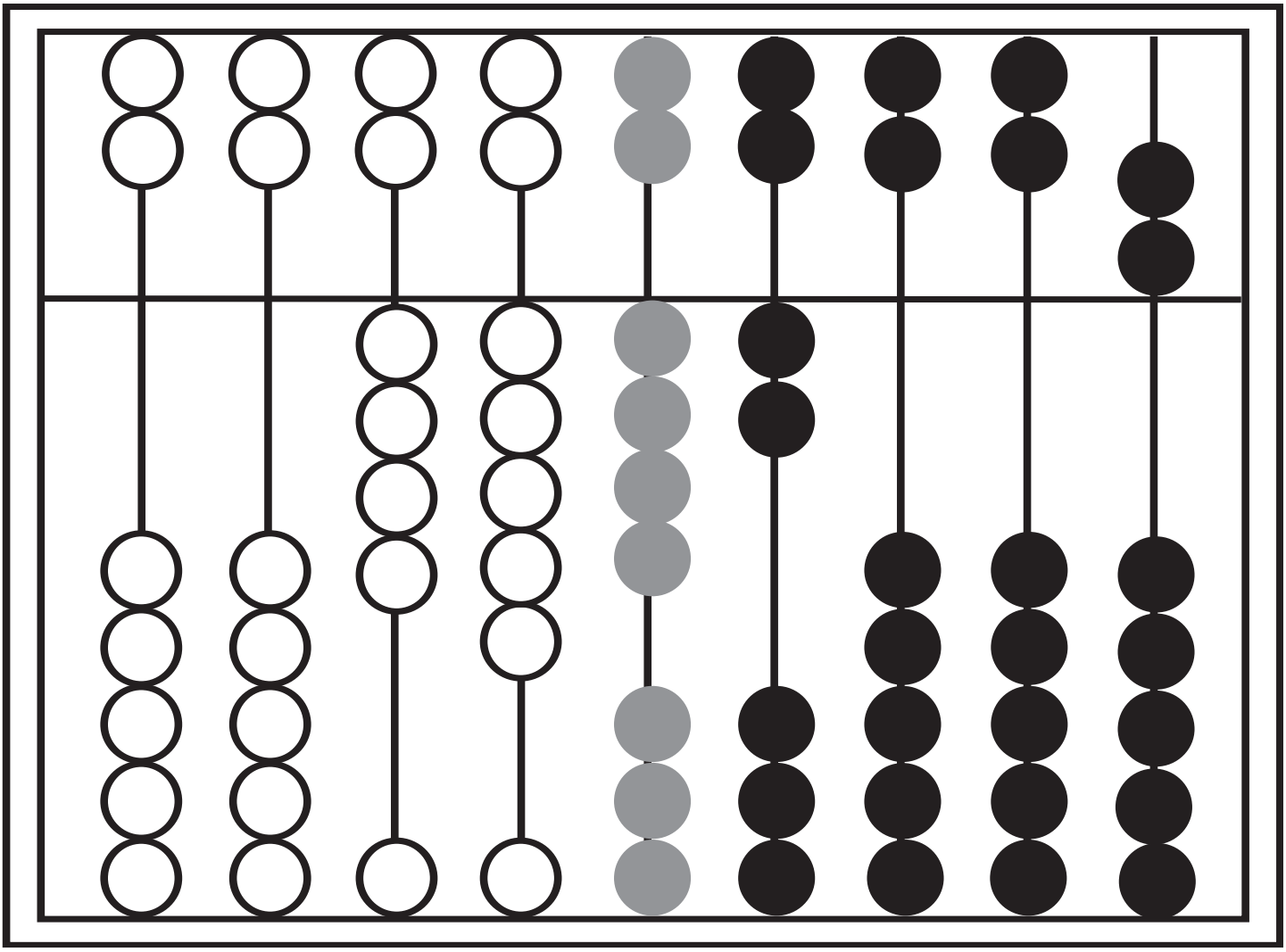
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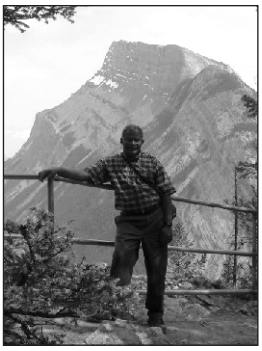
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Edited by Trevor Brown

▲ From the Editor's Desk

Trevor Brown



This month's editorial first appeared in the New Zealand Mathematics newsletter, November 2003. It is reprinted here as we believe that the article examines many of the issues that are somewhat problematic and with which we wrestle. These issues are invariably raised when we discuss the teaching of mathematics with colleagues. The challenge is an on-going one, how should we be teaching mathematics and how would we like it to be viewed.

I was wondering, do you enjoy mathematics? Does anyone? I occasionally meet people who tell me that they were good at mathematics at school and they really enjoyed the subject but ask them if they do any now and they look at you with a perplexed expression. It's as though people don't do mathematics in real life. Everyone knows that mathematics has lots of applications but surprisingly few people actually make use of the mathematics they were taught in school, except in a very simplistic way.

For example, I was talking to someone recently about a new concrete driveway they had put in.

"How did you decide how much sand and cement you would need?", I asked.

"I just guessed a couple of metres and bought a dozen sacks of cement", he replied.

"Was it enough?" I responded.

"No, I had to buy some more."

"Did you have enough then?", I asked.

"Yeah, too much actually. I gave what was left over to my brother. He's building a barbecue."

There wasn't much mathematics used there, just trial and error.

Of course, a good number of professional mathematicians must enjoy the subject. About the same proportion as those who enjoy their work in other professions, I expect, but are there any others who enjoy mathematics and do it in their spare time? For relaxation perhaps, or mental stimulation like doing a crossword? Very few, I suspect. One of the most rewarding times I had as a head of department was when two others in my team were so involved with the subject they used to research it and DO it at home. Wow!

How would you feel if you were forced to do typing every day? Or car maintenance? When you were a student, how did you feel about being forced to study English? What did you think of teachers who forced you to leap over a vaulting horse in the gym. How about people persuading you to investigate mathematics every second?

Mmm! Maybe as teachers we could show a little more compassion towards our students and at least try to make the subject more interesting, bearing in mind that many of them don't want to be there. Maybe we cold show our own enthusiasm, or pretend to if it's lacking, and ginger the lessons up with problem-solving exercises (investigations), which many people enjoy, or historical anecdotes at least. Just a thought.

Fermat's Little Theorem

Fermat's little theorem gives a condition that a prime must satisfy:

Theorem. If P is a prime, then for any number a, (a^P – a) must be divisible by P.

Let's check:

$2^9 – 2 = 510$, is not divisible by 9, so it cannot be prime.

$3^5 - 3 = 240$, is divisible by 5, because 5 is prime.

▲ The Lore of Infinite Numbers

Infinite decimals are infinitely more interesting than finite decimals.

How do you respond to the student who does not believe that

$$0.9999\ldots = 1$$

The most convincing answer I know is to let

$$x = 0.9999\ldots$$

Multiplying both sides by 10 gives

$$10x = 9.9999\ldots$$

because multiplication by 10 shifts the decimal point one place to the right.

Then we subtract x from 10x and the infinite decimals cancel out, leaving

$$9x = 9$$

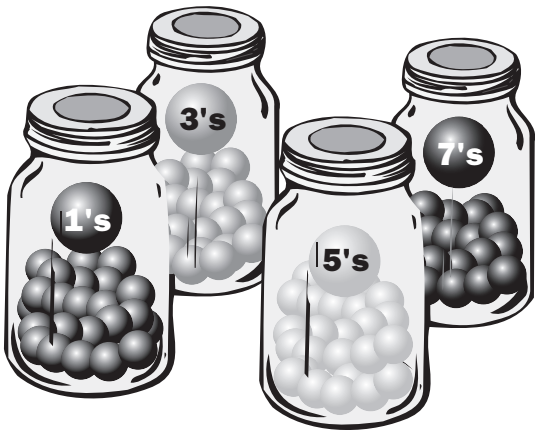
and therefore $x = 1$.



▲ Some Challenges from *NRICH*

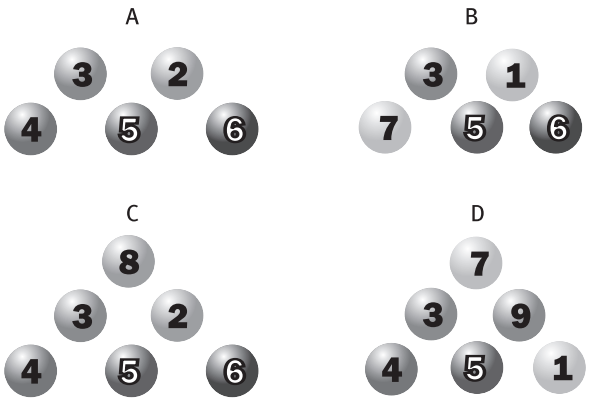
Four Jars

Four jars contain a large number of 1s, 3s, 5s and 7s. Pick any ten numbers from the jars above so that their total is 37.



Odds and Evens

There are some sets of coloured balls with numbers on them:



A set is mixed up and two balls taken at random.
For example:

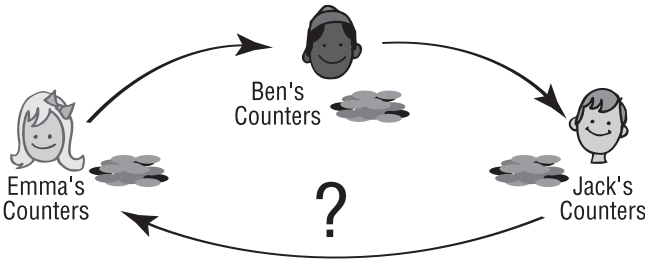


The numbers on the balls are added together:
4 + 5 = 9

The aim is to get as many even numbered totals as possible. Which set is most likely to give you even totals? Try them!

Ben's Game

Ben, Jack and Emma were playing a game with a box of 40 counters - they were not using all of them. They each had a small pile of counters in front of them. All at the same time Ben passed a third of his counters to Jack, Jack passed a quarter of his counters to Emma and Emma passed a fifth of her counters to Ben.



They all passed on more than one counter.

After this they all had the same number of counters.

How many could each of them have started with?

Remembering Leonhard Euler

> *"Gentlemen, that is surely true, it is absolutely paradoxical; we cannot understand it, and we don't know what it means. But we have proved it, and therefore we know it must be the truth."*
>
> Benjamin Pierce, a Harvard mathematician, after proving Euler's equation, $e^{i\pi} = -1$, in a 19th-century lecture.

Sunday, April 15, was the 300th birthday of Leonhard Euler (pronounced "oiler"), one of the most important mathematicians ever to have lived. His works help form the foundation of nearly all areas of mathematics, including calculus, number theory, geometry, and applied math.

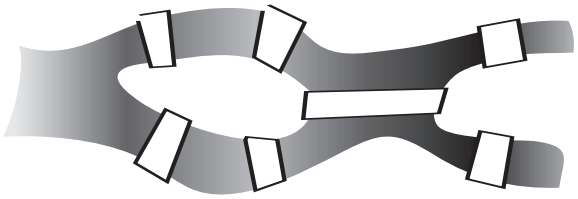
One of the many discoveries for which he is famous is the equation $e^{ip} = -1$. In a 1988 poll, readers of the journal *Mathematical Intelligencer* chose this equation as the single most beautiful equation in all of mathematics. The equation weaves together four seemingly unrelated mathematical numbers, e, p, i, and –1, in an astonishingly simple way.

The following two tasks showcase some of the problems that he analyzed and subsequently solved.

Konigsberg Bridge Problem

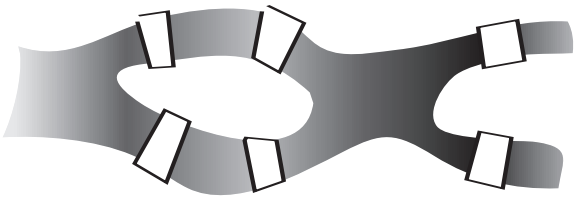
In Konigsberg, Germany, a river ran through the city such that in its center was an island, and after passing the island, the river broke into two parts. Seven bridges were built so that the people of the city could get from one part to another. A map of the center of Konigsberg is shown below:

On Sunday afternoons people went for a stroll



along the bridges and wondered whether or not one could walk across the bridges in such a way that would involve crossing each bridge exactly once. Here are some of the problems resulting from this question.

1. Is it possible to cross each bridge just once?
2. What If you were to remove one of the bridges, would you be able to now cross each bridge just once? Does it matter which bridge you remove?



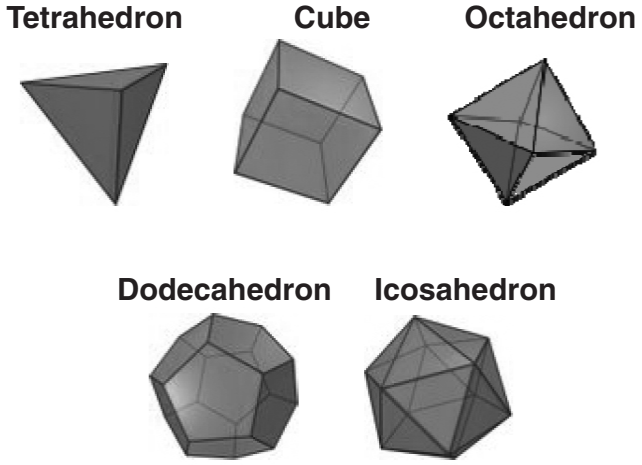
3. Now what if you were to add one bridge, two bridges, three bridges…to the original configuration, would you be able to cross each bridge just once?
4. How many bridges are needed to cross the island?

Platonic Solids

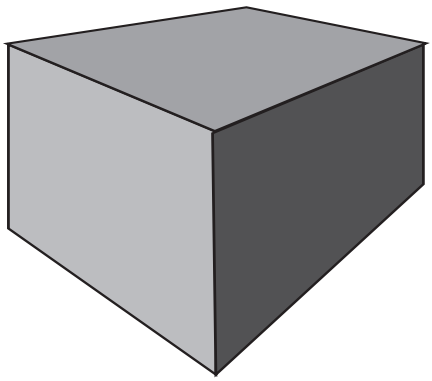
The five Platonic solids are the regular tetrahedron (four equilateral triangle faces), cube (six square faces), regular octahedron (eight equilateral triangle faces), regular dodecahedron (twelve regular pentagon faces), and the regular icosahedron (twenty equilateral triangle faces).

Prove that no more than five regular (convex) polyhedra exist.

The Five Convex Regular Polyhedra (Platonic solids)



Cuboids



Find a cuboid (with edges of integer values) that has a surface area of exactly 100 square units. Is there more than one?

Can you find them all?

Can you provide a convincing argument that you have found them all?

▲ Street Sequences

from NRICH

LET'S LOOK AT A STREET!

Most of the streets around here have their numbers going in order from one side of the street to the other - so that odds end up on one side and evens on the other.
So here's what is could be like:

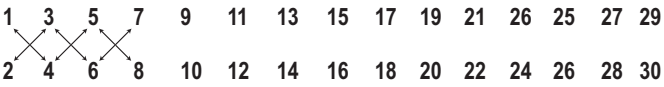


Looking from above the numbers could appear as:

1 3 5 7 9 11 13 15 17 19 21 26 25 27 29

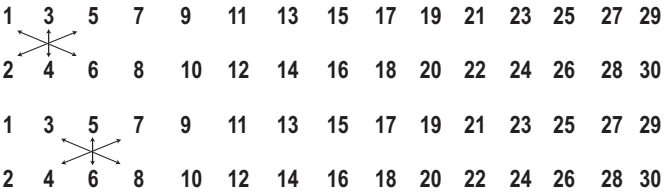
2 4 6 8 10 12 14 16 18 20 22 24 26 28 30

Well you can imagine walking down this street and adding the house numbers in various ways. You could start with adding in pairs across the street:



and carry on as far as you can.
Explore the answers you get for these additions.
What do you notice?
Can you explain why?

If you want to go a bit further with this you could change the grouping of houses in different ways. How about:



1 3 5 7 9 11 13 15 17 19 21 23 25 27 29

2 4 6 8 10 12 14 16 18 20 22 24 26 28 30

1 3 5 7 9 11 13 15 17 19 21 23 25 27 29

2 4 6 8 10 12 14 16 18 20 22 24 26 28 30

But maybe the houses are already grouped in some way - like all semi-detached:

1 3 5 7 9 11 13 15 17 19 21 23 25 27

2 4 6 8 10 12 14 16 18 20 22 24 26 28

Then one side of the street the totals are: 4, 12, 20, etc., and the other side are: 6, 14, 22 etc.

You could continue this and explore it further.

Some streets have terraces of three houses together:

1 3 5 7 9 11 13 15 17 19 21 23 25 27 29

2 4 6 8 10 12 14 16 18 20 22 24 26 28 30

and some have fours:

1 3 5 7 9 11 13 15 17 19 21 23 25 27 29 31

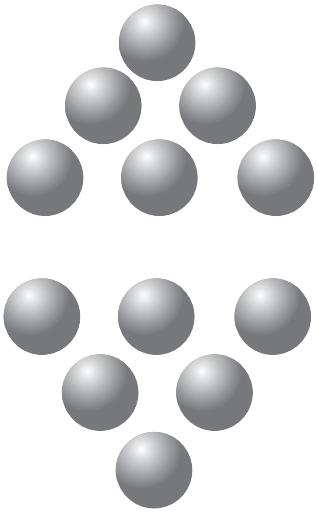
2 4 6 8 10 12 14 16 18 20 22 24 26 28 30 32

You can explore the addition of these groups too!

▲ Recreational Mathematics

Investigation 1

In the diagram there are 6 discs arranged in the shape of a triangle.



It's not too difficult to show that, by moving three discs only the 6 discs can be made to point in the opposite direction, like the discs shown below.

The question is though what happens with 15 discs? If you put 15 discs in a triangular shape with 5t rows, what is the smallest number of discs that you can move that will point the triangle in the opposite direction?

Is there a pattern to predict how many moves are needed?

Investigation 2

Could 29 982 998 be the product of three consecutive whole numbers?

Investigation 3

Could 39 983 998 be the sum of three consecutive whole numbers?

Investigation 4 - Inequalities printed from *nrich*

A bag contains 12 marbles. There are more red than green but taking the green and blue together exceeds the number of reds. The total number of yellow and green marbles is more than the total number of red and blue. Find exactly how many of each colour there are in the bag. How many different solutions can you find?

Investigation 5 - Eggs in Baskets from *nrich*

There are three baskets, a brown one, a red one and a pink one, holding a total of 10 eggs.

The Brown basket has one more egg in it than the Red basket.

The Red basket has 3 fewer eggs than the Pink basket.

How many eggs are in each basket?



Investigation 6 - 'How Many Eggs?' from *nrich*

Peter, Melanie, April and Jack received a total of 38 chocolate eggs.



Jack had one less than Peter.

Peter had 5 less than Melanie.

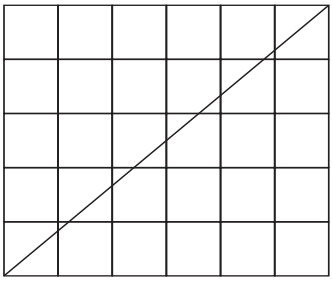
April had half as many as Melanie.

Peter had 2 more than April.

How many eggs did each person have?

Investigation 7

On squared paper draw a rectangle five squares by six squares.



Draw a diagonal of this rectangle.

How many squares does the diagonal pass through?

Do this for other rectangles.

Predict the number of squares passed through for an n by m rectangle.

Investigation 8

$$5 = 2^2 + 1^2$$

$$29 = 5^2 + 2^2$$

Fermat, a French mathematician, showed that certain prime numbers are the sum of two square numbers.

For example, the two prime numbers 5 and 29 can be written as shown above.

For each of the prime numbers less than 100, find out whether or not it can be written as the sum of two square numbers.

What else can you find out about the prime numbers which are the sum of two squares?

Investigation 9

$$7 + 19 + 23 + 29 = 78$$

$$2 + 53 + 23 = 78$$

$$37 + 41 = 78$$

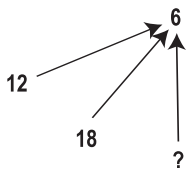
Select four prime numbers less than 50.Find their sum.

Can you now get the same sum with three prime numbers/

Can you get it with two?

Write the numbers from 1 to 50 as the sum of prime numbers, using the least number of primes in each case.

Investigation 10



The set of divisors of 12 is: {1, 2, 3, 4, 6, 12}

So 12 has six divisors.

This is shown on the graph above by an arrow from 12 to 6.

Check that an arrow should also be drawn from 18 to 6.

Try to find some other numbers with six divisors.

Draw arrow graphs showing the number of divisors of other numbers.

Investigate the relation between numbers having the same number of divisors.

Investigation 11 - Triangles and String

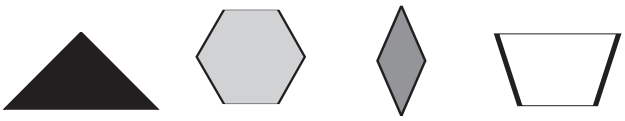
First, find all of the possible triangles I could make with this 12-cm loop of string where all of the sides of each triangle are integers.

Second, explain whether each triangle is scalene, isosceles, or equilateral, and also whether it is acute, right, or obtuse.

You'll need to carefully explain how you know you've correctly answered both parts of the question, and to include any calculations you might have done.

Bonus: Which of the possible triangles in the problem has the greatest area? Make sure you explain how you reached your conclusion.

▲ This Month's Concrete Materials: Pattern Blocks



1. Make the following pattern with your Pattern Blocks: red, red, blue, green, red, red, blue, green.
   a) When there are 14 blue blocks, how many green blocks will there be? How many red blocks? Organize your work so that you can explain your solution to the rest of the class.
   b) When there are 30 red blocks, how many blue blocks will there be?
   c) When there are 12 green blocks, how many blocks will there be altogether?
   c) Make a pattern with red, blue and green blocks that obeys all of the following rules:
      • the pattern repeats after 5 blocks;
      • when there are 15 blocks there are 3 red blocks;
      • when there are 9 red blocks there are 18 blue blocks.
2. Line up the following pattern blocks: yellow hexagon, a blue rhombus, 1 red trapezoid, 1 green triangle.

   How many ways are there to line up these 4 pattern blocks?

   Rearrange the 4 blocks and record the new arrangement.

   Find all possible "colour orders" for these 4 blocks.

   Be sure to record your work in an organized way so that you can explain your thinking later.

   How can you be sure that you have found all possible ways?

   If you add the orange square, how many ways would you predict there are to order 5 different blocks?
3. Make the following pattern: red, red, green, red, red, green, ...

   What colour will the hundredth block be?

   How many red blocks will there be after 100 blocks?

   What colour will the 487th block be?
4. Start with one green triangle.

   Make a larger triangle by adding three more green triangles.

   This is called "growing triangles".

   If you had 100 green triangles, how many growing triangles could you make?

   Use triangular paper to help you work out your solution.

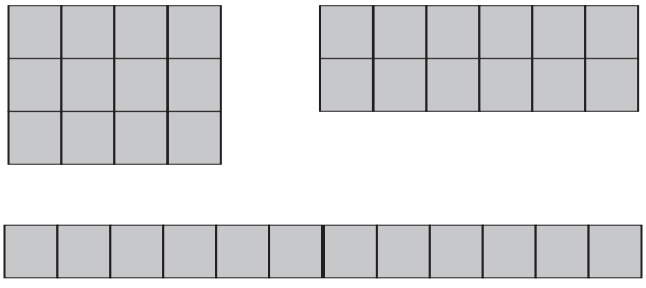
   How many triangle blocks would you need to make the first eight growing triangles?

   Make up a new problem about growing shows a different pattern block.

   Can all the pattern blocks be used to make a series of growing shapes?

▲ Colour Tiles

With twelve square tiles you can make just three different rectangles.



How many tiles do you need to make just five different rectangles? two different rectangles? etc.