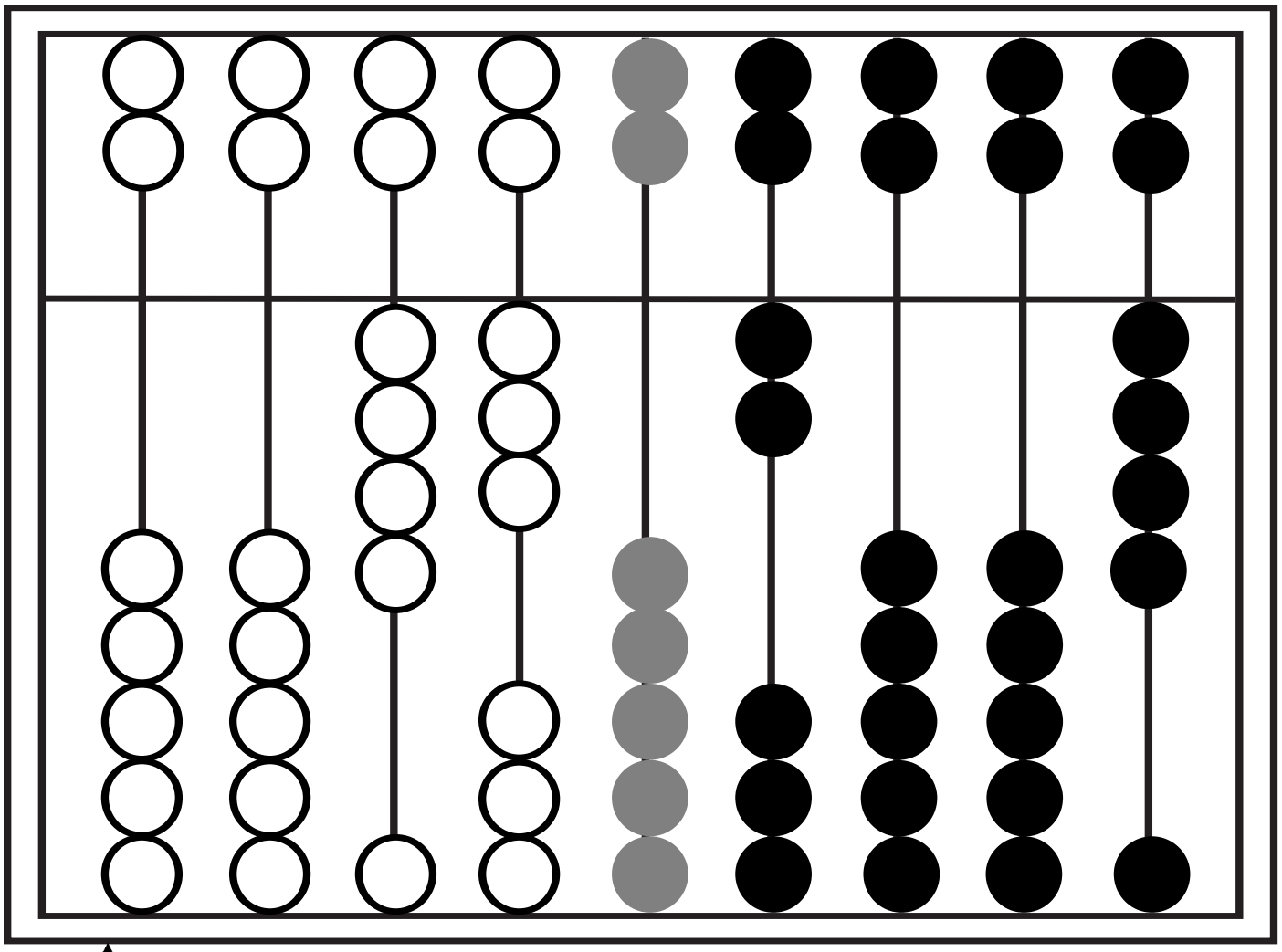
Abacus

Volume 43 2004



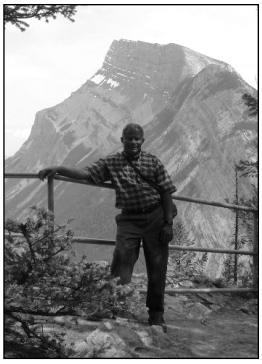
ONTARIO ASSOCIATION FOR MATHEMATICS EDUCATORS

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Edited by Trevor Brown

From the Editor's Desk

"The mathematical experience of the student is incomplete if he never had an opportunity to solve a problem invented by himself."
How to Solve It by George Polya



Welcome back to what we hope will be an exciting and stimulating year! Every year presents its own set of challenges and adventures and we know that this year will not be the exception.

First of all, as the new editor of *The Abacus*, I would like to take this opportunity on behalf of the membership of the OAME, to express our deepest gratitude to the previous editor Frances Schatz. Frances has worked tirelessly to produce a magazine of the highest quality. Many teachers have used the many practical ideas to enrich their teaching and empower their students. We shall miss the many puzzles she consistently provided and hope that she will continue to supply us with the many innovative and creative puzzles that were regular features of the magazine. We wish her the very best!

This leads us to a closely related issue. This journal thrives because it receives regular contributions from its readers. We know that there are many articles, ideas and gems just waiting to be published. Ideas and starters that have been used with students that have caused the students to say "Wow!" or "Aha!". We need these ideas and gems. Please send them in.

As you can see, in this issue we are looking at what one may call "Points of Departure" – ways of developing the mathematical eye and engaging and empowering the students we teach. You will notice that many "What if…?" questions that are posed. We would like the students to take control of their mathematics by asking "What if…?" questions as the asking of these questions will deepen their understanding of mathematics.

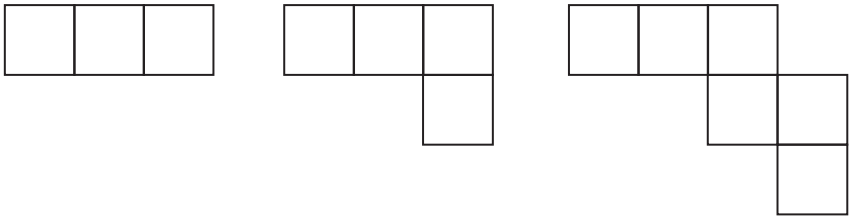
In this issue the focus is on **Number Sense and Numeration**. In the next issue we shall be using the same approach to look at different ways of modeling mathematical ideas and concepts. You may want to send us some of the mathematical models that you have used to enrich and deepen your students' understanding of mathematical concepts.

Have a wonderful school year!

The Mathematical Eye Column

Maria thinks that when you put squares side by side as shown below, the perimeter will always be an even number.

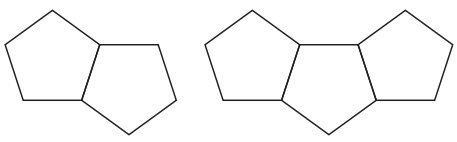
Do you agree? Give reasons for your answer.



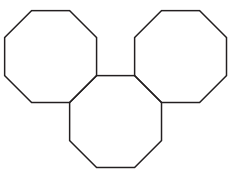
What if we were to use equilateral triangles?



regular pentagons?



regular hexagons?



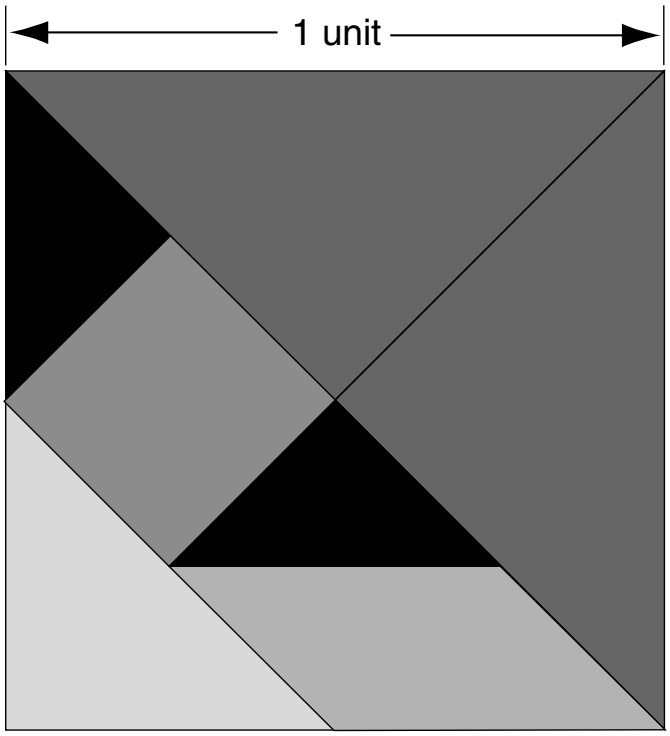
▲ Points of Departure: Tangrams

At a recent meeting of the Ontario Mathematics Educators' Executive, participants were asked to design some rich and engaging tasks involving tangrams. Many challenging questions were posed and we shall be highlighting some of these of in subsequent issues of The *Abacus*.

Here are some to get you started.

1. Given that the length of the side of the tangram is 1 unit.
   a Determine the area of each of the seven pieces.
   b In how many different ways can you determine the areas?

   Check your responses with others.
   c Determine the perimeter of each of the seven pieces.
   d What if the length was 2 units? 3 units?...

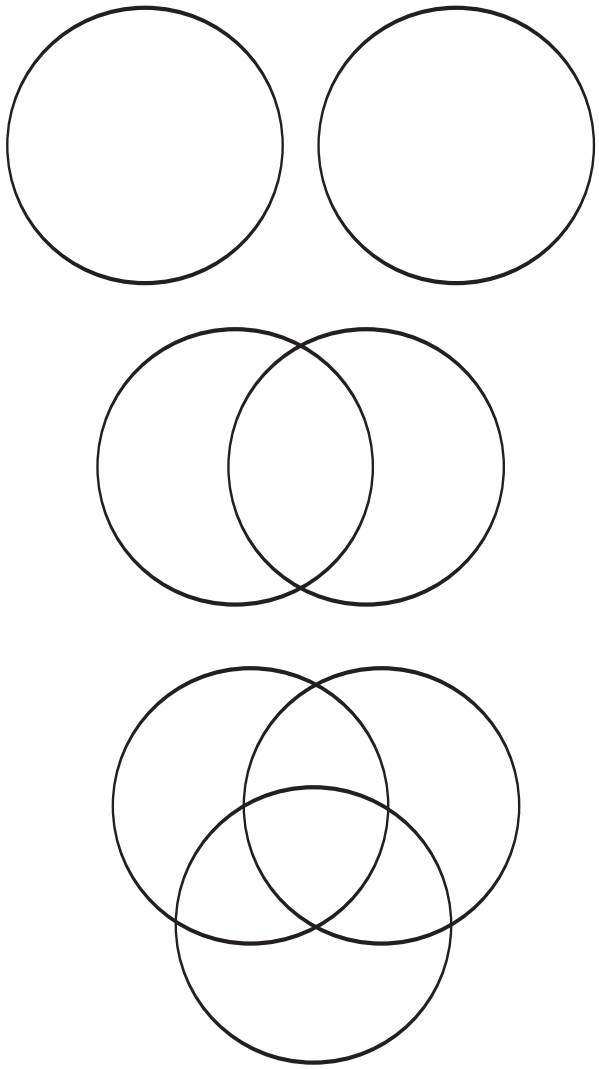


2. How many triangles can you make that are similar to the small triangle?

   Show that they are similar.

3. How many regular polygons can you make with the tangram pieces?

4. How could you sort all the seven pieces?
   a) two circles
   b) two intersecting circles
   c) three intersecting circles



5. What fraction of the large square is the parallelogram? square?...

▲ Points of Departure: The Hundreds Board

Some mathematicians have stated that one of the best gifts you can give to a student is a hundreds board. There are hundreds of different patterns that can be detected on the hundreds board.

Here are some of the questions that can be posed.

Notice that there are different configurations for the hundreds board. One can arrange the numbers on a 10 by 10 grid to show different arrangements. See the hundreds board/charts along the side.

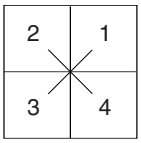
Points of Departure

1. Complete each of the following hundreds charts.
2. Circle the even numbers on each chart. Discuss the patterns formed.
3. Circle all the numbers that sum to 5 [for example 23, 32,41,etc] red, to 6 [for example 24, 33, 42, etc.] blue, 7 [for example 25, 34, 43, etc]green.

   What do you notice?

   What if…?

4. Sum the diagonal numbers in each configuration.
   Example, arrangement 2

   2 + 4 = 6

   1 + 3 = 4

   What do you notice?

5. Circle all the triangular numbers
   [1, 3, 6, 10, 15 …] in each configuration.

   Discuss the patterns formed.

6. Circle the numbers that are the factors of 4 red, 9 blue, 25 green, 49 yellow on one of the hundreds chart.

   What do you notice?
   4, 9, 25 and 49 are perfect squares. How many factors do they have?

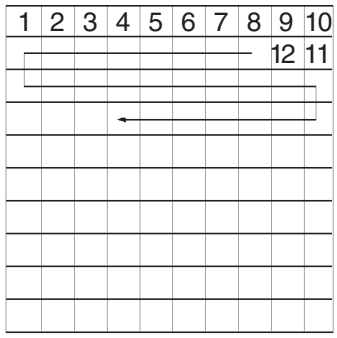
6. Circle the prime numbers on arrangement 2.
   What do you observe?

   To see what some mathematicians discovered, see Ivars Peterson's article. [Prime Spirals, May 6,2002, Ivars Peterson's *MathTrek*, MAA Online www.maa.org/mathland/matrek]

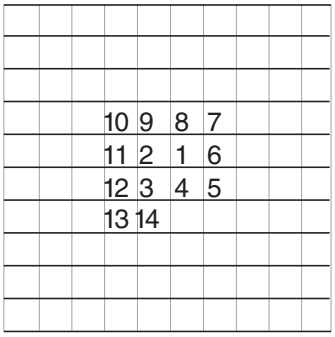
8. Send us some of your favourite "Hundreds Board" questions.

Investigating Hundreds Boards

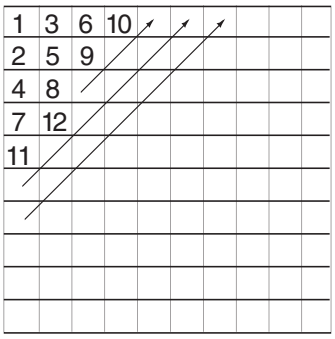
Arrangement 1



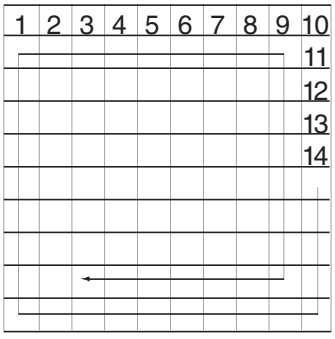
Arrangement 2



Arrangement 3



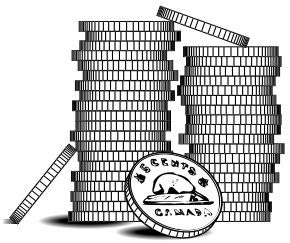
Arrangement 4



▲ Rich Learning and Assessment Tasks: Fractions

Many students experience problems when asked to perform operations on fractions. The following tasks are designed to deepen their conceptual understanding of fractions. All that will be needed for these are some coins, Cuisenaire rods and a calculator. We would like to see how your students solve these problems.

Task 1



Paula placed a number of coins on the floor.

One third of the coins show tails.

If Paula turns over 3 of the coins, then one fifth show heads.

How many coins are on the floor?

Show how you obtained your answer.

Task 2

Here are two questions that require a conceptual understanding of fractions.

a) Which of these two fractions is greater? How do you know?

$$\frac{333333}{444444} \text{ or } \frac{666664}{888885}$$

Make up a question similar to this and have another student solve it.

b) How does adding one to the digits of any fraction affect its size?

Compare $\frac{3}{5}$ with $\frac{4}{5}$

$\frac{3}{5}$ with $\frac{4}{11}$

Is the effect always the same?

Now ask some "What if…?" questions.

Task 3

For this task you will need some Cuisenaire rods.

a) Use your rods to determine which of the two fractions in each case is the greater?

i) $\frac{1}{2}$ or $\frac{2}{3}$ ii) $\frac{2}{3}$ or $\frac{3}{4}$

b) i) $\frac{1}{2}+\frac{3}{5}$ ii) $\frac{4}{5}-\frac{1}{2}$

iii) $1\frac{1}{2}+2\frac{2}{3}$ iv) $3\frac{2}{3}-1\frac{1}{2}$

Task 4

Using my calculator I divided two whole numbers under 100 and found the answer was 0.7397260274. I now cannot remember which two numbers they were. Investigate.

Task 5

What if you double the numerator of a fraction and halve the denominator,

What happens to the value of the fraction?

For example $\frac{3}{4}$

Double the numerator and halve the denominator $\frac{6}{2}$

Task 6

Consider the Fibonacci sequence:

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89,..

Take ratios of successive terms of the Fibonacci sequence: $\frac{1}{1}=$ $\frac{2}{1}=$ $\frac{3}{2}=$ $\frac{5}{3}=$ $\frac{8}{5}=$

$\frac{13}{8}=$ $\frac{21}{13}=$ $\frac{34}{21}=$ $\frac{89}{55}=$

What do you notice?

See *Math Hysteria*, Ian Stewart, Oxford University Press, 2004 ISBN 0-19-861336-9

▲ Technology Corner: Calculators and Number Sense

For this investigation you will need a calculator that has a constant.

Investigation 1

Enter the following operation on your calculator by pressing

10 – 10 = 0

1 should now be showing on the calculator screen.

Don't clear the calculator

Now press

1=

The following should now be displayed on your calculator "-9"

Now press 4 =

The following will now appear on the screen "-6"

Now we are ready for the investigation.

Enter the following and guess or determine what the answer will be before checking with your calculator.

1. Enter 9 – 9 = 0

   Guess / determine the answers when the following keys are pressed. Remember, do not press the clear button after you press the = sign.

   11 =

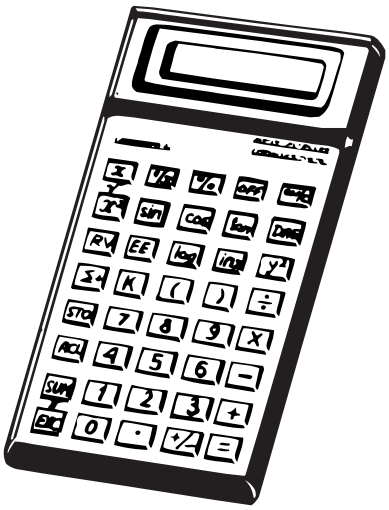
   15.5 =

   22 =

   100 =

   What do you notice?

   Discuss with a classmate.



Now here are some more for you to try.

Again, determine the answer before using the calculator to check.

2. Enter 17.2 – 17.2 = 0

   16.2 =

   14.2 =

   13 =

   19.95 =

3. (-2) – (-2) = 0

   (-15) =

   (-13) =

   8 =

   9 =

4. Now pose some "What if…?" questions of your own.

   For example, what if you were to divide the two numbers?

   You may want to send some of your "What if…?" questions.

▲ Investigating the Factors (Divisors) of a Number

The factors of 12 are 1, 2, 3, 4, 6 and 12

Here is an investigation dealing with the number of divisors of a given number.

As stated above, 12 has 6 divisors [1, 2, 3, 4, 6, 12].

1. How many divisors has each counting number?
2. Find numbers with an odd number of factors – three factors, five factors, seven factors… thirteen factors… What do you notice?
3. Find a number with thirteen factors, fifteen factors, seventeen factors,…
4. Is there an 'easy' way to determine a number with 4 factors? 6 factors?
5. Could the number 599p + 1 be a prime number if p is a prime number? Give reasons for your answer.
6. Marion conjectured that the product of two prime numbers is a number with 4 factors. Investigate.
7. Goldbach conjectured that every even number greater than 2 can be written as the sum of two prime numbers. Investigate.
8. Aluminumbach conjectured that every even number greater than 4 can be written as the sum of three prime numbers. Investigate.
9. Tinbach conjectured that every number can be expressed as a difference of two prime numbers.
10. Why isn't 1 a prime number?
11. Write your own conjectures and send them in.
12. Maria believes that -3 is a prime number. Do you agree or disagree? Give reasons for your answer.

Number of factors

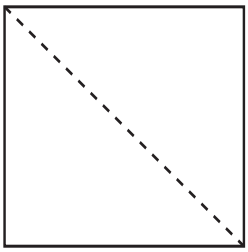
0	1	2	3	4	5	6	7	8	9
	1	2	4 [2^2]	6 [2x3]	16[2^4]	12	2^6	24	
		3	9 [3^2]	8 [2^3]	81 [3^4]				
		5	25 [5^2]	10 [2x5]	5^4				
		7		14 [2x7]					
		11		15 [3x5]					
		13		21 [3x7]					
		17		22 [2x11]					
		19		26 [2x3]					
		23		27 [3^3]					
		29		33 [3x11]					
		31		34 [2x17]					

▲ Pi - The Full Picture

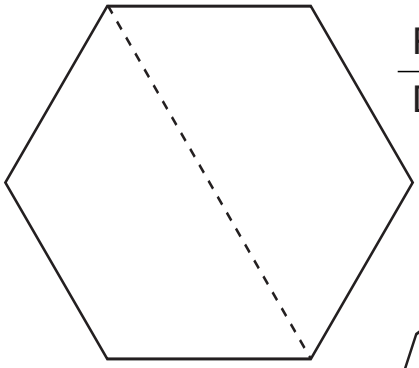
Investigate the value of Perimeter/Diameter for various regular polygons

1. Does your answer depend on the size of the polygon? Why?

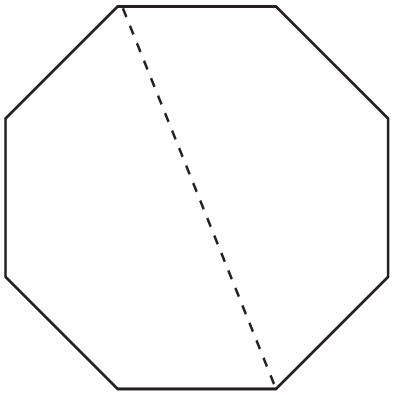
2. What happens to the answer as the number of sides of the polygon increases?



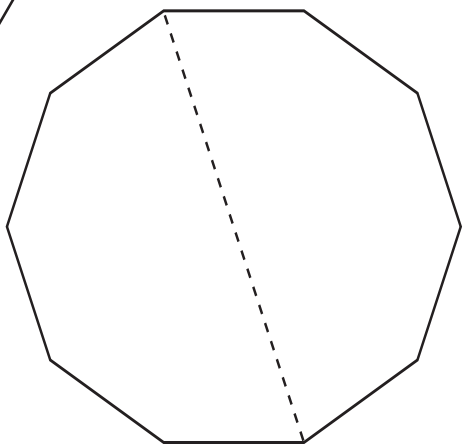
$\frac{\text{Perimeter}}{\text{Diameter}} =$



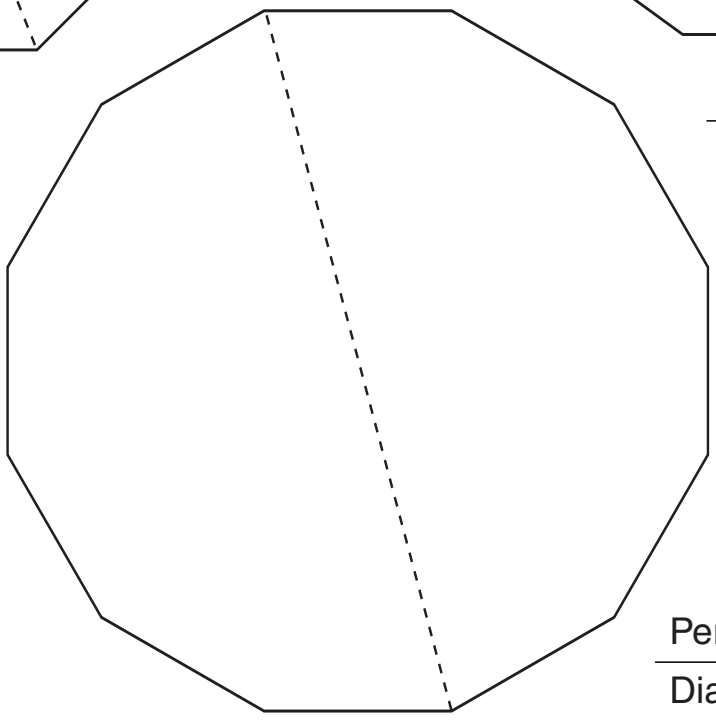
$\frac{\text{Perimeter}}{\text{Diameter}} =$



$\frac{\text{Perimeter}}{\text{Diameter}} =$



$\frac{\text{Perimeter}}{\text{Diameter}} =$



$\frac{\text{Perimeter}}{\text{Diameter}} =$