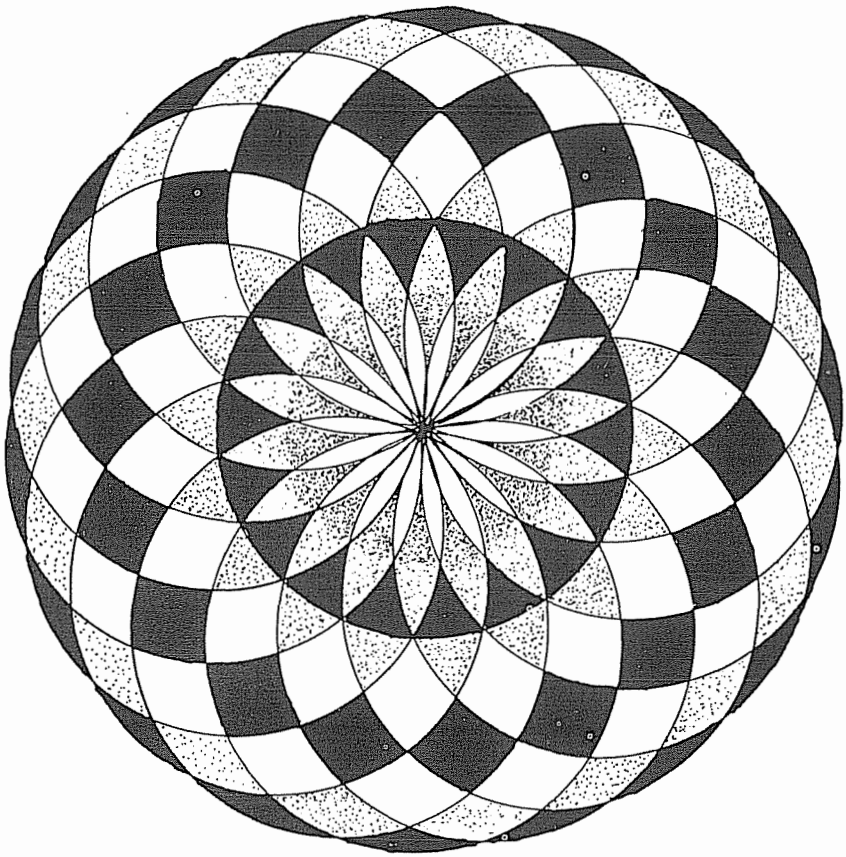
Volume II
Number 2

Abacus

A publication of the O.A.M.E.

December 1987



"Before LOGO"

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LARGE LAMINATED FLOOR GAME - SENIOR KINDERGARTEN

Barb Hunter
Robert Hopkins P.S.
Carleton Board of Education

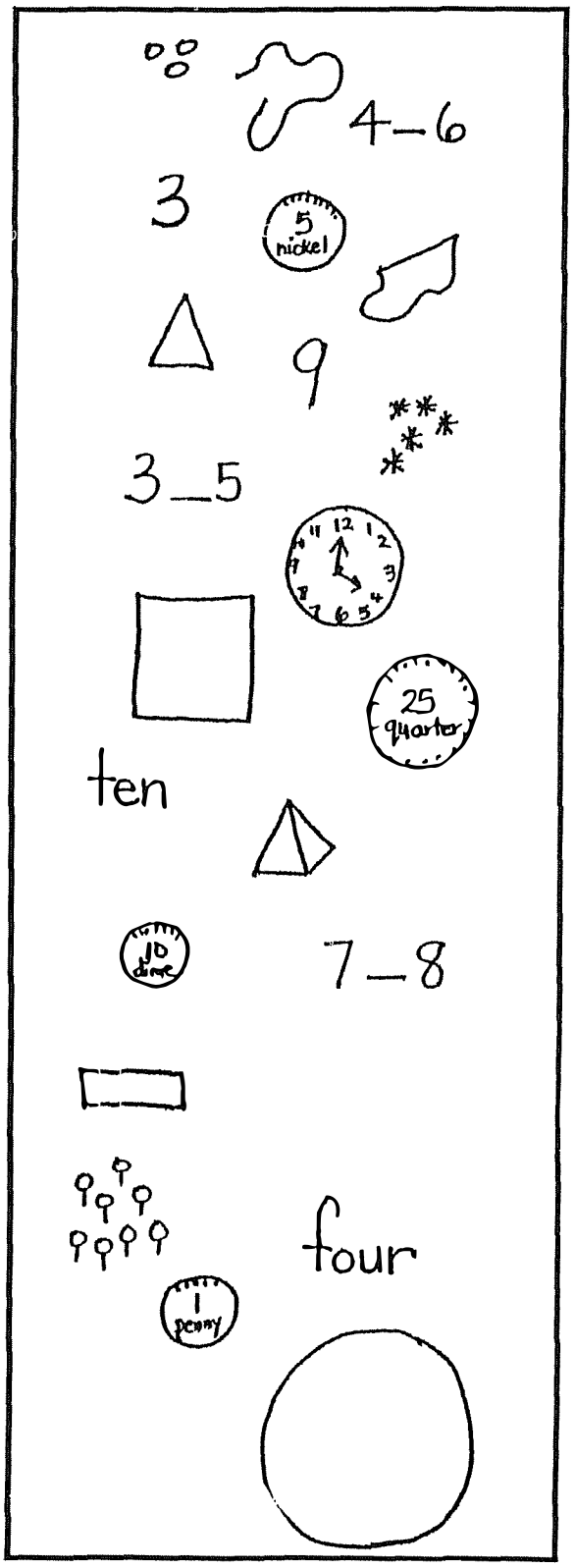
PURPOSE:

a game board to be used in unlimited ways for review of concepts previously presented in concrete stages.

Materials Needed:
- length of paper approximately 3 metres
- markers to colour

Draw on the Board:
- sets 1-10 (or more)
- numerals 1-10 (or more)
- sight words (one to ten)
- sequence of numbers
- solid and plane shapes
- clock with hour times
- money (coins)
- open/closed figures

**Make some objects large, some small.*



PLAYING:

<u>Teacher Directed</u>

a) Easy: - Teacher names object, child finds
- Child jumps from one object to another - naming
- One child describes, another finds
- Match a concrete example from your classroom - to the object
- Tactile - trace with your fingers.

b) More Challenging:
- comparisons (which is larger/smaller
- estimation - how many blocks needed to go across one end of the GAME BOARD
- how many erasers from the circle to the square
- measuring - standard/non standard
- clap as many times as the penny valuE of a dime
- which numeral shows the time on the ? clock

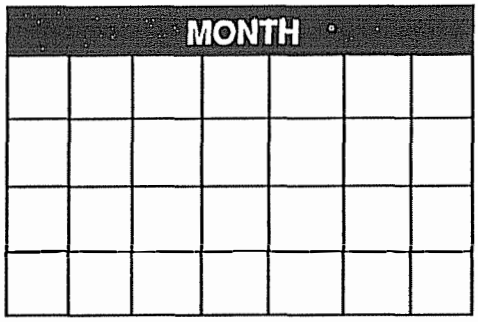
<u>Child Directed:</u>
- can play alone or with a friend (throw a bean bag - what did it land on?)

ACTIVITIES IN MATH USING THE CALENDAR:

Linda Loder
Area Resource Teacher
Ottawa Board of Education

MATERIALS

1. Large laminated "blank" calendar that can be written on
2. Small calendar stencils used as student activity sheets
3. Activity cards



SKILLS

1. Roman Numerals
2. Ordinal Numbers
3. Computional Skills

A FEW THINGS YOU CAN DO WITH THE CALENDAR:

1. On the laminated class calendar, put a "**Problem a Day** " in the squares which the class solves.

2. Activity Card Ideas.

Under each numeral in your calendar:
1. Print the ordinal words first, second, ...
2. Write the numerals in Roman Numerals.

Colour the days of the month
1. Use as few different colours as possible to colour each day.
2. No two days touching may be the same colour.

Cut Apart Your Calendar:
Then:
1. Make addition, subtraction multiplication and division problems.
2. See how many you can make.
3. Record your work on a sheet and file in your folder

1. Design math problems whose answers will equal days of the month - then have a friend try to match problems with the days:
E.g. 2 9 $2\overline{)8}$ 5
+3 -5 x3

2. Put each problem on a small card, then arrange in order 1-30 according to the answer.

1. Add the dates.
   a) Vertically
   b) Horizontally
   c) Diagonally
2. How many totals can you find?
3. What is your grand total?

1. How many squares can you draw in a given month?
2. Each section of each square must contain a day of the month. Your squares can be 2 x 2, 3 x 3, etc.
3. How many of each can you make? (Sections can overlap)

Calendar Game:
1. Start with today's date or any given date - the student follows the signal on the card and gives the date it would be

2. Some kids try to do all cards in a certain period of time.

1. Laminate a calendar sheet after mounting on construction paper.
2. Cut the daily squares in half diagonally and match correct at that time.
triangles to make squares.



Eg. of Cards:

1. Forward 2 weeks and 6 days.
2. Forward 2 months and 3 weeks.
3. Backward 6 months and 3 days.
4. Forward 5 weeks minus 4 days.
5. Backward 3 years, 7 months, 2 weeks, 3 days.
6. Backward 7 weeks and 2 days.

1. How many squares can you draw?
2. Use different coloured pencils or crayons to make your various squares

Magic Square

Douglas Biehler
Math Resource Teacher
Ottawa Board of Education.

Put the numbers from 1 to 9 in the box so that each row, column and diagonal add up to the same total. You must use each number once only, and you cannot repeat a number.

SOLUTION

8	1	6
3	5	7
4	9	2

Now look at the pattern of the numbers. To determine the pattern look at the numbers in consecutive order. Have the students develop the rules.

RULES

1. The number 1 is always in the middle of the top row.
2. Move up one space and one space to the right.
3. If you go outside the box, move to the end of that row or column.
4. If the box is taken, go to the space under the one you left. Corner box is the same as taken.
5. If you do it correctly, the last number will always be in the middle of the bottom row.

Now that you have developed the rules, you can do it for any square that is made from an odd number side matrix.

USING THE CALCULATOR

Dave Hunter, *Mathematics Consultant*
Ottawa Board of Education

The following sequence of exercises could be used as a centre for a grade 8 class. The general purpose is to illustrate how to integrate the use of the calculator into the measurement section of the course. The specific objectives are indicated at the top of the first exercise.
It is intended that two or three students would work through the exercises together, discuss their findings and eventually deduce the formula for the area of the unit.

USING THE CALCULATOR

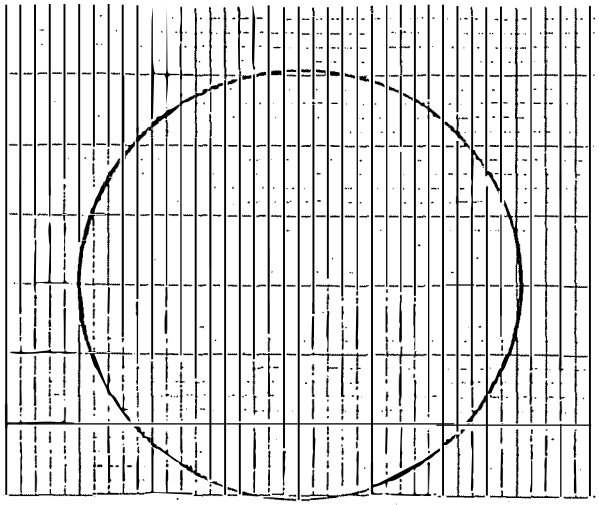
TOPIC: MEASURE THE AREA OF IRREGULAR FIGURES BY MEANS OF A SQUARE GRID. (GRADE 7-8).
INVESTIGATING THE RELATIONSHIP BETWEEN THE AREA AND THE SQUARE OF THE RADIUS FOR CIRCULAR OBJECTS (GRADE 8).

1. What happens to the area of a circle as you increase its radius?
2. What happens to the area of a circle as you decrease its radius?
3. In general, one could say that the area of a circle depends (in some way) on the size of its radius. In other words, the area of a circle is a function of its radius. Another way is to say that the area varies directly as the radius.

THE PURPOSE OF THE FOLLOWING EXERCISES IS TO SEE HOW THE AREA OF A CIRCLE IS RELATED TO THE SIZE OF ITS RADIUS.

EXERCISE 1.

The circle is drawn on a metric grid. Estimate the area in square centimetres by counting the number of squares that make up the area. Where the squares overlap the circumference, you must estimate the count.



Estimate of area is _____ cm^2

Find the ratio of the area to the square of the radius:
A: r^2 = _____,
therefore A = _____

EXERCISE 2

Estimate the area of the circle by attempting to fill in the circle with polygons. The accompanying figure shows a regular polygon with 8 sides (an octagon).

(a) Compute the area of one of the triangles by using a suitable formula

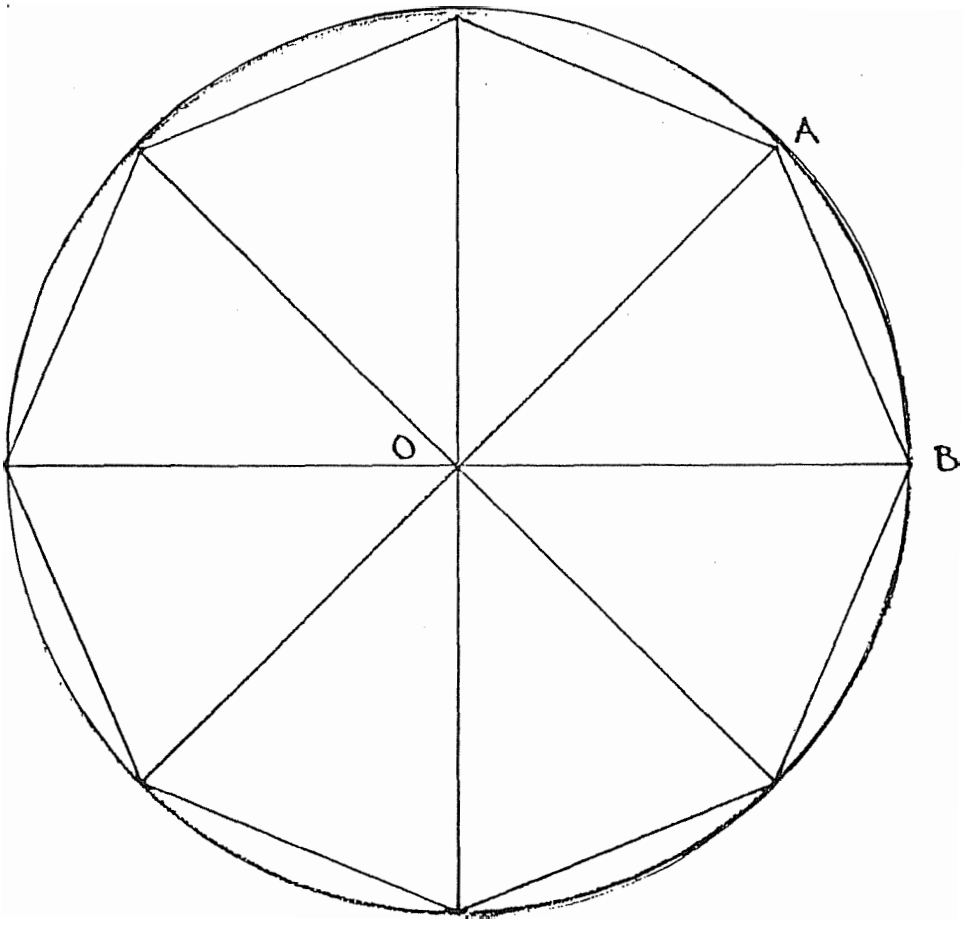
A = _____ cm^2

(b) Now estimate the area of the whole polygon. _____ cm^2

(c) Using this as an approximation for the area of the circle, compute the ratio of this area to the square on the radius:

$A : r^2$ = _____

Therefore, A = _____



EXERCISE 3

We can obtain a better approximation of the area of the circle by inscribing a twelve sided polygon (dodecagon) inside the circle as illustrated below.

(a) Compute the area of any one of the triangles by using a suitable formula.

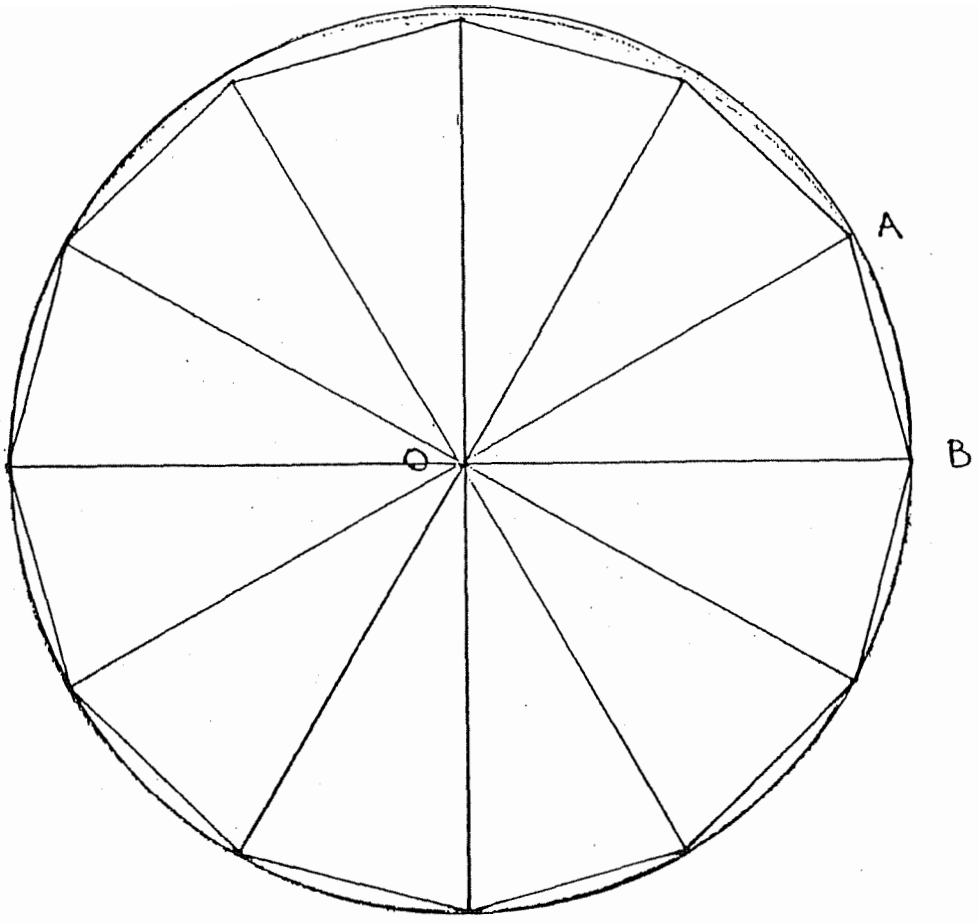
A = _____ cm^2

(b) Now estimate the area of the whole polygon. _____ cm^2

(c) Using this value as an approximation for the area of the circle, compute the ratio of the area of the circle to the square of the radius.

$A : r^2$ = _____

Therefore, A = _____



EXERCISE 4

To improve your estimate of the area of the circle even further, look at one last inscribed polygon containing six sides. This is illustrated in the following circle.

(a) Compute the area of any one of the triangles by using an appropriate formula.

$A =$ _____ cm^2

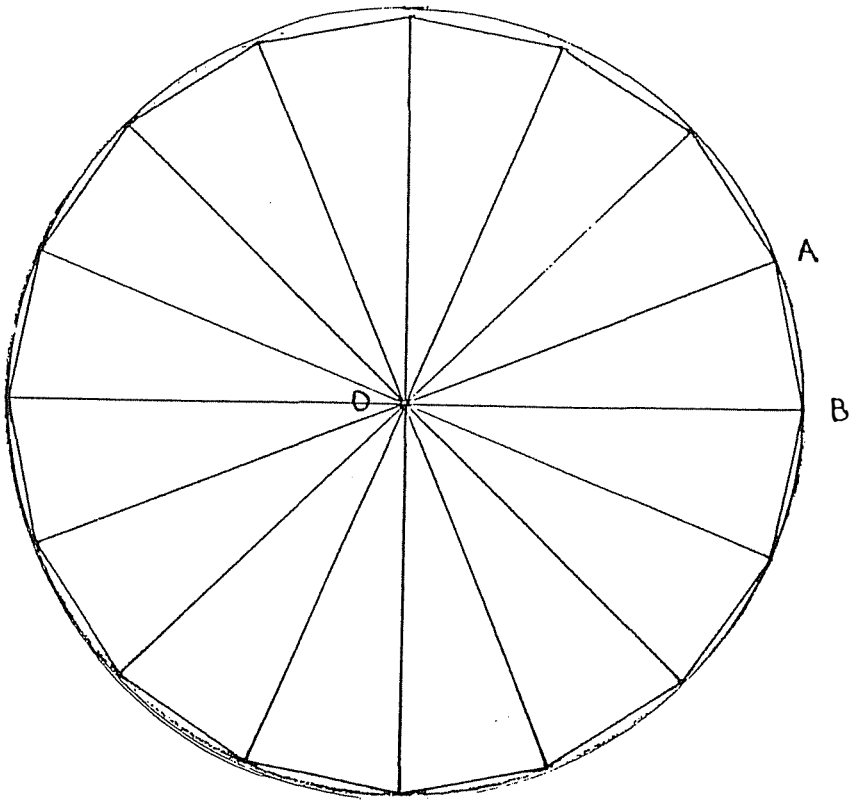
(b) On the basis of your calculation above, estimate the area of the entire polygon.

$A =$ _____ cm^2

(c) Using this value as an approximation for the area of the circle, compute the ratio of the area of the circle to the square of the radius.

$A : r^2 =$ _____

Therefore, $A =$ _____



CONCLUSION: THE RATIO OF THE AREA OF THE CIRCLE TO THE SQUARE ON ITS RADIUS SEEMS TO BE A CONSTANT VALUE. WE SHALL CALL THIS VALUE PI AND USE THE LETTER FROM THE GREEK ALPHABET TO DENOTE ITS VALUE. = 3.1 (APPROXIMATE).

SENIOR MATHEMATICS AND GRAPHING UTILITIES

by
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Graphing Utilities are available for virtually all types of computers, including ICON, Macintosh, Apple IIe, and Commodore 64. They can be applied to a wide variety of topics at all grade levels. By familiarizing themselves with only one piece of software, students can make widespread use of this tool at several grade levels. A Graphing Utility provides more time for conceptual learning through the elimination of the time consumed on the low level skills of plotting graphs by hand; a skill supposedly mastered prior to the study of advanced mathematics.

Here is a lesson which capitalizes on the advantages provided by Graphing Utilities, aimed at the MAT4A Relations and Functions unit. For this unit, the Provincial Curriculum Guideline states that "Computers with appropriate capabilities provide a means for illustrating the concepts of this strand", and that "The techniques of graphing using transformations should be consolidated and reviewed in a context such as the step or the quadratic function".

Note that with the computer doing the graphing, the function itself need not be a familiar one to the students. It is possible for instance, to use the cosine function. Why the cosine function ? For one thing, the relationships between the cosine function and its transformations, particularily stretches, are clearer than is the case with the quadratic function. It also provides an informal introduction to trigonometric functions will be studied later in the same course. In fact, logarithmic and exponential functions could also be also be used here, without having to actually define them. The objective is only to learn about how certain algebraic substitutions are related to transformations, not about the functions themselves.

MAT4A
FUNCTIONS AND TRANSFORMATIONS
Graphing Utility Exercise

Erase the screen after completing each investigation.

A Investigating $y = \cos(x + h)$

1. Graph $y = \cos(x)$
2. Graph $y = \cos(x + 1)$
3. What transformation of $y = \cos(x)$ would result in the graph of $y = \cos(x + 1)$? _____
4. Predict and verify the position of the graph of $y = \cos(x - 1)$
5. What transformation of $y = \cos(x)$ would result in the graph of $y = \cos(x + h)$? _____
6. Test your answer by experimenting with various values of **h**

B Investigating $y + k = \cos(x)$

1. Graph $y = \cos(x)$
2. Graph $y + 2 = \cos(x)$ *Note : Enter as* $y = \cos(x) - 2$
3. What transformation of $y = \cos(x)$ would result in the graph of $y + 2 = \cos(x)$? _____
4. Predict and verify the position of the graph of $y - 5 = \cos(x)$
5. What transformation of $y = \cos(x)$ would result in the graph of $y + k = \cos(x)$? _____
6. Test your answer by experimenting with various values of **k**

C Investigating $y = \cos(m*x)$, $m > 0$

1. Graph $y = \cos(x)$
2. Graph $y = \cos(2*x)$
3. What transformation of $y = \cos(x)$ would result in the graph of $y = \cos(2*x)$? _____
4. Predict and verify the position of the graph of $y = \cos(0.5*x)$
5. What transformation of $y = \cos(x)$ would result in the graph of $y = \cos(m*x)$? _____
6. Test your answer by experimenting with positive values of **m**

D Investigating n*y = cos(x) , n > 0

1. Graph **y = cos(x)**
2. Graph **2y = cos(x)** *Note : Enter as* **y = 0.5*cos(x)**
3. What transformation of **y = cos(x)** would result in the graph of **2y = cos(x)** ? _____
4. Predict and verify the position of the graph of **(0.5)*y = cos(x)**
5. What transformation of **y = cos(x)** would result in the graph of **n*y = cos(x)** ? _____
6. Test your answer by experimenting with positive values of **n**

E Summary

The graph of **y = cos(x + h)** is a horizontal translation of **-h** units of the graph of **y= cos(x)**

The graph of **y + k = cos(x)** is a vertical translation of **-k** units of the graph of **y= cos(x)**

The graph of **y = cos(m*x)** is a horizontal stretch of **1/m** units of the graph of **y= cos(x)**

The graph of **n*y = cos(x)** is a vertical stretch of **1/n** units of the graph of **y= cos(x)**

For equations which are in the form **y + k = cos(x + h)** or in the form **n*y = cos(m*x)** , the following are true:

The parameters **h** and **m** which describe horizontal transformations are located on the **x**-side of the equation.

Similarily, the parameters **k** and **n** which describe vertical transformations are located on the **y**-side of the equation.

When translations are involved, it is the **additive inverses** of **h** and **k** which determine the number of units to translate.

Similarily, for stretches, it is the **multiplicative inverses** or **reciprocals** of **m** and **n** which determine the stretch factor.

Vertically and Crosswise: Multiplication for the Mind

Students will be familiar with the traditional method of multiplying two numbers, say 13 and 27, together:

```
 13 — multiplicand
 27 — multiplier
 91
260
351
```

However, there is another way of multipling, called 'vertically and crosswise', (V. and C.). The above example should suffice to clarify the method:

1) Multiply the right hand most digit of the multiplicand , 3, vertically by the right hand most digit of the multiplier, 7, obtaining their product, 21; set down the 1 as the right hand most part of the answer and mentally 'carry-over' the 2.

```
 3
 *
 7
 1
(2)
```

2) Multiply 1 by 7, and 3 by 2 crosswise , add the two products, get 13 as the sum, add the 'carry-over', get 15; set 5 down as the middle part of the answer and mentally 'carryover' the 1.

```
1  3
   *
2  7
 5 1
(1)
```

3) Multiply 1 and 2 vertically , get 2 as their product, add the 'carry-over', get 3, and put it down as the last part of the answer.

```
1  3
*
2  7
3 5 1
```

Let's try a 3 digit problem, say 513 times 427. The first two steps are the same as for 13 times 27. Continuing with the remaining steps:

3) Multiply 5 by 7 crosswise, 1 by 2 vertically, and 3 by 4 crosswise, add the three products, get 49 as the sum, add the 'carry-over' from step 2 above, get 50; set down 0 next left of 51 and mentally 'carry-over' the 5.
4) Multiply 5 by 2 and 1 by 4 crosswise, add the two products, get 14, add the 'carry-over', get 19; set 9 down next left of 051 and mentally 'carry-over' the 1.
5) Multiply 5 by 4 vertically, get 20 as their product, add the 'carry-over', get 21; set 21 next left of 9051 to complete the answer.

```
3)  5 1 3      4)  5 1 3      5)    5 1 3  (Notice how we are
      *              *               *       multiplying
    4 2 7          4 2 7          4 2 7   symmetrically
    -----          -----          -----
      0 51           90 51       21 90 51  about the asterisk)
     (5)            (1)
```

Another example and comparison will demonstrate the full power of this method:

Traditional Method	V. and C.	V. and C. (left to right)
87265	8 7 2 6 5	(start on 8 7 2 6 5
32117	3 2 1 1 7	this side) 3 2 1 1 7
610855	2802690005	2478727575
872650		32396243
8726500		2802690005
174530000		
2617950000		
2802690005		

Observations:

1) V. and C. is versatile. We can multiply from 'right to left' or vice-versa. How ever, in the 'left to right' method 'carry-overs' are written down instead of remembered.
2) V. and C. (right to left) always yields a one line answer and there is no adding done on paper. V. and C. is neat and compact.
3) The mind is doing most of the work, not the pen.

References: *Jagadguru Swami Sri Bharati Krsna Tirthaji Maharaja, Vedic Mathematics. Varanasi: Banaras Hindu University, 1965.*

Elliott Applebaum *is a graduate of the Faculty of Mathematics of the University of Waterloo and a 1987 graduate of the Faculty of Education at Queen's University.* *This article was originally written as a contribution to the Gems module of the Student as Desktop Publisher Project. This collection of resource materials generated by Dr. William Higginson's Intermediate-Senior Mathematics course CURR 343B was produced with the assistance of a grant from the Apple Canada Education Foundation.*

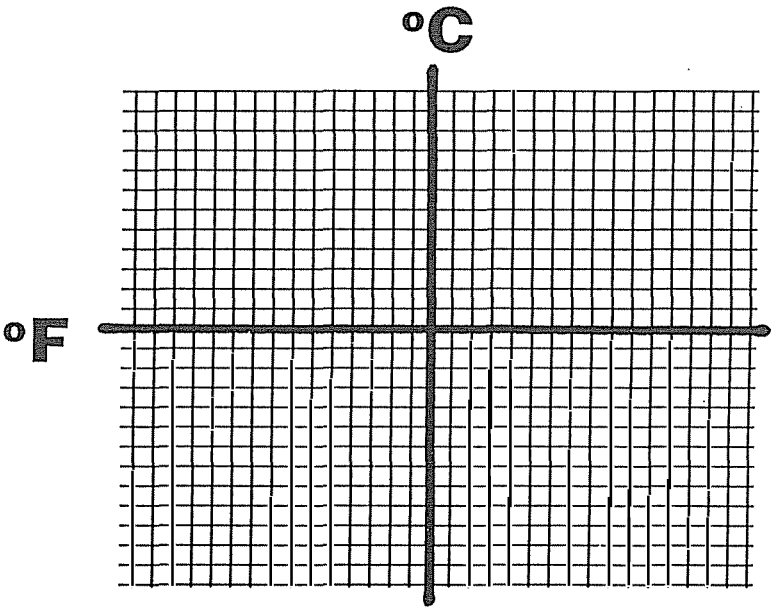
WINTER SCIENCE ACTIVITIES
(INTERMEDIATE)

Use both Fahrenheit and Celsius scales to record the temperature of each of the following:

1. *snow*
2. *snow and salt*
3. *the air*
4. *tap water*
5. *anti-freeze*
6. *alcohol*
7. *your hands*
8. *others*

Use the chart and graph your results on the large sheets of graph paper.

	^0F	^0C
SNOW		
SNOW & SALT		
AIR		
TAP WATER		
ANTI-FREEZE		
ALCOHOL		
YOUR HANDS		



1. *Is the graph a straight line?*
2. *What is its slope?*
3. *Where does it cut the Y-axis?*
4. *At what point are the Fahrenheit and Celcius readings the same?*

WINTER SCIENCE ACTIVITIES
(JUNIOR)

1. How long does it take to coast down the hill on the toboggan?
   How is the speed affected by:
   1. putting more riders on the toboggan?
   2. waxing the toboggan?

2. How far does the toboggan go? How is this distance affected by:
   1. the number of riders (i.e., increasing the weight) and
   2. waxing the toboggan.

3. Put coloured cloths on the snow.
   Record the temperature under each cloth.

4. Put the squares of metal, plastic and glass on the snow. *Around which one does the melting take place first?*

5. Build an igloo. Take temperature readings inside and out.

6. Put containers of (1) water (2) water and salt (3) water and anti-freeze in the snow. *Which freeze first? What happens to the container.*

7. Melt a container of snow. *How much water did you get?* (Primary)

