8 PAGE INSERT ELEMENTARY MATH

ONTARIO ASSOCIATION FOR MATHEMATICS EDUCATION

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FROM ARITHMETIC TO ALGEBRA – EXPRESSING GENERALITY USING MULTIPLE REPRESENTATIONS

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A Publication of the OAME/AOEM

ABACUS CO-EDITORS



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We'd like to share with you some memorable ideas offered by Dr. John Mason during the OAME Leadership 2009 conference. We hope that these ideas will provoke you to reflect on the ways that you think mathematically and that you provoke your students to think algebraically.

John explained, "Everything said is a conjecture that is to be tested in experience and to be modified as necessary. A conjecture is said in order to 'get it out' so it can be looked at clearly and closely. When we disagree with a conjecture, we can offer a potential counter-example or we can invite someone to modify their conjecture."

He stated, "Every child comes to school having displayed the powers necessary to think algebraically. The issue is not whether learners can generalize, but what teachers can do to stimulate students to use and develop the power of generalization, effectively and appropriately."

John concluded, "A lesson without the opportunity for learners to generalize mathematically, is not a mathematics lesson."

Expressing generality is a significant aspect of mathematics teaching and learning. Therefore, this issue of the Abacus will focus on the different ways that students can learn to express generality. These different ways include the use of manipulatives or concrete models, labeled diagrams, table of values, numeric expressions, algebraic expressions, graphic representations, and verbal statements.

LINKS TO LITERATURE AND MANIPULATIVES



PAT MARGERM is an independent literacy and mathematics consultant with more that 30+ years of K to 8 teaching experience with students, in-service and pre-service teachers. She is a long-time OAME board member and works in various aspects of mathematics education.

In this issue, I chose manipulatives and children's literature that provides opportunities for students to make mathematical generalizations. Perhaps, in thinking about next year, take a second look at your use of manipulatives and picture books to find other opportunities to generalize. I would be delighted to hear your ideas.

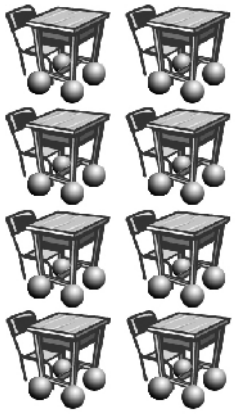
RESEARCH SUMMARY – SPECIALIZING AND GENERALIZING

- "Relationships can be described and generalizations made for mathematical situations that have numbers or objects that repeat in predictable ways."
- "Mathematical situations and structures can be translated and represented abstractly using variables, expressions, and equations." - Big Ideas from Randall, 2005, pp.17, 18

Tennis Ball Problem:

How many tennis balls are needed for all the legs of 8 desks?

How many tennis balls are needed for all the legs of any number of desks?



Whenever a generality is encountered, Mason (2005, 2008) suggests that it is valuable to try to see the particular in the general; that is, to seek specific examples to justify the generalization. Specializing or particularizing includes offering examples that support, contradict, challenge, or extend what is being generalized. On the other hand, the opposite action of specializing is generalizing. A generalization can be located within a collection of particular mathematical examples. Whenever a particular object or collection of objects is encountered, it is worthwhile trying to see the general through the particular, by asking what stays the same and what can be changed or varied. For the Tennis Ball problem (from Issue 3), what is staying the same and what is changing?

From the Particular to the General

Equivalent Numeric Expressions

Solutions to open-routed quantity problems have the same answer, but can have different numerical methods or numerical expressions. For example, solutions to the Tennis Ball problem have the same answer, 32, but different numeric expressions: 4 + 4 + 4 + 4 + 4 + 4 + 4 + 4 = 4 x 8 = 4 x 4 + 4 x 4. To think about generality, ask yourself this question, "From these 3 equivalent numerical expressions, which numerical expression has the best potential for determining the number of tennis balls for any number of desks?"

Quasi-Variable Numeric Expressions

Fujii and Stephens (2001) and Fujii (2003) argue that the use of generalizable numerical expressions can provide an important bridge between arithmetic and algebraic thinking. Generalizable numeric expressions which include quasi-variables provide a way to think conceptually about a variable as a changing quantity. *Quasi-variables* are numbers within a number expression or a group of number expressions that indicate an underlying mathematical relationship which remains true for a set of numbers used. Quasi-variable or generalizable numerical expressions are often written in an uncalculated form in order to disclose the relationships between the numbers involved. Let's revisit the Tennis Ball problem to illustrate these ideas.

Number of Desks	1	2	3	4	5	6	7	8
Number of Tennis Balls	4	8	12	16	20	24	28	32
Quasi-Variable Numerical Expression	4 x 1	4 x 2	4 x 3	4 x 4	4 x 5	4 x 6	4 x 7	4 x 8

In the table of values, the quasi-variable numerical expressions include 4 x 1, 4 x 2, 4 x 3, … 4 x 8. The critical numbers are (the number of desk legs (4) multiplied by the number of desks (ranging from 1 to 8)) and the relational elements (4 desk legs per 1 desk). When the critical numbers are organized in expressions like these, they provide students with an expression that represents the underlying arithmetic

structure of the situation. Quasi-variable thinking is shown when a student notices that the number of desk legs (4) stays the same and the number of desks (ranging from 1 to 8) changes in reference to the structural properties of the numerical expression (4 x 1, 4 x 2, 4 x 3, … , 4 x number of desks). Therefore, generalizable numerical expressions can provoke students to identify and discuss generalizations long before they learn formal algebraic notation (Fujii, 2003).

Multiple Representations of a Function

According to Van Dyke and Craine (1999), it is important that students see the explicit connections among various representations of a function; that is, they realize that a function can be represented by a real world context, a table of values, a graph, a verbal statement, and as an algebraic expression. Further, Brizuel and Ernest (2008) reported that students' understanding about a problem and its associated function can be enhanced by moving across four stages of representational notation: verbal representations and instantiations with manipulatives, initial written/algebraic notations using words or pictures, use of tables, and graphical representations. Let's revisit the Tennis Ball problem to consider different representations of a function.

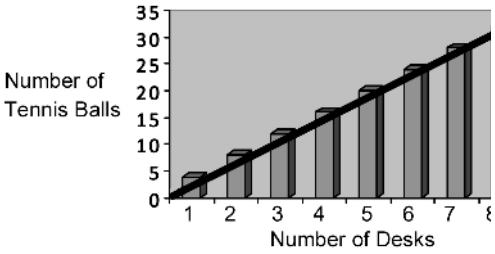
Tennis Ball Problem

Table of Values and Numeric Expressions

Number of Desks	Number of Tennis Balls	Quasi-Variable Numeric Expression
1	4	4 x 1
2	8	4 x 2
3	12	4 x 3
4	16	4 x 4
5	20	4 x 5
6	24	4 x 6
7	28	4 x 7
8	32	4 x 8
d	4*d*	4 x *d*

quasi-variable is the changing variable quantity

Bar and Line Graphs - Tennis Ball Problem



Generalized or Algebraic Equation

number of tennis balls (b)
= 4 x number of desks (d)

Pictorial Real Context



Verbal Statement

For each desk, 4 tennis balls are needed, so for 8 desks, 32 balls are needed.

From the General to the Particular

According to Mason (2005, 2009), a pattern or a functional relationship is inherently indicating aspects that are changing and aspects that are staying the same. Such a generality is often difficult to articulate in words or in symbols. Using clouds or starbursts to represent the unknown, changing quantity is a strategy for developing a generalization without relying on the particulars. Let's use the Hockey Cards Sharing problem to think through this idea.

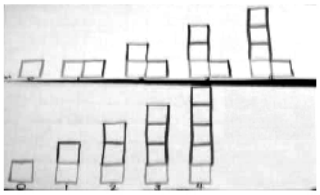
Hockey Cards Sharing Problem:

If Josie gives Marc one of her cards, she will then have one more than twice as many cards as Marc then has. If Marc started with 12 cards, how many did Josie start with?

Josie	1 + 2 x ☆	1 + 2 x 12 = 25	Josie gave 1 card away, so she started with 25 + 1 = 26 cards.
Marc	☆	12 cards	Marc started with 12 cards.

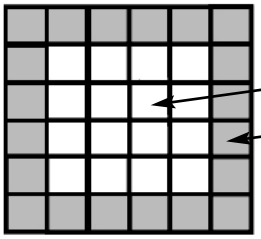
To generalize the relationship between the number of hockey cards Josie and Marc have, we don't have to know first the actual number of cards Marc has. However, once we know a particular (like Marc has 12 cards), it can replace the starburst and allow a numerical value to be calculated.

LET'S DO MATH - EQUIVALENT NUMERIC EXPRESSIONS



Getting Started:

Challenge students to create two different growing patterns using square colour tiles. Have the students record their pattern rule. Have students do a gallery walk (walkabout) to notice growing patterns that are similar and different to their pattern. Prompt students discuss how they determined similarities and differences between the patterns. Highlight the idea that growing patterns with the *same pattern rule* could *look different* when represented.



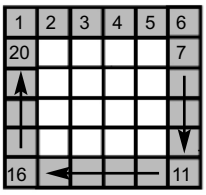
Problem 1: Pool Border Problem - Part A

How many 1 x 1 square unit, grey tiles are needed to make a border around a pool that has an area of 4 x 4 square units? Show 2 different solutions.

Anticipating Student Responses

What mathematics do you see in each sample of student work?

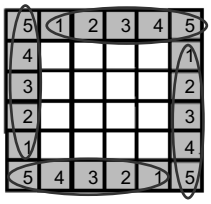
Anticipating Student Responses



Solution 1:

1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 = 1 x 20 = 20

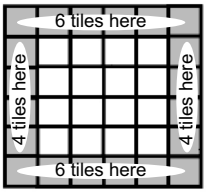
Count the grey tiles 1 by 1, in a clockwise direction, around the white tiles of the pool.



Solution 2:

5 + 5 + 5 + 5 = 5 x 4 = 20

Make groups of 5 around the outside of the pool tiles.There are 4 groups of 5.

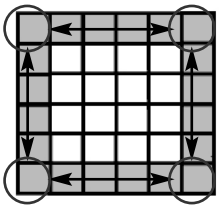


Solution 3:

6 + 6 + 4 + 4
= 6 x 2 + 4 x 2 = 20

Count the top and bottom and two sides. Top and bottom -> 6 x 2 or (4 + 2) x 2.
Two sides -> 4 x 2.

Total tiles -> 6 x 2 + 4 x 2 = 20.



Solution 4:

4 x 4 + 1 x 4 = 20

Each arrow shows 4 tiles (4 x 4); each circle shows 1 corner tile.

Total tiles -> 16 + 4 = 20.

Coordinating Student Discussion for Learning

Why might solution 1 be chosen first for student discussion, followed by solutions 2, 3, and 4?

- Solution 1 - direct counting by ones; it is a solution that works in particular cases, but is difficult to use for pools of varying sizes (especially large numbers). In each case, counting would have to restart and continue until the last grey tile was counted.
- Solution 2 - counting 4 groups of 5 tiles; this particular case for a pool 4 x 4 shows that there is 1 more tile (for 1 corner) added onto each side length (number of tiles). So a calculation could show side length + 1 or 5 taken 4 times, 5 x 4 = 20.
- Solution 3 - focusing on the lengths of the top and bottom together with the corner tiles and then considering the lengths of the remaining tiles on the two sides. The top and bottom have lengths equal to the pool side length + 2 (for two corners on top and two on bottom) and the sides are the pool length. So, particularly, for a 4 x 4 pool the number of tiles needed is (4 + 2) x 2 + 4 x 2 = 20.
- Solution 4 - thinking about the corners separately from the sides. The focus is on the number of tiles for the 4 lengths then adding 4 corner tiles. Particularly, 4 tiles for each of 4 sides and 4 corner tiles 4 x 4 + 1 x 4 = 20.

Is it true that the 4 different numeric expressions are equivalent? Explain.

These 4 numeric expressions are equivalent, as they have the same final result, 20. Yet, the structure of each numerical expression represents a different way of viewing the particular combinations of pool tiles.

LET'S DO MATH – FROM THE PARTICULAR TO THE GENERAL

Getting Started:

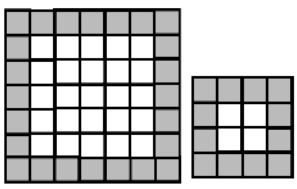
Have the students examine their numerical solutions to the Pool Border Problem – Part A. Ask the students to decide which numerical expression has the greatest potential for generalizability; that is, which numerical expression could be used to determine the number of border tiles for any size square-shaped pool.

Problem 2: Pool Border Problem – Part B

How many 1 x 1 square unit tiles are needed to make a border for any square-shaped size pool?

a) Which numerical equation that determines the number of tiles for the 4 x 4 square unit pool might be useful? Make a conjecture.

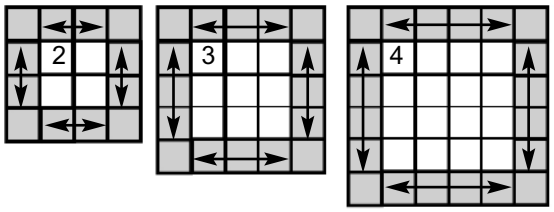
b) Test and defend your conjecture using 3 different square shaped pools. Use a labeled diagram, a table of values, and/or a graph.



Anticipating Student Responses

What mathematics do you see in each student sample work?

Sample Responses:

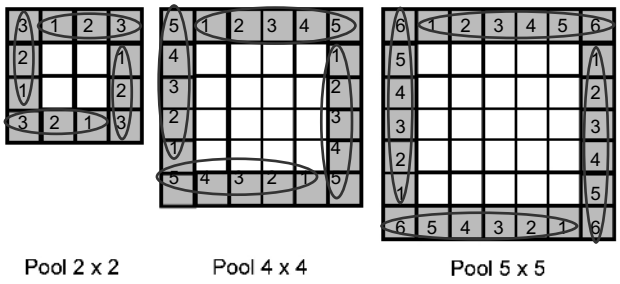


Solution 1:

Pool A is 2 x 2 – border is
2 + 2 + 2 + 2 + 4 = 12

Pool B is 3 x 3 – border is
3 + 3 + 3 + 3 + 3 + 4 = 16

Pool C is 4 x 4 – border is
4 + 4 + 4 + 4 + 4 + 4 = 20



Pool 2 x 2 Pool 4 x 4 Pool 5 x 5

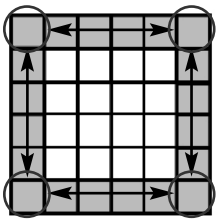
Solution 2:

For 2 x 2 pool: 3 + 3 + 3 + 3
or 3 x 4 = 12

For 4 x 4 pool: 5 + 5 + 5 + 5
or 5 x 4 = 20

For 5 x 5 pool: 6 + 6 + 6 + 6
or 6 x 4 = 24

Solution 3:



Number of tiles on one side of pool times 4 sides plus 4 corners.

Size of pool tiles x tiles	Number of tiles in border	Quasi-Variable Numeric Expression
	1 x 1	1 x 4 + 4
2 x 2	2 x 4 + 4	2 x 4 + 4
3 x 3	3 x 4 + 4	3 x 4 + 4
4 x 4	4 x 4 + 4	4 x 4 + 4
5 x 5	5 x 4 + 4	5 x 4 + 4
6 x 6	6 x 4 + 4	6 x 4 + 4
7 x 7	7 x 4 + 4	7 x 4 + 4
8 x 8	8 x 4 + 4	8 x 4 + 4
d x *d*	*d* x 4 + 4	*d* x 4 + 4

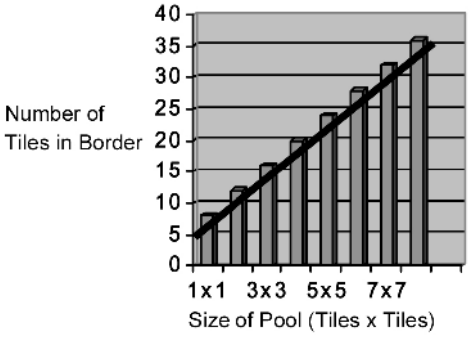
1 x 4 + 4

quasi-variables

variable – changing quantity

Solution 4:

Pool Border Problem		
Size of pool tiles x tiles	Number of Tiles	Coordinate Pairs
1 x 1	8	(1,8)
2 x 2	12	(2,12)
3 x 3	16	(3,16)
4 x 4	20	(4,20)
5 x 5	24	(5,24)
6 x 6	28	(6,28)
7 x 7	32	(7,32)
8 x 8	36	(8,36)



Coordinating Student Discussion for Learning

What are the mathematical relationships between the different representations (e.g., labeled diagrams, table of values, graph)?

- Solutions 1, 3 and 4 - similar counting methods. They show the relationship of tiles as being 4 groups of particular side lengths plus 4 corners.
- Solution 1 - repeatedly adding the lengths of the sides (number of tiles for each of 4 sides of a square) and then adding on the 4 corner tiles.
- Solution 3 - takes those particular cases (side lengths 2, 3, and 4) and extends the pattern to cover pools of many sizes. The “varying” quantity could represent the lengths of the sides of the pool and the “constant” quantities are the number of sides (always 4 for square) and the 4 corners again; using quasi-variable.
- Solution 4 - a table of values includes length of one pool length and the number of tiles to

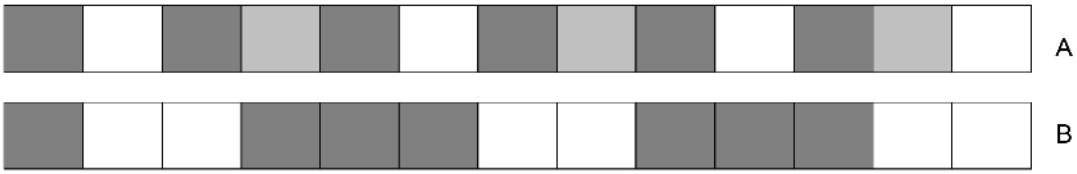
generate coordinate pairs (x,y); the graphical representation shows that the function y = 4x+4 has a constant growth or rate of change (i.e., a linear relation).

- Solution 2 – shows that there is 1 more tile (for 1 corner) added onto each side length (number of tiles). So, a calculation could show (side length + 1) or 5 repeated 4 times, 5 x 4 = 20. For a 5 x 5 pool, there would be 4 sides each (5 + 1) or 6 tiles long, 6 x 4 = 24. For a 1 x 10 pool, there would be r groups of (10 + 1) or 11 for 11 x 4 = 44. For any pool or, in general, number of tiles = (side length + 1) x 4.

LINKS TO MANIPULATIVES

Using Colour Square Tiles

Here is a pattern of coloured squares that is repeated at least two times. What do you think is the original pattern? Predict the colour of the 100th square and the position of the 100th white square



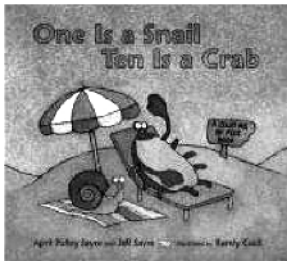
Pattern A is a familiar type of pattern where students manipulate square tiles to make repeating patterns that are "read" from left to right. It might be more challenging to identify the original pattern core or unit in Pattern B. Try reading Pattern B out loud. What do you notice? Ask students to predict the colour, shape or position of the 5th term, 10th term, 100th term. Such questions often provide a reason for organizing data into a table, as there are often not enough manipulatives to continue to model the pattern to physically see the 100th term. "We don't have enough blocks to continue our pattern. We have 20 blocks in our pattern. The 20th block is blue. What colour will the 40th block be? How do you know?"



Using Pattern Blocks

Pattern blocks can be used to create a growing pattern so that students can investigate the concept of constants and variables. Ask "What is staying the same? What is changing?"

LINKS TO LITERATURE



One is a Snail, Ten is a Crab

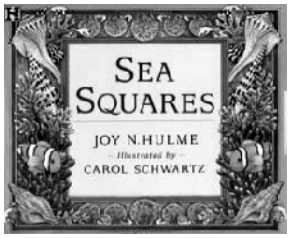
Teachers and students can develop equivalent numerical expressions for pictures in books, such as *One is A Snail, Ten is a Crab* by April Pulley Sayre. In this counting book, the number, 50, is described using words like "ten dogs and a crab".

Students can be challenged to create numeric expressions to depict the picture showing 50, such as: 4 x 10 + 10; 4 + 4 + 4 + 4 + 4 + 4 + 4 + 4 + 4 + 4 + 4 + 10; 4 x 5 + 4 x 5 + 10; 2 x 4 + 2 x 4 + 2 x 4 + 2 x 4 + 2 x 4 + 10; and so on.

After investigating numerical expressions for 50, students could explore other numbers of feet that could be made by combining dogs and a crab, as recorded in the table.

Number of dog feet	Number of crab feet	Total number of feet
4 x 0	10	10
4 x 1	10	14
4 x 2	10	18
4 x 3	10	22
4 x 4	10	26
4 x 5	10	30

Ask the students what they notice about the data in the table (e.g., the number of crab feet stays the same, the total number of feet increases by 4 which is the number of feet on one dog).



Sea Squares

Students can also explore "growing patterns" using books such as, *Sea Squares* by Joy N. Hulme. Each page shows a number of sea animals with the same number of characteristics. "3 clowns with frowny faces … Have 9 stripes in fishy places."

- Ask the students to develop numeric expressions for the number of stripes on the fish, such as: 3 + 3 + 3; 3 x 3; 2 x 3 + 3.
- Ask the students if the numerical expressions are equivalent 2 x 3 + 3 = 3 x 3 True or False?
- Ask the students, "If three fish have 9 stripes, how many stripes would 6 fish have?" Other similar questions with the 1:3 relationship could be asked. Keep track of their responses to the number of fish and the number of stripes in a table.

Number of Fish	3	6	10	30	4	8	1
Number of Stripes	9	18	30	90	12	24	3

LET'S DO MATH – FROM THE GENERAL TO THE PARTICULAR

Getting Started

Have the students write numerical expressions for the following statements. ☆ can be used to represent an unknown or changing quantity.

PROBLEM SITUATIONS	I	YOU	CARA	SOLUTION
a) I have 4 more things than you do.	☆ + 4	☆		4 + ☆
b) You have some things and I have 3 more things than you do; how many altogether?	☆ + 3	☆		☆ + ☆ + 3 2 x ☆ + 3
c) I have 5 fewer things than you do.	☆ - 5	☆		☆ - 5
d) If I have 4 more things than you do, and you have 2 more things than Cara has, how many more things do I have than Cara?	☆ + 2 + 4	☆ + 2	☆	☆ + 6

Problem 3: Card Problem

When Josie gives Marc 3 of her cards, she has 6 more than twice as many cards as Marc has then.

a) If Marc started with 10 cards, how many did Josie start with?

b) If Josie started with 81 cards, how many did Marc start with?

Anticipating Student Responses

PROBLEM SITUATIONS	MARC	JOSIE	CARDS STARTED WITH
Cards Josie started with (gave 3 away) to create this relationship	☆	☆ ☆ +6	☆ ☆ + 6 + 3
a) Mark has 10 cards, Josie started with?	10	10 + 10 + 6	10 + 10 + 6 + 3
b) Josie started with 81 cards, Marc started with	☆	☆ ☆ + 6 + 3 = 81	81 – 9 = 72, 72 ÷ 2 = 36

Coordinating Student Discussion for Learning

- Using ☆ to represent an unknown or changing quantity changes the focus from finding the unknown quantity to first establishing the relationship between unknown (changing) and constant (not changing) quantities.
- Only 1 variable or unknown quantity, represented as ☆ needs to be denoted, as the other quantities are in relation to the ☆ .

NEXT STEPS FOR YOUR PROFESSIONAL LEARNING

Application to Your Classroom

Questions Teachers Can Ask Students

Mason (2004) reminds us that students' experiences in the mathematics classroom frames the view students will have about mathematics. The questions and prompts used, and the responses to these which are accepted, become the model of mathematical behaviour for the learner. In fact, the kinds of mathematical questions that a teacher poses affects how students discuss and see mathematics throughout the lesson. Some mathematical behaviour or mental activities that constitute algebraic thinking include: exemplifying, specializing, comparing, reversing, justifying, generalizing, conjecturing, and varying. Jefffcot et al. (2004) developed general questions and prompts to provoke such algebraic thinking that they classified into six categories of mathematical activity.

MATHEMATICAL ACTIVITIES	GENERAL QUESTIONS AND PROMPTS
Exemplifying,	Give one or more examples of … ; Is … an example of ….?
Specialising	What makes … an example?; Find counter-example of …
Completing, Deleting,	What must be added, removed, altered, in order to allow, contradict …?
Correcting	What can be added, removed, altered, without affecting …? What needs to be changed so that …?

MATHEMATICAL ACTIVITIES	GENERAL QUESTIONS AND PROMPTS
Comparing, Sorting, Organising	What is the same and different about …? Sort or organize the following according to … What is the connection between …
Changing, Varying,	What do you get if you change …? What if …? If this is the answer to similar
Reversing,	Altering question, what was the question? Do … in two (or more) ways.
Generalising,	What do you notice? Of what is this an example? What happens in general?
Conjecturing	What can change and what has to stay the same so that … is still true? Do you think it will always happen?
Explaining, Justifying, Verifying, Refuting	Explain why … Is it always true that …? How would you explain …?

Questions Students Can Ask Themselves

Driscoll (2004, p. 92, 106) offers several questions that teachers can use to activate students' habits of mind for developing generalizations, as well as some questions that students could ask themselves as they prepare a convincing argument for their generalization.

- When I do the same thing with different numbers, what still holds true? What changes?
- Is there information here that lets me predict what's going to happen?
- Does my rule work for all cases?
- Am I doing the same steps over and over? What are they?
- Can I write down a mechanical rule that will do this job once and for all?
- How can I describe the steps without using specific inputs?
- What do I know for sure?
- What do I think may be true?
- What do I want to be able to show?
- What did I do to convince myself that I covered all cases?
- Did I check for mistakes?
- What will be some key things to emphasize in my explanation?
- What examples can best make my case?
- What will be communicated my argument: words? A picture? A table? An equation?
- Am I prepared to have them test my rule or any case they select?
- What counter arguments should I anticipate?

SUGGESTED READINGS

Some readings used in the Research Summary and in the development of the problems and solutions are listed below:

- Brizuel, B. and Ernest, D. (2008). Multiple notational systems and algebraic understandings: The case of the "Best Deal" problem. In J. Kaput, D. Carraher, and M. Blanton (Eds.), *Algebra in the early grades* (273-302). Mahwah, NJ: Lawrence Erlbaum Associates/Taylor & Francis Group.
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- Mason, J. (2009). OAME Workshop. Power point from OAME Leadership Conference: *Developing Perspectives on Learning and Teaching Mathematics*. Toronto: OAME.
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CLOSING

Our next series of Abacus issues will focus on fractions from a K to 8 perspective. We will investigate representations and comparisons of fractions, decimals, and percents.

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