8 PAGE INSERT ELEMENTARY MATH

ONTARIO ASSOCIATION FOR MATHEMATICS EDUCATION

Abacus

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INVESTIGATING CAPACITY AND VOLUME

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How would you measure the amount of toothpaste in a tube? How does a manufacturer decide on the size of a net for the rectangular prism that will contain the toothpaste tube? What size does the packing box need to be to hold 36 units of toothpaste? Capacity measurements are used to determine the amount of toothpaste and the size of the net of a rectangular prism. The volume that the toothpaste box takes up determines the size of the packing box. In this example, capacity and volume measurements are interconnected.

So, to develop students' understanding and use of capacity and volume to solve problems, what kinds of tasks should teachers use and in what sequence from grades 1 to 6?

In this issue, the Research Summary and problems (with multiple solutions) focus on strategies for determining volume and capacity measurements using direct and indirect comparisons and non-standard and standard units. The problems we provide are designed for use within a three-part, problem-solving lesson. Consider solving these volume and capacity measurement problems yourself first, before examining the solutions we provide. When we engage in solving the mathematics problems that we will used in our mathematics lessons, we are deepening our conceptual understanding of the mathematics we teach; that is, we are developing a mathematical sense of the range of thinking possible. As well, we are compelled to reflect on our choices of learning materials and teaching strategies, in order to develop our students' understanding of volume and capacity.

LINKS TO LITERATURE AND MANIPULATIVES



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During the busy family season that has just passed, I had many experiences in estimating capacity. I think that many of you will relate to experiences in which you "eye ball" left over food in serving bowls and plates and compare them to storage containers. It seems that there is usually just a little bit too much food for the capacity of the container that you have chosen. What do you do – throw it away or eat it?

Students also have similar experiences in choosing containers for storing toys or manipulatives or to pack lunches in reusable containers. Think about how you might involve the students in solving problems using capacity estimation the next time that you reorganize your classroom. Ask students to estimate quantities of math manipulatives and choose an appropriate container from a range of choices. In this issue, we take a closer look at the common centicube, as well as examining some real life connections to volume and capacity.

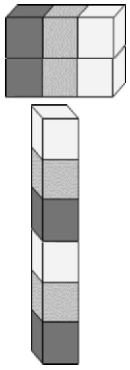
RESEARCH SUMMARY – ABOUT CAPACITY AND VOLUME

How are volume and capacity measurements similar and different? Both measurements require students to structure space in three dimensions. Capacity and volume describe attributes of 3-D objects. According to Small (2009), "capacity units are used to measure the amount of a liquid as well as the size of the inside of an object as if it were a container that could hold liquids. Volume units tell about the size of an object and standard volume units are expressed in terms of length units in three dimensions" (p. 152). For example, the capacity of a container may be 2 cups of rice or 500 mL of water, while the volume of a bicycle is the amount of "space" that that it takes up in cubic centimetres in a storage closet.

Developing an Understanding of Capacity and Volume

Like linear and area measurements, there are phases for developing understanding about and standard uses of capacity and volume measurements (Baroody, 1998; Clements & Sarama, 2009; Lehrer, Jaslow, & Curtis, 2003; Small, 2008, 2010):

- **Identifying the attribute** – As with other measures, students require practical experience to begin to form the concept of an object taking up space or how much can fit into something. Learning experiences include: building structures with blocks; filling and emptying containers of similar shape and different capacity (and vice versa) with objects, sand and water; and using language such as, "it takes up more space, there's no space left, it's full, it's empty, it can hold more" focuses attention on the attribute of volume and capacity.
- **Comparing and ordering** – To determine which structure takes up more space or which containers hold more, prompts for comparison and ordering. For example, the volume of objects can be measured directly by putting the object inside another object, such as putting different objects inside the same size rectangular prism and seeing which object takes up more space in the prism. Though the capacity of containers can sometimes be measured directly by placing one container inside another, most comparisons are made indirectly by pouring from one container to another container to see which holds more. Two matched amounts are the same when one amount is poured into a container of a different shape, the amounts show that volume is *conserved*. A *transitive relation* for capacity or volume is illustrated in this example: if container A > B and B > C, then it implies A > C.
- **Non-Standard units** – When a comparison of two objects or two containers requires how much more one container holds, then a unit of capacity and/or volume is required. Measuring the capacity and volume using non-standard or informal units includes using familiar units such as cupfuls or cubic units to focus on the process of repeatedly using a unit as a measuring device. For capacity, filling containers with water, same size objects like marbles or cubes and for volume, building different objects with same size blocks, like cubes prompt thinking about the comparative advantages of different choices of unit, as well as the iteration of same size units. Further, students are often surprised that structures with the same volume look differently. Also, learning about the appropriateness of units of measure is introduced; for example, a cup is more appropriate unit of measure than a spoon when measuring the capacity of a 2 L container.

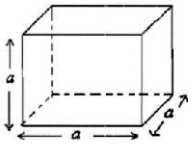


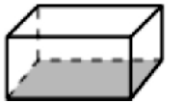
- **Standard units** – When students have used different non-standard units for the same volume, resulting in different number of non-standard units, then there is a need for consistency in the units used, so that communication about the measurement is more accurate. So, typically, litres, ten millilitres, and one millilitre are used for estimating and measuring capacity, in relation to everyday objects, such as the litre milk carton, ice-cream container and yoghurt containers. For volume, size of a cubic centimetre and a cubic metre are used for estimating and measuring volume of rectangular prisms, in relation to everyday objects, such as the volume of food packages, like raisin boxes, cereal boxes and photocopy boxes.

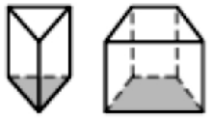
Applications of Capacity and Volume Measurement in Elementary Curriculum

When the students are able to measure efficiently and effectively using standard units, their learning experiences can be directed to situations where they develop measurement generalizations and formulae.

- Transitioning from capacity to volume measurement is developed as students investigate the relationship between millilitres and cubic centimetres. In Ontario, this transition occurs in the grade 5 curriculum. Students could find out that a container that holds 1000 centimetre cubes can also hold one litre of water. From that observation, students deduce that one cubic centimetre and one millilitre represent the same amount of space.
- Relating volume to capacity through displacement occurs when an object is completely immersed in water and it displaces an amount of water that is equal to its volume. Comparing the volumes of two objects occurs by immersing each object in the same amount of water and comparing the changes in the water level. The more the water level rises, the greater the volume of the object.
- Using centimetre cubes, students construct different size rectangular prisms, determine its volume in cubic centimetres, and then develop a generalization for calculating the volume of a rectangular prism. Student strategies are typically diverse, such as counting all cubes, counting horizontal layers, or counting vertical layers of cubes. The goal is to have students be flexible with different ways of partitioning and layering cube arrays to determine the volume of rectangular prisms, leading from a layering strategy to a slices of decreasing thickness (e.g., height of 4-1/2 rather than 4 or 5 units). Thus, relating area and volume contributes to seeing that calculating the surface area of a rectangular prism (area of the base) is used in calculating the volume of a rectangular prism.

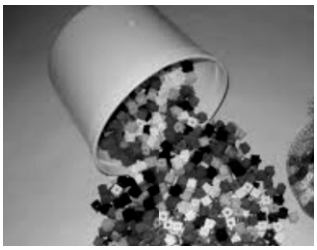






  - Volume (cube) = *a* x *a* x *a* = *surface area of one square face* x *a*
  - Volume (rectangular prism) = *l* x *w* = *surface area of the base* x *height*
  - Volume (triangular prism) = *area of base of triangle* x *height*
  - Volume of (any prism) = *area of base* x *height*

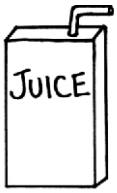
LINKS TO MANIPULATIVES: USING CENTICUBES



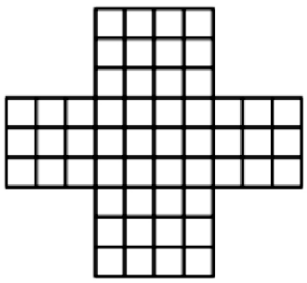
Most schools have access to centicubes. They are versatile manipulatives and are useful as standard units for investigating measurement concepts and solving problems. The linear dimensions of one centicube are 1 cm x 1 cm x 1 cm. Each cube has a volume of one cubic centimetre or 1 cm^3 and has a capacity of 1 mL. The centicube has a mass of one gram or 1 g.

Ask students how the cubes can be used to measure how much a container will hold. Watch to see if students demonstrate understanding of the measurable attribute and the volume of the centicubes by filling the container.

Ask students to use the centicubes to create a box for 125 mL of juice. Compare the dimensions of the rectangular prisms that are created using 125 centicubes e.g., 1 x 1 x 125, 5 x 5 x 5. How would the dimensions of the juice box change to hold 250 mL of juice? How do you know?



Here is the net for an open box. How many cubes will it hold? How do you know? Students may solve the problem by:



- making the box using centimetre grid paper and filling the box with cubes and counting the cubes one by one.
- making the box using centimetre grid paper, filling the box with cubes and counting the cubes by grouping in 2s, or 10s.
- using 12 cubes to make the first layer or the bottom of the box. Then adding two more layers of cubes 12 cubes + 12 cubes + 12 cubes = 36 cubes.
- multiplying the area of the bottom of the box by the number of cubes in the height so 12 cubes x 3 cubes high = 36 cubes.
- multiplying the dimensions for the area by the number of layers so 3 cubes x 4 cubes x 3 layers = 36 cubes.

LESSON LEARNING GOALS

Identify the curriculum expectations (MOE, 2005) grade-before and grade-after the grade specific learning goal:
- to see the development of mathematical knowledge, skills, and strategies across grade levels
- to determine the mathematical focus of the problems chosen for Before (activation), During (lesson), and After (Practice) parts of the lesson

BEFORE (Getting Started) – 5 to 10 minutes

- Activating students' mathematical knowledge and experience using a prompt or problem that is directly relates to the mathematics in the lesson problem.
- Includes student responses to prompt/problem to highlight key ideas/strategies.

AFTER (Consolidation) – 20 to 25 minutes

- Teacher selects 2 or more solutions for class analysis and discussion in a sequence (1st, 2nd, 3rd, etc.) based on mathematical relationships between the solutions and the lesson learning goal.
- Students (authors) explain and discuss their solutions with their classmates.
- During whole class discussion, solutions are organized (often re-organized) to show mathematical elaboration from one solution to the next and towards lesson learning goal.
- Teacher mathematically annotates (math terms, math symbols, labelled diagrams, concise explanations) on and around solutions to make mathematical ideas, strategies, and models of representation explicit to students.

Cube Problem A:

100 cubic centimetre cubes were dropped into 190 mL of water.
What would the water level be after the cubes sunk to the container bottom?



Solution 1:
1 cm^3 = 1 mL, so
100 cm^3 = 100 mL
190 mL of water will rise to 290 mL.

Solution 2:
Water at start 190 mL.
100 cm^3 displaces 100 mL water
190 mL + 100 mL = 290 mL

1 cm^3 = 1 mL
100 cm^3 displaces 100 mL water

Water Displacement - cubes take up space of water, so water level rises

Non-Standard Units of Measurement
Related to shape of unit of measure
- 1D -> length-> line → linear units
- 2D -> area ->square → square units
- 3D -> volume -> cube → cubic units

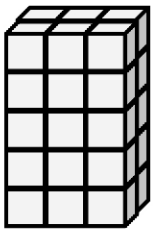
How are millilitres (mL) related to cubic centimetres (cm^3)?
- *drop 10 centicubes in 50 mL of water and water level is 60 mL*
- *60 – 50 = 10 mL (centicubes have a volume of 10 mL)*
- *space taken up by 10 centicubes in the water is 10 mL*
- *10 centicubes = 10 cubic centimetres (10 cm^3) = 10 mL*

DURING (Working on It) – 15 to 20 minutes

- Understanding the problem – Teacher asks, "What information from the problem are we using to make a plan to solve it? Explain." Teacher records in a list below the problem, the information that students identify.
- Students solve the problem on chart paper (landscape) with markers (visible for whole class discussion) in pairs or in small groups.
- Teacher circulates to record different student solutions, in addition to the ones the teacher anticipated.

Cube Problem B:

Interlocking cubes **are used to make a rectangular prism made up of 2 cubes x 3 cubes x 5 cubes.**
What is the volume of the 30 cubes?

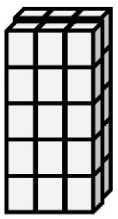


What information will we use to solve this problem?
- *30 cubes = (2 wide x 3 long x 5 high) cubes*
- *cubes make a rectangular prism*
- *calculate the volume of the 30 cubes in millilitres or cubic centimetres*

Solution 1:
- Count cubes in rectangular prism
- Measure water (200 mL)
- Put in 30 cubes.
- Read water level (about 450 mL).
- Water displaced by 30 cubes is difference of 450 mL - 200 mL = 250 mL

Volume of 30 cubes is 250 mL.

Solution 2:
-> 1st layer
2 x 3 = 6 cubes or 6 cube units
-> 5 layers of 6 cubes = 30 cubes



3D measurement is cube units, so volume of rectangular prism is 30 cube units

Water Displacement by Rectangular Prism

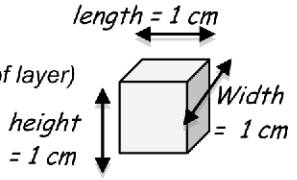
Counting Number of Cube Units

Note: Mathematical annotations include mathematical vocabulary, symbols, elaborations of mathematical details from solutions, labels describing the method/strategy, and questions to further thinking. All annotations are records of students' mathematical discussion.

Grade 4	Grade 5	Grade 6
– estimate, measure using concrete materials, and record volume, and relate volume to the space taken up by an object (e.g., use centimetre cubes to demonstrate how much space a rectangular prism takes up)	– determine, through investigation using stacked congruent rectangular layers of concrete materials, the relationship between the height, the area of the base, and the volume of a rectangular prism, and generalize to develop the formula (i.e., Volume = area of base x height)	– determine, through investigation using a variety of tools and strategies (e.g., decomposing rectangular prisms into triangular prisms; stacking congruent triangular layers of concrete materials to form a triangular prism), the relationship between the height, the area of the base, and the volume of a triangular prism, and generalize to develop the formula (i.e., Volume = area of base x height)

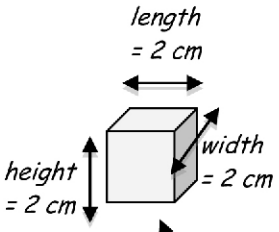
Volume of 1 Centicube

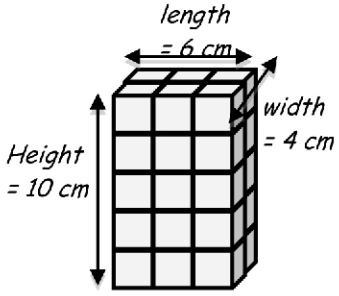
-> 1 cm^3
-> area (base) = length x width
= 1 cm x 1 cm = 1 cm^2
-> height of centicube (height of layer) = 1 cm
-> volume of centicube = 1 cm^2 x 1 cm = 1 cm^3



Volume of 30 Cube Rectangular Prism

Volume of 1 Cube





Solution 3:

-> length of edge measured 2 cm
-> Area of base (square face) = 2 cm x 2 cm = 4 cm^2
-> height (layer) length = 2 cm
→ Volume (1 cube) = 4 cm^2 x 2 cm = 8 cm^3
Each cube is 8 cm^3.
-> 30 cubes in the prism, so volume (30 cube rectangular prism) = 8 cm^3 x 30 = 240 cm^3

Solution 4:

→ measured the rectangular prism made of 30 cubes:
- length is about 6 cm
- width is ~ 4 cm
- height -> 10 cm
→ Area (base rectangular prism) = 4 cm x 6 cm = 24 cm^2
→ Volume (30 cube rectangular prism) = area base x height = 24 cm^2 x 10 cm = 240 cm^3

Same Size Square Units Iterated Around Border

Direct Measurement 3 Linear Dimensions Volume Formula

AFTER (Highlights/Summary) – 5 minutes

- Teacher and students revisit the student solutions for key ideas, strategies, and models of representation that are related to the lesson learning goal.
- Teacher lists key ideas, strategies, and models of representation separately, so the students can see how the mathematical details from their solutions relate explicitly to the lesson learning goal.

Highlights/Summary:

How are 250 mL, 30 cubic units, and 240 cm^3 Related?

They are different ways to show the volume of the rectangular prism made up of 30 cubes:

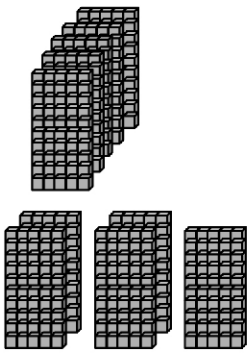
- *250 mL (water displaced) = 250 cm3 (1 mL = 1 cm^3)*
- *30 cubic units calculated using one cube unit using three <u>linear dimensions</u> of 2 cm x 2 cm x 2 cm = 8 cm^3 → then 30 cubic units, that is 8 cm^3 x 30 = 240 cm^3*
- *240 cm^3 calculated using the volume of a rectangular prism formula: <u>area of base x height (length of layers)</u> = 6 cm x 4 cm x 10 cm = 24 cm^2 x 10 cm = 240 cm^3*

AFTER (Practice) – 5 to 10 minutes

- Teacher chooses 2 or 3 problems, similar to the lesson problem for student to solve in pairs as a scaffold and individually.
- Problems could vary by number (choice, size), problem contexts, or what is unknown or needs to be solved
- Students are asked to solve these problems using a strategy different from the one they used for the lesson problem.

Rods Problem:

Twenty five rods were joined to make a prism 5 cm x 5 cm x 10 cm. When the prism was submerged in 300 mL of water, the water level rose to 525 mL but only 9 cm of the 10 cm tall prism were under water. What would the volume of 9/10 of the prism be?



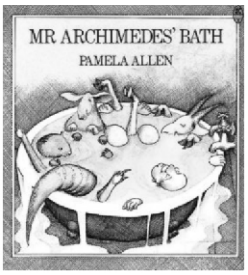
Solution 1:

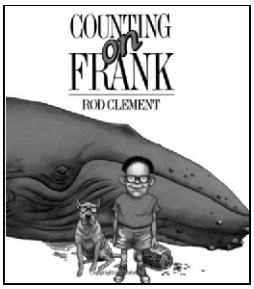
The volume is the area of the base x the height.
Each prism has a base of 5 x 5 = 25 cm^2
The height underwater is 9 cm.
The volume of the prism = 25 x 9 = 225 cm^3.

Solution 2:

525 – 300 = 225 mL
1 mL = 1 cm^3
225 mL = 225 cm^3

Note: A classroom board is longer proportionally than these 2 pages. Due to the space constraints on these pages, the mathematical annotations are recorded above the solutions with arrows, rather than on and around the solutions.





LINKS TO LITERATURE:

Mr. Archimedes has a problem. When he takes a bath with his friends, Kangaroo, Wombat and Goat, the water always overflows and makes a mess. Which of his friends is causing the problem?

A dog named Frank and his master ask questions that involve measurement estimation about everyday objects. Read the book, *Counting on Frank*, to your students. Discuss all of the different things the boy measures in the book.

- Students could measure a unit of water, freeze it - take it out, put the frozen part in another cup of water and measure the change in capacity.
- Compare the results.
- Look in the book for all of the ways the boy finds volume – 24 Franks in his bedroom, ten humpback whales in the house, 1/10 of his dad in the portable television, peas in the dining room, and 745 jellybeans in the jar.
- Have students investigate the accuracy of the estimates in the book.
- Have them pick a new object and write an addition to the book in which the boy and Frank use the new object to fill another object in a house.

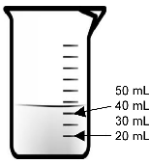
LET'S DO MATH – COMPARING CAPACITY AND VOLUME

Before (Getting Started)

Centicubes and Millilitres Problem A:

Which is greater – the volume of the centicubes or the amount of water in the beaker whose units are shown in 10 mL units?

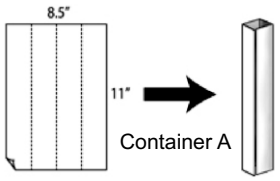


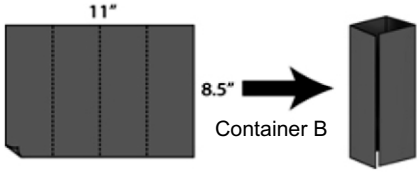


Solution 1	Solution 2
There are 6 + 10 + 6 centicubes with a volume of 22 cm^3. The beaker has about 48 mL of water in it.	There are 8 + 10 + 6 centicubes or 24 cm^3 which is the same as 24 mL because 1 cm^3 = 1 mL. There are about 48 mL of water that would take 48 centicubes to make the same amount. The volume of water is greater.

During (Working On It)

Filling Paper Containers Problem B:





Which is true?

☐ Container A holds more

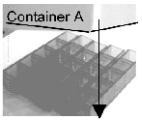
☐ Container B holds more

☐ Container A has the same capacity as Container B

Two tubes are made by folding two sheets of same-sized paper as shown in the pictures. One is folded along the longer side, the other along the shorter side. Predict whether filling the containers with a material you can measure (e.g., rice, sand, beans) will result in equal capacity in the two containers or one being greater than the other.

After (Consolidation)

Anticipating Student Responses:

Solution 1	Solution 2	Solution 3
They must hold the same amount because both containers are made with the same amount of paper. Volume is area of the *base* x *height*. Papers have equal areas so equal volumes.	 I filled container A (taller one) with lentils. I measured 3 cups or 750 mL or 750 cm^3. When I poured the lentils from the tall container A to the shorter container B, there was lots of unfilled space. So container B must hold more than container A.	 I made a base with 5 x 5 centicubes. Container A fit over my base. Height = about 28 centicubes. A shape 5 x 5 x 28 centicubes = 700 centicubes or 700 cm^3. Container B fits over a base 7 x 7 and is about 22 centicubes tall. That makes 7 x 7 x 22 = 1078 centicubes or 1078 cm^3. Container B holds more.

Solution 4
Volume (A) = 5.2 x 5.2 x 27.9 = 754.416 cm^3 Volume (B) = 7 x 7 x 21.5 = 1053.5 cm^3 Container B has a greater volume.

Coordinating Discussion for Student Learning:

Why might solution 1 be chosen first for student discussion followed by solution 2, and then solution 3?

- solution 1 -> students have responded as many young math students thinking the capacities will be the same because the paper size is the same. The mis-matched conception is that it is the paper size matters rather than the arrangement of space. The shapes have different dimensions once the paper is folded.
- solution 2 -> Students measured capacity using non-standard measure (i.e., lentils). They filled container A and found that it held less than container B by filling the taller container and pouring the lentils into the shorter container. The lentils did not fill container B making it have the greater capacity.
- solution 3 -> Students filled the containers with centicubes and found the space taken up by container B to be greater. The capacity is the same as the volume in this case.
- Solution 4 -> Students measure the dimensions of the length, width and height and found volume container B to be greater than volume container A.

LET'S DO MATH – BATTER TO BROWNIES

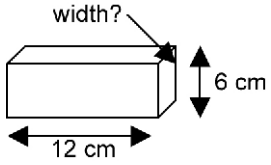
Before (Getting Started)

Missing Dimensions Problem A:

The box is 12 cm long and 6 cm high. How wide is the box if it holds 360 mL?

Solution 1

Box = 360 mL = 360 cm^3
V = *l* x *w* x *h*
360 = 12 x *w* x 6
360 = 72 w
w = 5 cm



Solution 2

Box is 360 cm^3
12 x 6 x 6 = 432 cm^3 too big
12 x 6 x 4 = 288 cm^3 too little
12 x 6 x 5 = 360 cm^3 just right
The width is 5 cm.

During (Working On It)

Brownie Problem B: Glynis makes a great brownie recipe. Doubling the recipe results in 1024 mL of batter poured into two 6 cm square baking pans. Will the two pans of brownies cut into squares 4 cm x 4 cm fit into a 1.3 L storage container with a base of 12 cm x 12 cm?

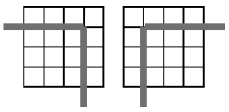
After (Consolidation)

Anticipating Student Responses:

Solution 1

Two batches of brownie mix is 1024 mL.
The storage container holds 1.3 L = 1300 mL
1300 mL is greater than 1024 mL so the brownies should fit in the container.

Solution 3

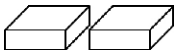


Each baking pan makes 16 brownies.
9 brownies fit on each layer in the storage container.

Height of each brownie	Height of storage
V = 1024 = l x w x h	V = 1300 = l x w x h
1024 = 32 x 16 x h	1300 = 12 x 12 x h
h = 1024 / 512 = 2 cm	h = 1300 / 144 = 9 cm

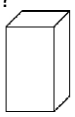
The picture shows that 4 layers are needed so if each brownie is 2 cm high, they will not fit.

Solution 2



Base of baking pans 16 cm x 16 cm

Batter is 1024 mL
1 batch = 524 mL = 512 cm^3
One pan will make sixteen 4 cm x 4 cm brownies. Two pans will make 32 brownies. How deep will the batter be? What is height of each brownie?
V(baking pan) = 512 = *l* x *w* x *h* = 16 x 16 x *h*
512 = 16 x 16 x *h*
512 = 256 x *h*
h = 2 cm
Nine brownies, each 2 cm high, 4 cm long, and 4 cm wide will fit on the bottom layer of the storage container. How tall is the container?
1.3 L = 1300 mL
V = *l* x *w* x *h* = 12 x 12 x *h*
1300 = 144 x *h*
h = 9 cm



1.3 L storage container

So, 3 layers of brownies 2 cm high will fit into the 9 cm tall storage container. That means 3 x 9 = 27 brownies will fit and the double batch made 32 brownies. Eat 5!

Coordinating Discussion for Student Learning:

Why might solution 3 be chosen first for student discussion, followed by solution 1, and then solution 2?

- solution 3 -> shows base of bake pan and base of storage container – nine brownies fit on each layer of storage – using volume formulas and capacity equivalences determine the height of both the brownies and the storage container
- solution 1 -> without respect to the shape and size of individual brownies the one volume (brownie mix) fits into the other volume (storage container) so it looks like the brownies should fit but the shapes of the brownies once baked prohibits this
- solution 2 -> solution shows step-by-step development of the size of the brownies, the layering of brownies and the storing of them in the container

NEXT STEPS FOR YOUR PROFESSIONAL LEARNING

Application to Your Classroom

- Try these capacity and volume measurement problems yourself first and then with your students.
- To suit your students' learning needs, you might adjust the numbers (e.g., 1-digit, 2-digit, or 3-digit whole numbers, decimal numbers to tenths, hundredths) and the number and type of calculations needed to measure and calculate capacity and volume (e.g., addition, subtraction, multiplication)
- Think about how the context of the problem evokes visual images and serves as a tool for understanding and determining capacity and volume measurements (e.g., people capacity in a gymnasium, space different size textbooks take up in a student desk, comparison of different size juice boxes, in terms of how much juice they hold and how much space they take up on the grocery shelf)
- Provide students with non-examples of using different volume and capacity measurement units (e.g., A rectangular prism is the linear dimensions of 10 cm length x 0.5 m width x 0.2 m high. Clark says that the volume of the rectangular prism is 10 cm x 0.5 m x 0.2 m = 1 m^3. Jeannelle says that the volume is 10 000 cm^3. Who is right? How do you know?)
- In preparation for your class discussion of student solutions, decide which solution you will discuss first, second, and third by referring to the developmental learning sequence for determining proportional relationships to solve problems (e.g., counting cubic units by ones, counting layers of cubic units, calculating by multiplying the surface area of one layer/slice by the number of layers/slices)
- Think ahead of the kinds of mathematical annotations you will record on and around the student solutions to make explicit key mathematical ideas, strategies for determining area measurements. when the students discuss and analyze each other's solutions (e.g., relating number of cubic units in an array to the surface area of the 3D figure; relating cubic units to its measurement units)
- Choose 4 student work samples to analyse further in terms of their mathematical details.

SUGGESTED READINGS

Some readings used in the Research Summary and in the development of the problems and solutions are listed below:

- Anderson, N. C., Gavin, M. K., Daily, J., Stone, W, & Vuolo, J. (2005). *Navigating through measurement in grades 3-5*. Reston, VA: National Council of Teachers of Mathematics.
- Baroody, A. (1998). *Fostering children's mathematical power: An investigative approach to K-8 mathematics instruction*. NJ: Lawrence Erlbaum Associates.
- Clements, D., & Sarama, J. (2009).*Learning and teaching early math: The learning trajectories approach.* New York: Taylor & Francis.
- Lehrer, R., Jaslow, L., & Curtis, C. (2003). Developing an understanding of measurement in the early grades. In D. Clements & G. Bright. (Eds.), *Learning and teaching measurement (2003) Yearbook* (pp. 100-122). Reston, VA: National Council for Teachers of Mathematics.
- Randall, C. (2005). Big ideas and understandings as the foundation for elementary and middle school mathematics. *Journal of Mathematics Education Leadership, 7*(3), 9-24.
- Small, M. (2008). *Making math meaningful to Canadian students, K-8*. Toronto: Nelson Education.
- Small, M. (2010). *Big ideas from Dr. Small, Grades K-3*. Toronto: Nelson Education.

CLOSING

Our next issue will focus on key notions of time and distance/time ratios, how teachers need to know these notions for teaching, as well as, ways that teachers can use these mathematical ideas and pedagogical strategies through problem solving-based teaching and learning strategies.

CALL FOR *ABACUS* SUBMISSIONS

Send your teaching strategy ideas, problems and student solutions to the *Abacus* Co-Editors via email

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