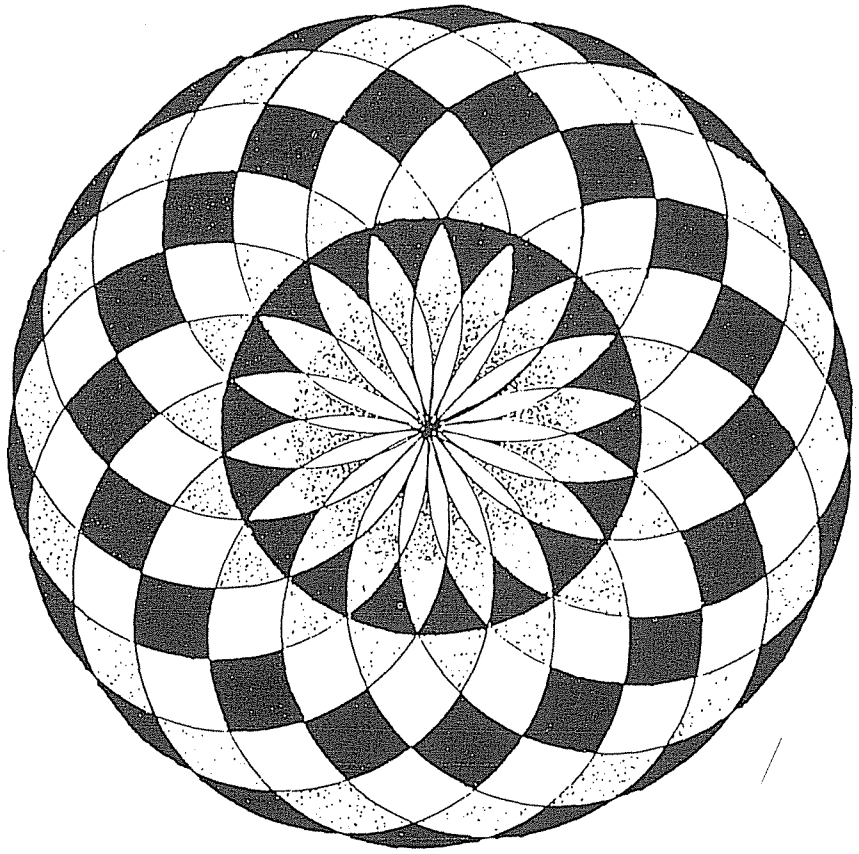
VOLUME III
NUMBER 2

Abacus

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"Before LOGO"

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The CHRISENDAVE Exploration

Grade: 11-13
Stream: General or Advanced
Subject: Algebra
Units: Powers, Patterns or Sequences
Time: 20-60 minutes

Dave Jellicoe
(1988 graduate OEE Program
Queen's Faculty of Education)

The following exploration guides you through a series of interesting observations discovered by two non-mathematicians. The discovery of the patterns took, not training or genius but curiosity and labour. Many of the observations we made have been explained by modern mathematicians but several remain puzzles of the number system.

Math problems and their solutions are often shown in books. We hope that this exploration will illuminate the method of problem recog- nition and the path to the solution.

How the problem was noticed.

A student was looking at the sequences of numbers from 1 to 9. He decided to square them and sum the digits of each square and continue to sum the digits until a single digit was obtained. For example, $7 = 49$ gives $4 + 9 = 13$ which similarly gives $1 + 3 = 4$. The resulting pattern was un- exciting and would have gone unnoticed if he had not carefully continued the operation for many more numbers. Table 1 shows the result of his work.

Table 1

sequence	1	2	3	4	5	6	7	8	9	
square	1	4	9	16	25	36	49	64	81	
sum digits	1	4	9	7	7	9	4	1	9	*
sequence	10	11	12	13	14	15	16	17	18	
square	100	121	144	169	196	225	256	289	324	
sum digits	1	4	9	7	7	9	4	1	9	*
sequence	19	20	21	22	23	24	25	26	27	
square	361	400	441	484	529	576	625	676	729	
sum digits	1	4	9	7	7	9	4	1	9	*

An interesting pattern appears by doing these simple manipulations of squaring and summing of digits. Several questions come to mind. Notice the rows with asterisks.

1. Why does the pattern repeat after 9?
2. Why are only the numbers 1, 4, 7 and 9 represented?
3. Is the pattern a product of the operations performed or a characteristic of the number system?
4. Would this pattern occur if other operations were used?

The plot thickens

These questions all occurred to us at the time but being non- mathematicians we could not answer them. (Can you?) What we could do is explore question #4. Our first attempt was to try different powers instead of squaring. The first nine numbers of the pattern for cubes are shown in Table 2.

Table 2

sequence	1	2	3	4	5	6	7	8	9	
cube	1	8	27	64	125	216	343	512	729	
sum digits	1	8	9	1	8	9	1	8	9	*

Take this opportunity to make two charts. Make one chart nine columns across by five or six rows. Fill in the array to find out whether the pattern repeats for the cubes in the same way it did for the squares. Make another chart, nine columns by twelve rows. In this array, fill in the pattern for the powers 2 through 12. You already have the numbers for squares and cubes. (see the rows with asterisks)

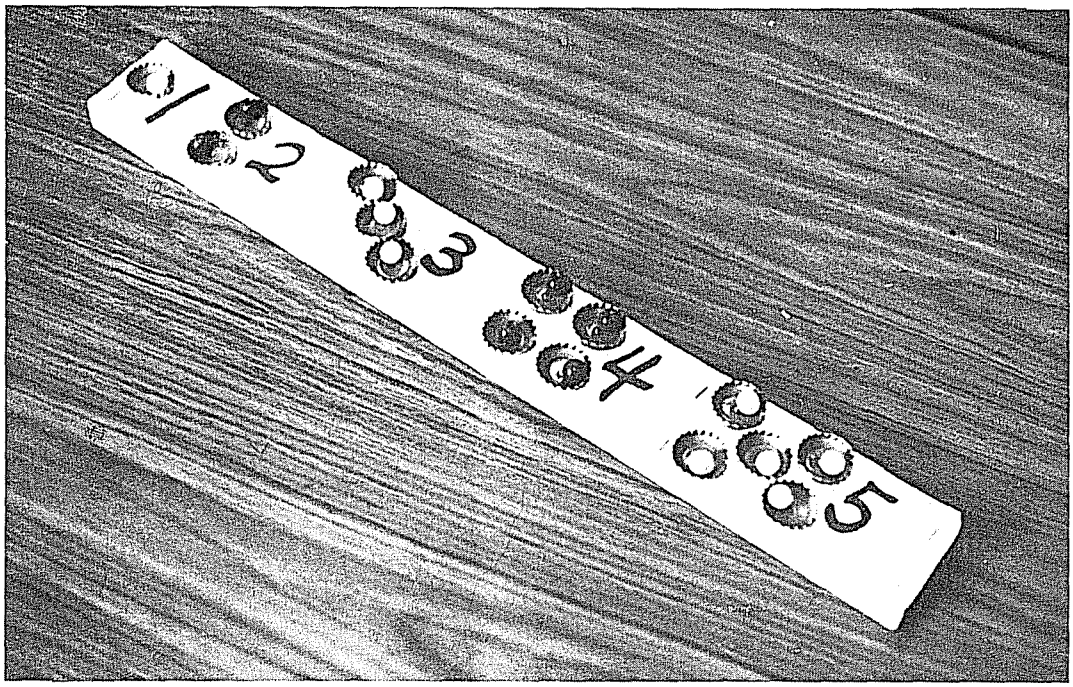
The original exploration was done in much the same way. We tried something and then extended it until a pattern formed. It just happened that the patterns formed early enough and blatantly enough that even we could see them.

Look at the chart and find as many patterns as you can. Make a list of the patterns you see and then, with the help of a small group or your teacher, try to explain the logic behind some of them. I'm told that this is what math is all about! If that is so, then I guess we're not non-mathema- ticians after all. If you've studied your array enough continue reading, if not go back and look at the array in a different way. The following list contains the questions we came up with at this point.

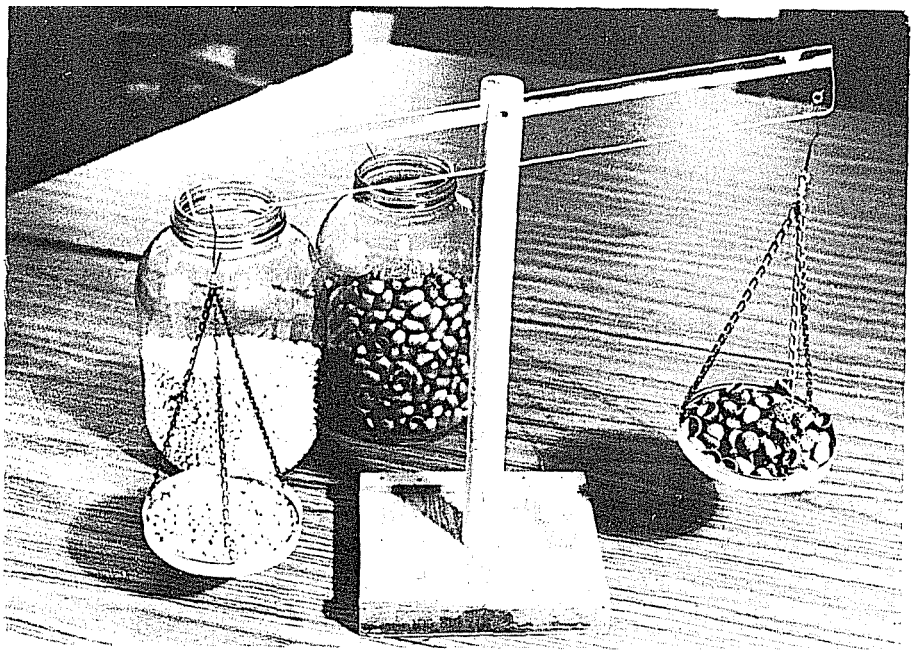
1. Why are the numbers in columns 1, 3, 6 and 9 constant?
2. What is the reason for the pattern for column 4 and 7 and why are only numbers 1, 4 and 7 found?
3. Notice that seventh powers yield almost 1 through 9. What limits this and is there a tendency toward it in the patterns?
4. Why does the row pattern repeat after the seventh power?
5. How are the row patterns for powers 2 and 8, 3 and 9, and 4 and 10 related?
6. Why do the numbers 3 and 6 never appear in the patterns?
7. What is the explanation for the palindromic structure in columns 2, 4 and 6? (same forward and backward, i.e. 147741)
8. Why does column 8 alternate between numbers 8 and 1?
9. Why does column 2 read 487512 and column 5, starting at the fifth power read the same digits in reverse, 215784?

The study of this problem is in its infancy. If solutions are not your forté, then ask more questions. The secret is that original and valuable math can be done by anyone. Just try!

Some Learning Materials for a MAKE & TAKE Workshop

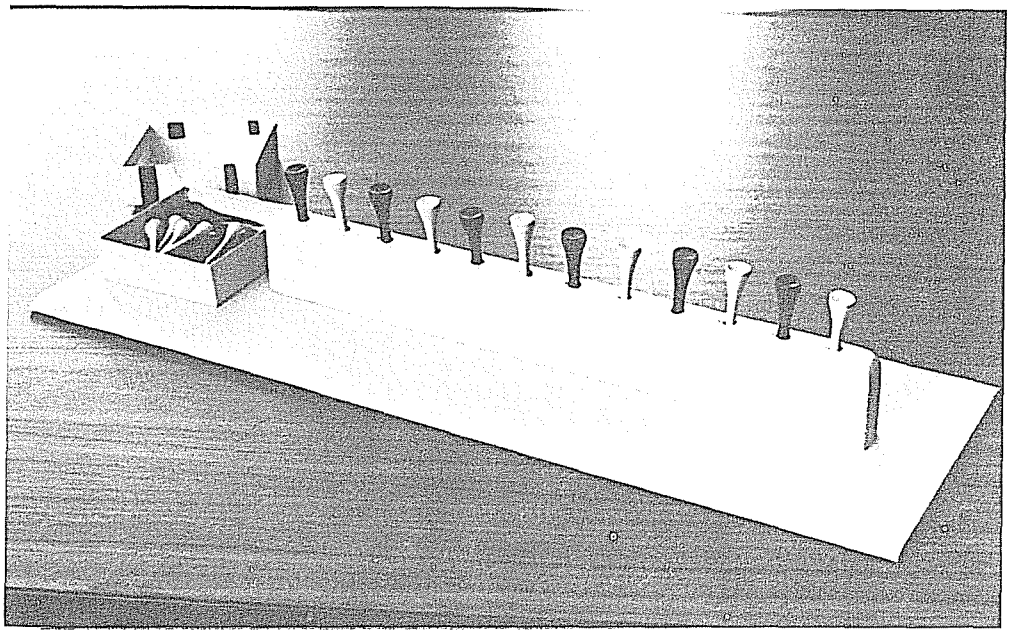


Nail bottle caps to a board. Children place coloured marbles to match the symbol.

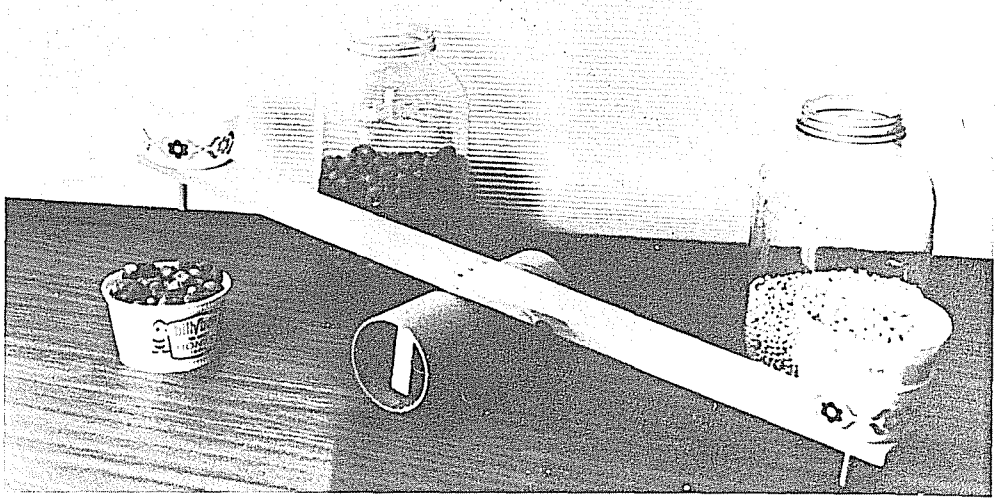


Simple primary balance from scrap materials

Learning Materials (cont'd)

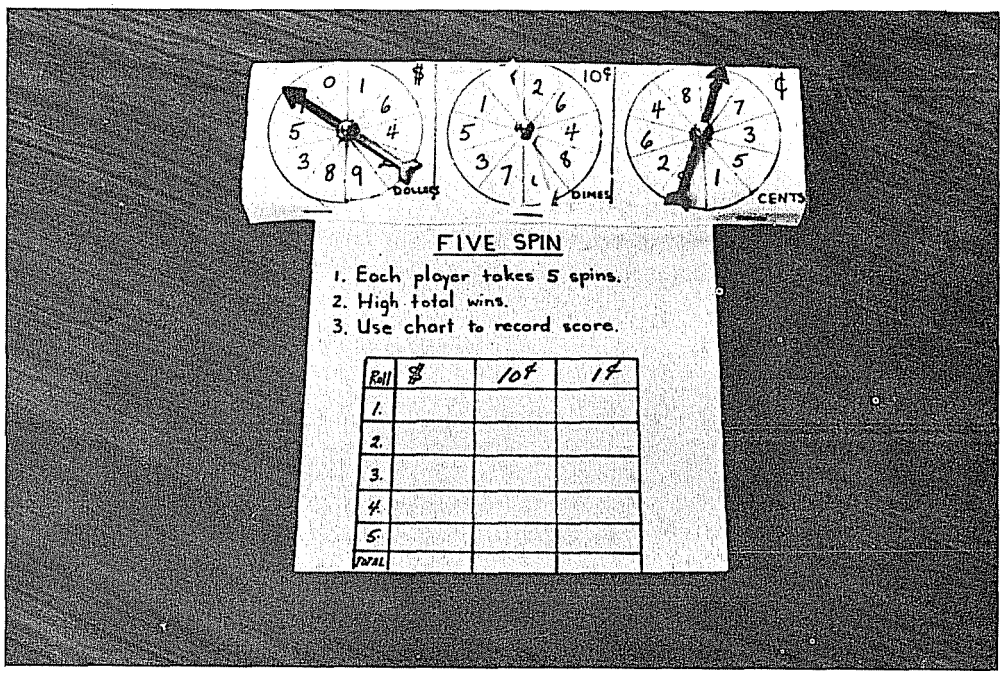


Place golf tees in holes drilled in piece of 1X2 wood. This device can be used for number facts, skip counting, notions of multiplication and division.

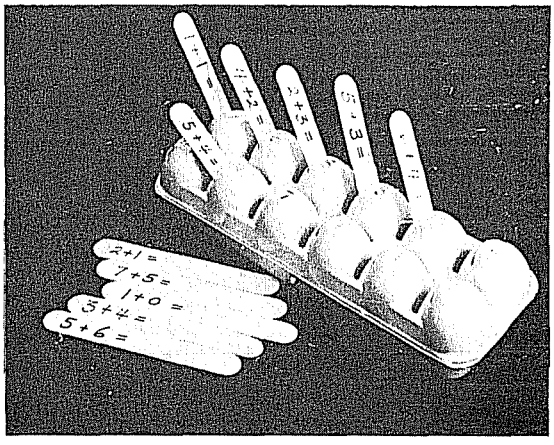


A simple primary balance
Which weighs more a cup full of beans or a cup full of chestnuts?

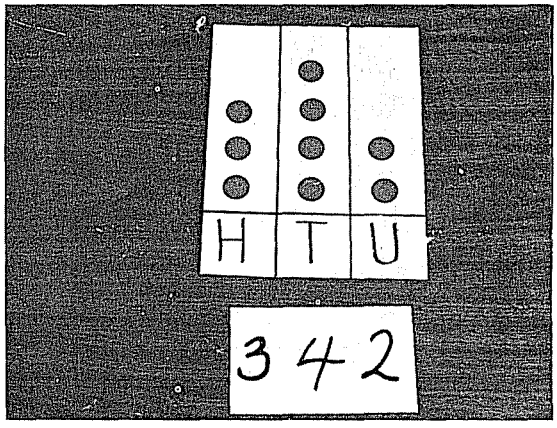
Learning Materials (cont'd)



Five Spin helps pupils learn about place value, money, decimals and addition

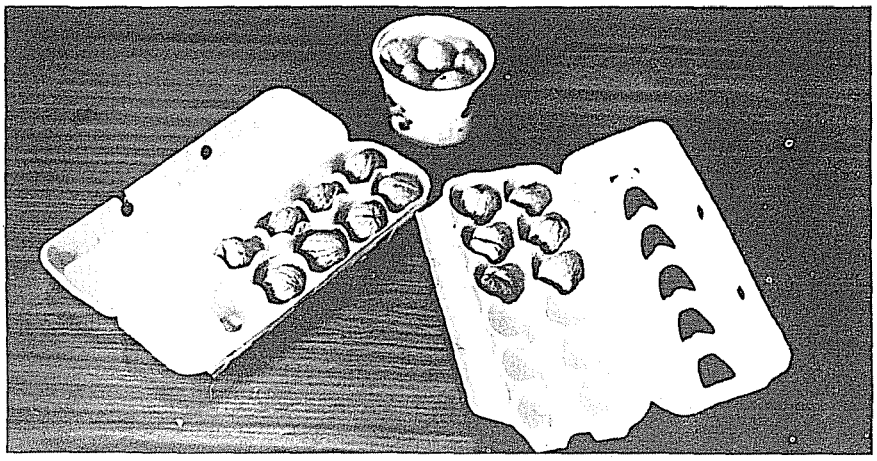


Questions are written on tongue depressors. Answers are written next to slots on the egg carton



The stickers on the card represent the number of hundreds, tens, and units (ones). Children place the correct symbol under the card

Egg Carton Arithmetic



Fill an egg carton with English walnuts.

Questions:

How many in a dozen?
How many in half a dozen?
How many in a quarter of a dozen?
How many in a twelfth of a dozen?

How many twefths are there in a quarter of a dozen?
How many twefths in a third of a dozen?
How many twefths in a third of a dozen plus a quarter of a dozen?

This exercise provides good examples of equivalent fractions and common denominators for addition and subtraction.

Remove some walnuts.

Questions:

How many walnuts are in the container?
How many more are needed to make a dozen?
How many spaces are in the carton?

Count the number of spaces by two.

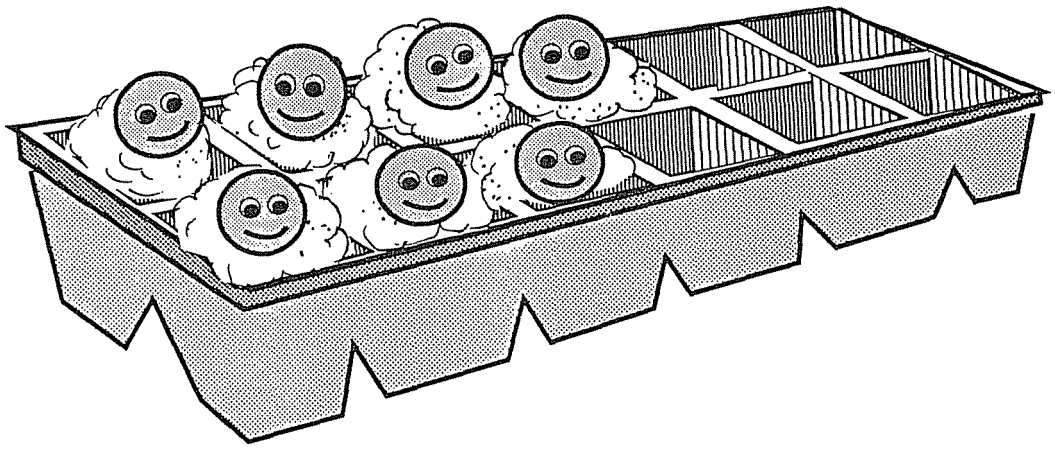
How many "twos" are in the twelve?

Count the number of spaces by three.

How many times is three contained in twelve?

Count by fours and sixes.
This exercise provides good illustrations of multiplication and division.

Egg Carton Math (continued)



Cut two sections from the end of the carton so that there are ten places left. Fill each container with a different number of cotton balls. Glue happy face stickers on them to make little fluffy creatures.

Questions:

How many spaces are in the carton?
How many fluffy creatures are in the carton?
How many more fluffy creatures do you need to fill the ten spaces?

The answers will give the addition facts of ten.

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needed!

EDITOR(S) FOR

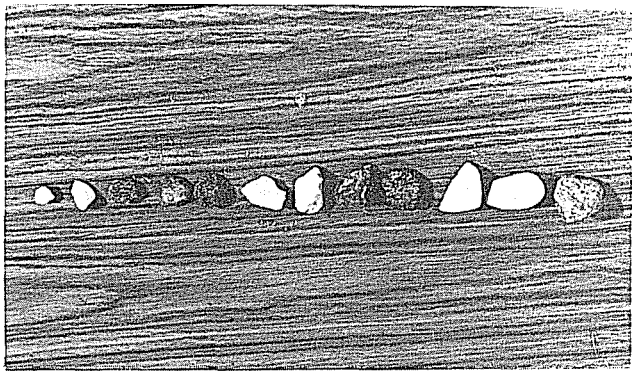
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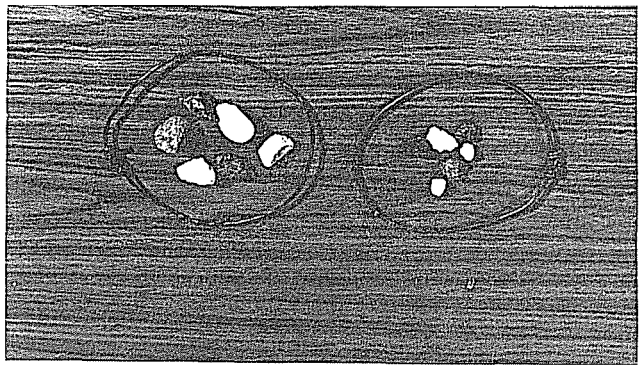
(16 page centre-section of the Gazette) for a two year term,
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Apply to: Jack Weiner
Department of Mathematics and Statistics
University of Guelph
Guelph, Ontario

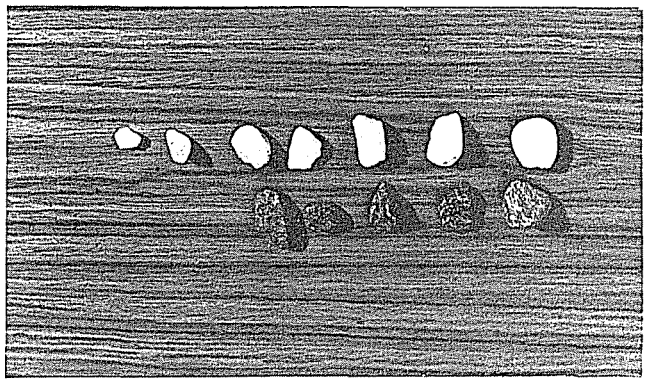
Sorting Materials



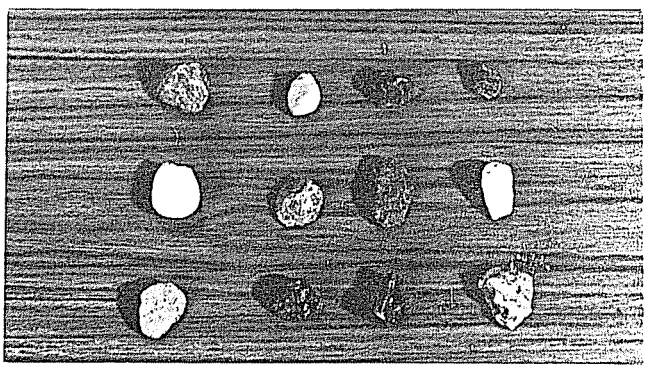
Take a handful of stones.
Sort them.
Why did you sort them in this way?
How many are in each group?



Match the stones.
Which group has more?
How many more are in the larger group?



Take twelve stones.
Put them in pairs.
Count to 12 by two.
Put them in rows of 3.
Count to 12 by 3.
How many groups of 3 do you have?

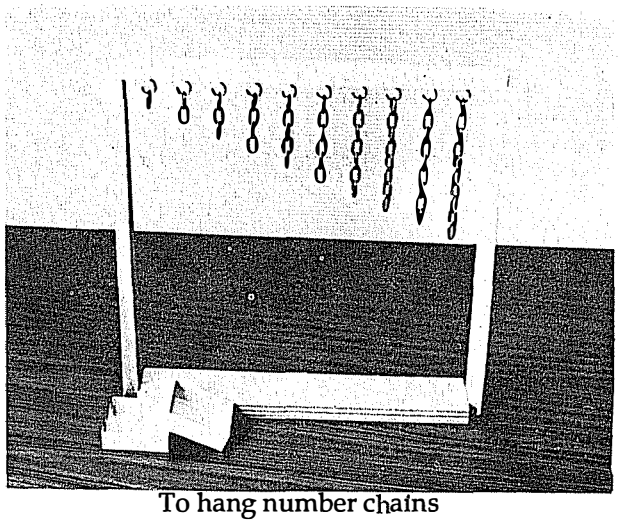


Take 12 stones.
Share them with 2 friends.
How many did each of you get?

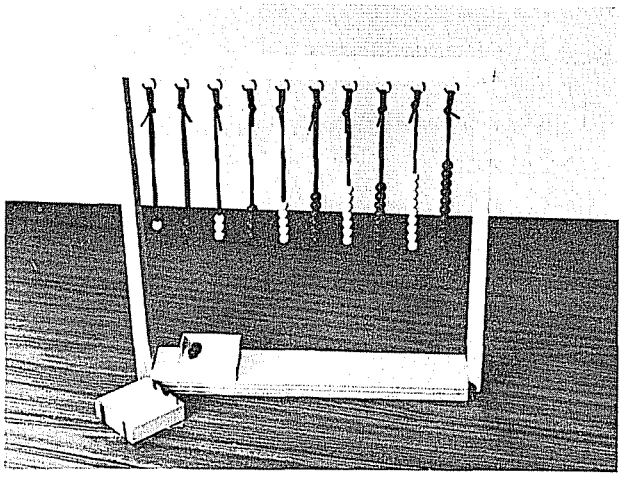
Take 12 stones.
How many times can you remove
a group of 3 stones?
How many times is 3 contained in 12?

Hooked Frame

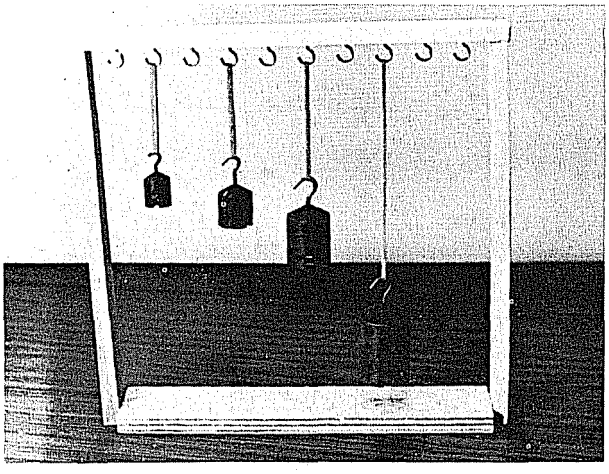
A row of hooks screwed into the top of a wooden frame has many uses in the classroom.



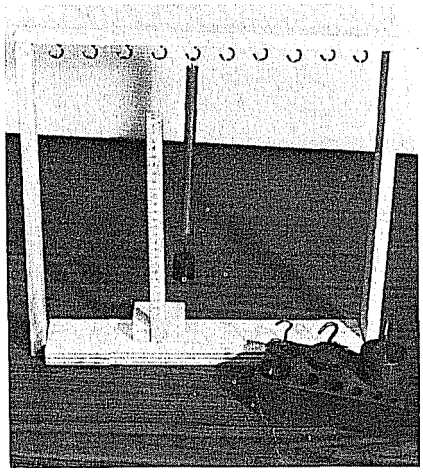
To hang number chains
(primary)



To hang threaded beads
(primary)



To experiment with elastic bands
(Junior-Intermediate)



To experiment with springs
(Junior-Intermediate)

Calendar Math

Before the winter break ask your class to keep the 1988 calendars and to bring them in when class resumes in January.



Some activities:

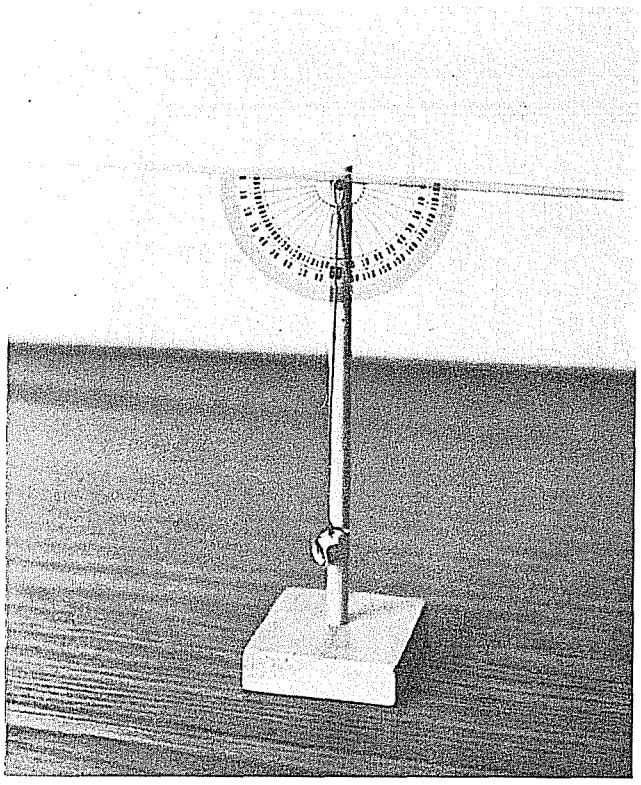
How many months in a year?
What months have 30 days, 31 days?
What is the shortest month?
What is today's date?
What is the first day of the month?
What is the second day?
What is the last day?
Have children make up more questions ....

8	9	10
15	16	17
22	23	24

Ask the class to choose a block of nine squares.
Ask them to add 3 numbers in each diagonal.
What did they find?
Why is this so?
Ask them to add all numbers in the block.
Multiply the middle numbers (e.g. 16 by 9). Compare answers.
Have them explain their answers.

Build a Simple Astrolabe

(Intermediate Grades)



Materials

- scrap wood
- soda straw
- protractor
- string and weight

Construction

- Glue the soda straw to the base of the protractor
- Attach the protractor with one nail as shown in the picture
- Attach the string with the weight to the nail

Operation

To measure the angle of elevation of the North Star:

- Place the astrolabe on a solid surface
- Tilt the soda straw and protractor until the North Star can be seen through the straw
- Count off how many degrees the protractor has turned
- Average the results of everyone in the class
- Look in the school atlas to find the latitude of your town
- Compare this to your class average

The North Star and Latitude

PURPOSE

To prove that the angle of elevation of the north star is equal to the degree of latitude of the observer.

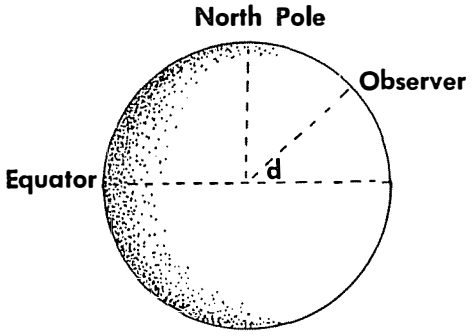


Figure 1

OPERATION

Angle *d* in Figure 1 shows the angle of latitude of the observer. Angle *a* in Figure 2 shows the angle of elevation of the north star above the horizon. Prove that angle *a* equals angle *d* . In Figure 3,

$$\angle a + \angle b + \angle c = 180°$$
$$\angle b = 90°$$
$$\angle a + 90° + \angle c = 180°$$
$$\angle a + \angle c = 90°$$

But $\angle d + \angle c = 90°$

Therefore $\angle a = \angle d$

Thus the angle of fix on the north star equals the angle of latitude of the observer

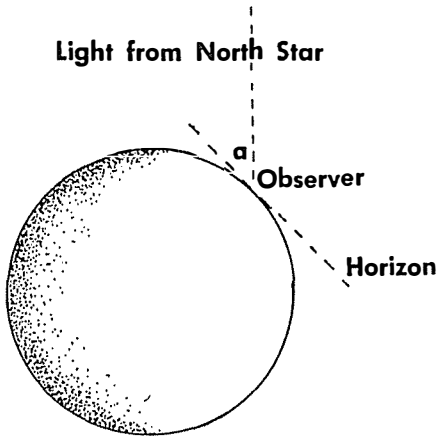


Figure 2

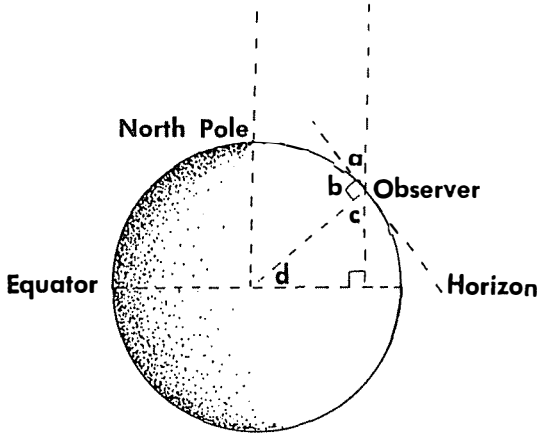


Figure 3

Slide-rule for Integers

PURPOSE
To add and subtract with integers.

MATERIALS
Two wooden sticks, or strips of Bristol board. (A bundle of hardwood flooring will provide enough slide-rules for a whole class.)

CONSTRUCTION
Mark integers on the slide-rules as illustrated in the diagram.

OPERATION
Addition
Example: (-4) + (+7) = (+3). See Figure 1

Place the zero of the top slide over the first addend as shown in the illustration (0 over the -4). Look along the top slide to the second addend (+7). The answer is found under the addend on the bottom slide, (+3).

Subtraction
Figure 1 can also be used to demonstrate subtraction: (+3) - (+7) = (-4).

Locate the subtrahend (+7) on the top slide. Push the rule so that the subrahend (+7) on the top slide is above the minuend (+3) on the bottom slide. Look along the top slide to locate the zero (0). The answer is found under the zero on the bottom slide, (-4).

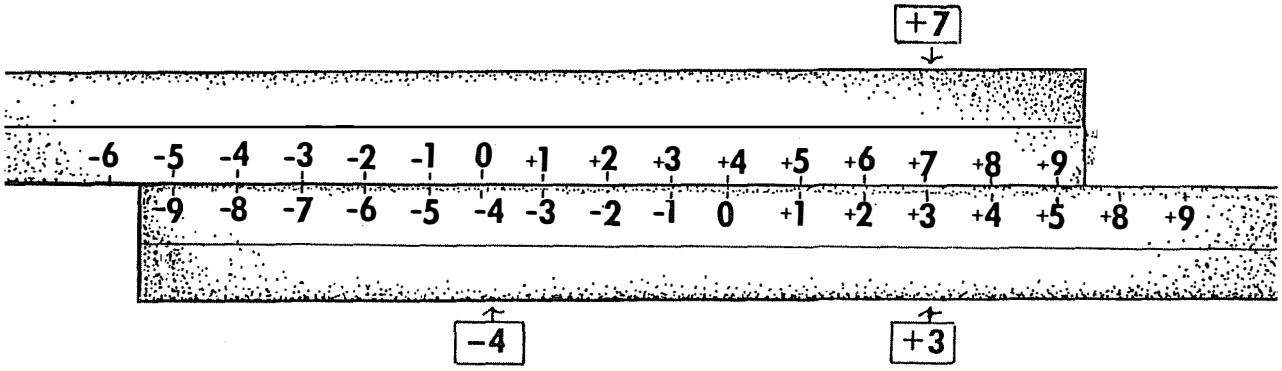
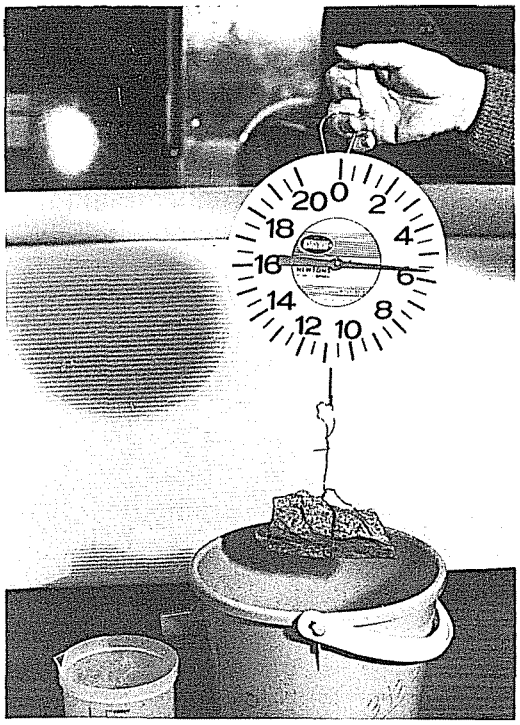


Figure 1

Buoyancy

(Intermediate Grades)

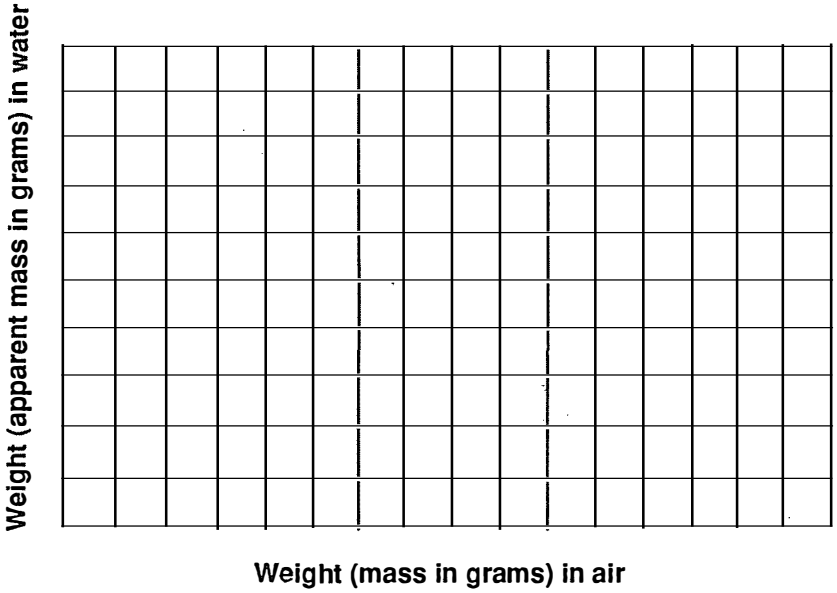


Procedure I

- Fill the overflow pail up to the spout with water
- Weigh the stone in air (mass in air)
- Weigh the stone when it is immersed in water (apparent mass in water)
- Collect the water that overflowed *(This is the volume of the stone)*
- Weigh the water that overflowed
- What did you find out?

Procedure II

Weigh several pieces of the same type of rock (e.g. limestone) in air and in water.
Make a graph of your findings.



Multiplication as Repeated Addition

Put rows of dots on one side of a strip of paper.
Put multiplication facts on the back.
Roll up the strip. *(from left to right)*
Let the children unroll the strip to reveal multiplication as repeated addition.

