Volume 13
Number 5

Abacus

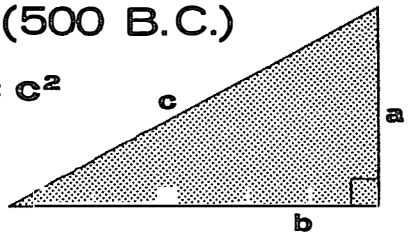
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July, 1986

A PROBLEM TO BE SOLVED

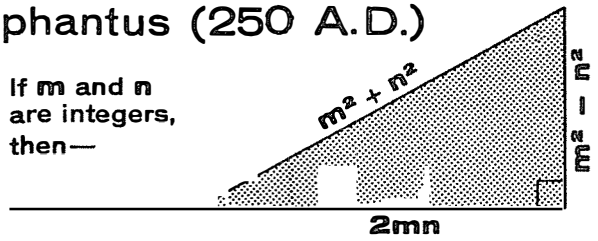
Pythagoras (500 B.C.)

$a^2 + b^2 = c^2$



Diophantus (250 A.D.)

If m and n are integers, then—



Fermat (1637)

Impossible if n greater than 2 (a, b, c, integers)

$c^n = a^n + b^n$

No proof yet discovered!

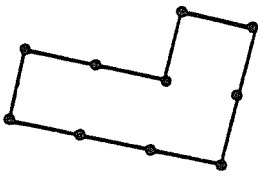
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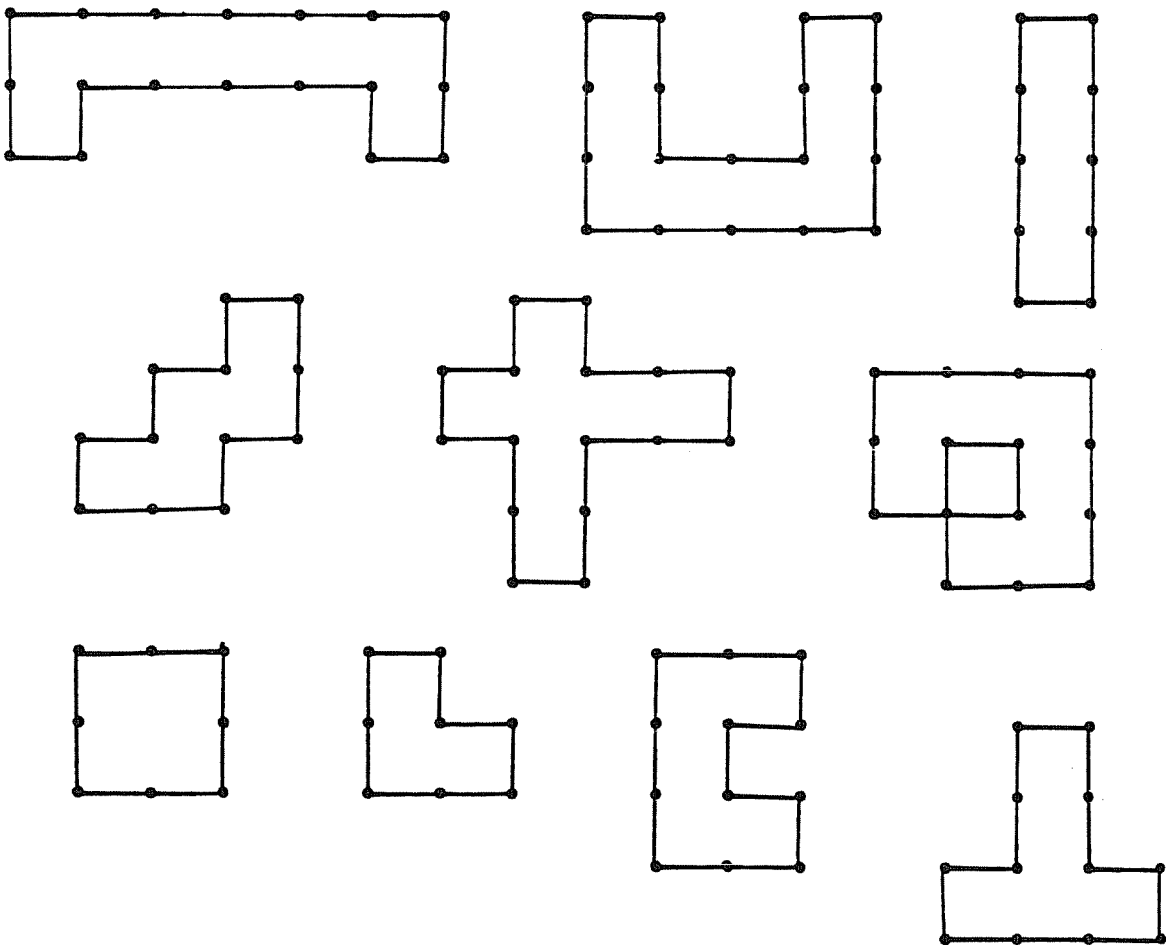
VISUALIZATION

The use of mirrors on diagrams provides a valuable lead into work on line symmetry. Marion Walter's delightful books for young children are the starting points for this activity.

Use a mirror on this ⟶



to make these:



FROM THE LIST

1. From this list 1, 2, 3, 4, 5, 6 make four choices and write them in the spaces below.
2. Use +, -, x, and ÷ signs and arrange your selections to indicate all the whole numbers 0 through 41.
3. In each example you must use all four of your choices once and only once.

FOUR CHOICES: _____, _____, _____, _____

0 =	21 =
1 =	22 =
2 =	23 =
3 =	24 =
4 =	25 =
5 =	26 =
6 =	27 =
7 =	28 =
8 =	29 =
9 =	30 =
10 =	31 =
11 =	32 =
12 =	33 =
13 =	34 =
14 =	35 =
15 =	36 =
16 =	37 =
17 =	38 =
18 =	39 =
19 =	40 =
20 =	41 =

Extensions?

MAKING A SYSTEMATIC LIST

Place some coins in a small container.
Divide the group/class into two teams.
Shake the container and announce to both teams the total value of the coins.
For example: "These coins total 51¢."
The teams take turns guessing what coins might be inside.
One point is scored by a team if it gives a new combination that equals the same value.
Three points are awarded to a team that identifies the coins exactly.

You may want to use a feedback form similar to that shown below so the students know what has already been guessed.

25¢	10¢	5¢	1¢
1	2	1	1
2			1
	4	2	1

Any extensions?

BELL RINGING

In bell ringing with three bells, there are six different orders in which they can be rung for example, (1, 2, 3), (2, 3, 1) etc.
The aim is to produce all six orders before repeating one of them.
No bell is allowed to move up or down more than one place from one order to the next (so (1, 2, 3) cannot be followed by (2, 3, 1)).

Investigate.

ESTIMATION

38 x 76

Round Up

40 x 80

3200 is an overestimate

So I'll adjust down:

3200^{-}

32 x 43

Round Down

30 x 40

1200 is an underestimate.

So, I'll adjust up:

1200^{+}

72 x 89

Down Up

70 x 90

You can't really tell.

So, I'll just say...

6300

46 x 65

Since they are both close to the middle, I'll round one up and one down:

3000

Up Down

50 x 60

Now try these:

a) 25 x 35 b) 37 x 48 c) 82 x 31 d) 52 x 47

e) 74 x 65 f) 92 x 87 g) 22 x 99 h) 31 x 71

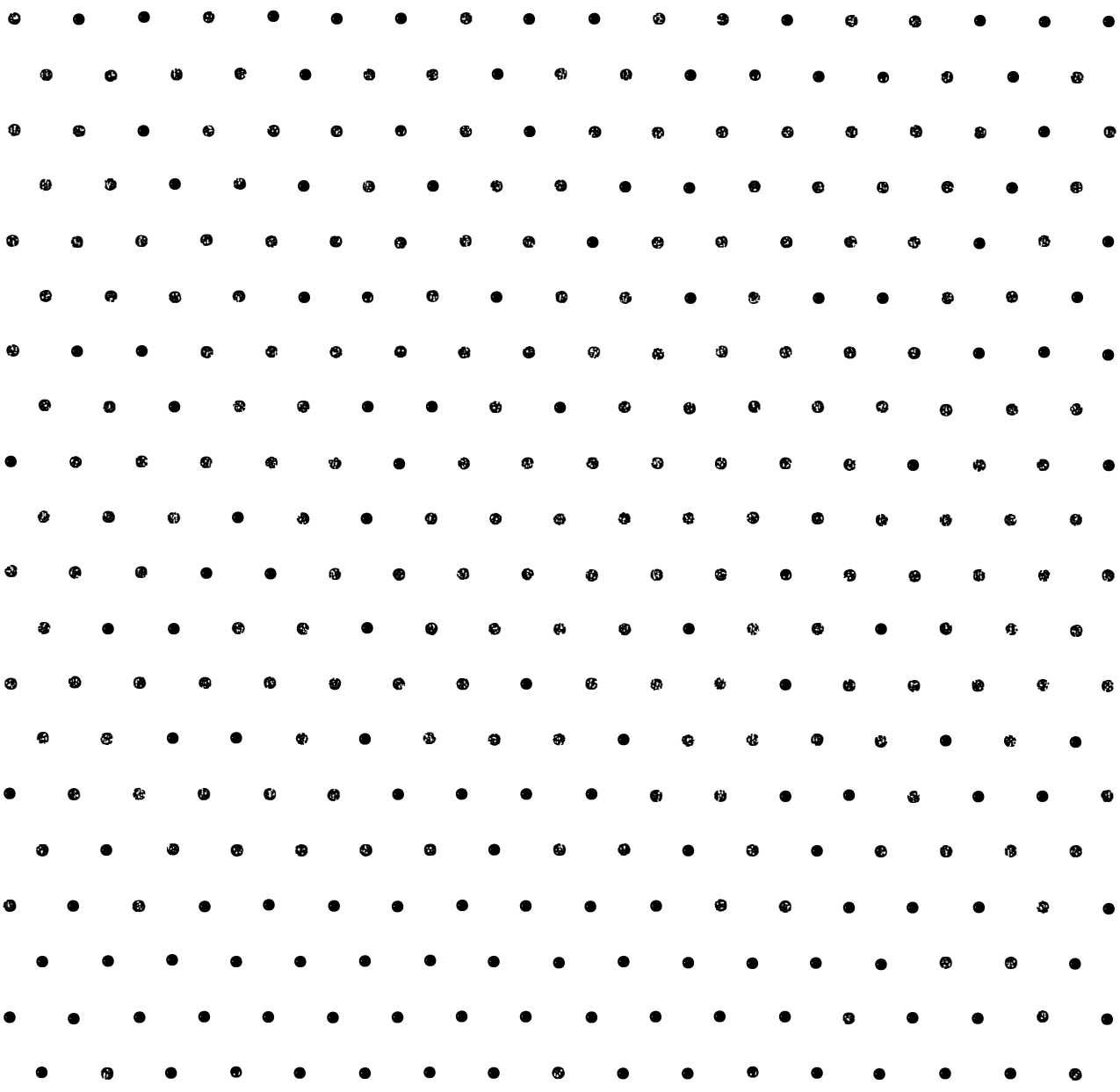
POLYGONS ON ISOMETRIC GRAPH PAPER

How many different triangles can you form by connecting three dots? four dots?

How many different quadrilaterals can you form by joining four dots? five dots?

Investigate for other polygons.

Set boundary conditions.



COMPUTER CORNER – Game of Life

This game, invented in 1970 by the British mathematician John Conway, is played by one person on a (theoretically) unbounded grid of squares. Each cell on the grid represents an organism which can, at any time, be either alive or dead. Which organisms are alive changes from generation to generation according to the number of neighbouring cells that are alive according to the following rules:

1. The neighbours of a given cell are the eight cells that are adjacent to it vertically, horizontally, or diagonally.

2. If a cell is alive but has either zero or one neighbouring cells alive, then in the next generation the cell dies of loneliness.

3. If a cell is alive and has four or more neighbouring cells also alive, then in the next generation the cell dies of overcrowding.

4. A living cell with either two or three living neighbours remains alive in the next generation.

5. If a cell is dead, then in the next generation it will become alive if it has exactly three neighbouring cells that are already alive.

6. All births and deaths take place at exactly the same time so that dying cells can help give birth to another but cannot prevent the death of others by reducing overcrowding nor can cells being born either preserve or kill cells living in the previous generation.

Write a program that will play the game of Life, on a bounded grid. Your program should take as input an initial configuration of living cells and the number of generations that you wish to observe. Output should be displayed on the screen of the computer.

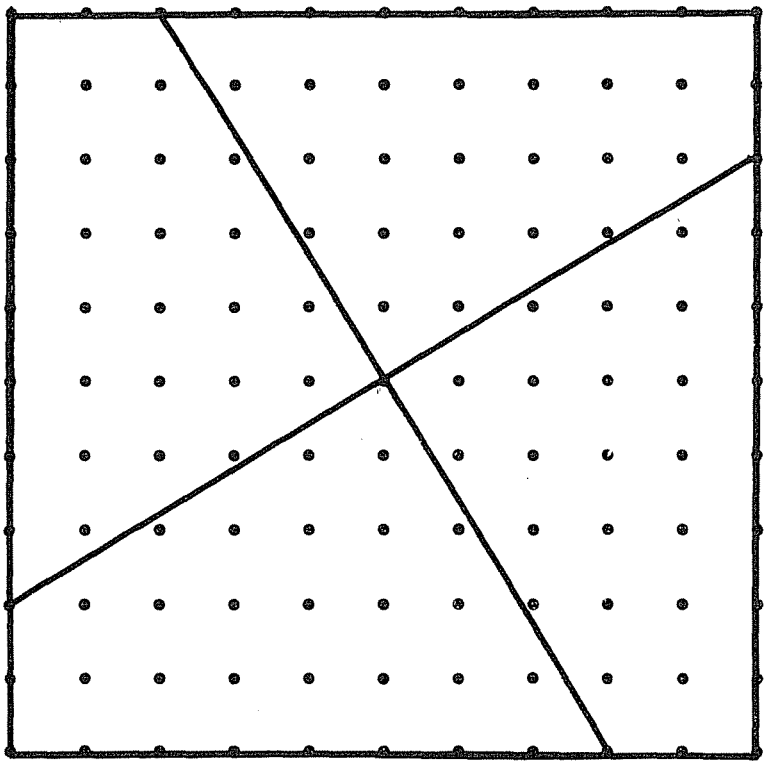
Programming Suggestions

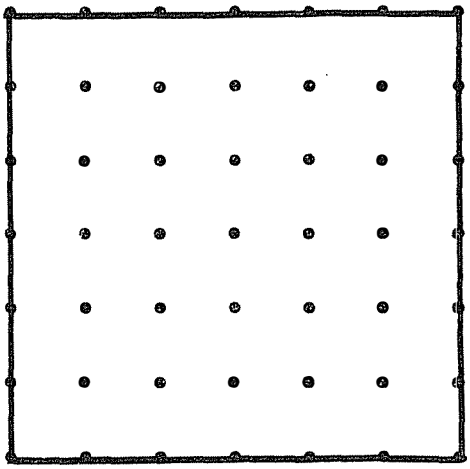
1. A grid of width 40 and height 20 should give enough room to display interesting patterns yet still be small enough for the computer to process the generations reasonably quickly. The array used for the grid should consist of characters with a blank indicating a dead cell and some special character (an asterisk, for example) indicating a live cell.

VISUALIZATION

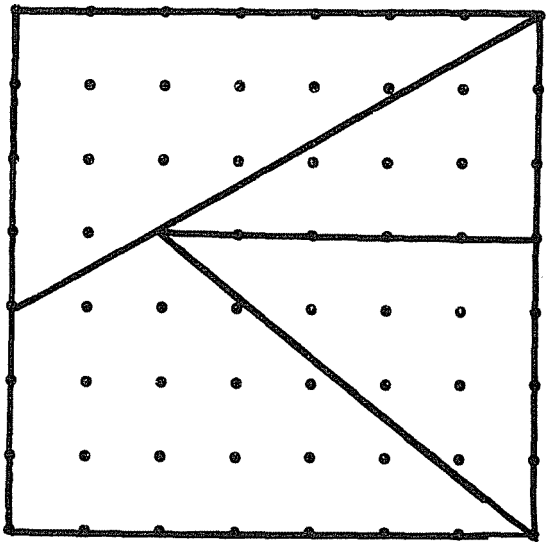
Cut out the ten-centimetre and the six-centimetre square shown below. Working clockwise, we mark a point on each side of the ten-centimetre square two centimetres from the corner. We then join the points on opposite sides.

Now cut along the lines and reassemble the four pieces and the six-centimetre square to make one large square.





Cut a square into four pieces of any shape, jumble the pieces up and ask another person to reassemble them to form the square.



KAPREKAR'S NUMBER

There is something very special about 6174. Select any four-digit number whose digits are not all equal and arrange the digits to form the greatest number possible, that is, put the digits in decreasing order. Then form the reverse and subtract the two numbers.

Continue this process of forming the greatest possible number from the difference and subtracting its reverse. The original number in the example below is 4218. The process ended in five steps with 6174.

Can you find a four-digit number whose digits are not all equal for which this process requires more than seven steps to produce 6174?

a) Write a computer program to help you investigate this question.

b) Use you program (in part a) to see if these are "special numbers" when this process is carried out for three-digit or five-digit numbers.

EXAMPLE

Number Selected: 4218

STEP	MAXIMUM NUMBER	REVERSE	DIFFERENCE
1	8421	1248	7173
2	7731	1377	6354
3	6543	3456	3087
4	8730	0378	8352
5	8532	2358	6174

Additional extensions?

POINTS OF DEPARTURE

Consider the transformation.

$$\frac{a}{b} \longrightarrow \frac{a + 2b}{a + b}$$ where a, b are whole numbers.

example:

$$\frac{1}{1} \rightarrow \frac{3}{2} \rightarrow \frac{7}{5} \rightarrow \frac{17}{12} \rightarrow \cdots$$

Investigate transformations of this kind.

Can you devise a strategy for tic-tac-toe that will ensure that you:

a) never lose?

b) always win?

O	O	X
	X	
		X

The rules of the game could be changed - for example, the players could be free to play an 'O' or a 'X' on each move and the first player to complete a line of three noughts or three crosses would be the winner.

Investigate the consequences of this and other changes in the rules.

Find the largest equilateral triangle within a regular hexagon, the largest square within a regular hexagon...

Investigate other such problems.

Extend to polygons within polyhedra, example, find the largest square and regular hexagon within a cube.

FROM THE DESK OF RAYMOND SMULLYAN: "To Mock A Mockingbird"

In a certain flower garden, each flower was either red, yellow, or blue, and all three colours were represented. A statistician once visited the garden and made the observation that whatever three flowers you picked, at least one of them was bound to be red. A second statistician visited the garden and made the observation that whatever three flowers you picked, at least one was bound to be yellow.

Two logic students heard about this and got into an argument. The first student said: "It therefore follows that whatever three flowers you pick, at least one is bound to be blue, doesn't it?"

The second student said: "Of course not!"

Which student was right, and why?

There is a certain club called the Barber's Club. The following facts are known about it.

FACT 1: Every member of the club has shaved at least one member.

FACT 2: No member has ever shaved himself.

FACT 3: No member has ever been shaved by more than one member.

FACT 4: There is one member who has never been shaved at all.

The number of members of this club has been kept a strict secret. One rumor has it that there are fewer than a thousand members. Another rumor has it that there are more than a thousand members

Which of the two rumors is correct?

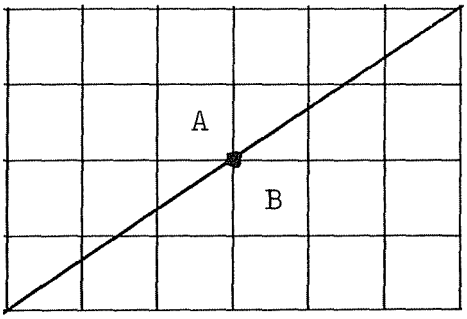
G.C.F. REVISITED

A rectangular floor has 94 tiles on one side and 118 tiles on the other side. How many tiles will be crossed by a diagonal from one corner of the floor to the opposite corner?

a) Understanding the Problem:

The diagonal of the square-tiled floor crosses a tile if it partitions the tile into two regions. In this 4 by 6 rectangle of tiles, the diagonal does not cross squares A and B.

How many squares does it cross?



b) Devising a Plan:

Simplifying and drawing pictures (using graph paper) is a natural way to approach the problem.

c) Carrying Out the Plan:

Form a table similar to the one shown below and record the number of squares crossed for simplified versions of the problem.

Use your observations to form a conjecture about the number of squares that will be crossed in larger rectangles.

Length

Width	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4						
2		2								
3										
4						8				
5										
6										

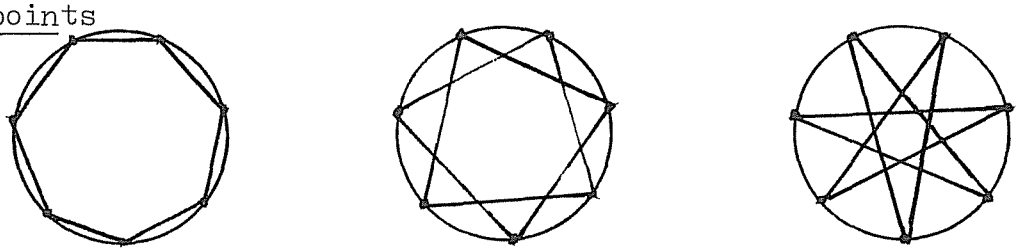
MATHEMATICIANS AT WORK

In 1809 Napoleon invaded Austria and put Europe in a turmoil. That same year, a French mathematician by the name of Louis Poinsot spent much of his time quietly doodling points and lines. While Europe was in flames, Poinsot calmly traced stars and circles with compass and ruler. Poinsot could not know that his idle speculations on the nature of polygons would eventually help electronic computers to solve complex problems.

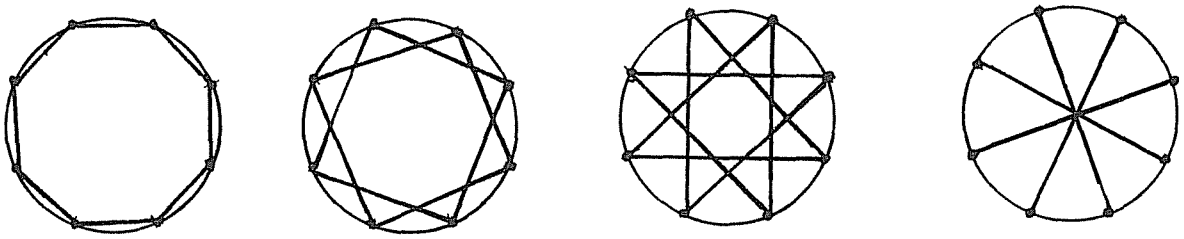
Let us look over Poinsot's shoulder as he draws his doodles. He begins by drawing a number of points at evenly spaced intervals around the circumference of a circle. He then joins the points with line segments. First he joins the points consecutively. Then he joins every other point, then every third point, and so on. Soon Poinsot's notebook is filled with stars and decorative designs of all shapes.

DOODLES

7 points



8 points



9 points ?

Generalizations?