ABACUS

A PUBLICATION OF THE
ONTARIO ASSOCIATION for MATHEMATICS EDUCATION

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WRITE TO THE SECRETARY TREASURER

1975 - 76 EXECUTIVE

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From the Editor

We are starting the third year of the Abacus. During the first two years Brock Rachor gave us a very good start and the new editor hopes that this publication will continue to be interesting and stimulating to ALL involved in mathematics education. We look forward to the variety of material that YOU the reader and mathematics educator always thought about publishing ... so do not hesitate, but forward your articles, problems (new and not so new), questions etc. Your contributions will make this publication a success.

I hope that you had an enjoyable summer, and are now full of energy and ideas to make 1975 - 76 a successful year in mathematics education.

Andy Czempinski

Please send all correspondence to:

Editor of Abacus

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Interested in Comparative Education?

The 4th InterAmerican Conference on Mathematics Education will take place in Caracas, Venezuela, December 1-6, 1975, and will be focused on mathematics at the elementary and secondary levels. For information, write to:

Claude Goulin, Ass. Prof.,
Faculty of Education,
Universite Laval,
Quebec, P.Q.
G1E 7P4

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Teaching is ... "an art so great and so difficult to master that a man or a moman can spend a long life at it, without realizing much more than his limitations and mistakes, and his distance from the idea " ...

W. L. Phelps

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Plan Ahead! Come to Detroit Next Year!

The National Council of Teachers of Mathematics will have its regional Name-of-Site Convention in Detroit on March 4-6, 1976. At this time the program committee is formalizing the convention program. A major emphasis is being placed upon the use of the calculator in the classroom. There will be speakers addressing the topic of calculators for all levels of K-12 Mathematics Curriculum.

Over two hundred different sessions are being planned. Some major sessions are listed below:

OPENING GENERAL SESSION

Phillip S. Jones
University of Michigan

Topic: "1876 ± 100"

ELEMENTARY GENERAL SESSIONS

Joseph N. Payne
University of Michigan

Topic: "What Changes Should Come in the Curriculum as We Move to the Metric System."

Lola Jane May
Winnetka Public Schools
Winnetka, Illinois.

Topic: "Mathematical Left Overs"

SECONDARY GENERAL SESSIONS

John C. Egsgard
Twin Lakes Secondary School
Orillia, Ontario

Topic: "Proofs That Euclid Did Not Have."

Zalman Usiskin
University of Chicago

Topic: "The Teaching of Applications of Mathematics."

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My only timepiece is a wall clock. One day I forgot to wind it and it stopped. I went to visit a friend, whose watch is always correct, stayed a while, and returned home. There I made a simple calculation and set the clock right. - How did I do this when I had no watch on me to tell how long it took me to return from my friend's house?

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A challenge to some of your students.

The following two problems are from Mathematics Olympiad in Europe bout 10 years ago. Solutions will be published in the next issue.

- If a, b and m are positive integers such that $a^2 + 2mb^2$ is the square of a positive integer, then $a^2 + mb^2$ is the sum of squares of two positive integers.

- How many digits has the number $2^{2^{16}} + 1$?

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You may try this one as well

- A Test of Divisibility by 11

It is not always necessary to test divisibility by actual division. We already know that if the digit sum of a number is divisible by 9, the number is divisible by 9. You also know that a number ending in 0 is divisible by 10; a number ending in 5 or 0 is divisible by 5; but do you know this simple test of divisibility by 11?

Add the digits in even positions (second, fourth, and so on), and add the digits in odd positions (first, third, and so on). If the difference between sums is 0 or a multiple of 11, the number is divisible by 11. Otherwise it is not.

Let us introduce the term modulus, which is similar to "remainder". Thus 18 has a remainder of 7 when divided by 11, and 18 = 7 mod 11 ("seven modulo eleven") = -4 mod 11. The digits 0,1,2,...,9 are, of course, 0,1,2,...,9 mod 11; but 0,10,20,...,90 are, by actual division, 0,10,9,...,2 mod 11 = 0,-1,-2,...,-9 mod 11; and 0,100,200,...,900 are again 0,1,2,...,9 mod 11, and so on.

The sum of two numbers has the same modulus as the sum of the moduli of the numbers, so a given number:

$$N = a + 10b + 100c + 1000d + \ldots;$$
$$N \bmod 11 = a \bmod 11 + 10b \bmod 11 + 100c \bmod 11 + 1000d \bmod 11 + \ldots$$
$$= a \bmod 11 - b \bmod 11 + c \bmod 11 - d \bmod 11 + \ldots$$
$$= a \bmod 11 + c \bmod 11 + \ldots - (b \bmod 11 + d \bmod 11 + \ldots)$$

Since a,b,c,d, ... are the digits of N starting from the right end, our test of divisibility by 11 is correct.

Now tell me:

(a) If 37,a10,201 is divisible by 11, what is the digit a?
(b) If $[11(492 + x)]^2 = 37,b10,201$, what are digit b and number x?

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SEND YOUR SOLUTIONS TO THE EDITOR

Master Teachers ...

D.A. Johnson & G.R. Rising in Guidelines for Teaching Mathematics recommend these questions for self-evaluation.

Do my students know what I am trying to accomplish and why?

Do I do something each day to stimulate student interest in Mathematics?

Do I ask questions that require reflective thinking?

Do I ask students to give reasons for their answers?

Do my students feel secure enough to ask questions?

Is my classroom attractive and conducive to learning?

Are my assignments reasonable and purposeful?

Do I vary the requirements and assignments for different individuals?

Do I use materials outside of the text frequently?

Do my students know the basis for their grades?

Do I give instruction in how to learn mathematics?

Are my tests based on my objectives?

Do I provide ample time and instruction for the mastery of key concepts and skills?

Do I listen to students so that I know their problems and needs?

Have I used library and department facilities adequately?

Do I relate new ideas to past experiences?

How well do I know each of my students?

Do I encourage students to respect each other's opinions?

Do I ever attack new problems with my students?

Do I point out applications of and ways to transfer mathematical knowledge?

Do I encourage students to pursue original ideas, solutions or proofs?

Do I use illustrations, examples, devices that add meaning to concepts?

Do I present a personality and appearance that my students accept as desirable?

Do I use a variety of tests to improve my instruction as well as to measure the learning of my students?

Are routine activities administered efficiently?

Do I exercise care to present mathematical ideas correctly?

When did I last do anything new or different?

Have I learned any new ideas recently?

Have I shared with another mathematics teacher to improve our instruction?

Do my students appreciate mathematics more now than they did before they began my course?

Do I show my students the same courtesy I expect from them?

Do I have answers for my students and myself to justify what I am teaching?

The following Letter to the Editor appeared in the Toronto Globe and Mail, Sept. 9, 1975. Your comments are invited.

When one undertakes to comment on the teaching of mathematics, one should at least have a clear idea what mathematics is.

Mathematics is not arithmetic.

Adding, subtracting, multiplying, and dividing is arithmetic, not mathematics.

Adding, subtracting, multiplying and dividing bears about the same relationship to mathematics as the carpenter's hammering and sawing bears to architecture. Is the teaching and practice of architecture adversely affected by the development of power saws?

Neither is the teaching and practice of mathematics adversely affected by the development of the pocket calculator. The calculator has a beneficial effect on the study of mathematics. By the removal of tedious, dull and now unnecessary arithmetic drudgery, it permits attention to be focused on the structure and meaning of the problem rather than on the trivial details of computation. The computation cannot be avoided, of course, but if it can be made easier, quicker, and more accurate in one fell swoop, why not?

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Probability anyone?

An examination in mathematics consists of 45 questions, from which the candidate draws 3 at random. The candidate receives an A if he answers all three questions correctly; a B if he answers two questions correctly; a C if he answers one question correctly and F if he does not answer any question correctly. A candidate is sufficiently prepared to answer correctly 2/3 of all questions. What is the probability that he would receive at least a C standing? What is the probability that he would receive an A standing?

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$$123456789 = 100$$

There is only one way to insert 7 plus and minus signs between the digits on the left side to make the equation correct:

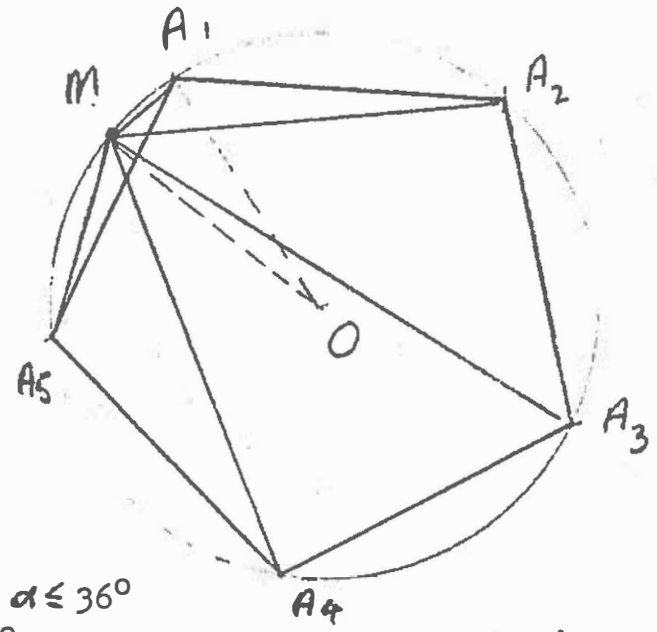
$$1 + 2 + 3 - 4 + 5 + 6 + 78 + 9 = 100$$

Can you do it with only three plus or minus signs?

Can you or your students suggest an alternative solution to this problem?

Given: circle, centre O and radius r, and inscribed pentagon $A_1A_2A_3A_4A_5$. Point M is on the minor arc A_1A_5.

Show that: $MA_1 + MA_3 + MA_5 = MA_2 + MA_4$



Solution:

Let $\angle MOA_1 = \alpha$

and assume that $\alpha \leq 36^\circ$

(Note: if $\alpha \geq 36^\circ$ the reasoning is analogical)

$\because \angle A_iOA_{i+1} = 72^\circ \quad (i = 1,2,\ldots)$

$\therefore \angle MOA_2 = 72^\circ + \alpha$

$\angle MOA_3 = 144^\circ + \alpha$

$\angle MOA_4 = 144^\circ - \alpha$

$\angle MOA_5 = 72^\circ - \alpha$

and Chord $MA_1 = 2r\sin\frac{\alpha}{2}$ Why?

Chord $MA_2 = 2r\sin(36^\circ + \frac{\alpha}{2})$

Chord $MA_3 = 2r\sin(72^\circ + \frac{\alpha}{2})$

Chord $MA_4 = 2r\sin(72^\circ - \frac{\alpha}{2})$

Chord $MA_5 = 2r\sin(36^\circ - \frac{\alpha}{2})$

Let $x = MA_1 + MA_2 + MA_3 - MA_4 - MA_5$

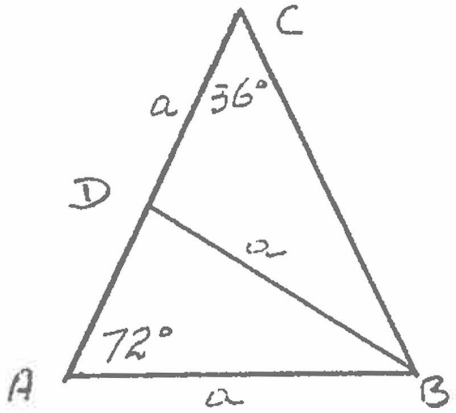
thus $x = 2r\left[\sin\frac{\alpha}{2} + \sin(72^\circ + \frac{\alpha}{2}) + \sin(36^\circ + \frac{\alpha}{2}) - \sin(36^\circ - \frac{\alpha}{2}) - \sin(72^\circ - \frac{\alpha}{2})\right]$

but $\sin(72^\circ + \frac{\alpha}{2}) - \sin(72^\circ - \frac{\alpha}{2}) = 2\sin\frac{\alpha}{2}\cos 72^\circ$

and $\sin(36^\circ + \frac{\alpha}{2}) - \sin(36^\circ - \frac{\alpha}{2}) = 2\sin\frac{\alpha}{2}\cos 36^\circ$

thus $x = 2r\sin\frac{\alpha}{2}(1 + 2\cos 72^\circ - 2\cos 36^\circ)$

Now consider ΔABC, where $\angle A = \angle B = 72^\circ$



and AC = BC = 1, AB = a.

Construct the bisector BD of $\angle B$; it follows that ΔABC and ΔBDC are isosceles, i.e. AB = BD = DC = a.

thus $\cos 36^\circ = \frac{BC}{2BD} = \frac{1}{2a}$

$\cos 72^\circ = \frac{AD}{2AB} = \frac{1-a}{2a}$

and $\cos 36^\circ - \cos 72^\circ = \frac{1}{2a} - \frac{1-a}{2a}$

$= \frac{1}{2}$

that is $x = 2r\sin\frac{\alpha}{2}(1 - 2\cdot\frac{1}{2})$

$= 2r\sin\frac{\alpha}{2}\cdot 0$

$= 0$

and $MA_1 + MA_3 + MA_5 = MA_2 + MA_4$ Q.E.D.

Your comments are invited on:

Statement of Aims and Objectives of Secondary School Mathematics Education.

Preamble: It is recognized that mathematics is a different "thing" to different people and that different people have different needs for mathematics; we, therefore, base the aims for our mathematics curriculum and program upon this promise.

Aims:

1. To develop in ALL students a mathematical literacy, a basic body of knowledge of mathematics useful to the individual and to his society.

2. To make all students aware that mathematics is a part of the total scheme of education and that it pervades all other human endeavours.

3. To stimulate and develop the student's curiosity so that he will recognize the use of mathematics and its inter-relation with other fields of human endeavour and, eventually, will be able to use mathematical methods and knowledge:

   i to understand natural phenomena,
   ii to investigate, interpret and make decisions in human affairs,
   iii to prepare for an avocation in which he utilizes mathematics as a tool,
   iv to understand how mathematics as a science and as an art, contributes to the cultural heritage and progress of society,
   v to communicate mathematical ideas clearly and correctly to others

4. To motivate students to pursue the study of mathematics in order to develop and create new mathematical concepts and ideas.
5. To provide a learning - teaching atmosphere of mutual trust and respect between students and teachers in order that learning and teaching is at an optimum and gives the maximum satisfaction to both educand and educator.

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Preamble: In an attempt to attain the proposed aims, we recognize that the instruction of mathematics today must be more flexible and adaptable to change than in the past, if it is to meet the increasing demands being made of the mathematical competence of our students, hence our objectives are:

Objectives:

1. To organize a program of instruction so that students have opportunities to learn the basic principles of mathematics useful to all. Within this program they should learn:

   i the basic principles of computation and estimation,
   ii the practical concepts of geometry (mathematics of space),
   iii the practical concepts of percent and its applications,
   iv the rudiments of "mathematics machines",
   v the basic concepts of statistics and probability.

2. To provide a program of instruction that will allow all students to learn:

   i how to analyse a problem situation,
   ii how to present the solution in a neat and logical format,
   iii how to examine critically their solution.

3. To develop in students the ability to apply the logic and methods of mathematics to distinguish fact from opinion, relevant from irrelevant, and experimental observation from proven theorem.

4. To stimulate students' curiosity and desire for a continuity of learning and exploring new ideas in order to create new ideas in mathematics. And to develop the reading skill, motivation and study habits essential for independent learning of mathematics.

5. To provide both students and teachers with the opportunity to challenge their own potential in mathematics beyond their minimum capabilities.

Conclusion: We recognize that these aims and objectives must in turn fit into the still broader socio-educational goals of the secondary school: self-acceptance, self-respect, and self-reliance; respect for the rights and privileges of others, appreciation of the intrinsic beauty of the world in which we live, acceptance of the full responsibilities to society, and full recognition of the complex and changing requirements of life of today and tomorrow.

Monday Morning Teaser

When driving to school, I had to stop at a railway crossing. I observed that the train passed me in T seconds. Later on I noticed that the same train crossed a bridge K metres long in t seconds; I also observed that the train travelled at constant speed.

Waiting for my first class to arrive, I calculated the speed of the train and its length. Can you do it?

(Note: for non-algebra students, give simple values for T, t, and K.)

Solution: Let l be the train's length and v its speed.
It passes the observer - that is, it covers a distance equal to its own length in T seconds.

$$v = \frac{l}{T}$$

It crosses the bridge, that is , it covers a distance equal to the sum of its own length l and K metres in t seconds.

$$v = \frac{l + K}{t}$$

$$l = \frac{KT}{t - T}$$

$$v = \frac{K}{t - T}$$

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DON'T FORGET ... SEND YOUR IDEAS AND CONTRIBUTIONS FOR THE NEXT ISSUE OF ABACUS ! ! !

Free Materials

The following materials are available free to OAME members

Elementary : "Boxes, Squares, and Other Things," A teacher's guide for a unit in informal Geometry
(cost to non-members $1.80)
from Mr. M. MacGregor, Sec. Treas. OAME

Secondary : Grade 9 Preparedness Test
Grade 13 Sample Examinations
A Handbook of Trigonometry
(cost to non-members $1. per item)
from Grand Valley Mathematics Association

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MENTAL ARITHMETIC MADE MODERN

Put these questions on slides to encourage pupils to make mental estimations.

1. If Jerry can ride his bicyle 4 miles in 20 minutes, how far could he travel in one hour if he travels at the same rate?
   (a) 20 miles (b) 8 miles (c) 25 miles (d) 12 miles
   (e) not given

2. Jack had 5 dollars. He spent $3.98 for a shirt. How much money did he have left?
   (a) $3.93 (b) $8.98 (c) $19.90 (d) $2.02 (e) $1.02

3. Jack bought a baseball glove for $1.69 and a baseball glove for $5.99. How much did both cost together?
   (a) $6.58 (b) $4.30 (c) $3.30 (d) $7.68 (e) not given.

4. Bill spent $6.76 of his $20. bill. Does he have enough money left to buy a coat costing $14.00?
   (a) Yes (b) No (c) not enough information

5. What is the best estimate for 4,491 divided by 81?
   (a) between 10 and 20 (b) between 100 and 200
   (c) between 60 and 70 (d) less than 10 (e) not given

6. What would be the best estimate for 249 x 8? (a) 2000
   (b) 1600 (c) 2400 (d) 2500 (e) not given

7. Jerry works part-time on Saturdays. He has earned $1.00, $1.50, 75¢ and $1.25 thus far. How much money has he earned altogether? (a) $11.25 (b) $5.00 (c) $4.50

8. A good estimate of 8 x $7.95 would be:
   (a) $80.00 (b) $56.00 (c) $64.00 (d) $100.00 (e) not given

THIRD ANNUAL CONFERENCE

Ontario Association for Mathematics Education

MAY 14, 15, & 16, 1976

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ST. CATHARINES TERCENTENNIAL

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