8 PAGE INSERT ELEMENTARY MATH

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FROM ARITHMETIC TO ALGEBRA – ALGEBRA AS FUNCTIONAL RELATIONSHIPS

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ABACUS CO-EDITORS



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It's interesting to notice that most people think of mathematics as being synonymous with arithmetic. If you ask someone to find mathematics in a scan of a newspaper or magazine page, they show surprise at how many numbers they notice; whole numbers, fractions, decimals, and percents are commonly found. Yet, they are less likely to notice algebraic generalizations or formulas.

If you are doing a mathematical calculation or process once, you probably don't need algebra. But if you are doing a mathematical process again and again, algebra provides concise language for describing and repeatedly carrying out a process or set of calculations. For example, there are perimeter and area formulas that can be used to calculate the length of fencing to enclose a rectangular field or to measure the area of carpet needed to cover a rectangular floor space. Algebra is also the language of relationships between quantities. For example, the relationship between the number of tennis balls can be used to cover the bottoms of 4 desk legs and the number of desks can used to figure out the number of tennis balls needed for 5 desks, 10 desks, 100 desks, or 252 desks. Though the value of algebra, especially as functional relationships is not as obvious to many of us as is the value of arithmetic, its usefulness is all around us. Yet, for those who don't know what to look for, algebra is often hidden from their view.

Therefore, in this issue, the problems and commentary focus on illuminating the idea of algebra as the study of functional relationships. Functional thinking focuses on understanding the kinds of mathematical situations where there is a relationship between quantities; that is, where one quantity depends on another.

LINKS TO LITERATURE AND MANIPULATIVES



PAT MARGERM is an independent literacy and mathematics consultant with more that 30+ years of K to 8 teaching experience with students, in-service and pre-service teachers. She is a long-time OAME board member and works in various aspects of mathematics education.

In addition to functional relationships, I will include ideas about the variable as an unknown value or placeholder, using manipulatives and children's literature. Research from Carpenter, Franke, and Levi (2003) describe how the idea of unknown values or placeholders contributed to students' learning about the relationships between addition and subtraction. For example, 21 – 7 = ___ and 7 + ___ = 21. Experiences with placeholders can provide opportunities to collect and organize numbers in charts in order to see numerical relationships and make generalizations.

RESEARCH SUMMARY – ABOUT FUNCTIONAL THINKING

According to Randall (2005), a *Big Idea* is a statement about a mathematical idea that is central to the learning of mathematics, and it links numerous mathematical understandings as a coherent whole. Through Big Ideas, he said that we can understand mathematics as an interrelated set of ideas, skills, and strategies. He described two Big Ideas about functional thinking.

- "Relationships can be described and generalizations made for mathematical situations that have numbers or objects that repeat in predictable ways" (p. 17).
- "Mathematical rules (relations) can be used to assign members of one set to members of another set. A special rule (function) assigns each member of one set to a unique member of the other set" (p. 18).

What is a Function?

According to Blanton (2008), a *function* describes how two (or more) quantities vary in relation to one another. A function can be depicted in words or mathematical symbols, and it can be portrayed through representations like graphs and tables. Although there are many types of relationships between quantities, functions are special because they reflect a particular type of correspondence between two quantities. That is, in a function, each value of one quantity (first quantity) corresponds or is related uniquely to the value of the second quantity.

Further, Small (2008) describes a function as a "relationship that leads to a particular output for a particular input. For example, if the function is 'double the number,' the input is the number to be doubled and the output is the double" (p. 9). Often such inputs and outputs are organized in a table to enable analysis and inferences about the relationships between the input and output quantities. A *function table* allows students to notice a relationship or function by listing the inputs in one column and corresponding outputs in the other. (See the Input-Output table to the left.)

Input	Output
1	5
2	7
3	9
4	11
5	13

This table represents the function as:

- a pattern – 5, 7, 9, 11, 13, 15, …
- an input/output rule - double the input and add 3.

The *input* represents the *position* or *term* in the pattern and the *output* represents the *term value* in the pattern.

Getting Started - Developing Number Relationships:

- "What number plus 17 gives 21?" or ☐ + 17 = 21
- 2 x ☐ = 34
- 3 x ☐ – 8 = 31

Similar to Carpenter, Franke, and Levi's (2003) research about variables as placeholders, Kieran and Chalou (1999) report on the use of a "cover-up equation-solving" technique for solving equations. The instructional sequence begins with posing one-operation equation problems then two-operation equation problems. They report that this approach helped students to learn to associate the sum or product of a number and an unknown as a single number. The unknown number or the empty box is later written as a variable, acting as a placeholder. This knowledge prepares them for writing two-operation equations. Such an approach aides students in seeing the overall structure of equations and in unpacking the structure in a systematic way. Think about how this structure of an unknown number in one- and two-operation equations builds a foundation for functional thinking.

Learning About Functions - Function Machines or Boxes:

To introduce the idea of functions, Cuevas and Yeatts (2001) suggest the use of function machines or function boxes. A function machine or box has input and output slots. A given value (input value) goes in and out comes an output value. The students have to figure out the "rule" or "function" that changes the number.

Marble Problem:

In a marble machine, the number of marbles inputted and outputted were recorded. One boy put in 1 marble and got back 3 marbles. One girl put in 9 marbles and got back 11 marbles, and so on. What is the rule?

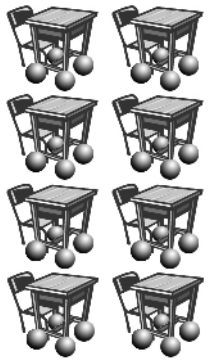
Some researchers recommend that the input quantities be recorded in numerical order (like the given example on page 1) to model the use of a function table. These tables are used extensively to develop generalizations of different functions. Moss, Beatty, Barkin, and Shillolo (2008) recommend that the input values be out of numerical sequence, so that students are provoked to read across the data values, rather than read down a column of values for a recursive pattern.

Input	Output
1	3
9	11
2	4
6	8
30	32
3	5
21	23

Learning About Functions - Building, Describing, and Reasoning:

Blanton (2008) describes functional thinking as a process of building, describing, and reasoning about functions. Such a process involves algebraic thinking because it includes making generalizations about how quantities are related.

Tennis Balls Problem:
How many tennis balls are needed for all the legs of 8 desks?



Through a simple computation, you can find the total number of tennis balls needed to cover the desk legs if you know the number of desks. You can either add 4 repeatedly (8 times) or multiply the number of desk legs (4) by the number of desks (8). This is a limited way of considering the relationship between the number of tennis balls and desks. What if you had a certain number of tennis balls and wanted to know the number of desk legs they could cover? What if you wanted to find the total number tennis balls for any number of desks? The functional relationship between the number of tennis balls and number of desks makes explicit how the change in one quantity (number of desks) produces a change in the other quantity (the number of tennis balls). So, finding the functional relationship between two quantities involves identifying and describing what that change looks like in general.

Furthermore, because the number of tennis balls depends on the number of desks, we say that the number of tennis balls is the dependent variable and the number of desks is the independent variable. For younger students, the independent variable is referred to as the *input* and the dependent variable as the *output*, where the output is dependent on the input. Organizing related quantities in a function table allows one to visually make meaning of the parts, so that relationships can be seen across the quantities (data). According to Blanton (2008), there are three common ways that students make sense of the numeric values from a function table: finding a recursive pattern, finding co-variation, and finding correspondence between quantities.

Finding a Recursive Pattern

This approach focuses on *looking down for a relationship within a sequence of values*. In the desk example, students might describe the recursive pattern as “add four every time” or “skip count by 4s starting at 0.” Finding the pattern involves looking for a relationship in a sequence of values, usually the dependent variable, while the independent variable (the number of desks) is often unnoticed by the student.

While recursive patterns are useful, they have limitations, such as:

- finding a particular value in a sequence requires knowing all the values before it
- for larger numbers it becomes cumbersome; might have to add four 199 times (200 desks).

Number of Desks (independent variable) (input)	Number of Tennis Balls (dependent variable) (output)
0	0
1	4 (+ 4)
2	8 (+ 4)
3	12 (+ 4)
4	? (+ 4)

Finding Co-Variation

This approach focuses on *looking down for a relationship within two sequences of values*. In the Tennis Balls problem, the learner notices that as the number of desks increases by 1, the number of tennis balls increases by 4.

The focus is shifted from looking down in one column to looking both down and across the columns. The function table prompts students to think about how the quantities change in relation to each other; that is, how the quantities co-vary.

Number of Desks (independent variable) (input)	Number of Tennis Balls (dependent variable) (output)
1 (+ 1)	4 (+ 4)
2 (+ 1)	8 (+ 4)
3 (+ 1)	12 (+ 4)
4	?

Finding Correspondence Between Quantities

This approach focuses on *looking across for a multiplicative relationship between corresponding quantities in the columns*. In the Tennis Balls problem, the student notices an operational correspondence between the quantities, such as what must be done to 1 (desk) to get 4 (tennis balls) is the same as what is done to 2 (desks) to get 8 (tennis balls). This function table shows a multiplicative relationship of "quadruple or multiply by 4" the number of desks to get the number of corresponding tennis balls.

Number of Desks (independent variable) (input)		Number of Tennis Balls (dependent variable) (output)
1	quadruple	4
2	⟷	8
3	x 4	12
4		?

The horizontal (or looking across) analysis between the quantities in the columns results in finding correspondence between the two quantities. The correspondence could be expressed in natural language, such as the number of tennis balls is quadruple the number of desks. In later grades, the correspondence could be expressed symbolically. For example, if number of the tennis balls is represented as B and the number of desks as n, then the function can be described as $B = 4n$. This symbolic representation is consistent with how we understand multiplication. In this case, B (number of tennis balls) is the product, 4 is the *multiplicand* or the size of the group and n is the *multiplier* or the number of groups.

LET'S DO MATH

Input	Output
	24
6	
13	39
	42
25	75
5.5	

Getting Started:

Have the students determine the missing values in the function table. Ask questions like, "How do you know?" so that the students are explaining and justifying the missing values. After students have explained how they determined the missing values, ask them to describe the functional relationship between the input and output values, using words and symbols.

Problem 1:

Soybeans (in kilograms)	Tofu (in kilograms)
2	5
4	10
?	20
100	?
?	400

Many Ontario farmers grow soybeans. So, a company in Simcoe, Ontario started to make tofu to sell to grocery stores. The table shows the different amounts of soybeans required to make specific amounts of tofu.

a) Determine the missing values in the table.

b) Describe the functional relationship used to complete the table of values.

Anticipating Student Responses:

What mathematics do you see in each student work sample?

Sample Responses:

Soybeans (in kilograms)	Tofu (in kilograms)
2	5
4	10
6	15
8	20
10	25
100	250
160	400

1. Students might want to reorganize the table and add more detail as follows. Soybeans (in kilograms) Tofu (in kilograms)
   - When I write the growing pattern I see that when there are 10 kg of soybeans I will have 25 kg of tofu.
   - So, for 100 kg of soybeans I will have 250 kg of tofu. (10 x 10 = 100 and 25 x 10 = 250)
   - I need 4 kg of soybeans for 10 kg of tofu, so if I want 40 times as much tofu (10 x 40 = 400), I need 40 times as much soybeans (4 x 40 = 160).
2. I notice vertical relationships 2 x 2 = 4 and 5 x 2 = 10.
   So, 5 x 4 = 20 and 2 x 4 = 8. So, 8 kg soybeans produces 20 kg of tofu.
   I know 2 x 50 = 100 so 5 x 50 = 250. 400/5 = 80 so, 5 x 80 = 400 and 2 x 80 = 160
3. I see horizontal relationships 5/2 = 2.5 or 2 x 2.5 = 5
   Also, 4 x 2.5 =10. All the rows need to be like that.
   ☐ x 2.5 = 20; 8 x 2 = 16 and 0.5 x 8 = 4 so 8 x 2.5 = 20 100 x 2.5 = 250
   ☐ x 2.5 = 400, so 100 x 2.5 = 250, need ☐ x 2.5 = 150 (250 + 150 = 400)
   Try 80 x 2.5 = 160 + 40 = 200 - too much.
   Try 60 x 2.5 = 120 + 30 = 150 - just right. So, 160 x 2.5 = 320 + 80 = 400

Coordinating Student Discussion for Learning:

Why might solution 1 be chosen first for student discussion, followed by solutions 2 and 3? The solutions are organized according to the three common ways that students make sense of the numeric values from a function table.

- solution 1 – focuses on finding a recursive pattern in one of the variables (looking down for a relationship within a sequence of values)
- solution 2 – focuses on finding co-variation (looking down for a relationship within two sequences of values)
- solution 3 – focuses on finding correspondence between the 2 quantities (looking across for a multiplicative relationship between corresponding quantities in the columns).

LINKS TO MANIPULATIVES

How Many Are Hidden?

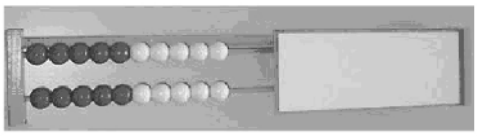
Most manipulatives can be used as objects to be hidden, covered, or masked.

Cubes

Total Number of Cubes	Cubes I can see	Cubes I can't see	Number Sentence

- Have students take a number of objects, such as 10 cubes. One partner covers part of the 10 cubes (e.g., hides 3 cubes under a bowl, a piece of paper, or in their hands). The other partner, seeing the 7 remaining cubes, determines the number of hidden cubes.
- Extend the challenge by having students use a different number of cubes.
- Collect and organize the numbers generated by each pair of students to make generalizations (e.g., when you don't hide any cubes, the number is the same as the one that you start with; when you hide all of the cubes you have zero cubes that you can see).

Rekenreks and Bead Strings



A teacher version of a rekenrek has a "cover" that can be used to hide some of the beads. Covers can be created for bead strings when working with larger numbers.

- Hide some beads behind the cover (using 1 row of 10 or 2 rows of 10) and ask students to determine the number of hidden beads.
- Ask students to compare strategies for determining the number of hidden beads, such as counting up from the number to 10, counting back from 10, adding, or subtracting.
- Ask students to make mathematical statements about the hidden number(s), such as " I know that it is 1 more than … I know that it is 10 less than …"

What's the Function?

Containers such as paper bags, egg cartons, or boxes can be used to show multiple sets of objects.

Pose questions like:



- "If there are the same number of cubes in each bag, how many total cubes are there? How do you know?" (e.g., 50 for 10 cubes in each bag; 45 for 9 cubes in each bag; 25 for 5 cubes in each bag)
- "What is the functional relationship between the number of bags and total number of cubes?" In this case, the number of bags stays the same, and the number of cubes in each bag changes; that is, 5 bags x number of cubes per bag = total cubes



- "What is the functional relationship between the total number of cubes and the number of bags, if there are 3 additional cubes with each set of bags?" the function can be described as "5 bags x number of cubes per bag + 3 cubes = total cubes"



What's the Functional Relationship?

- Ask students what they notice about the relationship between the numbers in the table (e.g., all of the numbers in the first column are multiples of 3).
- Have students identify the input and output numbers (e.g., input is 24 and the output is 4; input 60, output 10)
- Prompt students to describe the operation that is being applied to the input (first number) to get the output (second number) for all values in the table.

Total number of cubes	Number of cubes in each bag	Number of bags
24	4	
60	10	
42	7	
18	3	

LINKS TO LITERATURE

Picture books can be adapted to provide students with experiences with the placeholder and/or to collect and organize numbers in charts to see functional relationships and to make generalizations.

How Many Are Hidden?



In the story, "*A Dozen Ducklings Lost and Found*," a proud mother duck marches to the farmer's house to show off her ducklings. Some of her ducklings fall into post holes that the farmer has worked hard to dig. There are several opportunities in the book to build on the part/whole relationships, to use placeholders, and to collect and organize data.

- Early in the book, the eggs are hatching 1 by 1. The picture shows 4 chicks and 8 eggs. Ask the students to show with counters, bead string, or their fingers, how many chicks there will be when another egg hatches. Have students share their strategies (e.g., I know that the number that comes after 4 is 5, 4+1 is 5). Ask the students for ways that they could keep track of the number of eggs and chicks as eggs hatch. Prompt the students to make some generalizations about the number of eggs and the number of chicks (e.g., as the number of eggs decrease the number of chicks increase; some pairs of numbers are the same but represent different objects – 4 eggs and 8 chicks and 8 eggs and 4 chicks).
- When the mother duck counts only four of her babies, ask the students to figure out how many babies are missing. This question can be asked several times throughout the story, if the teacher "masks" or covers part of the picture to hide some of the ducklings. Ask the students to suggest equations to represent the mother duck's problem, such as 4 + ___= 12
- Mother Duck says that she has 12 ducklings or one dozen ducklings. The class can identify other objects that come in groups of a dozen. The class can develop a table to show how the total number of objects change as the number of dozens increase.

Number of dozens		Number of objects
1	12	12
2	12 + 12	24
3	12+12+12	36
4	12+12+12+12	48

What's the Function? What's the Functional Relationship?



In the story, "*One Watermelon Seed*," "Max and Josephine planted a garden. They planted one watermelon seed ... and it grew. They planted two pumpkin seeds ... and they grew. Max planted three eggplants ... and they grew … The seeds and the leaves, the stalks and the vines grew and grew and grew … One day they looked at their garden and saw there was plenty to pick … They picked ten watermelons, big and green. And twenty pumpkins, glowing orange. Max picked thirty eggplants, dark and purple, and forty peppers, shiny yellow."

What's the functional relationship between the number of seeds and the number of plants? Record students' observations of the number of seeds in relation to the number of garden plants in a table to discern the functional relationship between seeds and plants.

Number of Seeds	Number of Plants	Number Sentence
1 watermelon	10 watermelon	1 x ☐ = 10
2 pumpkin	20 pumpkin	2 x ☐ = 20
3 eggplants	30 eggplant	3 x ☐ = 30

LET'S DO MATH

Getting Started:

Input	Output
	23
35	
72	51
	27
141	

Have the students determine the missing values in the function table. Ask questions like, "How do you know?" so that the students are explaining and justifying the missing values. After students have explained how they determined the missing values, ask them to describe the functional relationship between the input and output values, using words and symbols.

Problem 2:

Johanna tried to record the mass of the pets in her mom's store, but they wouldn't sit still on the scale. Her mother says, "Hold the pets

Type of Pet	Mass of Johanna and pet (in kilograms)	Mass of the pet (in kilograms)
Tom the snake	30 + 6	6
Dan the BIG dog	30 + 50	50
Blake the dog	40	?
Simon the cat	36	?
Ralph the rabbit	33	?
Tally the guinea pig	32	?

and stand on the scale. And, by the way, the mass of the snake and the dog are listed on their collars." Johanna knows her mass is 30 kilograms. She records the numbers in the chart. How will these numbers help her complete the job?

a) Determine the missing values in the table.

b) Describe the relationship between the mass of Johanna and the pet and the mass of the pet.

Anticipating Student Responses:

What mathematics do you see in the students' sample work?

Sample Responses:

Solution 1

40 – 30 = 10; 36 – 30 = 6; 33 – 30 = 3; 32 – 30 =2

b) "Johanna and the pet weight 40kg; she weighs 30; so the dog weighs 10 kg; so the guinea pig weighs 32 kg – 30 kg = 2 kg

c) Subtract 30

Solution 2

dog 10, cat 6, rabbit 2, guinea pig 2
human and pet – human = pet
second value = first value – 30

Solution 3

I started filling in the table and I looked to see how the numbers are related. The mass of the animal is 30 kg less than the reading on the scale which is the mass of Johanna and the pet. The output is 30 less than the input.

Cat	36	6	Johanna and the cat mass 36 so cat 6
Snake		6	So snake will be the same 36 and 6
Rabbit	32		Rabbit is 4 less on left so right will be 4 less or 2

Coordinating Student Solutions for Learning:

Why might solution 3 be chosen first for student discussion, followed by solutions 2 and 3?

- solution 1 – focuses on the use of number sentences or equations to represent the calculations; each calculation is done separately; students have answered a specific example rather than describing the relationship; the statement can be used to pull out the number sentence structure and highlight number relationships as the discussion proceeds.
- solution 2 – focuses on numbers that match masses for the dog, cat, and pig, but the rabbit mass doesn't match the number relationship in the other solutions – discuss this difference; this solution includes word equations to describe the number relationships
- solution 3 – focuses on the use of a table (with verbal explanations inside the table).

NEXT STEPS FOR YOUR PROFESSIONAL LEARNING

Application to Your Classroom

Because functional thinking is representational thinking that focuses on the relationship between two (or more) varying quantities, Smith (2008) explained that the algebraic reasoning part of functional thinking occurs as students invent and/or apply representation to a generalization of a relationship among varying quantities. So, to nurture such mathematical thinking, he proposed six learning activities that engage students in a problem within a functional situation.

- Engage in some type of physical or conceptual activity (e.g., snow boots per student, markers per package, days per week)
- Identify two or more quantities that vary in the course of the physical activity, focusing one's attention on the relationship between these two quantities (independent and dependent variables)
- Make a record of the corresponding quantities, in a table, graph, or set of symbols
- Identify patterns in the records
- Relate the identified patterns with the actions involved in carrying out the activity
- Create a representation of the identified pattern in the relationship.

Alternatively, Blanton (2008) provided several suggestions for developing functional thinking problems.

- *Think of an arithmetic question* that involves counting a quantity that can change over time or vary in a natural way, such as the number of outfits that can be made out of 2 shirts with 1 pair

of pants or the number of cookies and drink boxes in a lunch bag. These questions can be answered using arithmetic and modelling with manipulatives.

- *Vary a Parameter or Quantity* in the task. The quantity varied will represent the independent variable in the function. For example, you might vary the number of shirts or the number of pants. So instead of 2 shirts, what if you had 3 shirts, 4 shirts, or 5 shirts? Or 10 shirts? Or 200 shirts? How many outfits would you get each time? A change in the number of shirts (independent variable) will result in a change in the quantity being counted; that is, the number of outfits (dependent variable). Also, changing the number of shirts will change the number possible outfits. With this task, the goal is to have students think about and describe (in words and/or symbolically) how a change in one quantity affects a change in the second quantity; that is, the relationship between the two quantities.
- *Use Numbers Algebraically* means that the numbers chosen for the quantities can be small enough at the start, so that students can discern a relationship between the quantities. But, the numbers could be later varied to large numbers, so that students are provoked to think about the structure and relationships in the numbers, rather than solving the problem using arithmetic. For example, if there are 100 shirts, students would be less patient or motivated to compute the total number of outfits through modelling and counting. Rather, they would be more interested in finding an operational relationship between the number of shirts and pants.

Suggested Readings

Some readings used in the Research Summary and in the development of the function-based problems and solutions are listed below:

- Blanton, M. (2008). *Algebra and the elementary classroom*. Portsmouth, NH: Heinemann.
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- Mason, J. (2008). Making use of students' powers to produce algebraic thinking. In J. Kaput, D. Carraher, & M. Blanton (Eds.), *Algebra in the early grades* (57-94). Mahwah, NJ: Lawrence Erlbaum Associates/Taylor & Francis Group.
- Moss, J., Beatty, R., Barkin, S., & Shillol, G. (2008). "What is your theory? What is your rule?" Fourth graders build an understanding of functions through patterns and generalizing problems. In C. Greenes & R. Rubenstein (Eds.), *Algebra and algebraic thinking in school mathematics* (155168). Reston, VA: National Council of Teachers of Mathematics.
- Small, M. (2008). *Big ideas from Dr. Small: Creating a comfort zone for teaching mathematics.* Toronto: Nelson Education.
- Smith, E. (2008). Representational thinking as a framework for introducing functions in the elementary curriculum. In J. Kaput, D. Carraher, & M. Blanton (Eds.), *Algebra in the Early Grades* (133-160). Mahwah, NJ: Lawrence Erlbaum Associates/Taylor & Francis Group.

CLOSING

Our next issue will continue to focus on the relationship between arithmetic to algebra, in terms of understanding functional relationships generated from corresponding arithmetic (table), algebraic (single-and-multi-variable expressions), and geometric (graphic) representations.

CALL FOR *ABACUS* SUBMISSIONS

Send your teaching strategy ideas, problems and student solutions to the *Abacus* Co-Editors via email