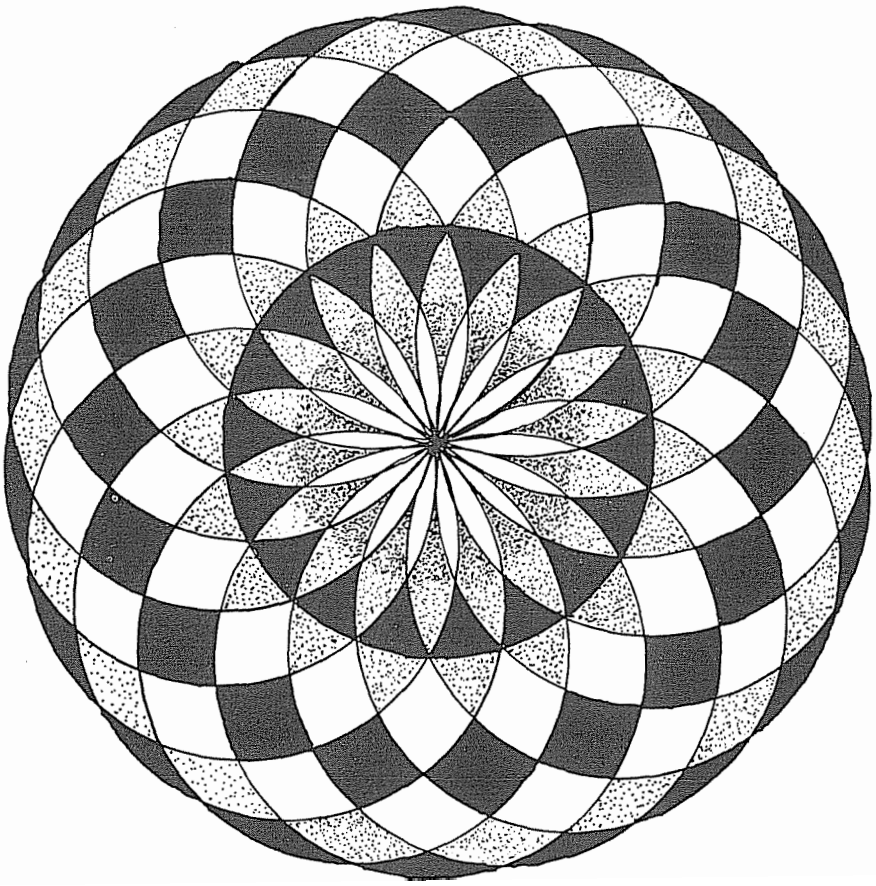
VOLUME III
NUMBER 3

Abacus

A publication of the O.A.M.E.

APRIL 1989



"Before LOGO"

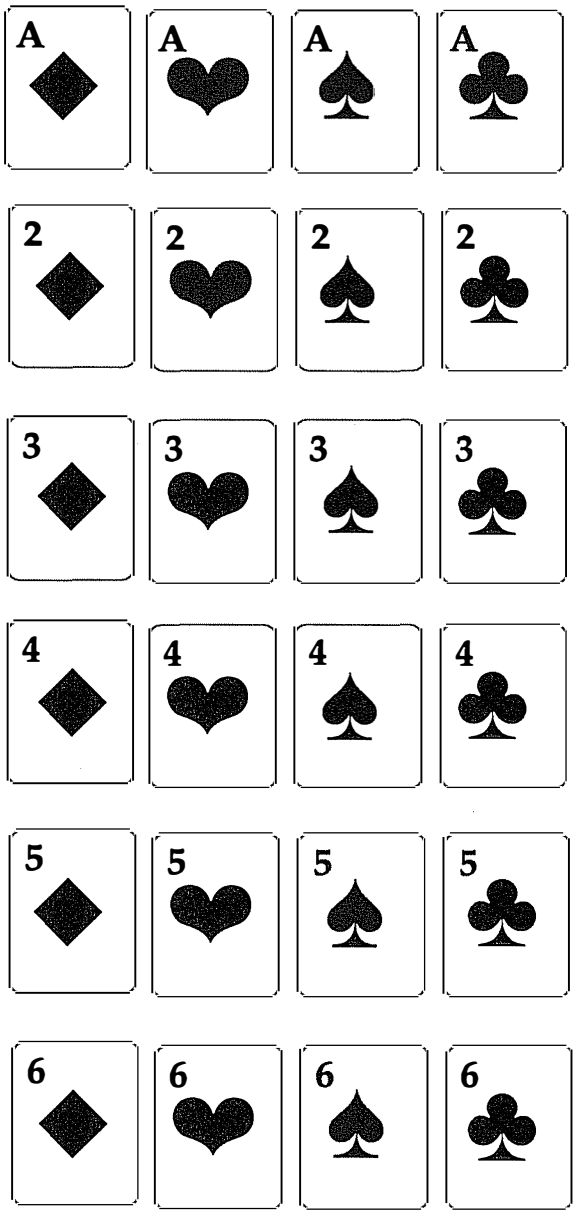
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The Australian Game of 31



From a deck of cards remove the 24 cards ace through six. Arrange the cards in the following fashion.

The game is played with two players. After deciding who goes first each player takes a turn placing a card face down adding the values of the cards turned as they do. Once a card is turned down it cannot be used again during the game. The player reaching 31 exactly first wins. Should a player be forced to go over 31 he/she loses.

- **Is there a strategy you can develop so that you can always win?**
- **Does it matter who goes first?**
- **Try the game and play a variety of partners.**

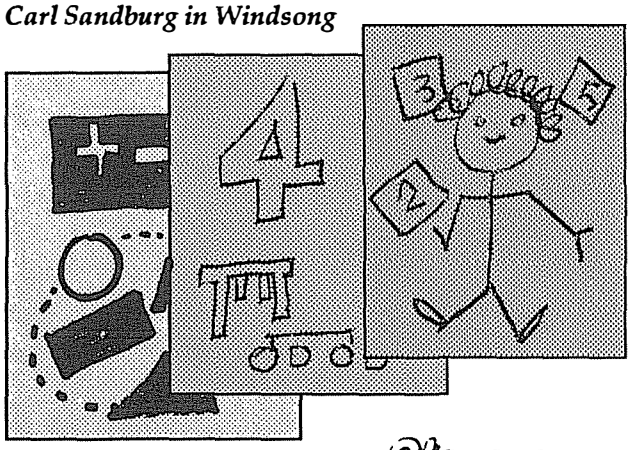
Arithmetic

- *Arithmetic is where numbers fly like pigeons in and out of your head.*
- *Arithmetic tells you how many you lose or win if you know how many you had before you lost or won.*
- *Arithmetic is seven eleven all good children go to heaven – or five six bundle of sticks.*
- *Arithmetic is numbers you squeeze from your head to your hand to your pencil to your paper till you get the answer.*
- *Arithmetic is where the answer is right and everything is nice and you can look out the window and see the blue sky – or the answer is wrong and you have to start all over and try again and see how it comes out this time.*
- *If you take a number and double it and double it again and then double it a few more times, the number gets bigger and bigger and goes higher and higher and only arithmetic can tell you what the number is when you decide to quit doubling.*
- *Arithmetic is where you have to multiply – and you carry the multiplication table in your head and hope you won't lose it.*
- *If you have two aninal crackers, one good and one bad, and you eat one and a striped zebra with streaks all over him eats the other, how many animal crackers will you have if somebody offers you five six and seven and you say No no no and you say Nay nay nay and you say Nix nix nix?*
- *If you ask your mother for one fried egg for breakfast and she gives you two fried eggs and you eat both of them, who is better in arithmetic, you or your mother?*

Carl Sandburg in Windsong

Children will enjoy rewriting this famous poem. Each can be written on construction paper and then illustrated by the children.

A class may embark on a cooperative poem based on Arithmetic to me is ... or Measuring to me is ...



How Do You Arrange the Cards So That the Following Will Happen?

Take a 13-card deck suit from a deck of cards and arrange them in such a way that the following will occur:

A "Holding the 13 card deck face down, take the top card and place it on the bottom of the deck. Turn the next card up on the table so that it will be an Ace. Place the next card on the bottom of the deck and turn the next card up on the table so that it will be a Two. Continue this procedure - i.e. place the next card on the bottom of the deck and the following card up on the table so that it follows in sequence 3, 4, 5, ..., 10, J, Q, K."

B "Holding the 13 card deck face down, take one card from the top and place it on the bottom of the deck. Turn the next card, which is the Ace, up on the table. Take the next 2 cards (separately) and place them on the bottom of the deck. Next turn the two up on the table. Continue this procedure - i.e. place the number of cards, separately, on the bottom of the deck which will suggest the next number in sequence which will be turned up on the table. Note that 11 cards must be placed separately on the bottom of the deck before the Jack is turned up; similarly 12 before the Queen turns up."

Answers: (may be given outright, hinted at, or left to the next Abacus)

Suggestions to Answers:

Clue 1

A _ A _ 2 _ 3 _ _ _ _ _ _ _ _

B _ A _ _ 2 _ _ _ 3 _ _ _ _

Clue 2

A 7 A _ 2 _ 3 _ 4 _ 5 _ 6 _

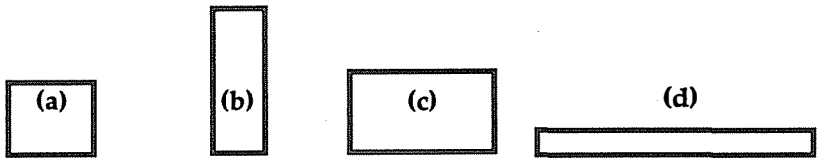
B 4 A _ _ 2 _ _ _ 3 5 _ _ _

Answer

A 7 A Q 2 8 3 J 4 9 5 K 6 10

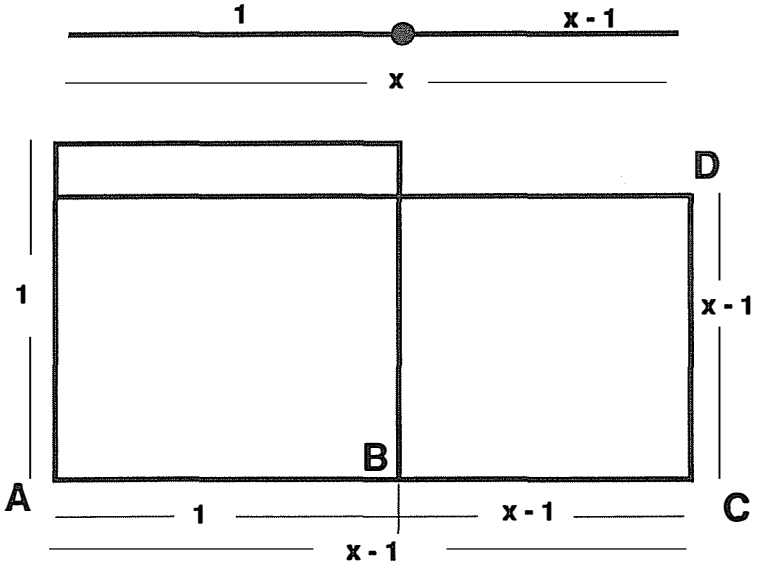
B 4 A K J 2 10 6 7 3 5 Q 9 8

The Golden Ratio



Which rectangle looks best to you? If you said (c) perhaps it is because its length and width have a particular ratio. Architects and artists often used rectangles of this shape because it was thought to be appealing to the eye. The Parthenon temple built in ancient Greece incorporates many examples of this structure. The ratio of length to width is called the "golden ratio" and is symbolized by the Greek letter ø. The value of ø is approximately 1.6 or 5:3. There are many illustrations which generate ø as indicated below:

(a) Euclid stated that "a given line can be divided into two parts so that the area of the rectangle whose length is the whole line segment and width is the shorter of the two parts, is equal to the area of the square whose side is the other longer part." Using this idea, the golden ratio ø evolves.



Let the given line be x and the part included in the square to be 1

From Euclid $(AC)(CD) = (AB)^2$

$(x)(x-1) = (1)^2$

$x^2 - x = 1$

$x^2 - x - 1 = 0$

Solving for $x = \frac{1 \pm \sqrt{5}}{2}$

A positive solution for $x = \frac{1+\sqrt{5}}{2} = 1.618033989$ or the golden ratio ø

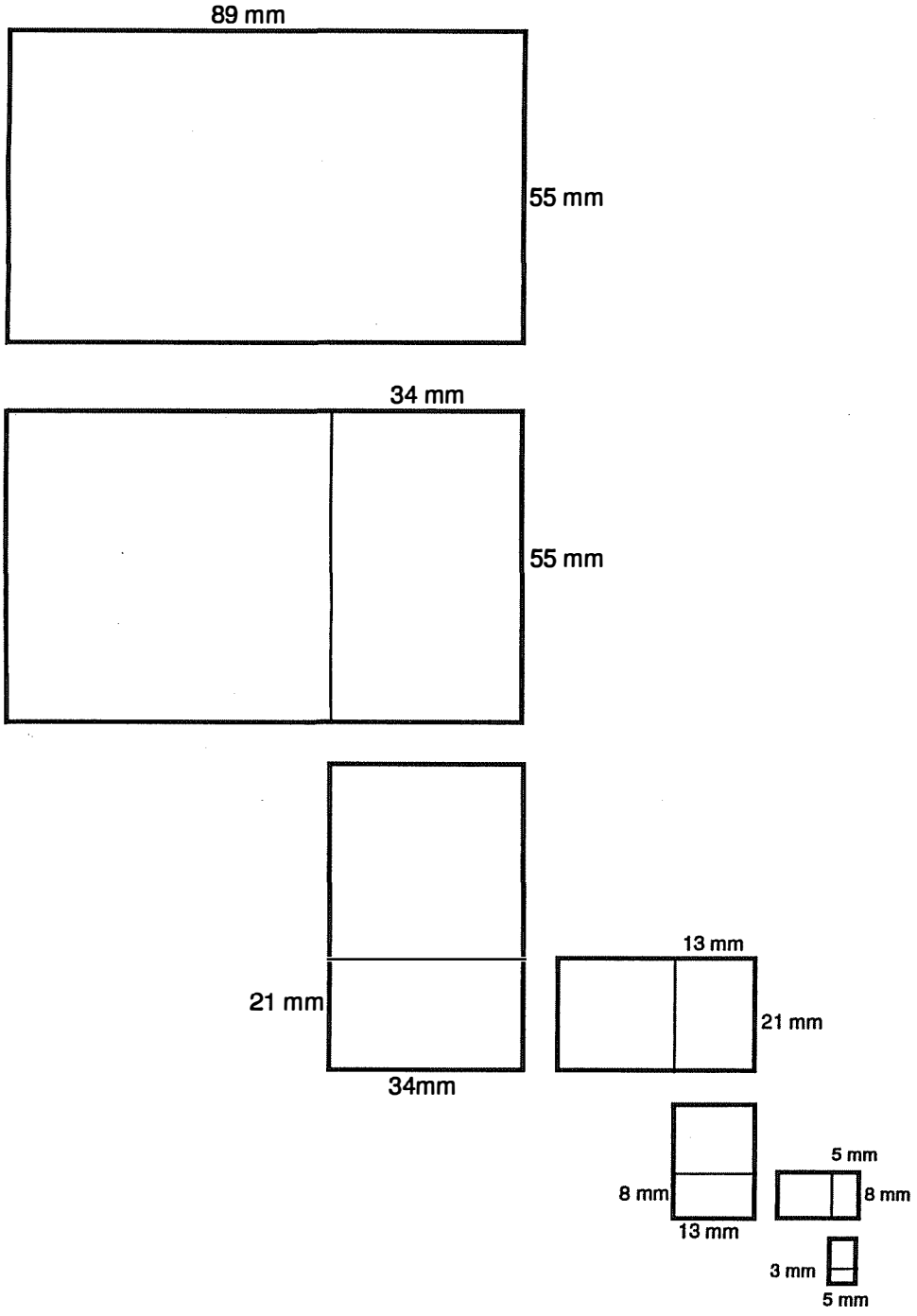
(b) The Fibonacci sequence of numbers is defined to be 1, 1, 2, 3, ... where each succeeding number is determined by adding the previous two numbers. Thus a few terms of the sequence are 1, 1, 2, 3, 5, 8, 13, 21, ...

An interesting aspect to consider is the ratio of consecutive pairs of numbers.

1 : 1 = 1.000
2 : 1 = 2.000
3 : 2 = 1.500
5 : 3 = 1.667
8 : 5 = 1.600
13 : 8 = 1.625
21 : 13 = 1.615
34 : 21 = 1.619
55 : 34 = 1.618
89 : 55 = 1.618
144 : 89 = 1.618

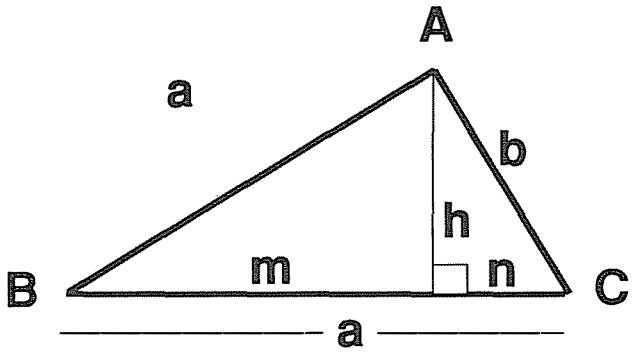
As the numbers increase in size the ratio of consecutive numbers approaches the golden ratio ø.

Consider a rectangle 89mm X 55mm where 89:55 ≈ 1.618.

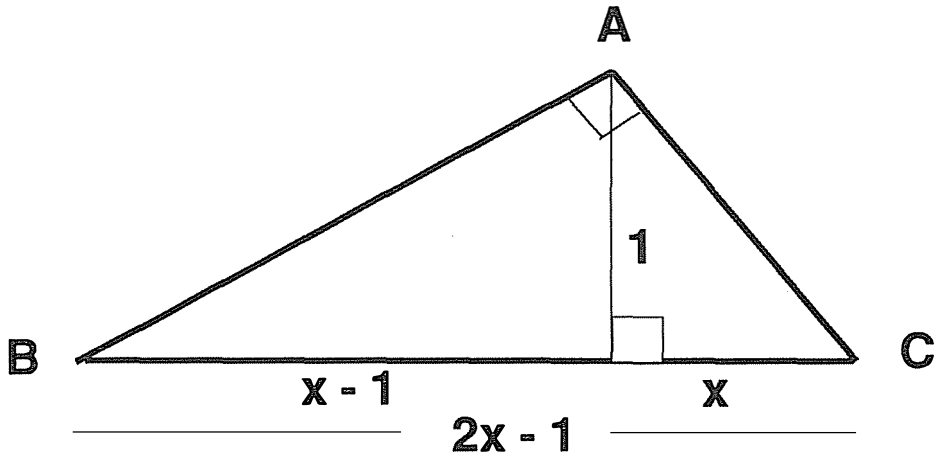


(c) Using right-angled triangles and the Theorem of Pythagoras the following can be obtained from the given diagram:

$m:c = c:a$ or $c^2 = ma$
$n:b = b:a$ or $b^2 = na$
$m:h = h:n$ or h^2



Suppose in the $h^2 = mn$ case that $h = 1$, $n = x$, $m = x - 1$ and $a = 2x - 1$ as below:



From above, since $h^2 = mn$,

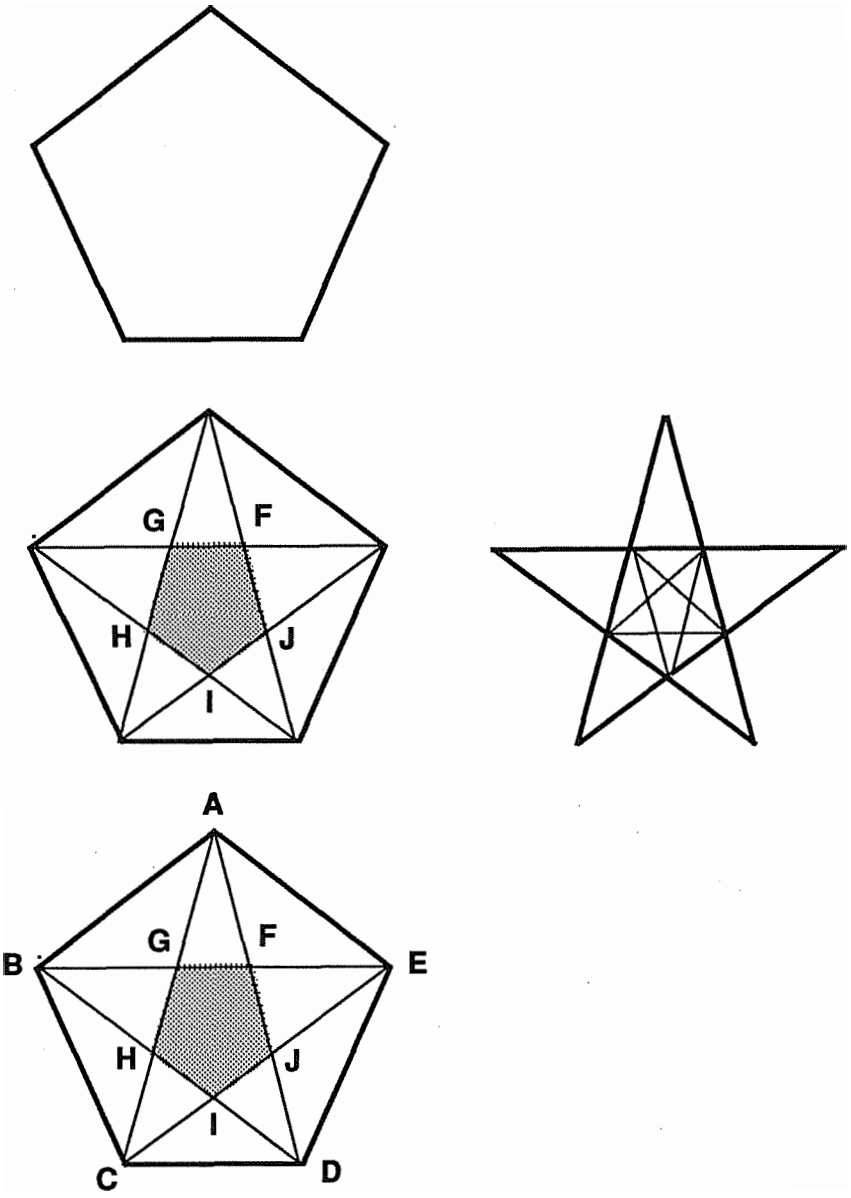
$$(1)^2 = (x - 1)(n)$$

or

$$x^2 - x - 1 = 0$$

and a positive value for x is ø.

(d) Take a regular pentagon and draw in the diagonals, thereby creating a new regular pentagon FGHIJ.



Note that the following ratios approximate ø

$$\frac{AD}{AJ} = \frac{BD}{BI} = \frac{CA}{CG} = \frac{DA}{DF} = \frac{EB}{EG} = \ldots = 1.618$$

$$\frac{AD}{DC} = \frac{BE}{ED} = \frac{CA}{AE} = \frac{DB}{BA} = \frac{EC}{CB} = \ldots = 1.618$$

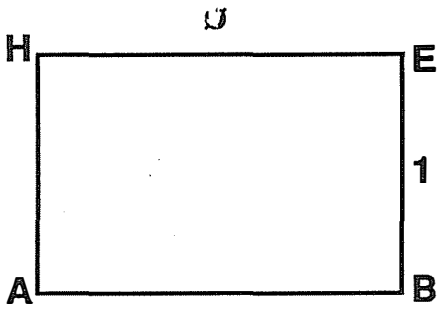
(e) Now $x(x-1) = (1)^2$ leads to $x^2 = x + 1$ or $x^2 - x - 1 = 0$.

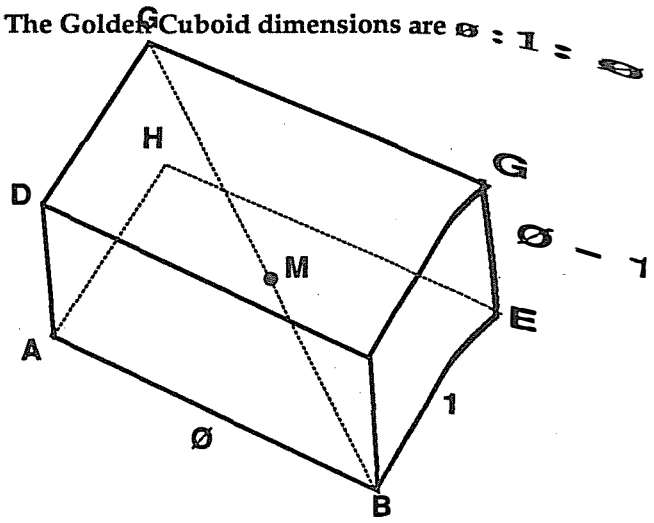
The solution is $x = \phi \approx 1.618$.

Since $x(x-1) = 1$, then $1/x = x - 1$

If $x = \phi$ then $\phi^2 = \phi + 1$
and $\phi^{-1} = 1/\phi = \phi - 1$

The Golden Rectangle dimensions are $\phi : 1$. The Golden Cuboid dimensions are $\phi : 1 : \phi$





The surface area of the Golden Cuboid is $2(\phi)(1) + 2(\phi)(\phi^{-1}) + 2(1)(\phi^{-1})$

$= 2\phi + 2 + 2\phi^{-1}$

$= 2\phi + 2 + 2(\phi^{-1})$

$= 4\phi$

The length of the diagonal BG of the Golden Cuboid is $\sqrt{(\phi)^2 + (1)^2 + (\phi - 1)^2}$

$= \sqrt{(\phi + 1) + (1) + (\phi - 1)^2}$

$= \sqrt{\phi + 2 + \phi^2 - 2\phi + 1}$

$= \sqrt{\phi + 2 + (\phi + 1) - 2\phi + 1}$

$= \sqrt{4}$

$= 2$

The radius of the sphere circumscribing the golden cuboid is

$MB = r = 1.$

The surface area of the sphere is $4\pi r^2 = 4\pi(1)^2 = 4\pi$. But the surface area is 4ϕ.

Thus the ratio of surface area of the golden cuboid to the surface area of the sphere that circumscribes it is

$4\phi : 4\pi$ or $\phi : \pi$

Magic Squares: Another Look!

If you have studied magic squares then you may have noticed the following patterns:

- any sequence of nine numbers can be arranged in the following order to make a "magic square".

8 th	1 st	6 th
3 rd	5 th	7 th
4 th	9 th	2 nd

In case you forgot - each row, column and diagonal must have the same sum.

- other arrangements which work are simple transformations of the above arrangement. That is

6 th	7 th	2 nd
1 st	5 th	9 th
8 th	3 rd	4 th

is the same as

4 th	3 rd	8 th
9 th	5 th	1 st
2 nd	7 th	6 th

and

2 nd	9 th	4 th
7 th	5 th	3 rd
6 th	1 st	8 th

A magic square constructed with the sequence of numbers 3, 6, 9, 12, 15, 18, 21, 24, 27 looks like this:

24	3	18
9	15	21
12	27	6

Of course each row, column and diagonal adds to 45.

What sequence of numbers is used to generate this magic square? What is the magic sum?

	20	
	40	
35		

Solve each of the following magic squares by discovering the sequence of numbers and their arrangement in the square. Write the magic sum.

Easy

		15
	27	
		23

Medium

13		
17		9

Hard

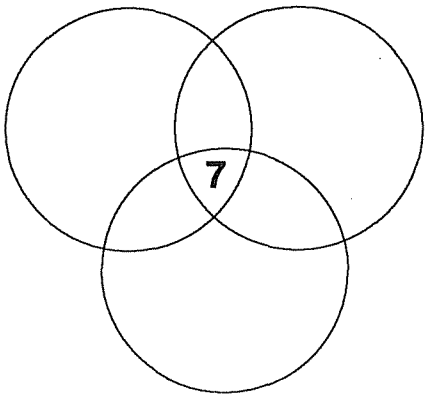
2		7
9		

Extension

Use the numbers from this sequence to fill each of the 7 sections created by this diagram so that the sum in each circle will add to the same total.

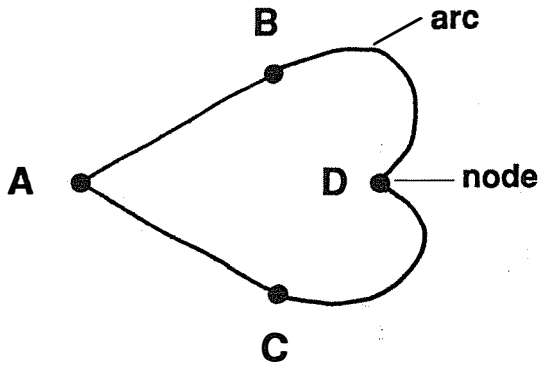
5, 6, 7, 8, 9, 10, 11

A hint is given!



Networks

This diagram is made of nodes and arcs.

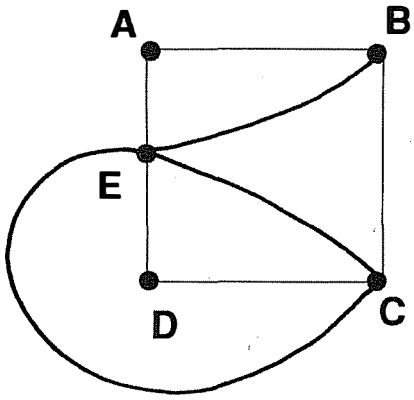


A, B, C, D are nodes

Nodes can be either even or odd

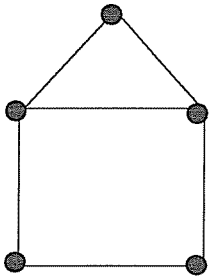
In this diagram nodes A and D are even while nodes B and C are odd. Explain why.

Which nodes are even, which nodes are odd in this diagram?

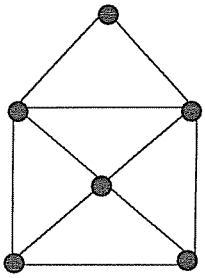


ODD/EVEN	WHY ?
A	
B	
C	
D	
E	

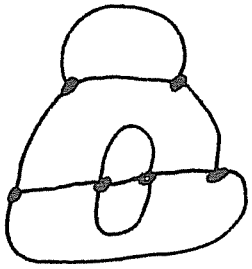
Some of the following diagrams can be traced with one continuous line without lifting the pencil off the paper or retracing any of the arcs.



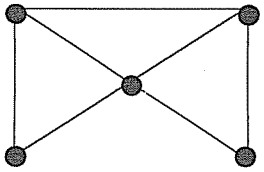
(a)



(b)



(c)



(d)

Complete the chart.

FIGURE	#ODD NODES	# EVEN NODES	Can be drawn with one continuous line ? Yes / No
(a)			
(b)			
(c)			
(d)			

Can you discover the relationship between odd, even nodes and whether the figure can be drawn with one continuous line? Write what you think the relationship is.

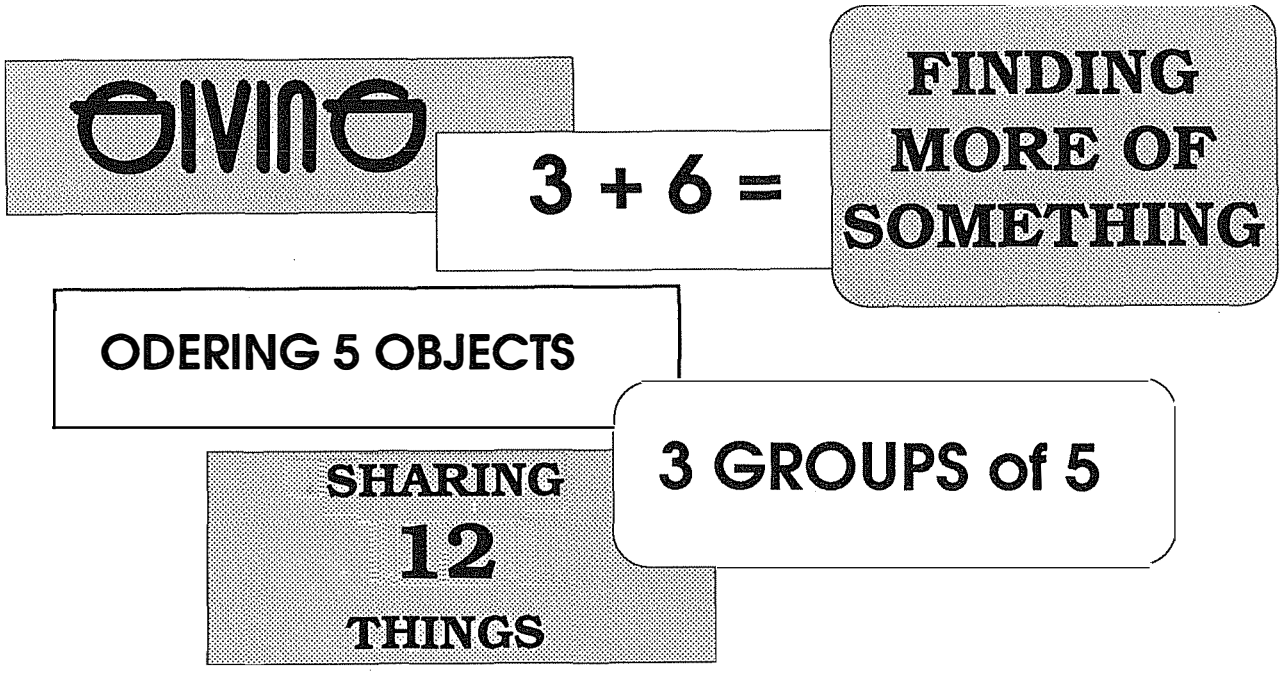
Create your own figures which can be drawn in one continuous line.

Problem Solving at the Puppet Theatre

Using materials commonly found around puppet theatres in primary classrooms young children can be encouraged to create and act out short (or not so short) story type problems.

Encourage children to work in groups of three or four. Allow plenty of time for them to create and plan. Provide sharing time for their production.

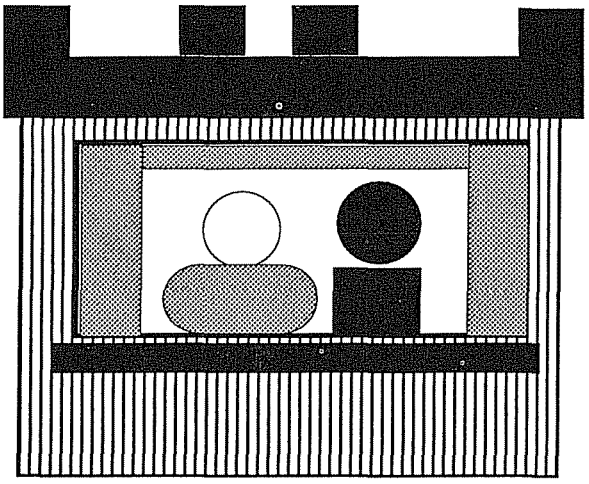
Cards similar to ones listed here can be used to generate story ideas.



Ask the children to create and act out a play which will finish with the posing of a question.

Many problem solving strategies will be used by the children in a natural way. As the teacher, sit back and enjoy the productions.

For older children, these types of activities lead to co-operative story writing.

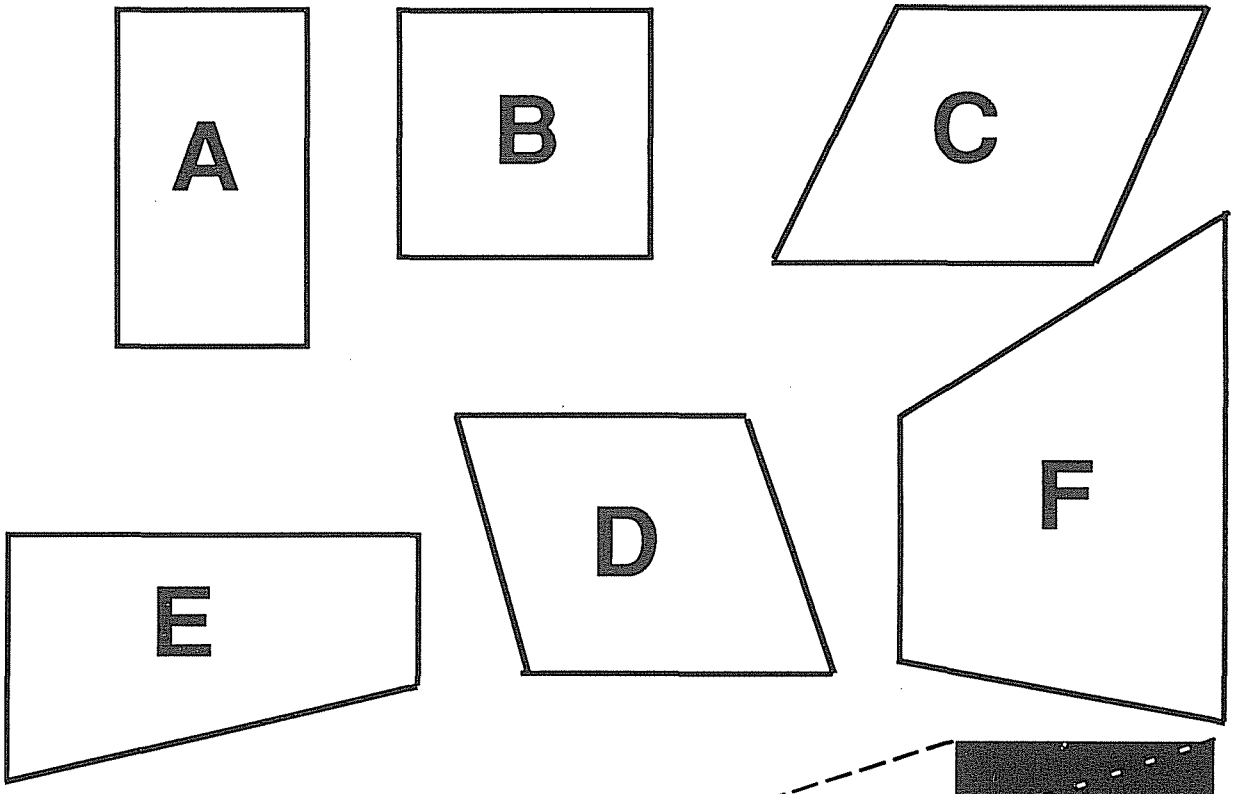


Quadshadows

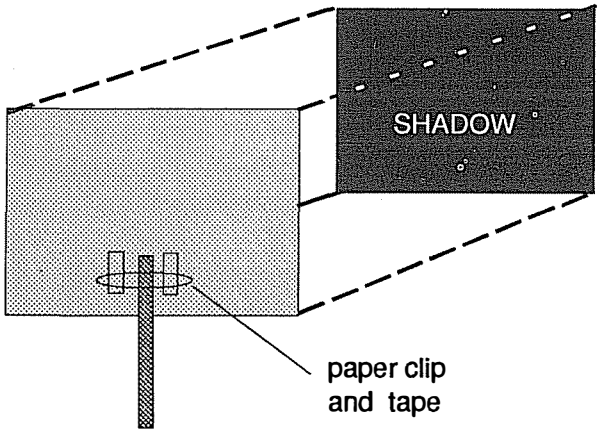
Ideas for this page came from a workshop James V. Bruni of H. H. Lehman College presented many years ago.

Enlarge and cut these 6 quadrilateral shapes out of Bristol board. Label each shape and attach a paper clip handle with tape.

Use each shape to create shadows with the following shapes:



a: square
b: rectangle
c: parallelogram
d: rhombus
e: rapezoid
f: irregular quadrilateral



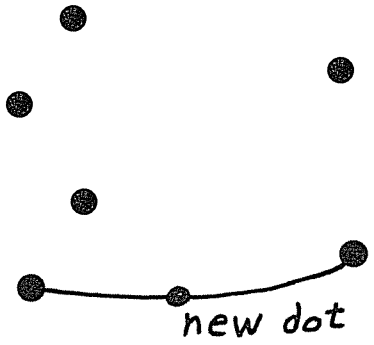
Sprouts and Super Sprouts

Sprouts

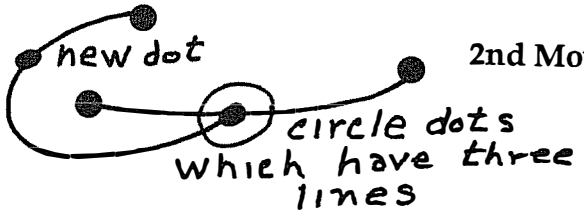
The invention of the game of sprouts has been attributed to Professor John Horton Conway and Michael Stewart Patterson who were at the time working on computer programming theory.

The game can be played with any number of dots but is best played using three dots by beginners.

- This game is for two players.
- A move consists of drawing a line joining one dot to another and then placing a new dot somewhere along the line.
- A line which is drawn can have any shape but it cannot cross itself, or cross a previ ously drawn line.
- No dot can have more than three lines ema nating from it. i.e.
- Players take turns drawing lines, placing dots
- Play continues until no further moves can be made. The last person to move, wins.



!st Move

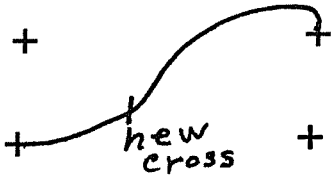


2nd Move

SuperSprouts

This game is played in a similar fashion to sprouts. Instead of using three dots, four crosses are placed on a page.

- A move consists of drawing a line joining the arms of any two crosses, and placing another cross somewhere along the line.
- Once the four arms of a cross have been used that cross becomes closed. (circle closed crosses)
- No line may cross itself or any other line
- Play continues until no further moves can be made. The last person to move wins.

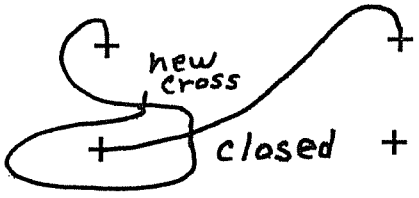


!st Move



2nd Move

Note: it has been said that the game of sprouts must last at least 2n moves where n is the number of starting dots. Prove this.



3rd Move