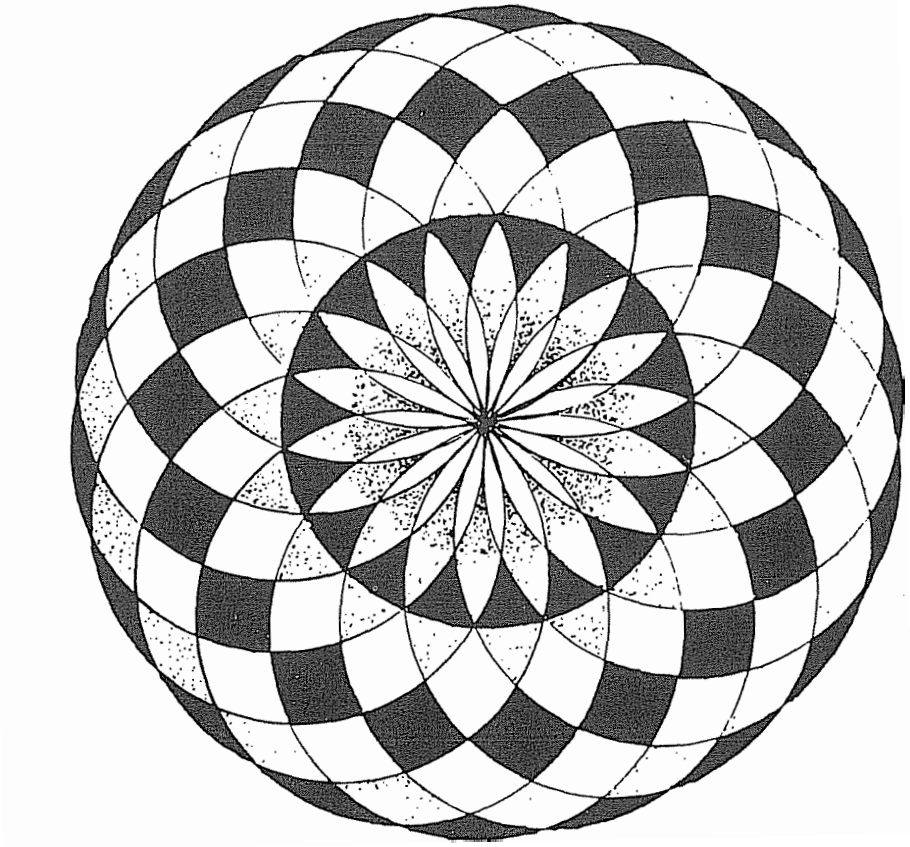
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"Before LOGO"

Prepared By

Ron Ennis, Ron Firth and
Colin Garnham on behalf
of the Pine Ridge Mathematics
Association.

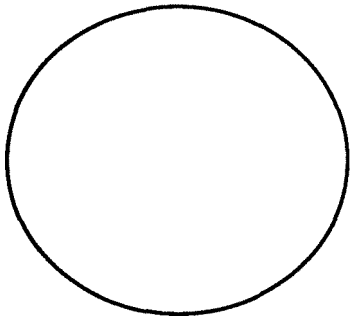
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Nelson, Canada

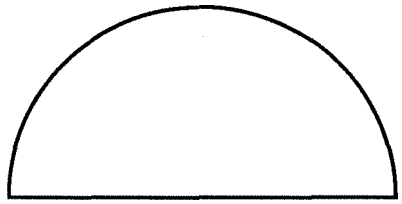
This paper-folding activity will permit your students to create

- an equilateral triangle
- a trapezoid
- a rhombus
- a regular hexagon
- a regular tetrahedron

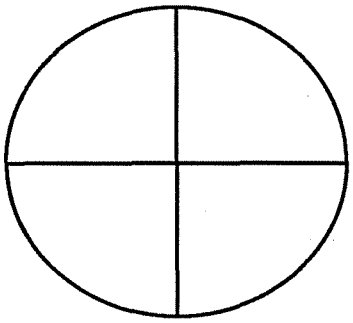
(a) Draw a circle on a blank piece of paper with a radius of 10 cm. Use a pair of scissors to cut out the circle.

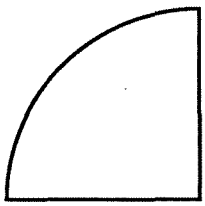


(b) Fold the circle in half creating a semicircle. The fold is the diameter of the circle.

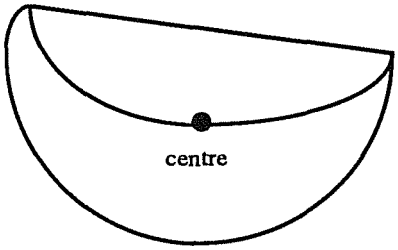


(c) Fold the semicircle in half. When the circle is "Opened Up", it has two diameters interesecting at the centre. Mark the centre with a dot. Remember: The radius is one-half the diameter.

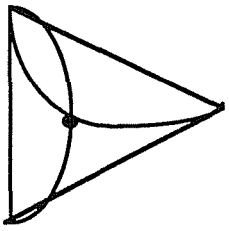




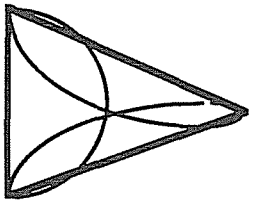
(d) Fold the circle so that a point on the circumference is at the centre of the circle.This fold will give you a chord of the circle.



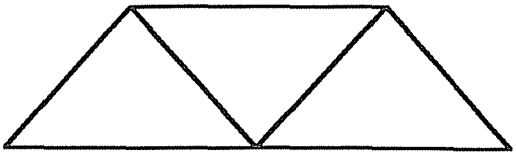
(e) Fold the circle again so that a second point on its circumference is at the centre of the circle and the chord that you form has an endpoint in common with the first chord.



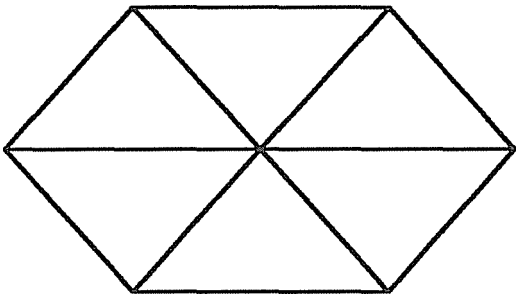
(f) Now fold the circle so that a third point on its circumference is at the centre of the circle. The chord, which you have now formed, has common endpoint with the first chord and a common endpoint with the second chord. You have now formed an equilateral triangle.



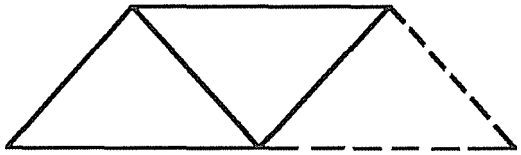
(g) Fold the triangle that you made in (f) so that a vertex is placed at the midpoint of an opposite side. You have now formed a trapezoid.



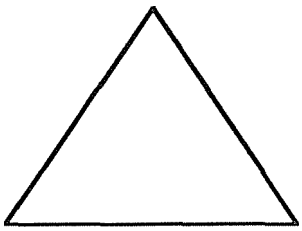
(h) By joining two trapezoids together using tape you can now create a regular hexagon.



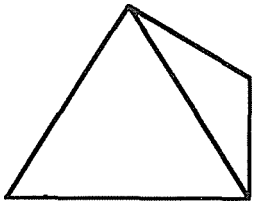
(i) Using the trapezoid of part (g), fold one of the triangles over the middle triangle. You have now formed a rhombus.



(j) Fold the second triangle that forms the rhombus in part (k) over one triangle. You have now formed an equilateral triangle that is similar to the one in part (f).



(k) Unfold the paper that you have in part (j). The paper should be made of four congruent triangles. They can be folded to form the regular tetrahedron shown below. Before you try the folding, make sure you know what a regular tetrahedron is.

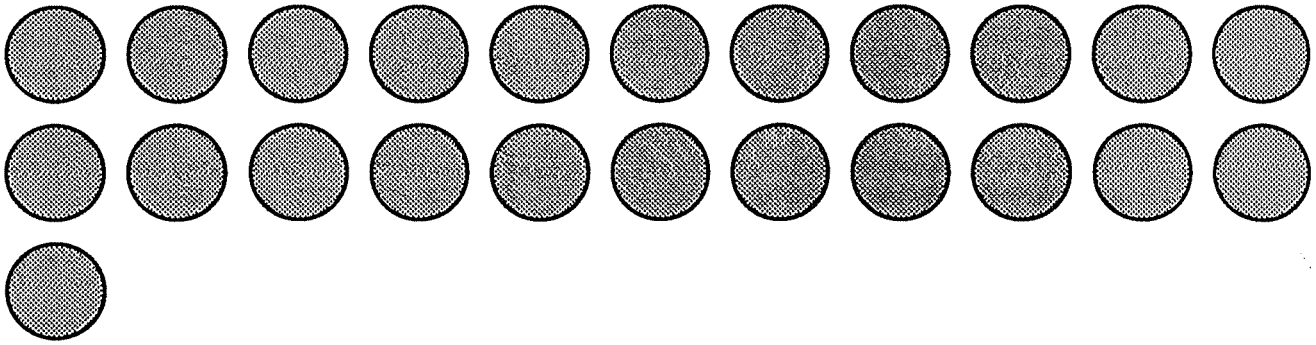


Manipulate the Date

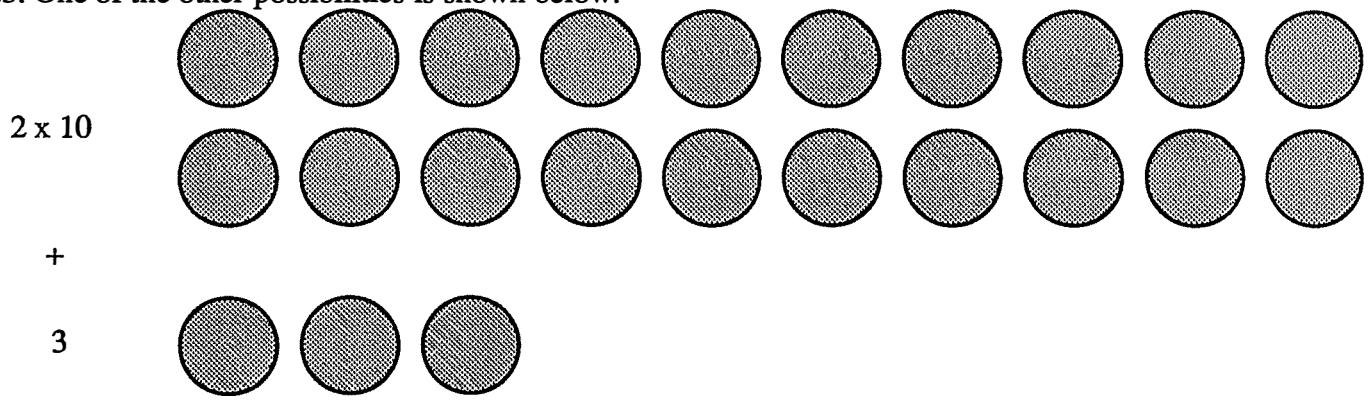
This section will help you to establish a permanent display in your class room that will enable your students to display the day's date. Since the date changes daily, ways in which the date is displayed will also change daily. It also has the added advantage that your students can now review some of their number facts. In other words, their skills with composite numbers, prime numbers, factors and possibly exponents.

To work with this manipulative display, you will need enough counters to display each day of the month. Thus, a maximum of 31 counters will be used by your students on any given day. Before beginning the display, review the idea that all numbers can be classified as prime or composite. A prime number has factors of only one and itself. A composite number can be written as the priduct of two or more prime numbers.

Now that your students have reviewed the basic skills (you will have reviewed the ideas necessary with exponents if they have progressed that far), you can go on to today's date and the pictorial representation. Suppose that today's date is Monday, October 23, 1989 (1989 10 23). Then, one array of counters can be used to display the date as shown below.



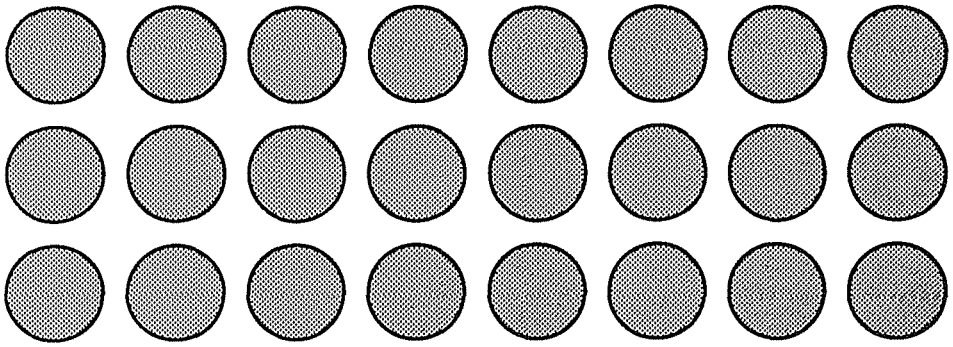
However, some of you more ingenius students may come up with other ways of representing the number 23. One of the other possibilities is shown below.



There are many solutions that your students may find. To encourage "Imaginative" solutions, you may wish to offer a prize for the most creative representation of the week. An example of a composite date is shown on the next page.

Suppose today's date is October 24, 1989 (1989 10 24). Now, your students are working with a composite number. There are a number of ways to show 24. Three of them are shown below.

Using Rectangular Numbers

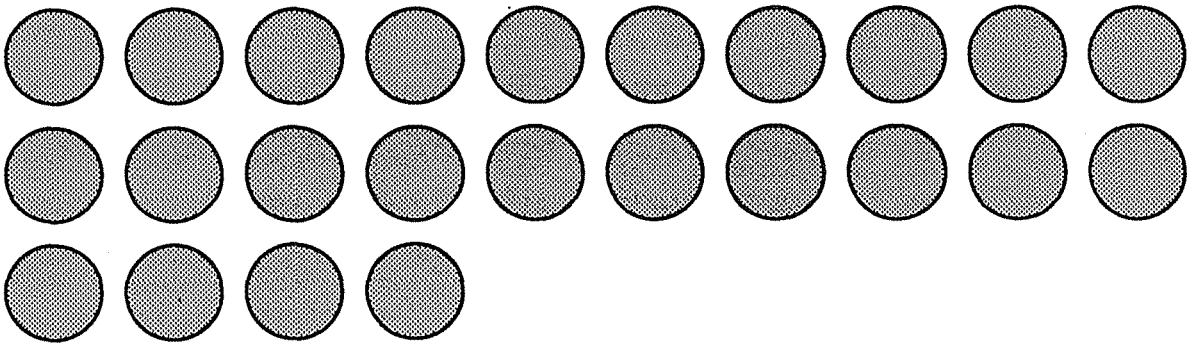


Using Base 10

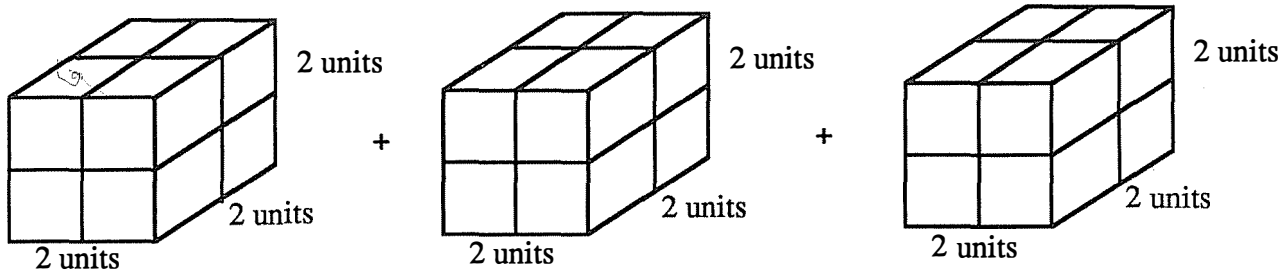
2 x 10

\+

4



As a Sum of Cubes



Problem Solving can also be introduced using this type of material. Have your students make up a problem involving a prime or composite number. Then, have them challenge a friend in the class to solve the problem.

Divingo

There are many games that your students can play in order to reinforce some of their basic skills. The game divingo is one of them. Divingo is a game that can be used to reinforce your students division skills and it can be played with any number of people. To play the game, you will need the following materials.

1. A set of expandable game sheets. Enough game sheets will be needed so that each player will have one sheet for each game played. A number of different game sheet forms should be made by varying the location of the divisors on each sheet.

2. A set of forty cards with digits on them. Each card will have one of the digits from 0 through 9 on it. Each digit will be used exactly 4 times. A deck of playing cards with all the face cards removed can be used as long as it is understood that each tens card represents zero.

To play divingo, you will use the following rules. A sample game sheet is provided on the next page.

1. The cards are well-shuffled and placed face down, in a pile, on a desk. One player turns over the top card and reads the number aloud.

2. Each player then chooses a square on his/her game sheet and writes the numeral in the chosen square. Once the numeral has been recorded, it cannot be moved to a different square.

3. Now, the next card is turned over and the number is read aloud. Once again, the students must record the number in the second square available. After the player has placed both numbers in the squares, they must then write the correct answer to the division question.

4. The player who completes the three division questions, either in a row, column or diagonal is the winner.

Try this game with your students. You may be surprised with the results. To encourage competition among your students, a prize could be offered for the first person to complete this "Tic Tac Toe" game. If your students are finding the game too easy, then you may wish to modify or extend the game as follows.

1. You could use remainders other than zero on your game sheet.

2. You could use a complete deck of cards where any Jack, Queen or King represents a wild card. Any wild card can be assigned any number that the student wishes and then placed anywhere on the game sheet.

3. You may even be able to change the game into a calculator activity if the divisors are changed to two digit numbers and the dividends are changed to three squares instead of the two currently presented.

8) □ □ 0 REM	10) □ □ 0 REM	6) □ □ 0 REM
3) □ □ 0 REM	5) □ □ 0 REM	7) □ □ 0 REM
4) □ □ 0 REM	9) □ □ 0 REM	2) □ □ 0 REM

Mathematics in Comics

The following conversation appeared in a Family Circus comic strip. As you can see, it is a conversation between a father and his son.

Father: Think of a number between 1 and 100. Then, multiply the number you have chosen by 99. Add the digits of your answer.

Billy: Okay. I did it.

Father: The answer to this problem is on the back of my shirt. The father's shirt has an 18 on the back.

Now, let's assume that Billy tries the trick on his mother. Billy tells his mother that the answer is on the back of Dad's shirt. By this time, Billy's father has changed his shirt. The number on the back of his shirt is now 7. Billy's mother informs him that he is wrong and that his trick did not work.

Poor Billy. His father did not make it clear that the sum of the digits is always 18. We hope that Billy will use his problem solving skills and check the problem with other numbers to see how the trick works. In other words, he did not "Look Back" at the solution in order to generalize the problem. If Billy had looked at the problem more closely, he would have realized that the sum of the digits, after multiplying any number between 1 and100 by 99, will have the sum of 18. The trick does not depend on the number on the back of Billy's father's shirt.

This "Magic Trick" has many applications and extensions. Calculators and computers can be used in conjunction with activities of this kind. A few of these applications and extensions are shown below.

1. You can use this trick with your class to promote using a calculator and then mental calculations to check the reasonableness of a calculation.

   Your students can use a calculator to calculate 57 x 99. The solution is 5643.
   Then, your students can add the digits 5 + 6 + 4 + 3. Since the sum is 18, the answer is reasonable.

2. You can also use this "Magic Trick" to provide exercises for your students in searching for a pattern. Encourage your students to look at a number of examples for finding the sum of the products for each of the following cases.

   (a) Multiply any number between 1 and 9 by 9. What is the sum of the digits in the product?
   (b) Multiply any number between 1 and 1000 by 999. What is the sum of the digits in the product?

3. For those of you who are teaching your students how to program a computer, a program can be written that can check the predictions in part 2. Remember, as your students search for a pattern, they are trying to predict the pattern. In using a computer program, the computer will check the sum of the digits in any product and check whether the results concur with the predicted pattern. This is where the computer can be of utmost advantage to you and your students. A student, and for that matter most teachers, would rebel at the thought of having to multiply, and then add the digits of each product, all the numbers between 1 and 1000 by 999. The computer will handle this job with ease and would verify or reject the prediction in a matter of seconds.

4. Your students must be careful as they arrive at their prediction of patterns. Some patterns cannot be extended past the found pattern without leading to a false prediction as shown below.

A number between 1 and 100, when multipled by 99, has the sum of the digits equal to 18. However, when the numbers between 100 and 1000 are multiplied by 99, the sum of the digits in a product does not always give a sum of 18. The following chart shows you some palindromes that do not have the required sum.

Number	Product	Sum of the Digits
101	9999	9 + 9 + 9 + 9 = 36
111	10 989	1 + 0 + 9 + 8 + 9 = 27
121	11 979	1 + 1 + 9 + 7 + 9 = 27
131	12 969	1 + 2 + 9 + 6 + 9 = 27

Palindromes are not the only numbers between 100 and 1000 with this property. In fact, there are many other numbers that have the same property. A few of these are shown in the following table.

Number	Product	Sum of the Digits
760	75 240	7 + 5 + 2 + 4 + 0 = 18
761	75 339	7 + 5 + 3 + 3 + 9 = 27
762	75 438	7 + 5 + 4 + 3 + 8 = 27
763	75 537	7 + 5 + 5 + 3 + 7 = 27

As you can see, a computer will find these exceptions rapidly if a prediction has been made that is not correct. However, the student would be seriously hindered if asked to show one example where the pattern fails. But, a new pattern does emerge that should be tried. Do you always get a multiple of 9 as the sum of the digits when you multiply by 9? or 99? or 999?

Mathematics and the Calendar

Julius Caesar, recognizing that the Roman calendar was out of step with seasons by almost two months, decreed that the year 46 BC would be 445 days in length. This "Year of Confusion" put the years back in step with the seasons. He also ordered that every fourth year would have an extra day so that this "Year of Confusion" would not be repeated. Thus, the first leap year was created.

However, by the sixteenth century, the calendar was once again running 13 d behind schedule. In the year 1582, Pope Gregory XIII decreed that the following year should be shortened by ten days and that three leap years would be omitted every four centuries.

The Gregorian Calendar is still used today. This is because the earth rotates about its axis once every 24 h. It also rotates about the sun once in a solar year. This takes 365 d, 5 h, 48 min and 46 s. In the Gregorian Calendar, one year is 365 d, 5 h, 49 min and 12 s or

365.2425 days

You can write this number as follows.

$$365 + 0.25 - 0.01 + 0.0025$$

or

$$365 + \frac{1}{4} - \frac{1}{100} + \frac{1}{400}$$

The fractions indicate how to adjust a regular year of 365 d using leap years.

$$365 + \frac{1}{4} - \frac{1}{100} + \frac{1}{400}$$

The fractions all indicate something different about the formula.

The $\frac{1}{4}$ indicates that you will add one extra day every four years.

The $\frac{1}{100}$ indicates that every 100th year is not a leap year.

The $\frac{1}{400}$ indicates that every 400th year is a leap year.

Leap years can be determined using divisibility tests. Remember, a number is divisible by 4 if the number formed by the last two digits is divisible by 4. A number that ends with two zeros is, therefore, always divisible by 4.

A number is divisible by 400 if the number ends in two zeros (in other words is divisible by 100) and, when the last two digits of the number have been removed, the last two digits are now divisible by 4. These facts about divisibility can now help you to determine which numbers are divisible by 400 and, as a result, which years in our calendar will be leap years. Some examples are shown below.

1. The year 1952 was a leap year because 52 is divisible by 4.
2. The year 1600 was a leap year because 1600 is divisible by 4.
3. The year 1700 is not a leap year because the number is divisible by 100 but not by 4 when the two zeros are dropped.
4. The year 1990 is not a leap year because 90 is not divisible by 4.
5. When is the next leap year?

Facts about leap years and how leap years are found can help you to integrate other parts of the curriculum into your mathematics course. As you know, there is a trend towards trying to integrate other areas of study into the mathematics course. Thus, you can now have your students invent a planet, or use a planet in the solar system, in order to write a short story about the calendar year. One such story is shown below.

Martian Calendar

On Mars, suppose that for every 688.3161 revolutions the planet makes about its axis, the planet will make exactly one trip around its sun. Now, as you can see, I can approximate 688.3161 to be about 688.3222 d in one Martian calandar year. Therefore, one Martian year can be expressed in the following form.

$$688 + 0.333 \ldots - 0.0111 \ldots$$

or

$$688 + \frac{1}{3} - \frac{1}{90}$$

Now, as you can see, the Martian calendar will have 688 d in one Martian year. Every third year will be a leap year because there will be one extra day. Every ninetieth year would not be a leap year so that the calendar keeps up with the Martian seasons.

Mathematically this becomes obvious because leap years can be found using divisibility tests. For example, leap years can be found by dividing by three. A number is divisible by 3 if the sum of the digits is a multiple of 3. The year that is not a leap year is a multiple of 90. Remember, a number is divisible by 90 if it is divisible by 10 and the sum of its digits is a multiple of 9.

PI: A Calculator Activity

You can use your calculator to estimate a value for π by finding approximations for the following expressions.

$t_1 = 2 \div [\cos(\frac{90°}{2})]$

$t_2 = 2 \div [\cos(\frac{90°}{2})\cos(\frac{90°}{4})]$

$t_3 = 2 \div [\cos(\frac{90°}{2})\cos(\frac{90°}{4})\cos(\frac{90°}{8})]$

$t_4 = 2 \div [\cos(\frac{90°}{2})\cos(\frac{90°}{4})\cos(\frac{90°}{8})\cos(\frac{90°}{16})]$

$t_5 =$

$t_6 =$

$t_7 =$

What limit do you think each successive term is approaching? These results come from the "Infinite Product" that was first discovered by the French mathematician Francois Viete in 1592.

$$\frac{2}{\pi} = [\cos(\frac{90°}{2})\cos(\frac{90°}{4})\cos(\frac{90°}{8}) \ldots]$$

This formula was discovered while calculating the perimeter of regular polygons with 4 sides, 8 sides, 16 sides, 32 sides etc. inscribed in a unit circle. The successive terms of this sequence approach the limit π much more quickly than the following infinite sequence.

$$\pi = 4[1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \ldots] \qquad \text{(Gregory, 1691)}$$

Your computer programming students could be asked to write programs that will find an approximation for π using the infinite product and the infinite series listed above.

Parts of words are sometimes combined to produce hidden messages. The following are such words.

Why would you like this word?

Take the first $\frac{1}{2}$ of HOLD. _____

Take the first $\frac{1}{5}$ of LITERATURE. _____

Take the first $\frac{1}{8}$ of DRAMATIC. _____

Take the last $\frac{1}{4}$ of SATURDAY. _____

Who is some people's hero?

Take the first 20% of HELLO. _____

Take the last 75% of BULK. _____

Take the first 25% of HOMETOWN. _____

Take the last 60% of BEGAN. _____

What is the home of the Toronto Blue Jays called?

Take the first $\frac{2}{3}$ of SKI. _____

Take the first 25% of YOUR. _____

Take the first $\frac{1}{2}$ of DOOR. _____

Take the last 40% of CHROME. _____

Concentration

Most of you have seen how to play concentration on television. Contestants are asked to match prizes, in pairs, so that they win the prize and, more importantly, reveal pieces of the puzzle. The puzzle is usually a pictorial representation of a well-known phrase.

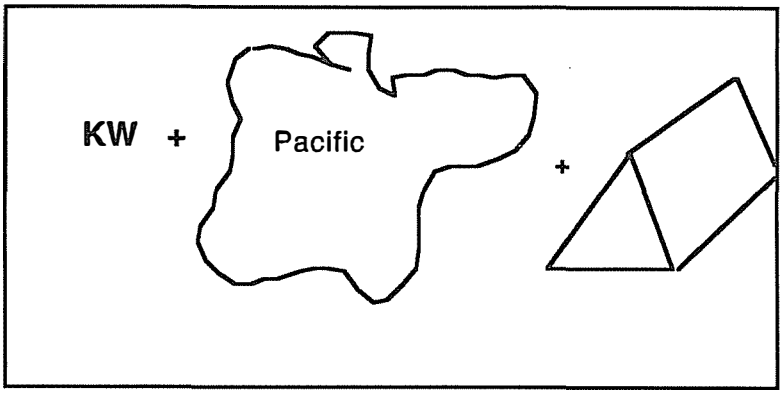
This game will be played the same way. Shown on the next page are a number of different puzzles. Each puzzle represents a well-known mathematical phrase or term. The object of this game will be to match up the correct number with the correct expression.

The expressions and the solutions are shown below on the 16 squares. The squares will fit exactly over the puzzles. Cut out the squares, label them 1 through 16, and place them over the puzzle. When a match is made, the pieces are taken away to reveal parts of the puzzle. The winner is the one who guesses the puzzle first.

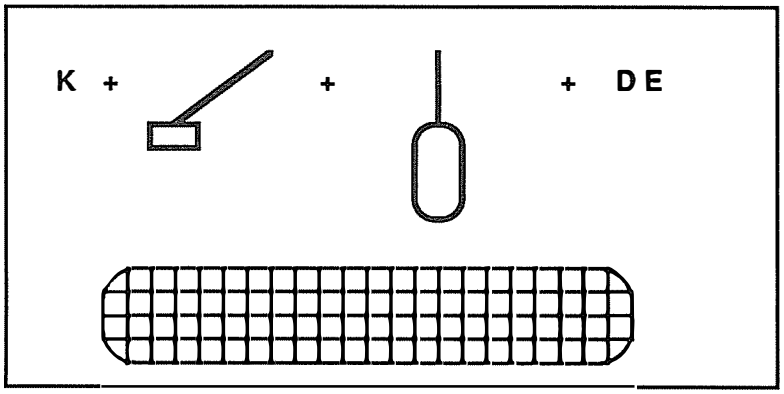
18	11	94	50
22 - 11 + 7	54 ÷ 18 + 8	99 - 55 ÷ 11	100 x 5 ÷ 10
0	3	15	6
33 - 15 x 2	17 - 16 ÷ 8	44 + 12 - 50	66 ÷ 33 - 2

It is important to note, however, that your students will have these expressions memorized after a short period of time. Thus, you will not only have to change the puzzles as your students solve them, but you will also have to change the expressions as your students solve them as well.

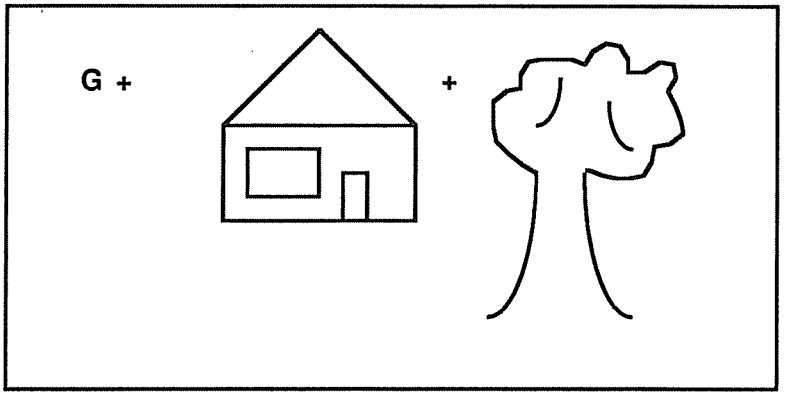
You may wish to make the expressions more challenging as your students gain experience and knowledge throughout the year.



Solution: KW + OCEAN + TENT = Quotient



Solution" K + HOE + OAR + DE + NET = Coordinate



Solution: G + HOME + TREE = Geometry

Newspaper Mathematics

Each week, a challenger similar to this one is published in the Saturday Edition of most newspapers. They can be used to review your students skills with addition and subtraction as well as improve your students problem solving processes and skills. Your students will soon learn that they will be looking for large or small sums as these types of challengers have a limited number of combinations that can be used.

See if your students can find the correct combinations. You may even be able to start a challenger club in your school. The members will bring in completed challenger puzzles each Monday morning and explain how they arrived at the correct answer. Shown below is an example of a challenger puzzle. To complete the puzzle, you will use the following information.

1. Fill each square with one number form 1 through 9.

2. Horizontal squares should add to the number on the right.

3. Vertical squares will add to the number at the bottom.

4. Diagonal squares should add to the square in the top right and the bottom right.

				16
		8		27
8				12
	4			25
			5	12
26	12	13	25	16