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8 PAGE INSERT ELEMENTARY MATH

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MAKING SENSE OF FRACTIONS – REPRESENTATIONS OF FRACTIONS

ABACUS CO-EDITORS



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Greetings! What have you noticed about the different ways that students solve fraction problems? We have noticed that students tend to conceive fractions as a part of a region within the contexts of pizzas or circles. What have you noticed? We've also noticed that students think of the whole as being synonymous with 1 whole. What have you noticed?

In this issue, we will be engaging together with problems that focus on different representations of fractions such as division, part of a region, part of a set of objects, or linear measure. Your engagement might include solving these fraction problems in more than one way and comparing your solutions to the different ones we present to you.

Also, consider taking time to jot down your own ideas about these problems, in terms of their mathematical detail. The mathematics inherent in these problems can be understood as being relative to a larger mathematics landscape of number quantity, relationships, and operations. So, gather a few colleagues together to explore representations of fractions. Enjoy!

LINKS TO LITERATURE AND MANIPULATIVES



PAT MARGERM is an independent literacy and mathematics consultant with more that 30+ years of K to 8 teaching experience with students, in-service and pre-service teachers. She is a long-time OAME board member and works in various aspects of mathematics education.

When you think about planning learning experiences for fractions, what manipulatives do you think about using? There seems to be many materials to choose from when you are planning fractions lessons. Fraction circles and fraction strips have been designed specifically for studying fractional concepts. Pattern blocks, tangrams, and geoboards can be used to study fractions, as well as other mathematical concepts. Have you found that some manipulatives are more effective than others as tools for modeling fraction concepts? What about children's literature books? Are there books focused on fractions? How can other math literature books be used to investigate fractions as well as other mathematical concepts? Let's find out!

RESEARCH SUMMARY – REPRESENTATIONS OF FRACTIONS

Meaning of Fractions

According to Small (2009a), "a fraction is a number that describes a relationship between a part (represented by the numerator) and a whole (represented by the denominator)." Key ideas include:

- a fraction is not meaningful without knowing what the whole is
- there are always two fractions involved in any single fraction situation: the part you are considering and the rest of the whole (e.g., whenever there is 3/4, there has to be a 1/4)
- fractions can be used to describe any partitions of a whole
- fractions can represent parts of regions, parts of sets, parts of measures, or ratio. These meanings are equivalent (e.g., 1/4 of a region is 1 whole divided into 4 equal parts).

Fractions as Division

A divided quantity or partitioned quantity usually refers to a whole quantity being divided into the same number of equal-sized parts. According to Fosnot and Dolk (2003), fractions can be represented as partitive and quotative division.

Cookie Sharing Problem A:

12 cookies are shared among 4 children. How many cookies does each child get?

12/4 = 12 ÷ 4 = 3 -> partitive division where you think about fair sharing 12 cookies with 4 children

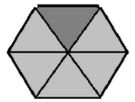
Cookie Grouping Problem B:

12 cookies are divided up so there are 4 cookies to a bag. How many bags are needed?

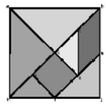
12/4 = 12 ÷ 4 = 3 -> quotative division where you think about how many groups of 4 there are in 12

Fractions as a Part of a Whole or as Part of a Set of Objects

"Equal parts into which the whole is divided are equal, but do not have to be identical. For parts of a set, this means that the members of the set do not have to be identical" (Small, 2009b, p. 44).



1/6 of 1 hexagon is 1 triangle (congruent)



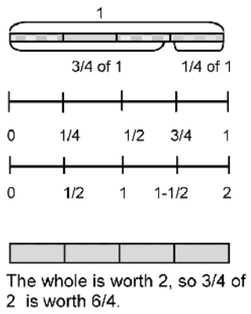
1/7 of 1 tangram is any 1 tan (some tans are not congruent)

Fractions as a Ratio and Rate

When a quotient of a divided quantity refers to a multiplicative relationship between two quantities (e.g., how many times larger or smaller is one than the other), the term, ratio is often used. According to Fosnot (2007), Small (2009), and Smith (2002), fractions are also relations. Small (2009) adds that a ratio is a comparison between two numbers with the same unit, while a rate compares quantities with different units. So, for the cookie problem, a ratio shows part of a whole (12 cookies: 3 cookies or 12:3), while a rate could be 12 cookies per 4 children (12/4) or a unit rate be 3 cookies per 1 child (3/1).

Fractions as Part of a Measure

Watanabe (2006) explains that the use of fractions in measurement contexts, such as linear or liquid (litre) measures helps students to understand fractions as quantities. Understanding fractions as quantities is a common challenge for students when their knowledge of fractions is focused on fractions as part of a whole. Thompson and Saldanha (2003) report that students who are primarily using the part-whole representation often have difficulty making sense of fractions greater than 1 (e.g., improper fractions, mixed numbers). So, Watanabe (2006) explains that with an emphasis on measurement, the idea that non-unit fractions (e.g., 2/3, 3/4, 4/5,) are a collection of unit fractions (e.g., 1/3, 1/4, 1/5) emerges.



The whole is worth 2, so 3/4 of 2 is worth 6/4.

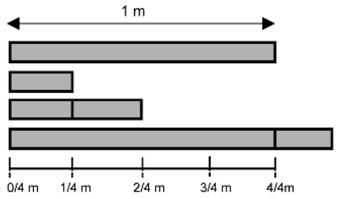
What does 3/4 mean? How do you know?

- 3 parts of a whole that is partitioned into 4 equal parts
- representation of measured quantities such as 3/4 L (litres) or 3/4 m (metres)
- 3 times of the unit obtained by partitioning 1 into 4 equal parts
- a quotient fraction (3 ÷ 4)
- A is 3/4 of B; if we consider B as 1 (a unit), then the relative size of A is 3/4

Paper Ribbon Problem:

A satin ribbon that is 1 metre long is partitioned into 4 equal pieces.

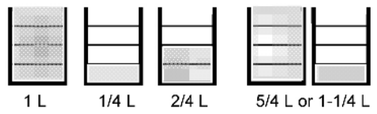
a) How much of one metre is 1 piece of ribbon?
b) How much of one metre are 2 pieces of ribbon?
c) How much of one metre are 5 pieces of ribbon?



Water Bucket Problem:

A water bucket that holds 1 litre of water is partitioned into 4 equal parts.

a) How much of one litre is 1 part of the bucket.
b) How much of one litre is 2 parts of the bucket.
c) How much of one litre is 5 parts of the bucket.



In addition Randall (2005, p. 13) outlines other key mathematical understandings of fractions:

- A fraction describes division.(a/b = a ÷ b, a & b are integers & b ≠ 0), and it can be interpreted on the number line in two ways. For example, 2/3 = 2 ÷ 3. On the number line, 2 ÷ 3 can be interpreted as 2 segments where each is 1/3 of a unit (2 x 1/3) or 1/3 of 2 whole units (1/3 x 2); each is associated with the same point on the number line.
- Each fraction can be associated with a unique point on the number line, but not all of the points between integers can be named by fractions.
- There are an infinite number of fractions between any two fractions on the number line.
- A decimal is another name for a fraction and thus can be associated with the corresponding point on the number line.
- Whole numbers and integers can be written as fractions (e.g., 4 = 4/1).
- A percent is another way to write a decimal that compares part to a whole where the whole is 100 and thus can be associated with the corresponding point on the number line.

LINKS TO MANIPULATIVES:

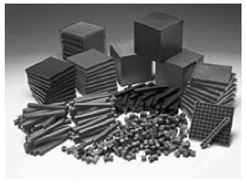
A fraction is not meaningful without knowing what the whole is.



- What might this idea look like using fraction circles?
- What is the fractional name for each of the following pieces.
- Did you have to visualize the whole circle in order to be able to name the fraction circle piece?



How would you name these fraction circle pieces? Are they both one fourth? Why?



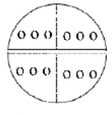
When we identify base ten blocks as units, rods, and flats, to specify its numerical value, we need to know the value of the whole.

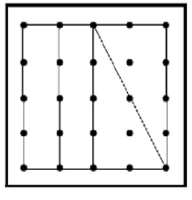
For example, a rod can have a value of 10, 1, 0.1 (1/10), or 0.01 (1/100) depending on which block has been identified as the whole.

Manipulatives make explicit the connections between fractions as part of a whole or part of a set as well as the idea of fractions as division.

- How can we share 12 in 4 equal groups?
- What is 1/4 of the set of 12?

Place 12 counters on a fraction circle divided into fourths to divide them into 4 equal groups.





We can use manipulatives to see that equal parts do not have to have identical shapes.

- Is this geoboard divided into fourths? How do you know?
- Which model of representation of fractions are you using?

LESSON LEARNING GOALS

- Curriculum Expectations (MOE, 2005) (not to put on the board with students, for board (bansho) plan only)
- Show relevant curriculum expectations the grade before and after the grade-specific learning goal, to see development of mathematical thinking across grade levels; Use the curriculum expectations to determine the appropriateness of the problems chosen for Before (activation), During (lesson), and Aftrer (Practice)

Grade 3	Grade 4	Grade 5
Divide whole objects and set of objects into equal parts and identify the parts using fractional names without using numbers in standard fractional notations	Represent fractions using concrete materials, words, and standard fractional notation and explain the meaning of the denominator as the number of the fractional parts of a whole or a set and the numerator as the number of fractional parts being considered	Represent ... fractional amounts with like denominators, including proper and improper fractions and mixed numbers using a variety of tools and using standard fractional notation

BEFORE (Getting Started)

- 5 to 10 minutes
- Activating students' mathematical knowledge and experience that is directly related to the mathematics in the lesson problem
- Use a smaller problem similar to previous lesson problem
- Use student work responses for class analysis and discussion to highlight key ideas and/or strategies

Licorice Problem A:
I have 18 m of string licorice.
How much do each of us get if
I have 1, 2, or 5 friends?



1 friend? -> 18 m ÷ 2 = 9 m -> 18/2 m



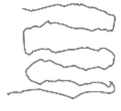
1 friend? → 18 m ÷ 2 = 9 m => 18/9 m
2 friends? -> 18 m ÷ 3 = 6 m -> 18/3 m
5 friends? -> 18 m ÷ 6 = 3 m -> 18/6 m

- dividing total length of licorice by the number of people who are sharing (self + friends)
- fractional amount represents division

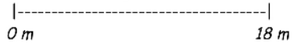
DURING (Working On It)

- 20 minutes
- Understand the problem, Make a Plan, Carry Out the Plan
- Understanding the problem – Ask, "What information from the problem will we use to make a plan to solve it? Explain."
- Teacher records in a list, below the problem, the information that students identify (We don't want different solutions to different problems; we want different solutions to the same problem. Students are learning to discern important information (i.e., problem definition))
- Students solve the problem on chart paper with markers (visible for later discussion) in pairs or in small groups
- Teacher circulates to record different student solutions, in addition to the ones the teacher has anticipated prior to the lesson

Licorice Problem B:
Five friends bought 18 m of red string licorice. They wanted to share it fairly. How much of the whole licorice does each friend get?



What information will we use to solve this problem?
18 metres long licorice



- share fairly among 5 friends - each friend gets the same amount; amounts in linear measurement unit, metres (m)

AFTER (Consolidation)

- 20 to 25 minutes
- Teacher selects 2 or more solutions for class analysis and discussion
- Teacher decides which solution to share 1st, 2nd, 3rd based on mathematical relationships between the solutions
- Students (authors) explain and discuss their solutions with their friends
- During discussion, solutions are organized to show mathematical elaboration from one solution to the next and towards lesson learning goal
- Teacher mathematically annotates on and around solutions to make mathematical ideas, strategies, and models of representation explicit to students, using students' ideas, relating students' informal vocabulary with formal vocabulary

- *denominator – number of fractional parts of the whole string licorice -> 5*
- *numerator – number of fractional parts considering → 1 part (out of 5 total parts)*

What measurements should we use to label this number line?

1 m divided by 5 people → 1 m ÷ 5 = 1/5 m
3m divided by 5 people → 3 m ÷ 5 = 3/5 m
→ 1 m ÷ 5 = 1/5 m, 1 m x 3 = 3 m, so 1/5 m x 3 = 1/5 m + 1/5m + 1/5 m = 3/5 m

18 ÷ 5 = 18/5 = 3 R3 or 3 + 3/5
-> Equivalent representations

Division and multiplication are inverse operations.
15 ÷ 5 = 5 x 3
18 - 15 = 15 ÷ 3

Solution 1
18 / 5 = 3 and some left over
5 x 3 = 15 and 18 – 15 = 3
15 fair share, 3 m left so not enough for 5 people, need 2 more

Solution 2
18 m / 5 = 3 m each and 3 m left over.
Split 1 m into fifths, so 1/5 of 1m
3m → 1/5 + 1/5 + 1/5 = 3/5
3 m and 3/5 m of red licorice for each



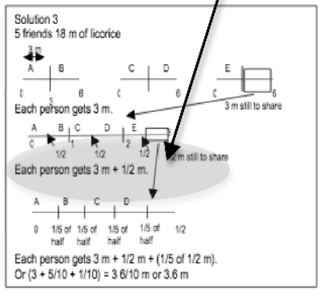
Can't Fair Share Whole Metres of Licorice

Divide 1 Metre by 5 people to get Fifths

What does the denominator and numerator mean for these fraction → 1/5 and 3 3/5?

- *tenths -> 0.1 or 1/10 or 1 ÷ 10*
- *1 m dividing by 5 people is the same as*
  - *→ 1 m ÷ 5 is the same as -> 1 m ÷ (10 ÷ 2)*
  - *→ fraction is 1/5 m*
- *18 m divided by 5 people is the same as*
  - *→ 18 m ÷ 5 is the same as 18 m ÷ (10 ÷ 2)*
  - *→ fraction is 18/5 m or 3 3/5 m*
- *18 m ÷ 10 = 1.8 m (with 10 people get less)*
- *18 m ÷ 5 = 1.8 m x 2 (with 5 people, get double of what 10 people would get)*

1/2 m is left over, so divide it by 5 people
→ 1/2 m divided by 5 people
-> 1/2 m ÷ 5
-> 1/5 of 1/2 m



Using Fraction of a Fraction
1/2 ÷ 5 = 1/5 of 1/2

Solution 4
Break 18 m into tenths
18 m /10 = 1.8 m
Have 5 people not 10 so each person gets 2 strips 1.8 m long.
1.8 m + 1.8 m = 3.6 m

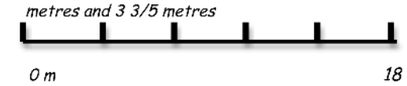
Using Friendly Fractions 1/10

AFTER (Highlights/Summary)

- 5 minutes
- Teacher revisits in the different student solutions the key ideas, strategies, and models of representation that are related to the lesson learning goal
- Teacher records key ideas, strategies, and models of representation separately, so the students can see the explicit focus of learning from their solutions, as it relates to the lesson learning goal

Highlights/Summary:

1. ***Fractions as Division & Part of a Measure:***
   - *18 m divided into 5 parts results in children getting 3 whole metres and 3 3/5 metres*



   - *if divide 1 whole by 2, fractional part is ½,*
   - *if divide 1 whole by 3, fractional part is 1/3*
   - *if divide 1 whole by 4, fractional part is ¼*
   - *Leftovers from Whole Number Division can be represented as fractions →18 ÷ 5 = 3 3/5*
2. ***Denominator*** *– total number of parts of a whole; for 1/5 > 1 whole has 5 parts*
3. ***Numerator*** *– number of parts of a whole: for 1/5 -> 1 part out of 5 total parts*

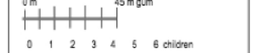
AFTER (Practice)

- 10 minutes
- Teacher chooses 2 or 3 problems, similar to the lesson problem for students to solve individually or in pairs as a scaffold)
- Problems could vary by number (choice, size), problem contexts, or what is unknown or needs to be solved
- Students are asked to use a strategy different from the one they used for the lesson problem to solve practice problems

Bubble Gum Problem:
6 children are sharing fairly 45 m of gum. What fraction of the bubble gum does each child get? Show at least 2 solutions.

Solution 1
45 m of licorice split into 6 equal pieces.
42 m as 6 x 7 m = 42 m
Each child gets 7 m of bubble gum but not all the gum is shared yet.
45 m – 42 m = 3m -> still share with 6 children. Each one gets 1/2 m more
1 m for 2 children
2 m for 4 children
3 m for 6 children
1 child gets 7 m and 1/2 m bubble gum.

Solution 2
45 / 6 = 7 R 3
Each child gets 7 m and a little more gum.



3 m split into 6 parts is 1/2 m each.
Each child gets 7 1/2 m of bubble gum .

Note: Mathematical annotations includes mathematical vocabulary, symbols, elaborations of mathematical details from solutions, labels describing approach/strategy, and questions to further thinking and practice. All annotations are records of students' mathematical discussion.

Note: A classroom board is longer proportionally than these 2 pages. Due to the space constraints on these pages, the mathematical annotations are recorded above the solutions with arrows, rather than on and around the solutions.

LINKS TO LITERATURE:



Piece = Part = Portion by Scott Gifford is an example of a picture book geared towards understanding representations of fractions. Written more as an informational text rather than as a story, it focuses on the key mathematical understanding that "decimal is another name for fractions" (Randall, 2005), as well as the understanding that "per cent is another way to write a decimal." The author displays and describes photographs of real life objects using fractions, decimals, and percents. For example one light on a traffic light is 1/3 or 0.333 or about 33%. Students could be challenged to identify fractions in real life objects in the classroom by labelling them with stick-on notes.



The Tangram Magician by Lisa Campbell Ernst is an example of a book that seems to be explicitly connected to geometry but because of the proportional design of the tangram pieces, it can also be used to explore fraction concepts.

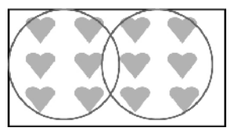
- What fraction of the tangram pieces are triangles?
- What fraction of the tangram is used to make the magician's head?
- What fraction of the area of the square is the large triangle?
- What fraction of the area of the magician is his head? How do you know?

LET'S DO MATH – FRIENDS PROBLEM

Before (Getting Started)

How could you fair share 12 cookies with 2 children? How much of the whole does each child get?

- fair share 12 cookies one at a time in 2 groups, until there none left; 6 cookies for each child or half of the whole
- set of 12 cookies, 2 groups means half of 12 or 6 out of 12 which is 6 cookies



During (Working On It)

A child exclaims, "Half of my friends are 7 years old." Show two possible groups of the child's friends.

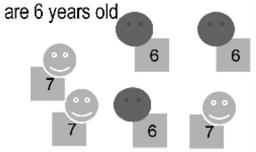
After (Consolidation)

Anticipating Student Responses:

Solution 1
Half of my friends are 7 years old.
One is 7 the other is 6.



Solution 2
6 friends – 3 are 7 years old and
3 are 6 years old



Solution 3

	age			age
Jim	7		Kye	5
Jane	6		Kyle	7
Jon	8		Kim	7
Jose	5		Kate	9
Jaq	7		Kerr	7

Half of my friends are 7 and other half of friends are not 7.

Coordinating Student Discussion for Learning:

Why might solution 1 be chosen first for student discussion, followed by solutions 2 and 3?

- solution 1 – divides one set of 2 objects into 2 fractional parts, identifying the fractional parts of one half part of the set as 1 -> Annotate on the solution, word "half"; draw rectangle around the 2 friends and label it "one whole of 2 objects"; divide the rectangle into 2 parts, showing each part of the whole as "one half" which is 1
- solution 2 – divides one set of 6 objects into 2 fractional parts, identifying fractional parts of one half part of the set as 3 objects -> Annotate on the solution, word "half"; draw rectangle around the 6 objects and label it "one whole of 6 objects"; divide the rectangle into 2 parts, showing each part of the whole as "one half" or "three sixths"
- solution 3 – similar thinking as solutions 1 and 2, but the table offers another representation of the set of 10 objects. Note: Sort the data in the table to show all 7 year olds in one column and others in the 2nd column. -> Annotate on the solution, word "half"; draw rectangle around the 10 objects and label it "one whole of 10 objects"; divide the rectangle into 2 parts, showing each parts of the whole as "one half" or "five tenths".

LET'S DO MATH – EQUIVALENT DESIGN PROBLEM

Before (Getting Started):



This design has a numerical value of 2 and 4/3 (2 wholes and 4 thirds of 1 whole).

Represent at least 2 shapes with equivalent areas and rename the shape each time.



1 hexagon is worth 1 of 1 whole

1 trapezoid is worth 1/2 of 1 whole

1 rhombus is worth 1/3 of 1 whole

Solution 1



3 1/3

Solution 2



4/2 and 4/3

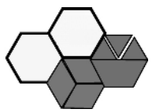
4/3 plus 4/2

1/2 + 1/2 + 1/2 + 1/2 + 1/3 + 1/3 + 1/3 + 1/3

= 1 + 1 + 1 + 1/3 = 3 1/3

During (Working On It)

If the hexagon is worth 1/3, what is the value of this design?

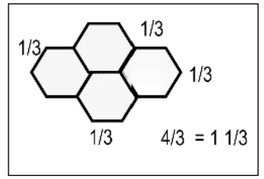


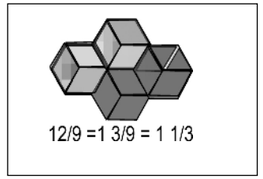
a) Show 3 different equivalent representations of this design.

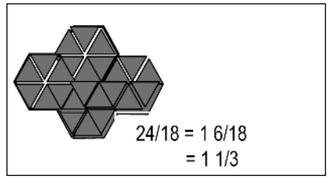
b) Justify your solutions. Use labeled diagrams.

After (Consolidation)

Anticipating Student Responses:







Coordinating Student Discussion for Learning:

Why might solution 1 be chosen first for student discussion, followed by solutions 2 and 3?

- solution 1 – the shapes of 3 hexagons can be seen in the original design. Students will see that 3 hexagons make 1 whole and the 4th hexagon makes a mixed number 3 1/3.
- solution 2 – one hexagon is 1/3. It takes 3 rhombuses to cover the hexagon so it would take 9 (3 x 3) rhombuses to cover 1 whole = 3 hexagons. This follows from solution 1. A justification could show 3 yellow hexagons and 3 rhombuses for 3 3/9 or 3 1/3.
- solution 3 – one hexagon is 1/3. It takes 6 triangles to cover the hexagon, so it would take 18 (3 x 6) triangles to cover 1 whole = 3 hexagons; this follows from solution 1; justification could show 3 yellow hexagons and 6 triangles for 3 6/18 or 3 1/3.

NEXT STEPS FOR YOUR PROFESSIONAL LEARNING

Application to Your Classroom

- Try these fraction representation problems yourself first and then with your students.
- To suit your students' learning needs, you might adjust the numbers (e.g., larger or smaller numerator and/or denominator) and/or the context of the problem.
- Think about how the context of the problem evokes particular representations of fractions (e.g., fraction as division, part of a region, part of a set of objects, or linear measure).
- Practise noticing the different strategies and the breadth of the mathematics your students use in their solutions. Take note of the number of solutions that show fractions as a part of a region, a set of objects, and as a linear model of representation.
- In preparation for your class discussion of student solutions, decide which solution you will discuss first, second, and third.
- Think ahead of what mathematical annotations you will record on and around the student solutions to make explicit key mathematical ideas, strategies, vocabulary, and fraction representations. These annotations are records of students' analysis and discussion about each other's solutions.
- Choose 4 student work samples to analyse further in terms of their mathematical details.

Suggested Readings

Some readings used in the Research Summary and in the development of the problems and solutions are listed below:

- Baroody, A. (1999). *Fostering children's mathematical power: An investigative approach to K-8 mathematics instruction.* NJ: Lawrence Erlbaum Associates.
- Empson, S. (2002). The development of students' knowledge of fractions and ratios. In B. Litwiller and G. Bright (Eds.) *Making sense of fractions, ratios, and proportions* (pp. 29-40). Reston, VA: National Council for Teachers of Mathematics, Inc.
- Fosnot, C. and Dolk, M. (2003). *Young mathematicians at work: Constructing fractions, decimals, and percents.* Portsmouth, NH: Heinemann.
- Fosnot, C. (2007). *Field trips and fund-raisers: Introducing fractions.* Portsmouth, NH: Heinemann.
- Randall, C. (2005). Big Ideas and Understandings as the Foundation for Elementary and Middle School Mathematics. *Journal of Mathematics Education Leadership* (7)3, 9-24.
- Small, M. (2009a). *Big ideas from Dr. Small: Creating a comfort zone for teaching mathematics, Grades K-3.* Toronto, ON: Nelson Educational Ltd.
- Small, M. (2009b). *Big ideas from Dr. Small: Creating a comfort zone for teaching mathematics, Grades 4 - 8.* Toronto, ON: Nelson Educational Ltd.
- Smith, J. P. (2002). The development of students' knowledge of fractions and ratios. In B. Litwiller and G. Bright (Eds.) *Making sense of fractions, ratios, and proportions* (pp. 3-17). Reston, VA: National Council for Teachers of Mathematics, Inc.
- Thompson, P. and Saldanha, L. (2003). Fractions and multiplicative reasoning. In J. Kipatrick, G Martin, and D. Schifter (Eds.), *A research companion in Principles and Standards for School Mathematics* (pp. 95-113). Reston, VA: National Council for Teachers of Mathematics.
- Watanabe, T. (2006). The teaching and learning of fractions: A Japanese perspective. *Teaching Children Mathematics*, pp. 368-374.

CLOSING

Our next issue will focus on key notions of comparing proper and improper fractions and mixed numbers, how teachers need to know these notions for teaching, as well as ways that teachers can use these mathematical ideas and pedagogical strategies through problem-based teaching and learning strategies.

CALL FOR *ABACUS* SUBMISSIONS

Send your teaching strategy ideas, problems and student solutions to the *Abacus* Co-Editors via email