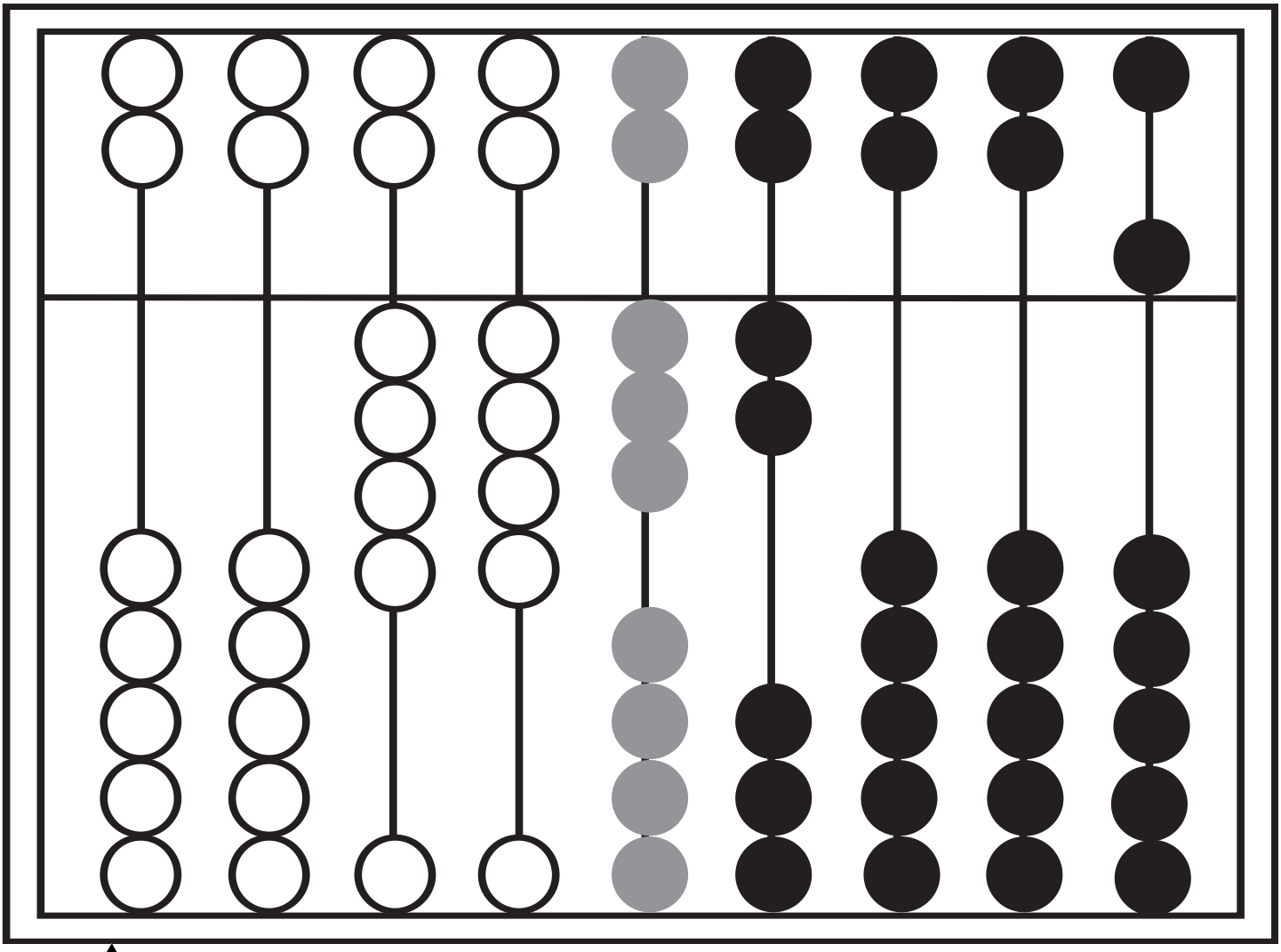
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# Abacus

Volume 44 # 3 • 2006



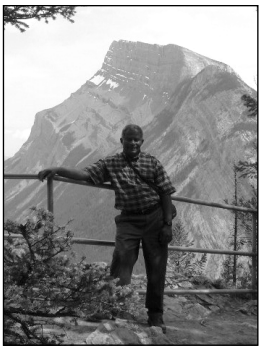
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Edited by Trevor Brown

# ▲ From the Editor's Desk

***Trevor Brown***



I recently came across an article entitled ***Thinking is Driven by Questions, Not Answers***. What really caught my attention were the three questions posed by Lynn Steen, former president of the Mathematical Association of America. The three questions were:

- What problems would encourage an 8th grade student to take more mathematics?
- What problems must a high school graduate be able to solve to succeed in the workplace?
- What problems will make teachers into better teachers?

I was so intrigued by these questions that I now find myself asking fellow math teachers to identify an area of mathematics that they think is beautiful. The responses are varied and at times very informative. Sometimes I get a blank look and at other times a very animated discussion normally ensues either about a teacher who was able to ignite the mathematical fire, or a book which showed the beauty and elegance of mathematics.

Is mathematics beautiful? If it is, then what makes it beautiful ?

Do our students or many of our friends see the creative and beautiful side of mathematics? If not, then what is the issue? Why not pose the question to some of your friends and observe their responses. We would be interested in hearing from you as to an area of mathematics that you believe is very beautiful and/or elegant, or responses that you have elicited from some of your friends.

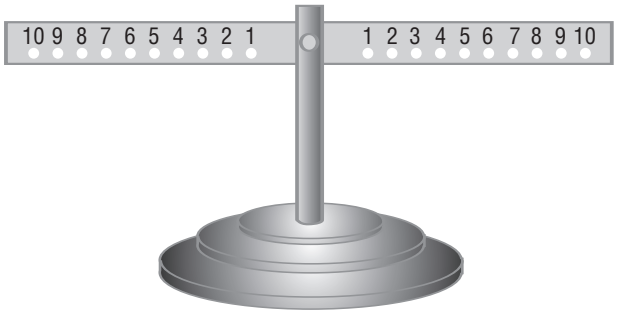
In this issue of ***The Abacus*** we would like at some 'beautiful' areas of mathematics. We have included in this issue the ***Candy for Everyone*** problem that was solved by Ron Lancaster. Writing in the January 10th issue of ***MathTrek***, Ivars Peterson wrote the following.

*"Ron Lancaster, a math teacher in Hamilton, Ontario, first encountered the candy problem in an article by James Tanton or Merrimack College, Massachusetts, in the September, 199? issue of* **Math Horizons**, *under the heading "Iterated Sharing". Lancaster found himself captivated by the problem, and he soon wrote a little program for his TI-83 graphing calculator to do some simulations and see what happens. It also occurred to him that the problem would make a wonderful class activity for middle-school math students, requiring a nice blend of conjecture, experimentation (or simulation), and proof. Lancaster worked with teachers Carly Ziniuk and Sharon Djordjevic of Bishop Strachan School in Toronto to try the idea out with several seventh-grade classes. "The students found the problem to be highly engaging and interesting," Lancaster reports. "We were amazed at the types of questions they asked and answered."*

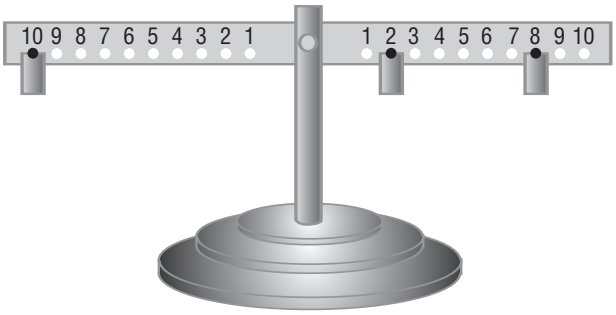
# ▲ Mathematical Balance

***Adopted from NRICH***

This is a number balance. It has masses suspended on the hooks.

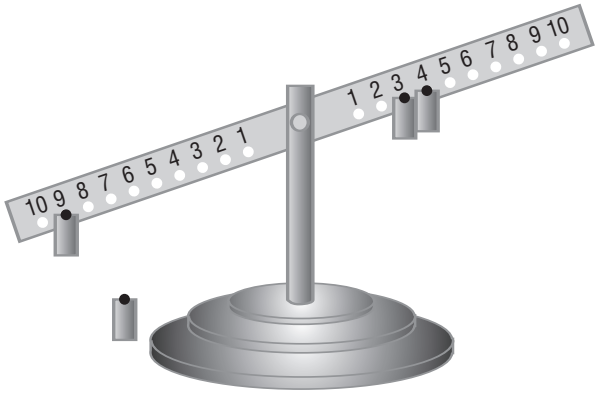


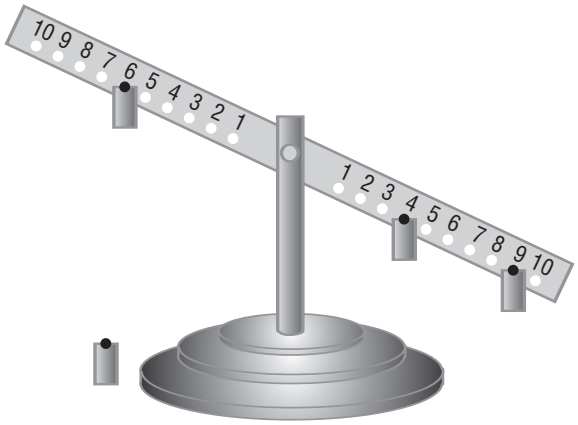

The masses are hung below the numbers. It balances equal numbers, for example like this:




Balance each of the mathematical balances by using one mass.

Could you use two, three, four… balances to balance each of the balances?



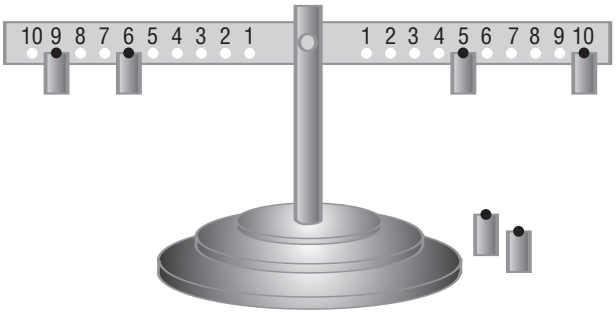




If you had two masses where would you put them to balance the Mathematical Balance?

What if you had three? four? five?... masses?

Why?

How many different ways can you make it balance with two masses?



# ▲ Probability: The Game of Pig

### Need: Number Cube [Die]

Each turn of the game consists of one or more rolls of the cube.

You keep rolling until you decide to stop or until you roll 1.

You may choose to stop rolling at any time.

### Scoring

If you choose to stop rolling before you roll 1, your score for that turn is the sum of all the numbers you rolled on that turn.

However, if you roll 1, your turn is over, and your score for that turn is o.

### Examples

- You roll 4, 5, 2, and then decide to stop.
  Your score for this turn is 11.



- You roll 3, 4, 6 and 1.
  The turn is over because you rolled 1, and your score for this turn is 0.



Each turn is scored separately. With each turn you play, you add the score for that turn to the total of your previous turns.

### Your Assignment

1. Play several turns as a whole group.

   As you play, talk about how you are deciding whether or not to roll again.

2. Split into two smaller groups and play some more turns.

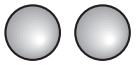
3. Make a list of some strategies that you found yourself using.

   Be prepared to discuss these with the entire class.

# ▲ Probability: Two and One

***Adopted from NRICH***

Two students are playing a game with three marbles, two blue ones and one red one.

Terry is red  and Ali is blue .

They toss up the marbles, which run down a slope so that they land in a row of three, like this:

.

The colour in the middle wins the round.

Is this a fair game?

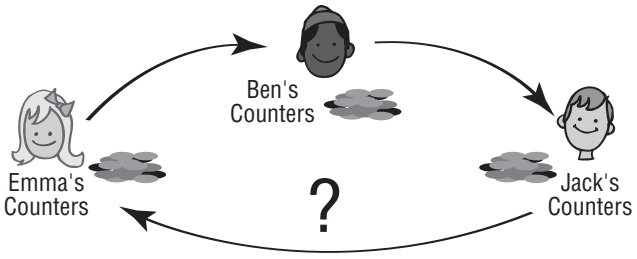
Who do you think will win?

Why?

# ▲ Ben's Game

***Adopted from NRICH***

Ben, Jack and Emma were playing a game with a box of 40 counters – they were not using all of them. They each had a small pile of counters in front of them.




All at the same time Ben passed a third of his counters to Jack, Jack passed a quarter of his counters to Emma and Emma passed a fifth of her counters to Ben.

They all passed on more than one counter.

After this they all had the same number of counters.

How many could each of them have started with?

# ▲ The Candy Problem (Iterated Sharing)

***Ron Lancaster***

A group of friends sit in a circle each with a pile of covered candy in front of them (Covered candy is used because each piece is going to be handled by many people before it is eaten). Some people have more than twenty pieces, other zero, and the rest some number in between. The distribution is quite arbitrary except for the fact that everyone has been given an even number of pieces. These friends now perform the following maneuver: each person gives half her candy to the person on the left (and hence receives a supply of candy from the person on the right). They do this simultaneously. Everyone then recounts her candy supply. If as a result of this operation some people end up with an odd number of pieces, those folk each take an extra piece of candy from a supply that was held in reserve. This boosts their piles back up to an even number of pieces and everyone is ready to perform the maneuver again.

What happens to the distribution of candy amongst these friends if this maneuver is performed over and over again? Will people be forever taking extra pieces from the centre and the amount of candy each person has will grow without bound? Or will the distribution stabilize or equalize in some sense? Or will one person end up with all the candy? Perhaps clumps of candy will move around the circle with each iteration, or perhaps some strange oscillatory pattern will emerge. Is it possible to predict what the result will be? What happens if, instead, each member of the circle eats any odd pieces of candy to bring his or her pile back down to an even number? What happens if the sharing pattern is varied, say each person gives half her candy to the person on the left and the other half to the person on his or her right?

This wonderful problem is from the article ***A Half-Dozen Mathematical Activities to Try*** with Friends by James Tanton, an Associate Professor of Mathematics at Merrimack College in Massachusetts. For the past several years he has run a math club for students at St. Mary's College of Maryland and this activity is one of his favourites.

## Solution and further discussion

No matter the initial distribution of candy or the number of people in the circle, the distribution of candy will eventually equalize, i.e. everyone will end up with the same amount. Suppose the largest amount of candy any individual posses is M pieces. Upon sharing, such as individual will give M/2 pieces to his or her neighbour on the left and receive no more than M/2 from the right (since M is the maximal number of pieces any one individual posses). They end up with no more than M pieces as a result. This individual will then be given an extra piece of candy only if his or her resultant pile is odd in number. But since M is even an individual with M+1 pieces will never result. Thus the number of pieces any individual can ever possess is bounded above by the maximal amount M in the initial distribution.

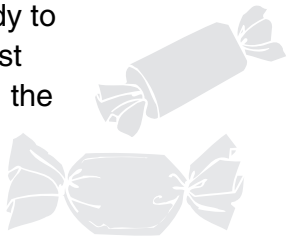


Now consider those people possessing the smallest amount of candy. Let m represent this amount. Upon sharing, such an individual will pass m/2 pieces to his or her left neighbour and receive at the very least m/2 pieces from the right. Only if the person to the right possesses the same minimal amount m of candy will his or her pile fail to grow, otherwise they will end up

with m + 1 pieces or more (and maybe even the addition of an extra piece!). Thus if there are k people sitting in a row each with m pieces, the first k – 1 people will stay at m pieces and the $k^{th}$ person will end up with more (unless everyone has m pieces, in which case the iteration has stabilized). Thus the number of people with a minimal amount of candy decreases on each iteration. Given the upper bound on the amount of candy one can possess, this fact forces the situation to stabilize!

Sharing all your candy with both your left and right neighbours can produce oscillatory patterns if the number of people in the circle is even. Imagine people changing seats between iterations, moving one seat to the right to follow their right pile. The situation is thus equivalent to each person simply giving half their candy to the person two places to the left. If there are an even number of people in the circle this breaks the analysis into two disjoint groups, each of which must stabilize (but not necessarily to the same number). For an odd number of people, the problem is equivalent to just one circle, which must stabilize.

There are a myriad ways to explore this problem further. What happens if thirds or quarters of piles of candy are shared with two or three people? What if candy is passed to different positions along the circle? What if different people pass candy to different places – say the first person one place to the left, the second person two to the left and so on. Must the situation always stabilize?



Returning to the original problem: Given the number of people in the circle and the initial distribution of candy, is it possible to predict how much candy everyone will end up with in the end?

Can you predict how many iterations it will take to reach the stable pattern?

# ▲ Mathematical Curiosities

## Multiplication by 11

Multiplication by 11 is easy!

To multiply by a 2-digit number add the two digits and place the sum in between!

25 x 11 = 275

31 x 11 = 341

57 x 11 = 627 - you need to 'carry' the 1.

Now what about a 3-digit number?

Can you figure out what's going on here?

253 x 11 = 2783

117 x 11 = 1287

532 x 11 = 5852

267 x 11 = 2937

## Hailstone Numbers

Start with any positive integer (an initial seed) and obtain a sequence of numbers by following these rules.

- If the number is even, divide it by two;
- If it is odd, multiply it by three and add one.
- Repeat

Let's look at an example to see what happens:

n = 5 > 16 > 8 > 4 > 2 > 1> 4 > 2 > 1 > …

n = 4 > 2 > 1 > 4 > 2 > 1 >…

I wonder…does every initial seed yield a sequence that eventually hits 1 (and then

repeats: 4, 2, 1, 4, 2, 1, …)?

The **Collatz conjecture** states yes, but this has never been proved. However, it has been shown true for every number ever tried! The numbers in the sequences bounce up and down, which is why they are sometimes called "hailstone numbers".

What if…?

# ▲ Patterning and Algebra: Moving Pennies

One day, while I was watching my favourite television program, I started to play around with some discs on the table.

I had three discs arranged in a triangle as shown in figure 1.



I realized that if I moved one disc, I could get the triangle to be facing upside down a shown below.



Now clearly, for more rows of pennies it would take more moves, so next I tried three rows of counters (three counters in one row, two counters in the next row, and one counter on top).

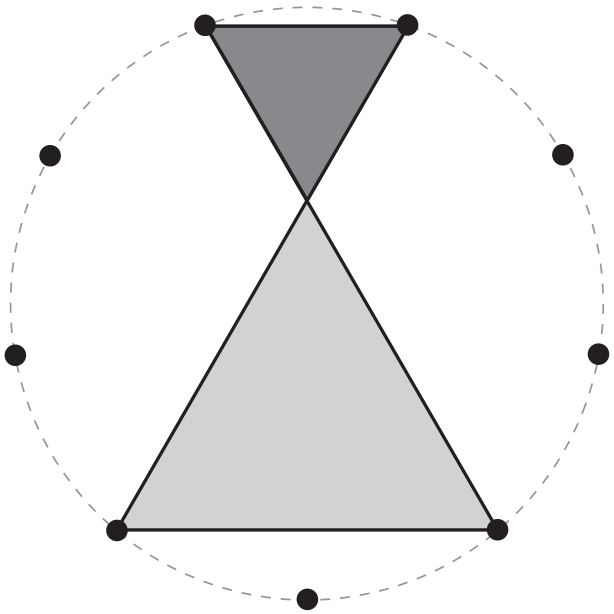
Now it took me two moves to get the triangle to be facing upside down.

1. First, if the number of moves is M and the number of rows is R, how many moves are needed for R = 1, 2, 3, …,8 to turn the triangle upside down?
2. Second, find an equation or equations to form a relation between R and M.

# ▲ Concrete Materials Corner: Circular Geoboard

***Adopted from NRICH***

On a nine-point geoboard a band is stretched over 4 pegs in a "figure of 8" arrangements so that one pair of opposite sides cross and the other pair are parallel.



How many different "figure of 8" arrangements can be made?

Which of these arrangements has the area of the lower portion nearest to double the area of the upper portion?

How many different triangles can you make on a circular pegboard that has nine pegs?

Make five different quadrilaterals on a nine-point geoboard. Work out the angles in each quadrilateral you make.

What do you notice?

