ABACUS

"Music is the pleasure the human soul experiences from counting without being aware it is counting." Leibnitz

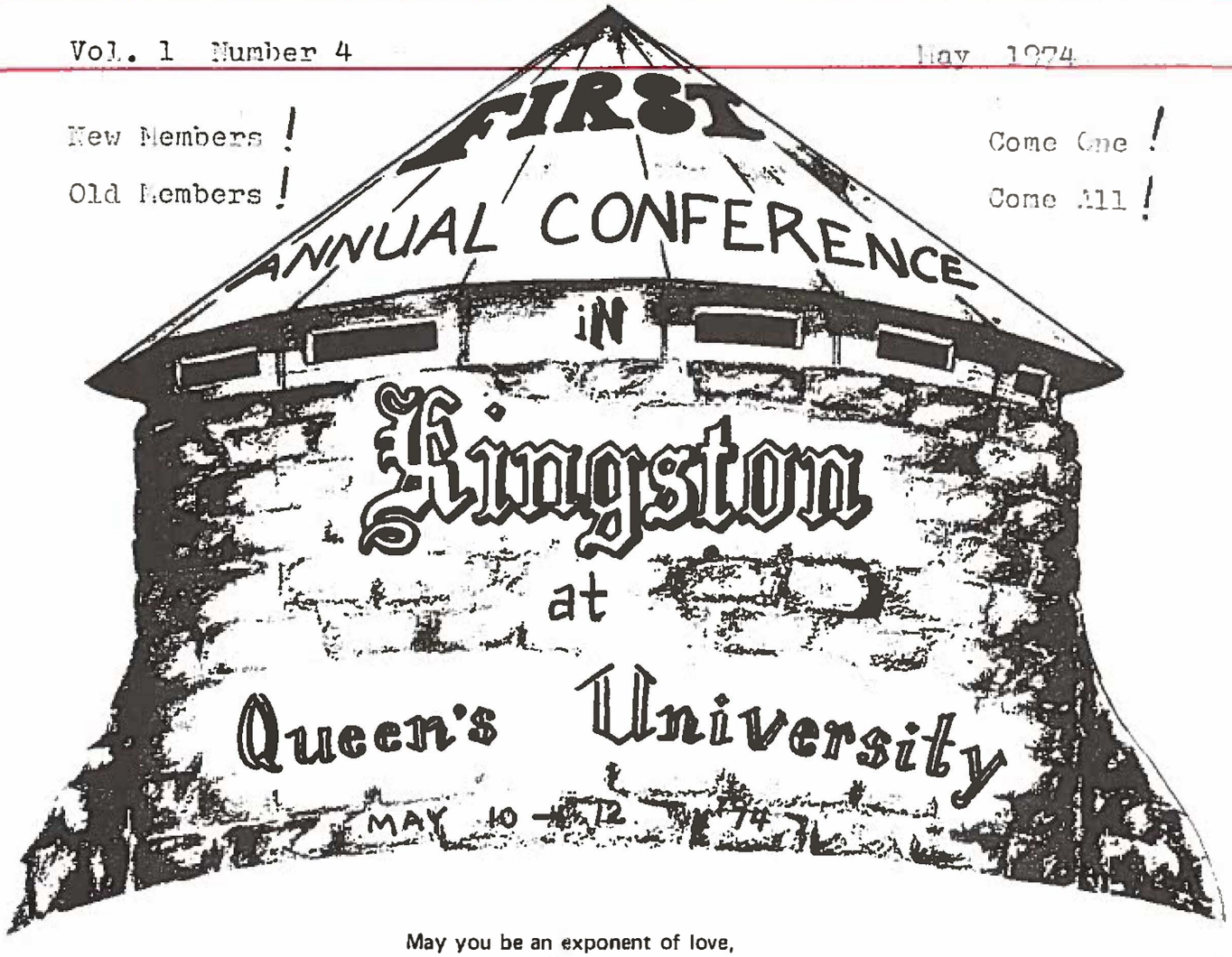
OAME

A PUBLICATION OF THE
ONTARIO ASSOCIATION for MATHEMATICS EDUCATION

Vol. 1 Number 4 May 1974

New Members !
Old Members !

Come One !
Come All !



May you be an exponent of love,
Your friends distribute over the earth,
Your success equivalent to your happiness;
Your gifts greater than your expectations,
And peace be a subset of the universe.

YOUR MEMBERSHIP IN OAME INCLUDES
THE ABACUS YOU CAN COUNT ON IT!
WRITE TO THE SECRETARY TREASURER

Plan now for London in '75

1973 - 74 EXECUTIVE

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FROM THE EXECUTIVE OF OAME

Welcome to this first birthday of the Ontario Association for Mathematics Education. It is just one year ago now that the Ontario Association of Teachers of Mathematics and members of the Ontario Mathematics Commission joined forces in OAME.

It has been a good year and with your help we may look forward to many good years ahead.



NOTICE OF MOTION

The executive of the Ontario Association for Mathematics Education hereby publish the following notice of motion to be duly presented at the Annual Meeting of OAME on May 12, 1974 in the city of Kingston.

ARTICLE VIII - ELECTIONS

1. Elections shall be held at each annual general meeting for the positions of Elected Council Members.

2. (a) Elections of Vice-President (the President-elect)
   Executive Members
   Secretary-Treasurer (if not a paid employee of the organization)
   shall be by mailed ballot.

   (b) Ballots shall be mailed by the Nominating Committee to all members five weeks prior to the annual general meeting. Ballots shall be counted by scrutineers appointed by the Nominating Committee twoweeks prior to the annual general meeting.

3. If the Secretary Treasurer is a paid employee of the organization the appointment is to be ratified at the annual general meeting.

SEND YOUR MATH GAME, puzzles, class activities, book reviews, ideas to:

The ABACUS EDITOR
c/o CITY MATH CENTRE
C.C. CARROTHERS SCHOOL
360 CHIPPENDALE CRESCENT
LONDON, ONTARIO

ABACUS ITEMS MAY BE REPRINTED WITHOUT FORMALITY EXCEPT CUSTOMARY ACKNOWLEDGEMENTS

The following recommendations were given almost unanimous support by the OAME Council and have been distributed on behalf of OAME to all Directors of Education in the province of Ontario and to the Program Consultants of Mathematics.

RECOMMENDATIONS OF THE COUNCIL OF THE ONTARIO ASSOCIATION FOR MATHEMATICS EDUCATION

RE

INTERMEDIATE DIVISION CYCLIC REVIEW MATHEMATICS SECTION

1. Mathematics should be taught as a distinct discipline. Its applications and inter-relationships with other studies should be stressed but mathematical concepts and skills should be studied as such.

2. The importance of competence in performing basic arithmetic operations should be recognized by providing appropriate time allowance in line with the student's needs.

   This competence should be maintained as the students progress through the curriculum by providing opportunities for numerical work at all levels.

3. The more common algebraic manipulations should be stressed, practiced and mastered in order to establish the student's competence in handling a variety of expressions involving symbols.

4. Students should be competent in the recognition and use of basic geometric and measurement concepts. Opportunity should be given for the application of arithmetic and algebra to these concepts.

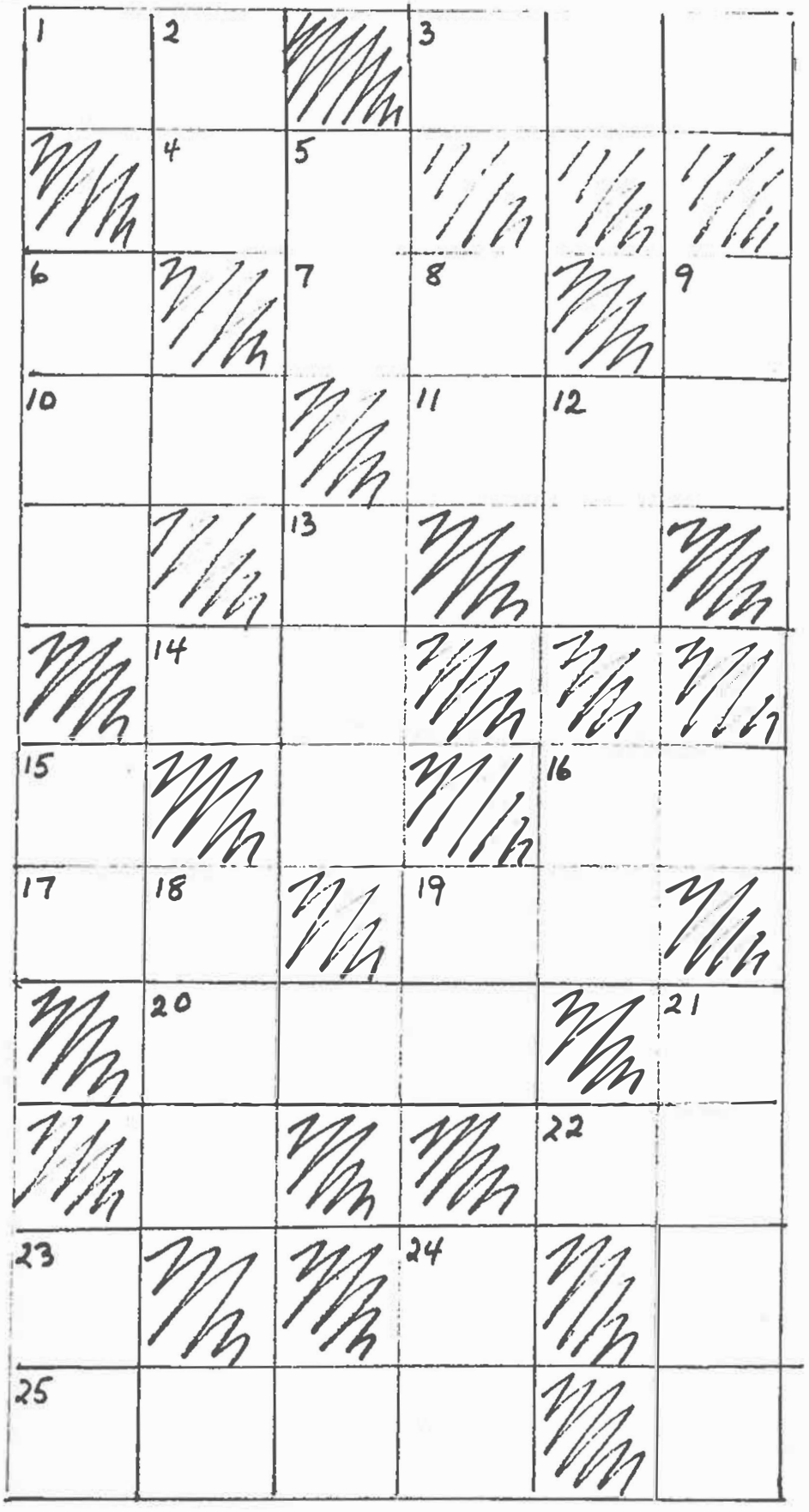
5. The guidelines should clearly indicate the sequential development of these mathematical skills and should recognize the variations in student ability and maturity.

6. Students should be given the opportunity to develop mastery in certain fields of mathematics rather than only a brief exposure to a wide range of topics. A proliferation of topics which could prevent skill development should be avoided.

7. Guidelines should be established which outline an essential core of topics and suitable additional topics for expansion and enrichment. This core content should include some topics which illustrate both structure of mathematics and mathematical reasoning.

8. The study of mathematics, appropriate to the student's program, should be required at least to the level of compulsory school attendance.

CROSS NUMBER PUZZLE



Across

1. 9 x 9
3. 1 century = _____ years
4. $\frac{17}{20} = \frac{\quad}{100}$
7. 4 feet = _____ inches
10. largest 2 digit numeral
11. 25 x 25
14. 4 x 16
16. 7 x 8
17. 7 x 4
19. 13 x 3
20. 79 x 10
22. 9 x 5
25. hour = _____ seconds

Down

2. 22 - 4
5. 6 x 9
6. 16 dozen
8. (12x7) + 2
9. 11 x 5
12. 6 gallons = ___ quarts
13. a dozen dozen
15. 26 x 2
16. 73 - 14
18. 35 x 25
19. 1/2 min. = ____ seconds
21. 73 x 49
23. 104 ÷ 8
24. 10,000 ÷ 1,000

AFFINE GEOMETRY

by

Kenneth Cummins

The underlying idea of "transformational geometry" or "geometry through transformations" is that different geometries are characterized by what properties remain invariant under different kinds transformations. The properties which remain invariant under rigid motion transformations in the plane {translation and rotation} make up the geometry of Euclid. A more recent approach to this geometry has been through transformations known as isometries which are length- and angle-preserving but which include translation, rotation and reflection. It can be shown that these three transformations can be based on reflections only. The isometries as a class preserve length of line segment, angle-size, parallelism, midpointness, cross-ratios and, indeed, all geometric properties except orientation. Although there are formulae which can express isometries there exist postulate systems, as we all know, from which can be deduced the same geometry as that which arises as the set of properties invariant under transformations which are isometries.

$$T: \begin{cases} \bar{x} = ax + by + c \\ \bar{y} = dx + ey + f \end{cases}$$

$$\{ae - bd \neq 0\}$$

which is called an affine transformation is not length- or angle- preserving but it does preserve parallelism, ratio of lengths of segments, equality of lengths of line segments if they are parallel and incidence properties of points and lines as well as other projective and topological properties. Postulationally affine geometry is a consequence of primitive statements on incidence and the Playfair postulate on parallels-- that through a given point P not on line L there can be drawn one and only one line parallel to L.

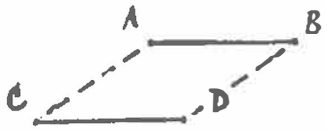
Intuition is the conception of an attentive mind, so clear, so distinct, and so effortless that we cannot doubt what we have so conceived.

- Descartes

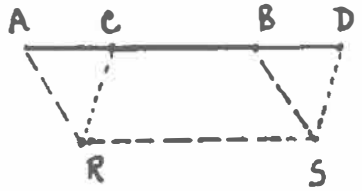
* *

It is interesting to note that many theorems in high school geometry are really affine theorems and hence we should be able to prove them in an affine postulate framework. For example, in proving the well-known theorem that the diagonals of a parallelogram intersect at their midpoints we use {as did Euclid} the congruence of angles and segments which properties belong to euclidean geometry. Since this theorem concerns only the affine properties of parallelism and midpointness it should be possible to prove it with affine methods. Herein lies a challenge.

In affine geometry we construct and define a very useful concept-- that of parallel congruence. The segments AB and CD are defined to be parallel congruent {A, B = C,D} if AB ∥ CD and AC ∥ BD.



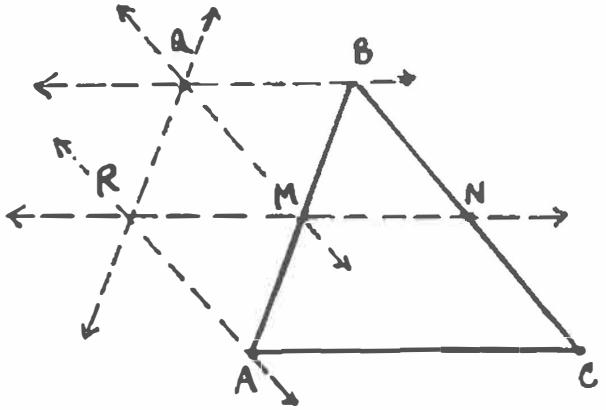
If the points A,B,C,D are collinear then A,B ∥ C,D is defined to mean that there exists a segment RS such that AB ∥ RS, AR ∥ BS, CD ∥ RS and CR ∥ DS. It follows immediately that if A,B ∥ C,D then A,C ∥ B,D; also parallel congruence is either proved or taken to be an equivalence relation. Too, we define P to be the midpoint of AB if A,P ∥ P, B.



Let us now use these ideas in proving that if a line id drawn through the midpoint of one side of a triangle parallel to another side then it bisects the third side.

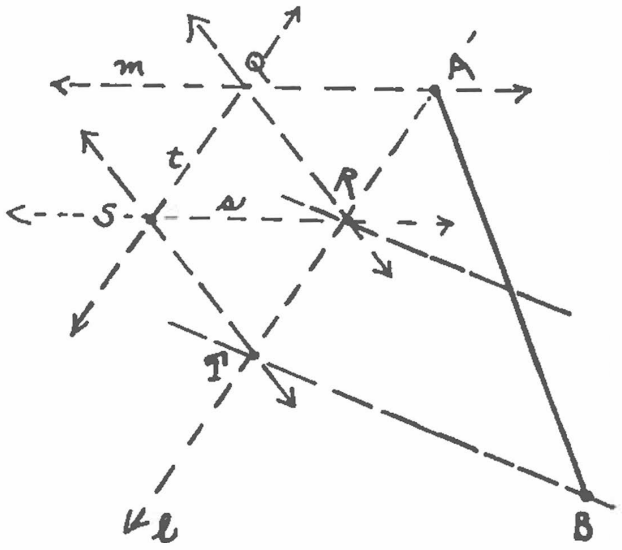
P. Let M be the midpoint of AB and let the line through M parallel to AC intersect BC at N. Through M and B draw lines parallel to BC and MN respectively to get point Q. Through Q draw a line

parallel to AB which meets MN extended in R. Draw AR. {The above steps are based on the Playfair postulate and on incidence postulates.} Now by definition of parallel congruence Q,R B,M and by definition of midpoint M B,M M,A and hence Q,R M,A. Now Q,M B,N and therefore RA BN NC and therefore R,A N,C and B,N N,C and hence N is the midpoint of BC. {The reader might devise an easier way to prove this.} Q.E.D.



The proof of the above theorem suggests a way to construct the midpoint of any given line segment by a method other than constructing a parallelogram with the given line segment as a diagonal. By repeated use of Playfair's postulate and the concept of parallel congruence we can do the following: Let AB be the segment to be bisected. Through A draw lines L and M as shown. On M take a point Q and through Q draw a line to meet L at, say, R. At R draw a line S parallel to M. Through Q a line T parallel to L and get point S. Through S draw a line parallel to QR and get point T. R is the midpoint of AT. Now draw TB and through R draw a line parallel to TB and where this intersects AB will be the midpoint of AB. This method can be extended to trisect and "n"-isect line segments. Incidentally all this can be done with the parallel ruler-- the tool of affine geometry.

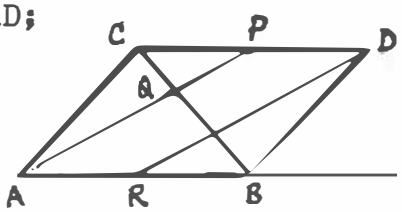
The reader might like to consult Chapter 9 of Prenowitz and Jordan's Basic Concepts of Geometry {Waltham, Mass.: Blaisdell Publishing Company, 1965} for basic ideas in affine geometry.



It might be interesting now to attempt to prove other affine theorems with the method demonstrated above. Some suggestions, other than the one not proved above, are:

{a} if in the parallelogram shown below P is the midpoint then Q trisects BC and also AP;

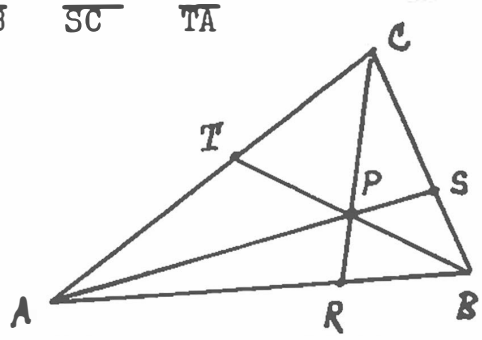
{b} if P and R are midpoints then AP RD;



{c} the medians of a triangle are concurrent;

{d} the theorem of Ceva which says that

$$\frac{AR}{RB} \cdot \frac{BS}{SC} \cdot \frac{CP}{TA} = 1$$



We see that Euclid did not need all his postulates for some of his very important theorems since part of his structure belongs to the less demanding affine geometry. It hence becomes a very fascinating challenge for students and teachers alike to try to prove these theorems with less strong postulate systems.

PATTERNS & PUZZLES

Young Lady to her Lover

I ask you, sir, to plant a grove
To show that I'm your lady love.
This grove, though small, must be composed
Of 25 trees in 12 straight rows
In each row, five trees you must place
Or you shall never see my face.

1. Three men stopped at a hotel one night. They took three separate rooms. The rate for the rooms was $10.00 each so the men paid a total of $30.00. The next day the owner of the hotel found that the rooms should have been rented for $25.00 and not the $30.00 that was paid so he gave a refund to the bellboy to take to the three men.

The bellboy not being too honest gave each man a $1.00 refund and kept $2.00 for himself.

After receiving the $1.00 refund, each man only paid $9.00 for his room - that made $27.00 for all three rooms. The bellboy kept the other $2.00 which totals $29.00. Where did the other dollar go?

2. A lady looked in her purse and noticed that she had 6 bills that totaled $63.00 - none of the bills were $1.00 bills. Can you figure out what the denomination of the 6 bills were?

$2x + 5y + 10z = 630$
$x + y + z = 6$

3. A man had two rare dogs. He sold one of them for $198.00 and made a 10% profit. The next day he sold the other for $198.00 but this time he took a 10% loss. On the two deals combined, did he show a profit or a loss?

Use the digits of 1974 (1, 9, 7 and 4 -- in that order). Use the arithmetic operations of addition, subtraction, multiplication and division. Use any mathematical processes like square root, exponent, place notation, factorial, greatest integer, etc.

Problem: Write the numerals 1 through 74

To get you started, the representation for the first ten numerals are:

1= $-1 -9 +7 +4$
2= $1 (-9 +7 +4)$
3= $(19-7) / 4$
4= $1 (9 + 7) /4$
5= $-1 + \sqrt{9} +7 -4$
6= $1 \cdot (9-7+4)$
7= $1 + 9 - 7 + 4$
8= $19-7-4$
9= $19+7 \cdot 4$
10= $\sqrt{-1+97+4}$

If the requirement that the digits be used in order is removed then it is possible to write the numerals differently

9= $4 (9-7) +1$
25= $41-7-9$

Angle θ	sine θ	
0°	$\frac{\sqrt{0}}{2} = 0$	90°
30°	$\frac{\sqrt{1}}{2} = \frac{1}{2}$	60°
45°	$\frac{\sqrt{2}}{2}$	45°
60°	$\frac{\sqrt{3}}{2}$	30°
90°	$\frac{\sqrt{4}}{2} = 1$	0°
	cosine θ	Angle θ

CHECKING THE WORLD WITH METRICS

Did you know that this page measures 28 cm. by 21½ cm.? Try measuring it for yourself.



What two whole numbers, neither containing any zeros, have a product exactly 1 000 000 000. ?

(Note: commas no longer separate in 3's)

CREATIVITY IN MATHEMATICS – GRADES 7-10

WENDALL C. HALL
North Eugene High School, Eugene, Oregon

Dr. Polya has said that the best teaching methods are those experienced in a math class. I would endorse that after my own experiences. Certainly "Mathematics is not a spectator sport!" To teach creativity you should engage in creative experiences.

Cut out a square, divide it into nine congruent squares, and number as shown. Cut on the solid lines (between 1-2 and 4-5 squares and between the 5-6 and 8-9 squares). Fold to form a cube. Can you do it more than one way? What is your time on each attempt? Can you write instructions on how to fold the pieces? How else might this square or other figures be cut to do a similar problem?

1	2	3
4	5	6
7	8	9

After you have tried this, let students try it. You should have additional insights about students' problems.

This activity is perhaps more of a problem-solving type than creative. But the solution patterns should convince students that there is more than "one way", and *that* is a prerequisite for trying new ways.

Creativity seems to involve these factors: activity, a willingness to do things differently, plenty of background via prior experience, and opportunity with adequate time. (Who can "create" in ten minutes or even one period?) And the creative spirit should permeate classroom activities and not be merely a word in the plan-book.

What can be done to create a different learning atmosphere? Use open ended assignments and challenge problems often. Don't just present material and talk; rather learn inquiry techniques so you can ask better questions which motivate further student activity. Answer student responses when possible as Dr. Maletsky suggests, with either "Yes" or "Yes, but . . . ", the latter rather than "No" or "Not correct". Avoid single answer questions on assignments when feasible. Ask students to observe, then explain. Give as much credit for solving a problem three different ways as three problems each one way!

These are some activities which I have used in Junior High and high school classes.

I. What is the ratio of black triangles to total triangles in each figure? Record your data. What is that of white to total? (Let students note the sum of the two ratios!) Graph these ratios on a cumulative basis. What do you predict on the graph of the white ratio? Could you do a similar study of figures other than triangles? (Note: This study could extend over units on ratios, percentages and graphing or be a later inclusive review.)



II. Study patterns and find applications of Pascal's triangle. Can you create other triangles based on a pattern somewhat like this? This relates very well to a probability unit or to a geometry unit and provides interesting graph. ("Graph the numbers in the second diagonal." Students will find graphing other diagonals interesting too/)

```
        1
      1   1
    1   2   1
  1   3   3   1
1   4   6   4   1
```

IDEAS AND MANIPULATIVES YOU CAN TRY

Solution Card Game

contributed by Al Ferenz

Objective: To learn to solve single algebraic equations.

Materials: Felt Pen
#63 Round Corner Cards or
3" x 5" Blank Index Cards cut in half

Deck: One each of two parts.

Part One: Suggested Sentence Cards.

$X+X=X$	$7X-21$	$X^2-8X+16=0$
$X^3-1=0$	$4X+2=-6$	$X/2+3=1$
$X^2-4X+4=0$	$X/3+1=0$	$(X+3)(X-4)=0$
$X^2+6X+8=0$	$X^2-2X-3=0$	$5X\pm2=12$
$7X-4=3$	$X(X+1)=0$	

Part Two: Suggested Value Cards.

Two each of the following:

$X=0$	$X=3$	$X=-2$
$X=1$	$X=4$	$X=-3$
$X=2$	$X=-1$	$X=-4$

Players: Two (2)

Rules:

1. Shuffle Value cards and place face down in middle.
2. Shuffle Sentence cards and deal 7 to each player.
3. Players turn all Sentence cards face up on table.
4. First player turns over Value card and matches it with one or more Sentences for which it is a solution. (If no match, second player tries with same Value card.)
5. Turns alternate. Winner is the first to match all his Sentence cards with Value cards.



ANNOUNCEMENTS

- 8th Annual Mathematics Seminar for Vocational Schools AT YORK HUMBER HIGH SCHOOL (99 Humber Blvd.) 9 MAY,1974.

(Send your Chapter News, Region Reports for inclusion in ABACUS)

ARTICLE CREDITS

Lady to Lover - T. Asbury, Toronto
Creativity- from Washington Math Newsletter
Solution Card Game- from Arizona ATM Newsletter
Brain Drain - from VECTOR, B.C.Math Teachers
Affine Geometry- Ohio TM Newsletter
Other items from London Math Council

BRAIN DRAIN

(Total Twelve)

This is an individual game for mathematical practice of sums to 12.

OBJECT: To line up the numbers horizontally so that each row of four numbers totals 12.

Cut four strips of cardboard or other stiff paper, each strip consisting of eight numbers (see diagram below). Tape, glue or fasten each of the strips so that they form a circle. Place these circles over a cylinder of rolled up construction paper so that the numbers can move freely.

1	4	3	2	3	5	4	3
1	2	3	1	5	2	4	3
1	2	2	3	3	5	4	3
1	5	4	2	3	4	3	5

A THOUGHT FOR YOUR STUDENTS:

WHAT'S YOUR WORTH?

Think about this for a moment: You have a plain iron or steel bar worth about $5 to start with.

Made into horseshoes, it is worth $10.50.

Made into screwdrivers or kitchen cutlery, it may be worth $250.

Made into needles, it is worth $3,250.

Made into balance springs for watches, it is worth $250,000.

The same is true for another kind of material -- you.
Your value is determined by what you make of yourself!

ADD TO MAKE ONE*

(2-4 Players)

Preparation. A pair of dice--one red and one green--and paper and pencil for recording scores are all the equipment needed. Make an answer key so players can check answers during play.

Directions.

1. The player who has been chosen leader rolls the dice. The red die names a numerator while the green die names a denominator.
2. The first player names the fraction indicated by the dice. Then he names another fraction that can be added to it to give a sum of 1. If he is correct, he scores one point.
3. Play continues with each player taking his turns in order.
4. The winner is the player who has the most points after a given number of rounds have been played or a given period of time has passed.

* This game appeared in GAMES FOR INDIVIDUALIZING MATHEMATICS LEARNING, Charles E. Merrill Publishing Company. Written by Leonard M. Kennedy and Ruth L. Michon, the book is basically a collection of mathematics games organized by mathematics topics such as counting, place value, addition, integers and logic; and games within any topic are further organized by performance objective.

"The teaching of mathematics reaches its highest potential when, in the process, the student learns how to direct his own education in this field of knowledge."

THINK METRIC

THE OAME ABACUS

"MATHEMATICS EDUCATORS SHARE THEIR IDEAS"

SARNIA MAY 6 19?? ONT



Robert A. Tuck,