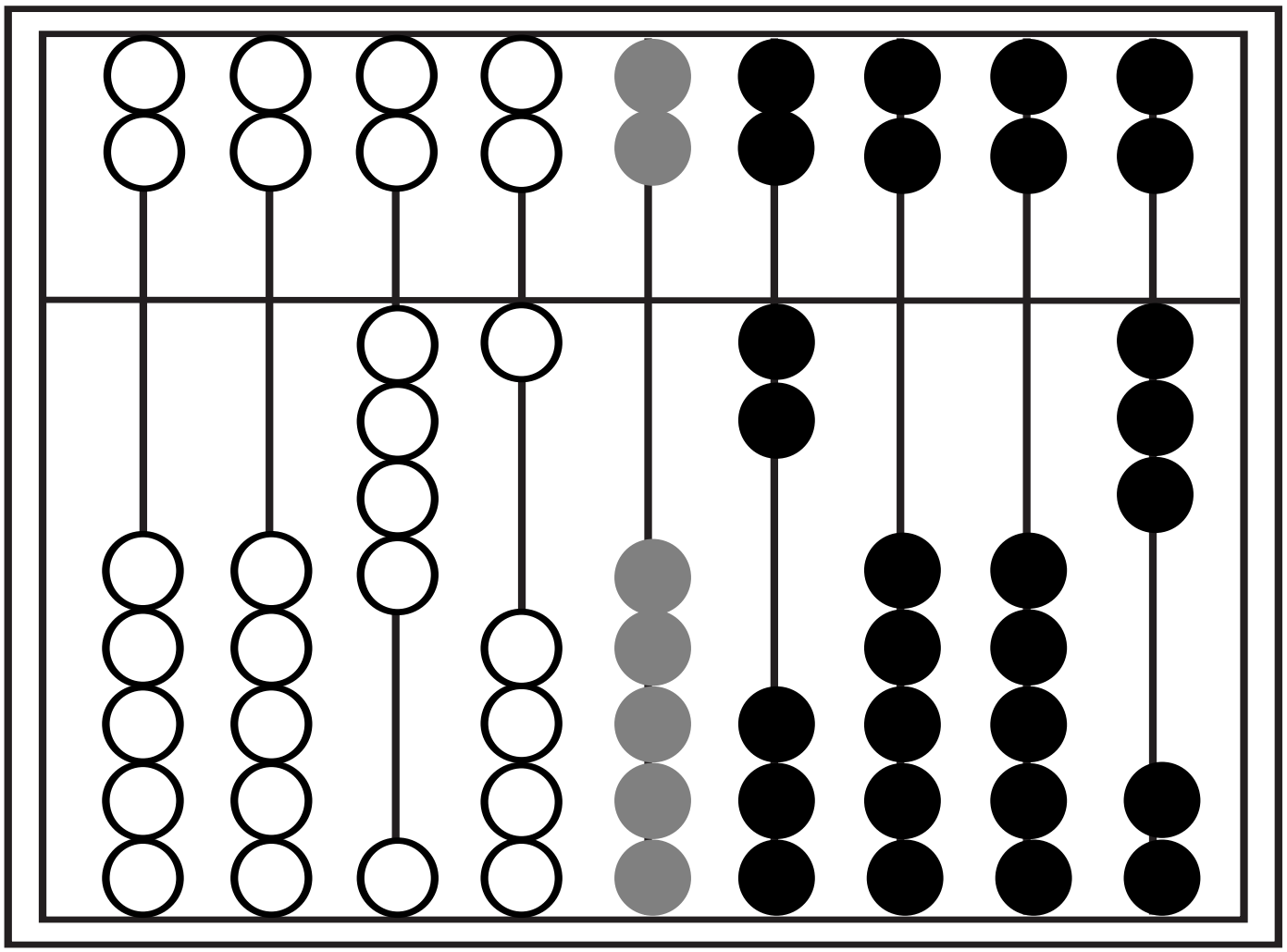
Abacus

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Edited by Frances Schatz

From the Editor's Desk

Since the last issue of *Abacus*, I've been experiencing a steep learning curve. Two former students encouraged me to set up my own web site. I have a new appreciation for what I find when I search for a topic on www. If any student questions the need to learn algorithms for solving mathematics problems (elementary or secondary), I will have an answer. The method for adding an item to a web site is a multi-step algorithm with lots of error possibilities that slow one to a snail's pace or maybe it shows me that I am a slow learner. At the same time I'm making great progress in the course 'Writing Children's Literature' that I am taking via correspondence for professional learning. My goal for retirement is to write stories/activities that involve mathematics and farming.

This issue features a toothpick puzzle from **Gill *'Mathman'* Dunn's** manipulatives kit. The puzzle leads into a unit on 3-D geometry for grades 5-6. Several teachers suggested that they would like classroom ready activities for this topic. One told me that she has access to a photocopier but no resources for 3D geometry. I am addressing that problem with *Marshmallow* and *Toothpick Skeletons*, *Paper Folding Nets*, and *Counting on 3-D Solids*. A bibliography of sources that I like is below.

References for 3-D Geometry

Bolt, Brian (1982). *Mathematical Activities.* Cambridge University Press, Great Britain.

National Council of Teachers of Mathematics (2002). *Navigating Through Geometry: Grades 3-5 and Grades 6-8.* NCTM, Reston, Virginia

Olson, Alton (1998). *Mathematics Through Paper Folding.* National Council of Teachers of Mathematics.

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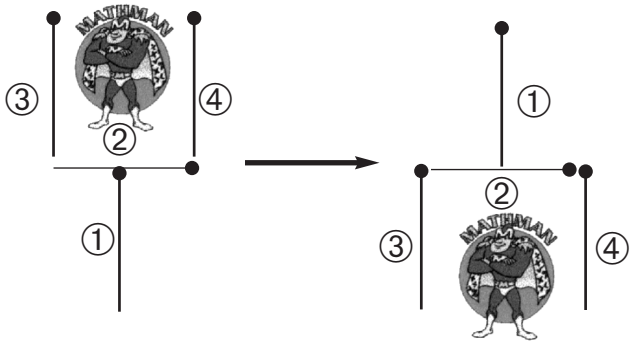
Snape, Charles & Scott, Heather (1995). *Puzzles, Mazes and Numbers.* Cambridge University Press, Great Britain.

Frances Schatz

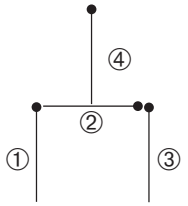
Teacher Notes
Toothpick Puzzle *by Gill Dunn*

Mathman hands out his signature puzzle to the participants at his workshop titled: *Using Manipulatives to Build Self Esteem.* It is part of one of the eight stations from his manipulatives kit. He writes:

In attempting to solve this problem, I have observed several participants doing the following:



The second requirement of the puzzle (keep the shape of the glass the same) has been fulfilled but Mathman is still inside the glass. We overlook the law of gravity here but many cannot get past this point and give up trying other strategies. I encourage them to try another transformation (without mentioning the word slide). Picking up the two sticks and moving them to a different location doesn't lead to the solution. The word, slide, makes the solution obvious. Slide stick ② to the right so that its end is at the top of ①. Now ④ is the stem of the upside down glass when you move ③ to its new position. Note that Mathman stays and is now outside the glass and the glass has maintained its shape.



Question: Do students have the problem that workshop participants had with this puzzle?

Observation: Students (especially young children) make "toothpick" mathematics exciting because they don't study the sticks; they just manipulate them and sometimes end up creating their own puzzles. Enthusiasm abounds as they challenge each other to solve their newly created puzzles.

[Editor's note: Mathman is collecting students' puzzle creations and he tells me he has some very interesting ones. Send your students' creations to me and I will be sure to forward them to him. A future issue will possibly feature these.]

Teacher Notes for 3-D Geometry

by Frances Schatz

The Ontario Curriculum, Grades 1-8 Mathematics, gives overall and specific expectations for Geometry and Spatial Sense: Grade 5 and Grade 6.

Overall Expectations

Students will draw and build three-dimensional objects and models.

Students will draw and construct three-dimensional geometric figures from nets.

Specific Expectations: Grade 5

Students will identify nets for a variety of polyhedra from drawings while holding 3-D figures in their hands.

Students will construct nets of cubes and pyramids using a variety of materials.

Students will sketch the faces that make up a 3-D figure by looking at a 3-D figure.

Specific Expectations: Grade 6

Students will identify nets for a variety of polyhedra from drawings by visualizing the 2-D faces of the 3-D figures.

Students will design nets of cubes and pyramids using grid and isometric dot paper.

Students will sketch the net for a 3-D figure by looking at a 3-D figure.

Before students can be comfortable with identifying nets for various polyhedra and designing 3-D figures, it is important that the students have experience with making skeleton polyhedra, paper folding, and looking at the properties of defined 3-D solids.

Marshmallow and Toothpick Skeletons

Preparation for the Activity:

1. Show and tell about samples or pictures of 3-D objects such as boxes that illustrate different possible 3-D solids. Encourage students to bring small 3-D items from home.
2. Use the show and tell items to develop the vocabulary needed for this unit.
   Vertex – the corner or point at which edges meet
   Face – any flat surface of a solid
   Base – the face on which the solid stands
   Edge – the line of a solid at which two faces meet
3. Count the number of faces, vertices and edges on each box.
4. Provide each group of two or three students with a baggy containing marshmallows and toothpicks and a paper towel. (Be sure to say 'No Eating' and 'Toothpicks are to stay on the towel when not being used for construction'.)
5. Copy the triangle, square, and pentagon using marshmallows for corners and toothpicks for edges. Let the students get comfortable with 2-D construction.
6. Now you are ready to go to the activity pages. Some groups could do the skeletons and some could do the paper folding of the nets. Each group should have a copy of each activity.

Notes and Observations During the Activities

1. The five Platonic solids are the cube, octahedron, dodecahedron, icosahedron, and the tetrahedron. These solids are perfectly symmetrical in that each face of a solid is identical to every other face of the solid. Have the students count the faces, edges, and vertices for each one. Identify the polygon face. How are the numbers for edges and vertices related to the number of marshmallows and toothpicks used? Record this information on the Counting on 3-D Solids chart.
2. How are the cube and the octahedron related?
3. How are the dodecahedron and the icosahedron related?
4. What other materials could be used to construct polyhedra?
5. For each shape add the number of faces to the number of vertices. How does this sum compare with the number of edges?
6. Do you think this may be a rule for other shapes?
7. Each platonic solid uses identical shapes for the faces. Can you create your own solid using a combination of the given shapes?
8. Using the isometric dot paper can you create other polygons that could be used in combination with the given polygons to make new polyhedra. (Teachers: the answer key for Counting on 3-D Solids includes another named solid.) Some students might be interested in creating prisms and pyramids.

A rectangle can be made using two squares side by side.

A parallelogram can be made using two triangles side by side.

A triangular prism is a solid in which two congruent, parallel triangles are joined by rectangular faces.

triangular prism



rectangular prism

A prism is a solid having two polygon-shaped faces that are parallel and congruent and other faces that are parallelograms.

A pyramid is a solid with any polygon for a base, the other faces being triangles meeting at a point (vertex).



pyramid

▲ Marshmallow and Toothpick

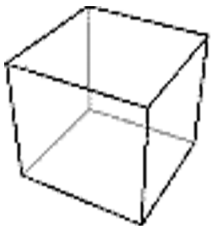
Goal: Construct the Platonic Solids as skeletons or crystal structures using marshmallows for corners (vertices) and toothpicks for edges.

Materials:

- Mini-marshmallows (white or coloured)
- Toothpicks
- Copy of the Platonic Solids nets (pages 7 and 8)
- Copy of the basic square, triangle, and pentagon shapes (page 5)
- Paper towel to keep sticky marshmallows and toothpicks off the desk.
- Copy of 'Counting on 3-D Solids'

Story

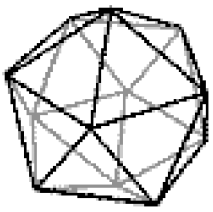
Polyhedra is a Greek word meaning "many faces". The Greek mathematicians determined that there were only five solids that could be made using 3-sided, 4-sided or 5-sided polygons. Mathematicians have since discovered other solid shapes; however, none are 'regular' meaning all faces the same. The philosopher, Plato, concluded that these five polyhedra must be the fundamental building blocks of nature's geometry. We will build skeletons of the five Platonic solids that form the basis for three-dimensional geometry.



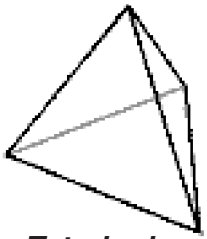
Cube



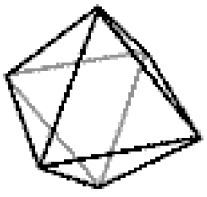
Dodecahedron



Icosahedron



Tetrahedron



Octahedron

Sample Constructions

Cube

1. Take four toothpicks and four marshmallows. Make a square.
2. Make another square.
3. Place four toothpicks vertically in the corners of the first square sitting on the towel.
4. Stick the other square on top. You now have your cube.
5. Note how many marshmallows and toothpicks you used. How many faces, edges and vertices are there?

Tetrahedron

1. Take three toothpicks and three marshmallows and make a triangle.
2. Set the triangle on the towel. Stick an additional toothpick into each marshmallow so that the opposite ends of the toothpicks meet above the centre of the triangle (it will look like a tent).
3. Join these three toothpicks with a marshmallow at the top.

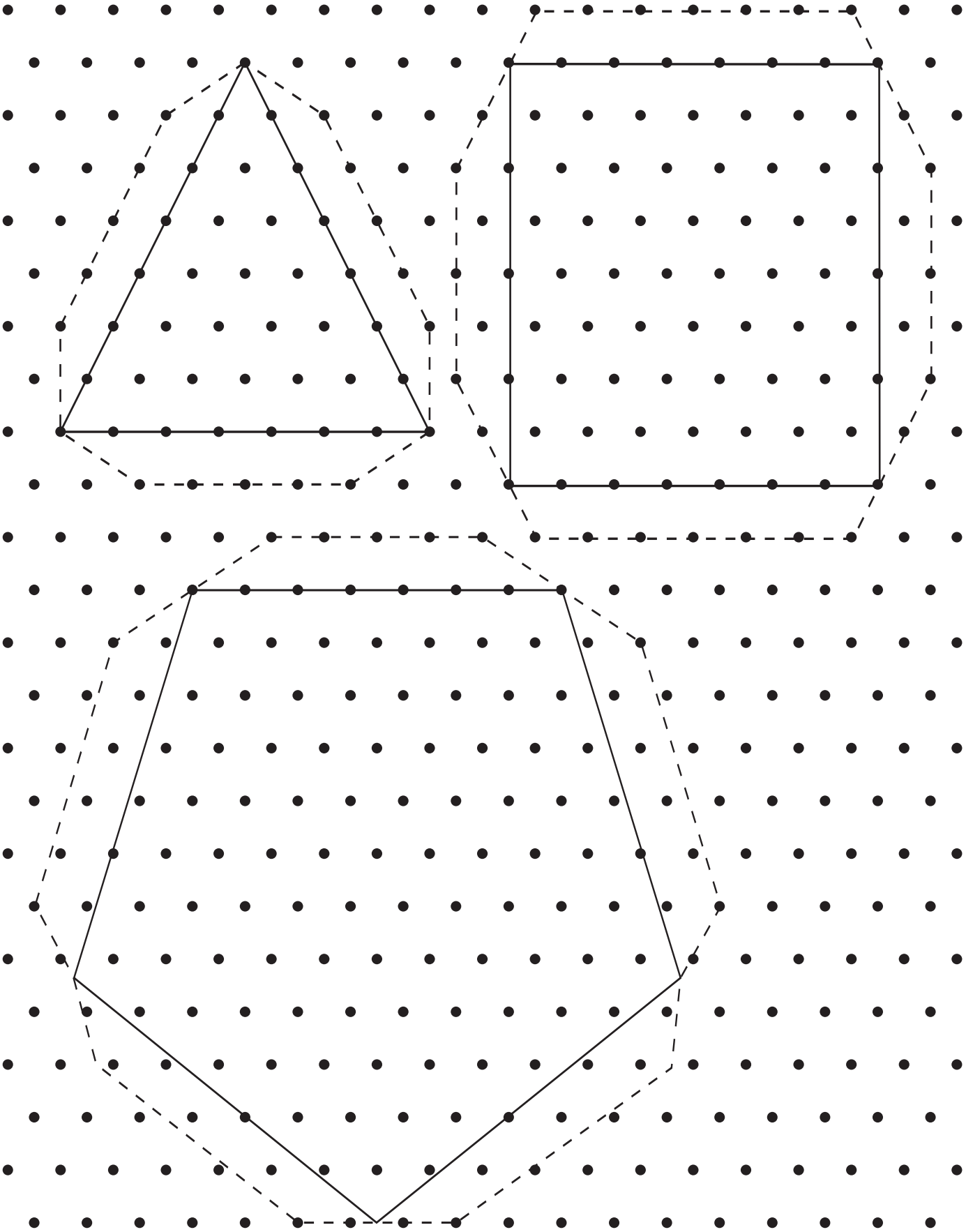
Use the nets to construct the octahedron (6 marshmallows and 12 toothpicks), the dodecahedron (30 toothpicks and ? marshmallows), and the icosahedron (12 marshmallows and ? toothpicks).

It's Your Turn

You will write a report for your teacher after you complete the chart.

1. Describe what you did to complete this activity.
2. Did you experience any problems while making your skeletons?
3. The 3-D solids you made have special names. Can you tell what the names mean?
4. Describe your favourite 3-D solid.

▲ Square, Triangle, Pentagon on isometric dot paper



▲ Paper Folding with Nets Activity

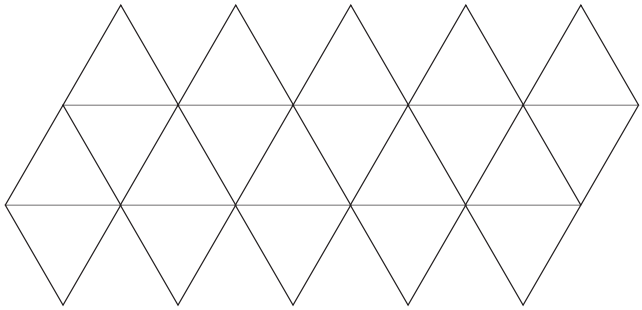
Goal: To construct polyhedron from nets by paper folding.

Materials:

- Coloured construction paper or any fairly stiff paper that will hold shape on folding.
- Copies of nets (see page …) and the basic polygons (page…)
- Sharp pencil, straight edge, and scissors
- Isometric dot paper
- Tape or glue to fasten edge tabs
- Copy of 'Counting on 3-D Solids'

Method

1. Make a copy of the 3-D shape you wish to make on your own isometric dot paper or cut-out if your teacher has provided copies for cutting. You can choose from cube, octahedron or dodecahedron. Cut around the tabs since these will help you join the edges. The dotted lines are tabs and the broken lines are fold markings.
2. Trace and cut out your net using the construction or similar paper.
3. To get started on the folding, match the upper and lower case letters that are shown if you start with the dodecahedron.
4. Make your own net for the tetrahedron. It has 4 triangular faces.
5. Make your own net for the icosahedron using 20 triangles. Use the octahedron net to get started. Here is the shape of the net.



It's Your Turn

You will write a report for your teacher after you complete the chart.

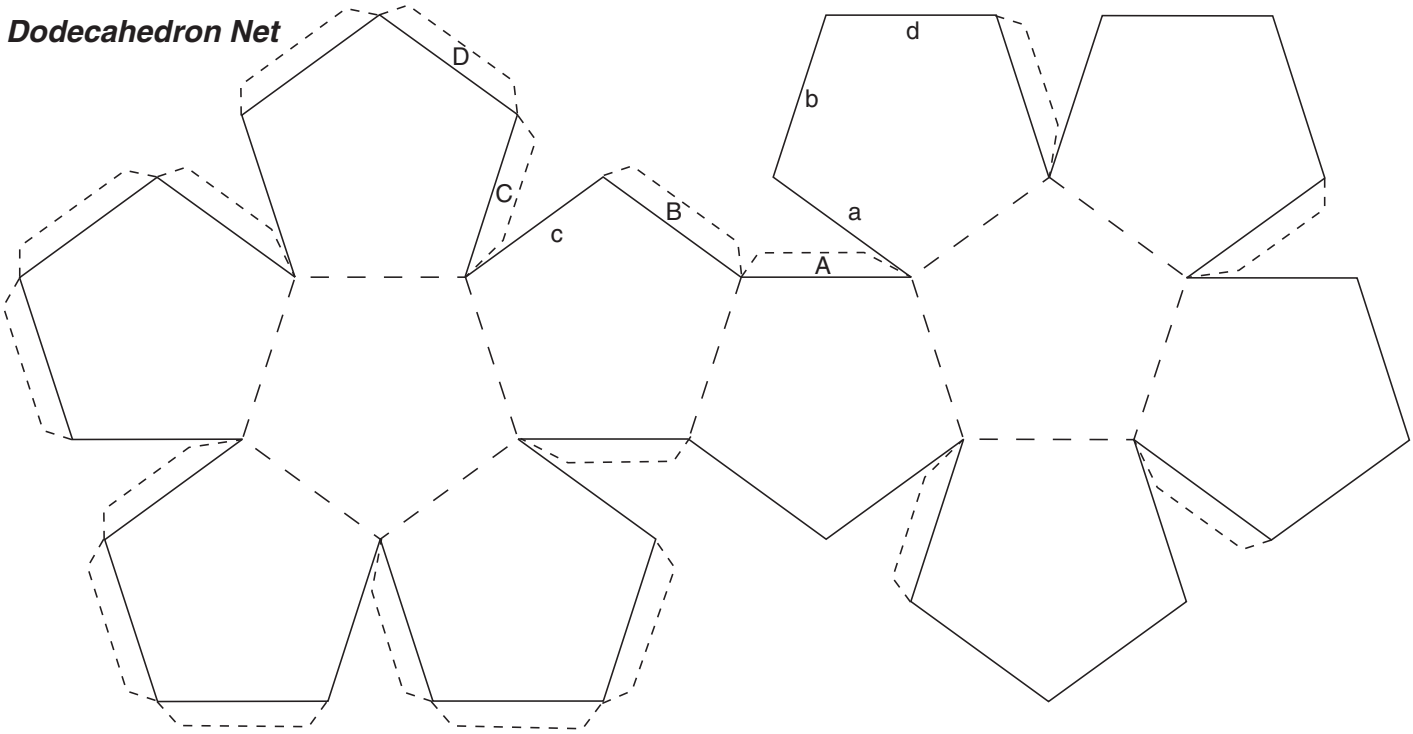
1. Describe what you did to complete this activity.
2. Name the shapes you completed. How are these two shapes alike? How are these two shapes different?
3. What have you learned about perfect solids and what makes a solid perfect?
4. Describe your favourite 3-D solid. Why is it important to study this one?

Counting on 3-Dimensional Shapes

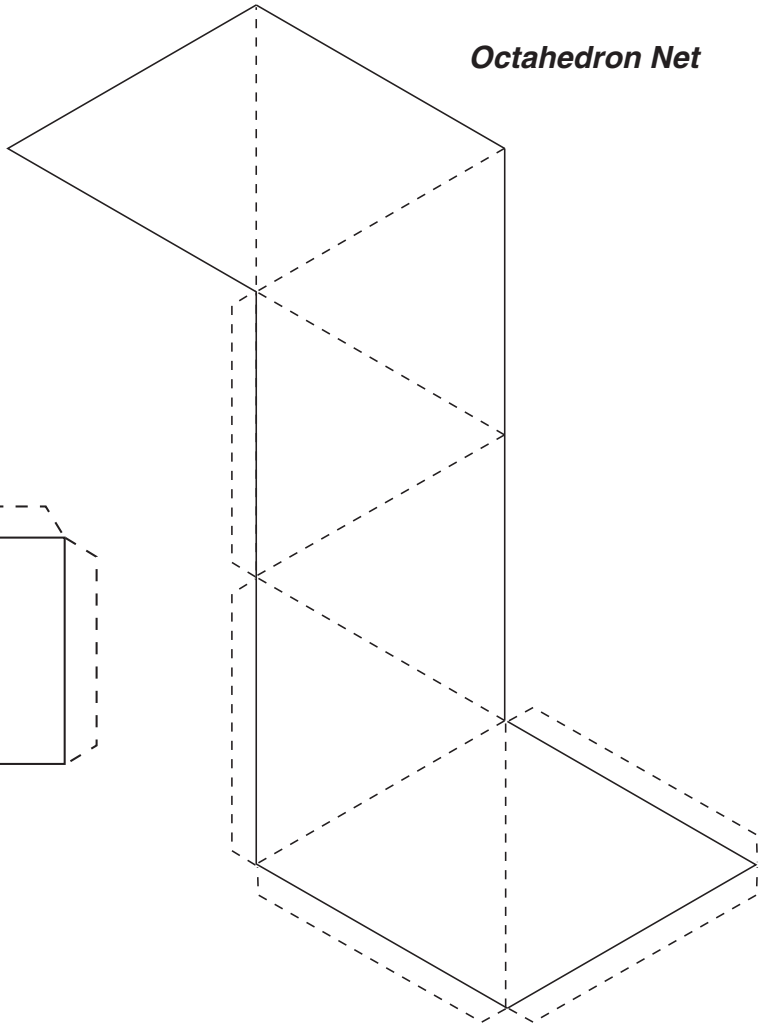
Name of Solid	Number of Faces	Number of Vertices	Number of Edges	Shape of Face	How Many Marshmallows	How Many Toothpicks
Cube						
Octahedron						
Dodecahedron						
Icosahedron						
Tetrahedron						

▲ Three Platonic Solids Nets

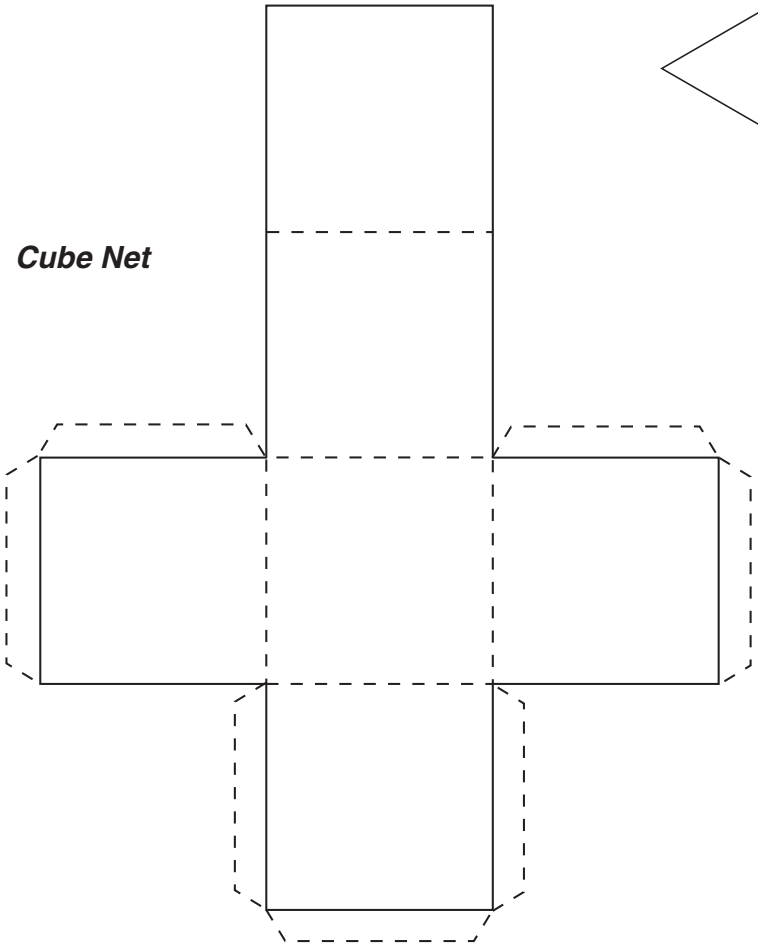
Dodecahedron Net



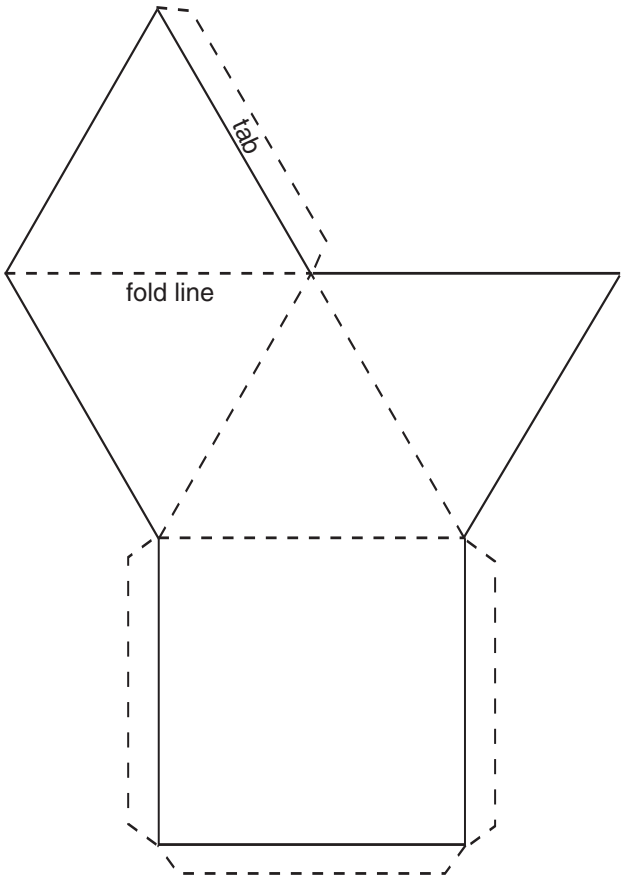
Octahedron Net



Cube Net

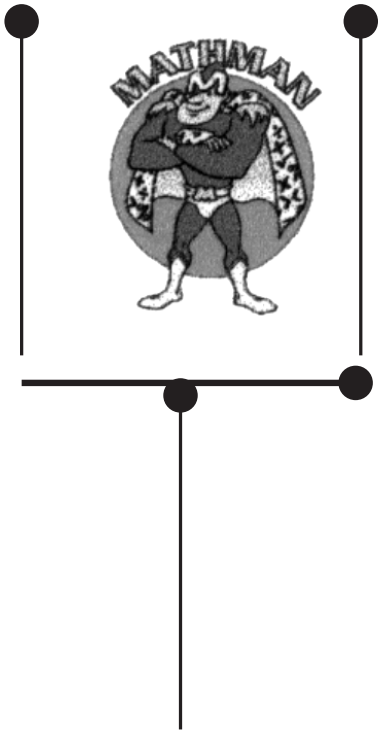


▲ Square Pyramid Net



▲ Mathman's Toothpick Puzzle

Move 2 skewer sticks so that Mathman is no longer inside this glass. You must keep the exact shape of this glass.



Counting on 3-D Solids Answer Key

Name of Solid	Number of Faces	Number of Vertices	Number of Edges	Shape of Face	How Many Marshmallows	How Many Toothpicks
Cube	6	8	12	square	8	12
Octahedron	8	6	12	triangle	6	12
Dodecahedron	12	20	30	pentagon	20	30
Icosahedron	20	12	30	triangle	12	30
Tetrahedron	4	4	6	triangle	4	6
Square Pyramid	5	5	12	square + 4 triangles	5	12