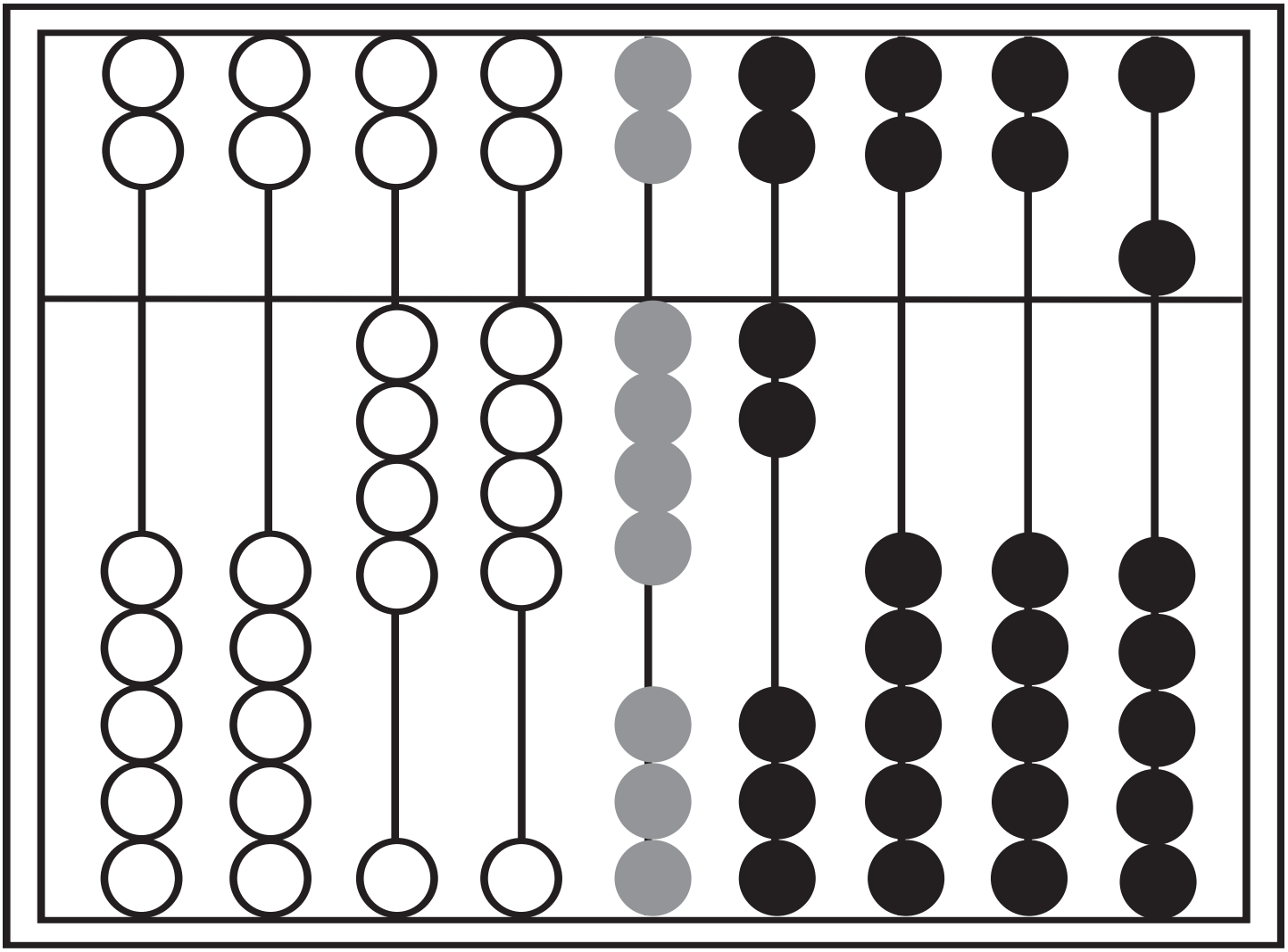
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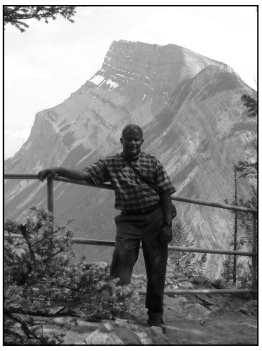
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Edited by Trevor Brown

▲ From the Editor's Desk

Trevor Brown



What do we want mathematics learners to be able to do?

This is a recurring question that mathematics teachers are continually asking, especially when a new curriculum is being introduced. Traditionally the question may have been answered with the statement, "We want them (students) to know the important mathematics listed in the curriculum." More recently the answer is or seems to be, "We want them (students) to know and to use the mathematics they have learnt." Not everyone, however, is satisfied with this response.

In the paper '**Habits of mind, an organizing principle for mathematics curriculum**' (Cuoco, Goldenburg, & Mark, 1995), the authors try to answer the question "Why do we teach mathematics?" The authors suggested that we are trying to have our learners develop *habits of mind and these seem to us be the ways of thinking that we would like learners to develop. The headings in their paper give some idea of what they see as the habits of mind for mathematics and as the paper can be accessed easily on the web we shall not go into it here. The list of habits is in the table below but the paper gives a little more meaning to each of them*

This list shown in Figure 1 is a bit daunting and overwhelming to many educators looking at it for the first time. The insight offer by John Mason at the Open University is particularly instructive here. He sees the most important 'habits' in mathematics as generalizing and conjecturing. He summed up how he saw these as important when he said that, "Lessons that are not imbued with generalization and conjecturing are not mathematics lessons, whatever the title claims them to be."

This focus on generalizing and conjecturing which is at the heart of both reasoning and problem solving/posing has much to commend it.

Students should be describers
Students should be tinkerers
Students should be inventors
Students should be visualizers
Students should be conjecturers
Students should be guessers
Mathematicians talk big and think small
Mathematicians talk small and think big
Mathematicians use functions
Mathematicians use multiple points of view
Mathematicians mix deduction and experiment
Mathematicians push the language
Mathematicians use intellectual chants
Geometers use proportional reasoning
Geometers use several languages at once
Geometers use one language for everything
Geometers love systems
Geometers worry about things that change
Geometers worry about things that don't change
Geometers love shapes
Algebraists like a good calculation
Algebraists use abstraction
Algebraists like algorithms
Algebraists break things into parts
Algebraists extend things
Algebraists represent things

Figure 1: Headings from* Habits of mind: an organizing principle for mathematics curriculum*. (From: Cuoco, Goldenburg, & Mark, 1995)

▲ Number Sense and Numeration

Consecutive Sums

We had three discs with numbers printed on one side. They were known to be 5, 8 and 11, as shown. Numbers were also printed on the reverse sides.



When the discs were tossed in the air it was found that the sums of the numbers showing when they landed were consecutive.

Find the numbers on the reverse sides of the discs.

Share your strategy with others in your group.

What other configurations will give us consecutive sums?

More About Primes

There are all sorts of unexpected relationships relating to prime numbers. The first few prime numbers are 2, 3, 5, 7, 11, 13, 17, 19, …and all of them, apart from 2, give remainder 1 or 3 when divided by 4.

First set
(remainder 1) = {5, 13, 17, 29, 37, 41,…}

Second set
(remainder 3 = {3, 7, 11, 19, 23, 31,…}

The interesting thing here is that all the first set and none of the second can be expressed as the sum of two integral squares, for example

$5 = 1^2 + 2^2$, $17 = 1^2 + 4^2$

What about 13 = ?

19 = ?

29 = ?

37 = ?

41 =?

Fractions

Choose any four numbers.

2 5 9 7

Write a fraction using any two of the numbers.

Write another fraction, another, another,…

$\frac{2}{5}$ $\frac{2}{9}$ $\frac{5}{9}$ $\frac{5}{7}$

A question may arise about using the same number twice.

If it does not…

- What fractions can you make using the same number twice/
- How many fractions can you make?
- How many different fractions can you make?
- What if you had started with three numbers? Five numbers? Six numbers?...
- Which is the largest fraction?
- Which is the least fraction?
- Which fractions are greater than 1? Less than 1? Equal to 1?
- Write the fractions in order of size

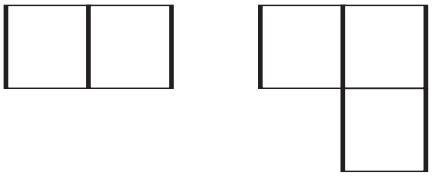
We investigated numbers of fractions for the numbers 2, 4, 6 and 8 and tabulated the results as shown to the right.

$\frac{2}{2}$	$\frac{2}{4}$	$\frac{2}{6}$	$\frac{2}{8}$
$\frac{4}{2}$	$\frac{4}{4}$	$\frac{4}{6}$	$\frac{4}{8}$
$\frac{6}{2}$	$\frac{6}{4}$	$\frac{6}{6}$	$\frac{6}{8}$
$\frac{8}{2}$	$\frac{8}{4}$	$\frac{8}{6}$	$\frac{8}{8}$

- How many different patterns can you see on the table?
- Equivalent fractions? Fractions greater than 1? Less than 1? Which pairs of fractions have a product equal to 1? How many pairs of fractions are there like this?
- Is it possible to decide on the number of fractions for any ‘n’ numbers?

▲ Colour Tiles in the Mathematics Classroom

Task 1: Perimeter



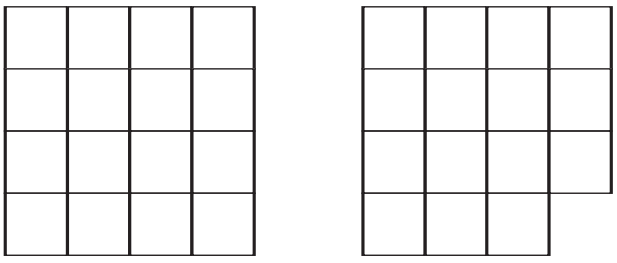
Perimeter = 6 units ***Perimeter = 8 units***

A unit square has perimeter 4 units. When two such squares are placed, non-overlapping, in a plane, the minimum perimeter of the shape they form is 6 units.

For three squares the minimum perimeter possible is 8 units.

- What is the minimum perimeter when (1) four units squares are placed, non-overlapping, in a plane? (ii) How about five squares? (iii) Six squares? (iv) 22 squares?
- Can you generalize the result for shapes formed from non-overlapping unit squares in a plane?

Task 2: More on Perimeter

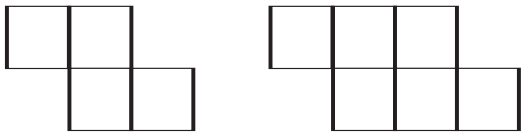


Perimeter = 16 units ***Perimeter = 16 units***
Area = 15 square units ***Area = 14 square units***

- What is the maximum number of squares can you remove from figure 1 before changing the perimeter ?

If it were a three by three its perimeter would be 12 units.

- In that case what would be the maximum number of squares you would have to remove before changing its perimeter?
- What if it were a 5 by 5? 6 x 6? 8 x 8?...n x n?
- While playing around with these squares I noticed that in each case the perimeter was an even number. The question I would like to leave with you is this.
- Is it always true that whenever you place squares side by side as shown below, the perimeter will always be an even number? If this is always true, can you state why?

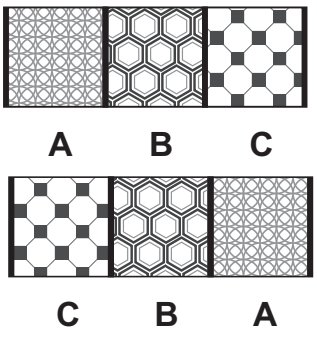


Task 3: Shunting Problem

Some tiles come in on one line. Tiles can be shunted one or more at a time into the sliding shown and out on the other line. The tiles are reassembled on the other line.

Try some shunting of three tiles.

- Can you end up with the original order?
- Can you end up with any order you want?
- How many operations are required each time?
- How can the shunting be recorded?
- What about five tiles? Six tiles?...?



Order ABC to CBA

Task 4: Tiles, Patterning and Algebra

In the diagram (Figure 1) there are 6 tiles arranged in the shape of a triangle.

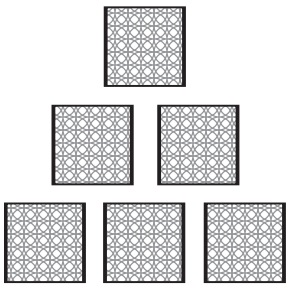


Figure 1

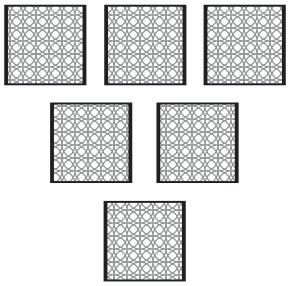
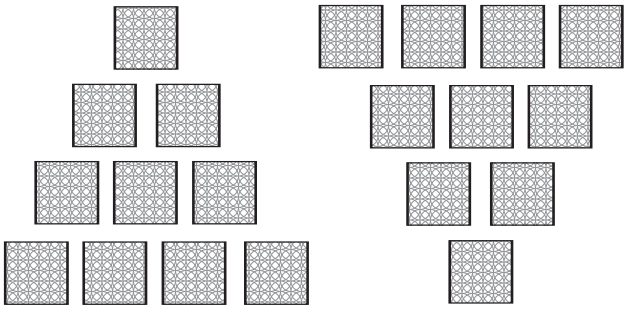


Figure 2

The question is though what happens with 15 tiles? If you put 15 tiles in a triangular shape with 5 rows, what is the smallest number of tiles that you can move that will point the triangle in the opposite direction?

It's not too difficult to show that, by moving two tiles only the 6 tiles can be made to point in the opposite direction (Figure 2)

- What if there were 21 tiles? 28 tiles?...?
- What are these numbers called?
- How many of them can you find?

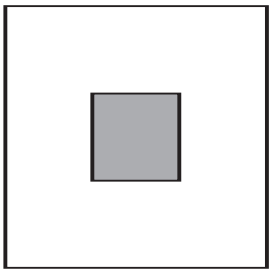




Task 5: Square Number

A square patio was tiled with square tiles all the same size. Some of the tiles were removed from the middle of the patio in order to make a square flower bed, but the number of the remaining tiles was still a square number:





- What were the dimensions of the patio and the flower bed?

Task 6: Tiles and Factors

- How many different rectangles can you make with 6 tiles?

We can see that we can make two different rectangles.



2 x 3 rectangle



1 x 6 rectangle

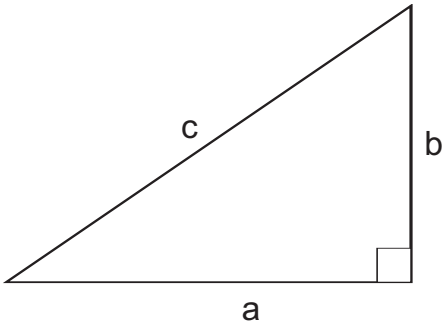
Notice that the factors of 6 are 1, 2, 3 and 6

- How many rectangles are needed to make three different rectangles? Four different rectangles? Only one rectangle? What is the most number of rectangles you can make from a given number of tiles?

▲ Mathematical Reasoning: Pythagoras' Theorem

Pythagoras' contribution to mathematics was that he (or one of his group), proved that in any right angled triangle, the area of the square on the hypotenuse is the same as the sum of the areas of the squares on the other two sides.

From the diagram $a^2 + b^2 = c^2$.

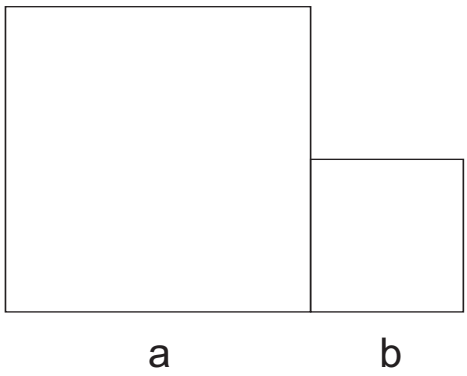


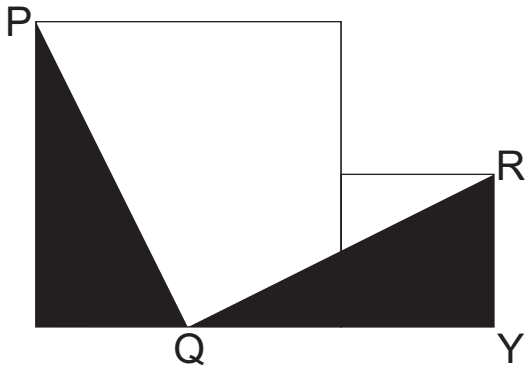
Proof is important in mathematics. It makes sure that we are not just guessing. This is one part of mathematics that sets it apart from other subjects.

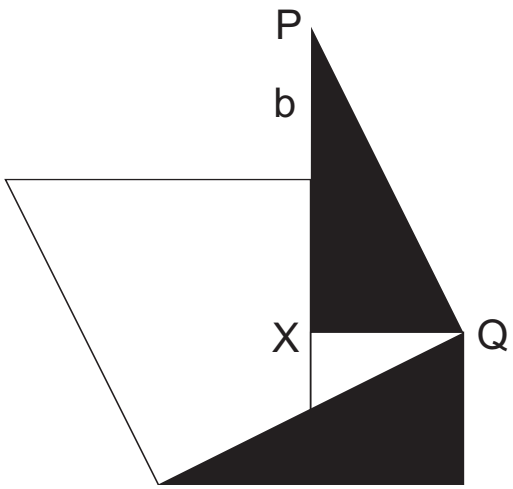
It turns out that there are over **100 proofs of Pythagoras' Theorem.**

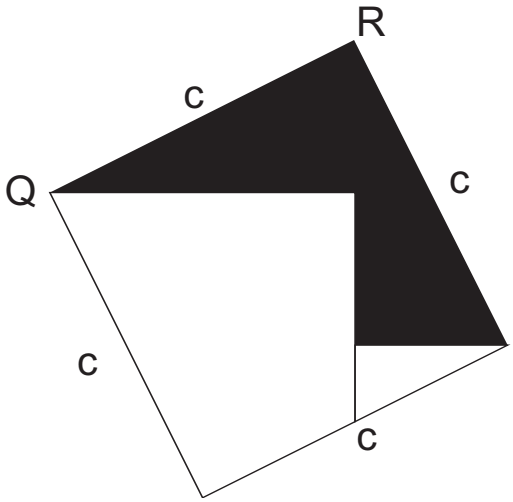
See if you can figure out how these two prove Pythagoras' Theorem.

Hint: They require only knowledge of area.









▲ Literature Corner- *One Hundred Hungry Ants* (From NRICH)

A Remainder of One

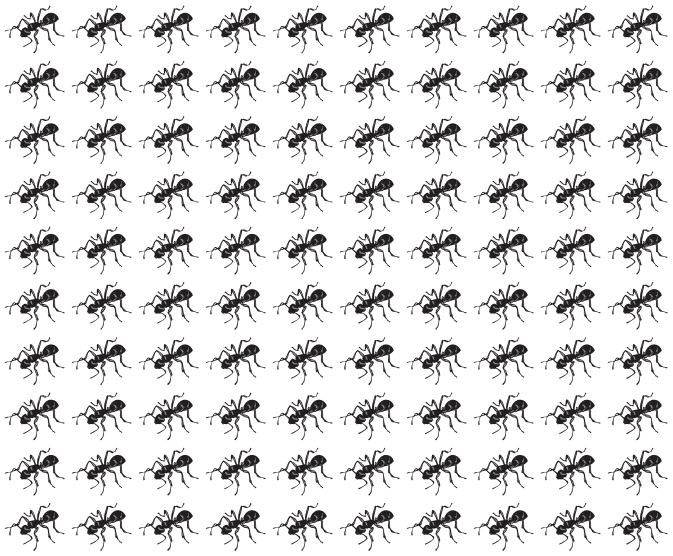
Simply read and enjoy the story the first time. Accept comments on the mathematics but avoid going into too much detail.

Re-read the story, focusing (verbally only to start with) on the division facts generated by dividing 25 by 2,3, and 4 with a remainder of one, then on 25 divided by 5 leaving no remainder.

Physically do the same division with the class. Can they be divided by 2,3,4,5, etc. Are there any remainders? Is the remainder always 1? (Warning - this is a noisy, fun activity!).

Individually or in pairs, the children can divide 25 (or the class number or another appropriate number) by 2,3,4,5, etc. Some children are likely to need concrete apparatus when they try this.

Discuss how to record what they have found out, either using the children's own method or by using the appropriate symbols. Some children can extend the activity by checking their calculation with an inverse calculation. They can then record it alongside their work as a check. Each pair or individual can compare their work with another pair or individual and discuss what they notice. Can they see a pattern in the remainders?



Revisit the calculations to discuss their meaning. For example, if 26 divided by 3 is 8 remainder 2, What does that actually mean in terms of lines of children (or bugs)?

Use the suggestions for activities above, but dividing 100 by 2, 4, 5, and 10. What happens if you divide 100 by other numbers?

- Use a pack of 0-100 cards (with 0-10 removed, or more as appropriate) and a pack of playing cards with the picture cards removed to investigate numbers to 100. Give each child one card from each pack and ask them to find out if the playing card number will divide exactly into the larger number. Some children will need concrete apparatus for support. Known multiplication facts can be a useful starting point.

Although most simple calculators do not have a constant button, you can create a sequence with the following key presses (e.g.) 6 + + =. Each time = is pressed, another 6 is added. For subtraction, use 6 – – =. If you want to start with a number other than zero, you need to enter the jump size, then + +, then the start number. So 8 + + 3 =,=,=,=,etc will generate the sequence 11, 19, 27 etc. 6 – – 100 =,=,=, etc will generate the sequence 94, 88,82 etc. So to find out if 8 will divide exactly into 100, enter 8 - - 100 = = etc. Remember that what is shown on the screen is what is 'left over'. As the display approaches zero, is the last number shown 8 or something else?

This is a useful activity to build up information on numbers, recognize patterns and to begin to develop divisibility rules.

▲ Divisibility Rules ~ simplified versions for numbers up to 10

1. All whole numbers are divisible by one.
2. A number is divisible by two if it is even.
3. A number is divisible by three if the sum of the last two digits is divisible by 3, e.g., for 27, 2 + 7 = 9, which is divisible by 3, so 27 is too.
4. A number is divisible by 4 if the number of tens is odd and the unit digit is 2 or 6, or the number of tens is even and the unit digit is 4, 8 or 0. Alternatively, a number is divisible by 4 if you can halve it twice without a remainder.
5. A number is divisible by 5 if the unit digit is either 0 or 5
6. A number is divisible by 6 if it is divisible by both 2 and 3.
7. There is no simple divisibility rule for 7.
8. A number is divisible by 8 if you can halve it three times without a remainder.
9. A number is divisible by 9 if the sum of the digits is 9 or a multiple of 9, e.g., 99 is divisible by 9 because 9 + 9 = 18, which is divisible by 9.
10. A number is divisible by 10 if the unit digit is 0.



