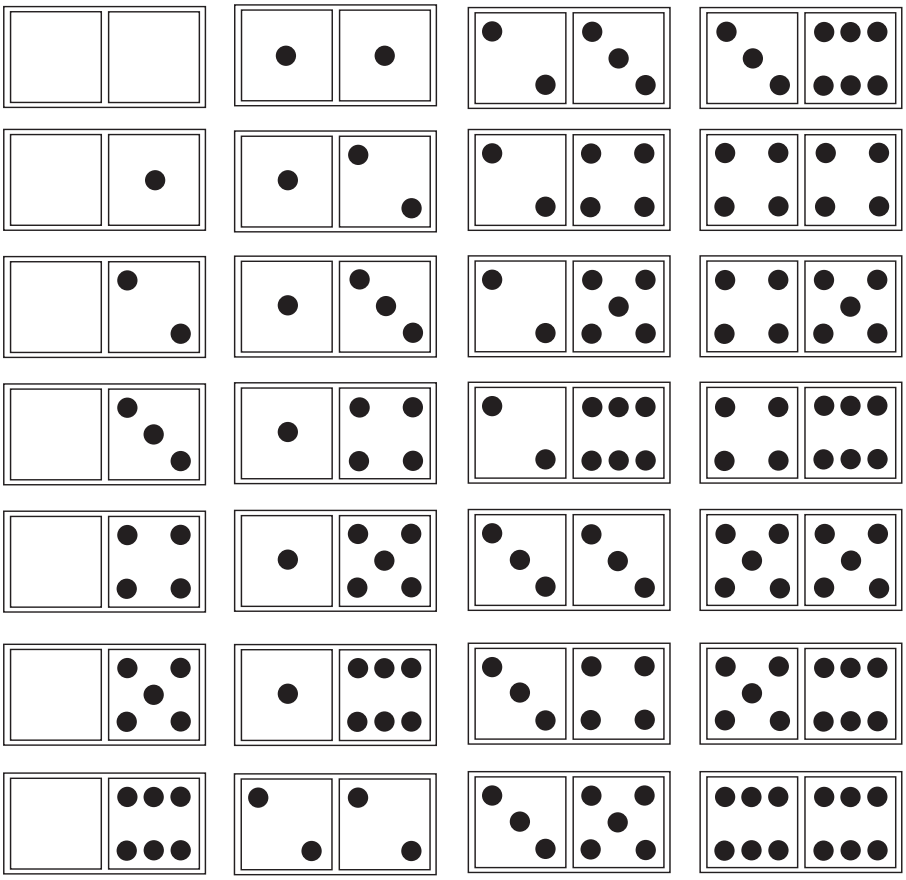


4. There are 28 dominoes in the standard domino set.

   If the domino set ended with a double seven rather than a double six, how many dominoes would there be in such a set? What if the highest domino was a double 8? double 9?...?

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5. How many spots are there in a standard set of dominoes?
   Try to determine the number of spots without physically counting them.



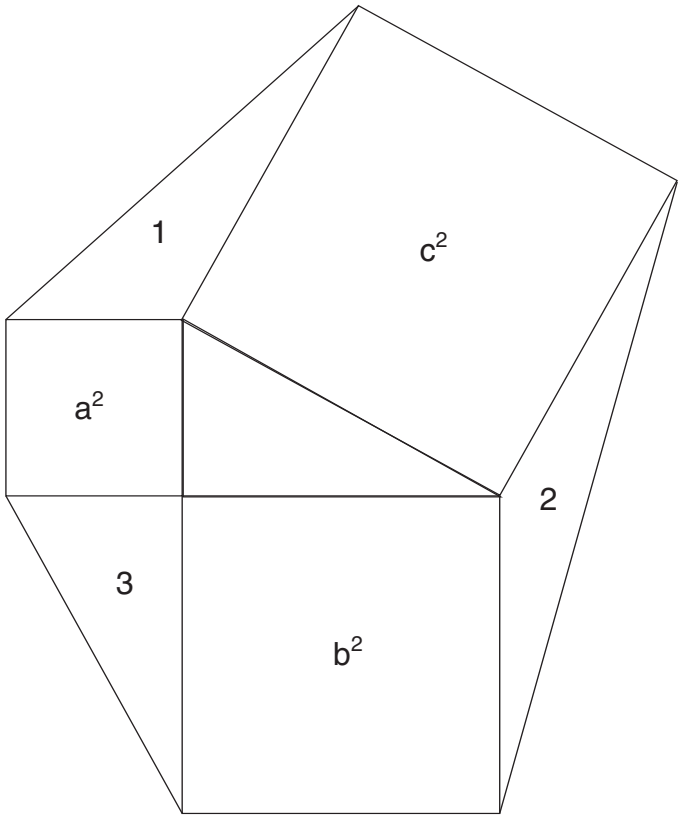
▲ TECHNOLOGY CORNER

The theorem of Pythagoras states that the area of the square built on the hypotenuse c of a right triangle is equal to the combined area of the squares built on the sides a and b of the triangle.

That is, $A^2 + B^2 = C^2$

Determine the areas of 1, 2 and 3 and compare these to the area of the original triangle.

What do you notice?



Maximum Area

A triangle is inside a circle. The circle's center is one vertex of the triangle, and two of the circle's radii represent the legs of the triangle.

What central angle is necessary to provide a triangle with maximum area? With minimum area?

