8 PAGE INSERT ELEMENTARY MATH

ONTARIO ASSOCIATION FOR MATHEMATICS EDUCATION

Abacus

Volume 49 • Number 4 • June 2011

UNDERSTANDING TIME AND DISTANCE/TIME RATE

TABLE OF CONTENTS

Abacus Editor Greetings 1
RESEARCH SUMMARY - About Time and Distance/Time Rate 2
Links to Manipulatives - Using Calendars and Concrete Time Lines 3
LET'S DO MATH: "On Time" Problems (with bansho plan). 4-5
LINKS TO LITERATURE 6
The Grouchy Ladybug by Eric Carle
LET'S DO MATH: "Passage of Time" Problems 6
LET'S DO MATH: "Track and Field" Problems 7
NEXT STEPS FOR YOUR PROFESSIONAL LEARNING. 7
- Application to Your Classroom
- Suggested Readings

A Publication of the OAME/AOEM

ABACUS CO-EDITORS



MARY LOU KESTELL works on K to 6 mathematics professional learning at the provincial level. She has spent more than 30+ years working in all aspects of Ontario mathematics education and is a past-president of both OAME and OMCA.

KATHY KUBOTA-ZARIVNIJ currently focuses



on K to 6 mathematics professional learning at the provincial level. She is a long-time OAME board member and OAME past president. In relation to her work in mathematics education, she uses complexity thinking to interpret mathematics teaching and learning for students and teachers.

More than just understanding numbers on the clock, the concept of time helps students learn about sequences, daily routines, calendars and general numeracy skills. Before kids can read time on a clock, they can understand the notion that activities happen in a flow or sequence, such as the past, present, future, before or after. They can also see that days are broken down into schedules and that days add up to months and years. These ideas can be developed when we have students draw pictures about doing routine parts of their day, such as getting dressed, playing outside or watching a television show. Next to the picture, they can record the time each activity normally occurs. Consequently, students begin to associate time with their day schedule. However, to further develop students' understanding of time measurement, elapsed time, and the distance/time rate (or speed) by solving problems, what kinds of tasks should teachers use and in what sequence from grades 1 to grades 6?

In this issue, the Research Summary and problems (with multiple solutions) focus on strategies for determining time, elapsed time, and distance/time using clocks, time lines, time schedules, and number lines The problems we provide are designed for use within a three-part, problem-solving lesson. Consider solving these time and distance/time problems yourself first, before examining the solutions we provide. When we engage in solving the mathematics problems that we use in our mathematics lessons, we are deepening our conceptual understanding of the mathematics we teach; that is, we are developing a mathematical sense of the range of thinking possible. As well, we are compelled to reflect on our choices of learning materials and teaching strategies, in order to develop our students' understanding of time and distance/time.

LINKS TO LITERATURE AND MANIPULATIVES



PAT MARGERM is an independent literacy and mathematics consultant with more that 30+ years of K to 8 teaching experience with students, in-service and pre-service teachers. She is a long-time OAME board member and works in various aspects of mathematics education.

Since this is the June issue of the Gazette, I anticipate that readers of ABACUS and others whose lives are planned around the school year may be using the calendar to count down the remaining days of the school year until summer vacation. Calendars are often used as a context for patterns and number sequencing, yet they are very useful tools for measuring time. In this issue, I take a look at some ways for using calendars in meaningful ways; that is, to understand stories, characters and informational text. Then I examine picture books to make explicit connections to time and use these connections to pose mathematics problems.

RESEARCH SUMMARY – ABOUT TIME AND DISTANCE/TIME RATE

Charles (2005) outlines several key time concepts: “a given time of day can be represented in more than one way … time duration for events can be compared using ideas, such as longer, shorter, and equal … time in minutes and seconds can be added and subtracted where one minute (1 min) is regrouped as sixty seconds (60 s) … the longer the unit of measure (e,g., hours vs. minutes), the fewer units it takes to measure …” (pp. 14-20). However, these time concepts are comprised of several mathematical details that require careful pedagogical considerations, such as sequencing of learning experiences, choice of student learning materials, and types of questions, prompts, and problems to solve.

Developing an Understanding of Time and Distance/Time Rate

According to Small (2008, 2010), concepts of time are introduced in three stages.

a) *Definition/Comparison:*
   - It includes comparing the durations of two events to see which takes longer without using units by waiting to see which event keeps going when the other event has finished. There is a parallel to measuring length, as a longer time event extends farther than a shorter time event. Time vocabulary describes one event as being longer, shorter, faster, or slower.
   - Organizing a sequence of pictures to tell a story provides experiences about what events happened first, then next, then next, and so on. Time vocabulary in this context includes words like, first, last, next, before, and after (Barnes, 2006).

b) *Non-standard Units:*
   - Non-standard tools to measure duration or elapsed time include sand timers of various sizes, pendulums or metronomes. For sand timers, students can estimate the amount of sand flowing to the bottom (e.g., half, quarters, thirds), the number of pendulum and metronome swings, or they can make their own time device (Kamii & Long, 2003).
   - By varying the length of the pendulum swing or mass of the pendulum weight, students can see how the size of the unit of time affects the resulting measurement of time (e.g., a longer string or heavier pendulum weight creates a larger unit of time or longer swing than a shorter string or lighter pendulum weight). There is the potential for students to realize that shorter units of time better measure short time periods and longer units of time better measure longer time periods.
   - Having students estimate time duration before measuring with non-standard units enables them to develop personal time referents (e.g., number of times a name can be written, number of hand claps, number of letters in an alphabet said in 1 minute).

c) *Standard Units:*
   - Students learn to tell time on analog and digital clocks. Yet, it is common to hear students read a digital clock without understanding the standard units, hours and minutes. So learning to read time on an analog clock provokes students to consider the key aspects of measuring time and the proportional relationships between the hand movements.
   - Key notions about reading the time on an analog clock include: (1) how the hand movements relate (i.e., as the minute hand (longer hand) travels around the clock, the hour hand (shorter hand) moves slower); (2) the hour hand does not point directly at the hour number except for time on the hour; (i.e., the number on the clock (i.e., 1, 2, 3, …., 12) corresponds to the hours not the minutes, and the number of minutes after the hour is counted by 5s for each number, 1, 2, …. 12); (3) different meanings for different hands (i.e., different words when different hands point to the name number on the clock – when the hour hand points to 6, we say “six,” when the minute hand points to the 6, we say “thirty”); (4) multiple names for the same time (e.g., if the time falls more than halfway past an hour, time can be read as either the number of minutes after one hour or as the number of minutes before the next hour -> 6:45 is 45 min after the 6th hour or 15 min before the 7th hour).
   - Because it is possible to have two times with the same name in a single day, morning and evening times can be distinguished using acronyms a.m. (ante meridiem – before noon) and p.m. (post meridiem – after noon) for a 12 hour clock, or extending the hours to 24 and reporting using 24-hour clocktime.

Concept of Distance/Time Rate (Speed)

Piaget (1970) explains that the concept of speed is constructed as a proportional correspondence between distance moved and time of movement—“the elaboration first of concrete and later of formal metrical operations” (p. 259). He described student learning the concept of speed (quantified motion) as when students first re-conceive motion as entailing changes in position simultaneously with changes in time, then coordinate the two dimensions of distance and time as changing in proportion to one another (Piaget, 1970, pp. 279- 280).

Applications of Elasped Time and Distance/Time Rate (Speed) in Elementary Curriculum

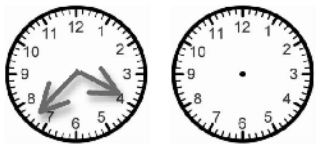
Kamii and Long (2003) state that there is no way to really show elapsed time to children because it is something that they have to see mentally. They suggest that a majority of students do not recognize operational time developmentally until third grade. However, textbooks show students "standard" ways of determining elapsed time such as counting the hours first, then the minutes, creating a sort of algorithm and subtracting, or counting only the minutes. Kamii and Long (2003) report that these strategies do not allow students to construct their own understanding of elapsed time naturally, instead, such experiences create misconceptions. Instead,

- prompt students to create their own understanding of elapsed time and ways to model it, by using a timeline with hours and minutes, or an analog clock with minute units and count the number of minutes a hand moves
- solve elapsed time problems by using hour benchmarks, counting back to determine start time when the end and elapsed times are given, counting up to determine elapsed time, determining end time by using a jump by minutes to a benchmark hour (Dixon, 2008)
- offer students meaningful opportunities to work with elapsed time in class, such as saying, "How much time do we have until lunch? Are we able to complete a 1 hour and 15 minute math lesson before lunch or would it be best to do a 45 min social studies lesson right now?"

Ways to have students consider 135 minutes differently (adapted from Jones and Arbaugh, 2004):

- How many hours is 135 min? Explain your thinking.
- Here are 3 different start times for a tour through museum exhibits. If each tour lasts 135 min, at what time will each of the three tours end? Show your work.
- Describe an activity that takes about 135 min to complete. Explain how you know that the activity takes 135 minutes. Compare this activity with another activity that either takes more time or less time than 135 min.
- Draw the hands on clock B so that the time shown is 135 min earlier than the time shown on clock A. Explain how you know where to put the hands.
- Who will complete the 200 km distance first, if both vehicles start at the same time: the bicycle at 30 km/hour or the car at 80 km/hour? Explain.
- If the bicycle has a head start of 100 km distance, will the bicycle (30 km/h) or the car (80 km/h) arrive at the 200 m distance first? How do you know?

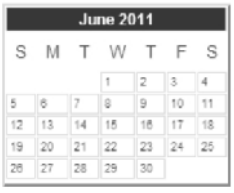
Museum Tours Begin
8:05 a.m., 1:20 p.m., 3:45



Clock A Clock B

LINKS TO MANIPULATIVES: CALENDARS AND CONCRETE TIME LINES

Calendars are charts that are organized into rows and columns. It can be challenging for students to understand that one day or square belongs to both a row and a column (e.g., this is under the Tuesday column so the day is Tuesday and its in the third row or third week of the month). Like hundreds charts, calendars can be cut into rows and connected together to form a number line or time line for the month. For example, while studying the life cycle of butterflies, students may read that on average, it takes between 4 and 10 days for a butterfly egg to hatch into a caterpillar. If the butterfly egg is laid on June 4, when might the egg hatch? Students can count the days on the calendar time line to visualize how much time will pass.



June 2011

S	M	T	W	T	F	S
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30		

SUN	MON	TUES	WED	THUR	FRI	SAT	SUN	MON	TUES	WED	THUR	FRI	SAT	SUN	MON	TUES	WED	THUR	FRI	SAT	SUN	MON	TUES	WED	THUR	FRI	SAT	SUN	MON
			1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27

Read more: From Egg To Butterfly | **eHow.com http://www.ehow.com/from-egg-to-butterfly/#ixzz1HFDIP0iz**

Students can also use reasoning to solve problems posed about the month. Today is Wednesday June 8. We started the month on June 1. What day of the week was June 1? How do you know?

I know that there are 7 days in one week. 8 – 7 = 1. So June 1 is Wednesday.

I can count backwards with the days of the week while counting to 1 on my fingers so

Wednesday Tuesday Monday Sunday Saturday Friday Thursday Wednesday. It was Wednesday!
1 2 3 4 5 6 7 8

The time line can be used like a number line to measure the number of days between events. Creating a "calendar" time line for books such as, *A Child's Garden: A Story of Hope* by Michael Forman, helps students to understand plot line as well as the boy's perseverance. "Over the following weeks, he cared for his secret garden." How many days might the boy have been watering and shading the plant?

LESSON LEARNING GOALS

Identify the curriculum expectations (MOE, 2005) grade-before and grade-after the grade specific learning goal:
- to see the development of mathematical knowledge, skills, and strategies across grade levels
- to determine the mathematical focus of the problems chosen for Before (activation), During (lesson), and After (Practice) parts of the lesson

BEFORE (Getting Started) – 5 to 10 minutes

- Activating students' mathematical knowledge and experience using a prompt or problem that is directly related to the mathematics in the lesson problem.
- Includes student responses to prompt/problem to highlight key ideas/strategies.

AFTER (Consolidation) – 20 to 25 minutes

- Teacher selects 2 or more solutions for class analysis and discussion in a sequence (1st, 2nd, 3rd, etc.) based on mathematical relationships between the solutions and the lesson learning goal.
- Students (authors) explain and discuss their solutions with their classmates.
- During whole class discussion, solutions are organized (often re-organized) to show mathematical elaboration from one solution to the next and towards lesson learning goal.
- Teacher mathematically annotates (math terms, math symbols, labelled diagrams, concise explanations) on and around solutions to make mathematical ideas, strategies, and models of representation explicit to students.

On Time Problem A:

Sasha's baseball game started at 6:30 p.m. and they played for 1 h and 35 min. What time did they finish?

Solution 1:

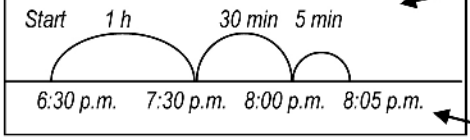
6 h 30 min
+ 1 h 35 min
7 h 65 min = 7 h 60 + 5 min = 8 h 5 min

6:30 p.m.
→ 6 hours after 12:00 noon
→ 30 minutes

60 minutes (min) = 1 hour (h)

30 minutes (min) = ½ hour (h)

Solution 2:





6:30 p.m.



8:05 p.m.

a.m. -> time after 12:00 a.m. (or 12 midnight)
p.m. -> time after 12:00 p.m. (or 12 noon)

DURING (Working on It) – 15 to 20 minutes

- Understanding the problem – Teacher asks, "What information from the problem are we using to make a plan to solve it? Explain." Teacher records in a list below the problem, the information that students identify.
- Students solve the problem on chart paper (landscape) with markers (visible for whole class discussion) in pairs or in small groups.
- Teacher circulates to record different student solutions, in addition to the ones the teacher anticipated.

On Time Problem B:

If you wake up at 7:30 a.m. and it takes you 15 min to eat breakfast, 5 min to brush your teeth, 25 min to wash and get dressed, 5 min to pack your backpack, and 35 min to walk to school, will you be there by 9:00 a.m.?

What information will we use to solve this problem?

- *Start time is 7:30 a.m.*
- *End time - arrive at school by 9:00 a.m.*
- *5 events in between 7:30 a.m. and 9:00 a.m.*
- *Each event takes period of time (e.g.,15 minutes)*

Solution 1:

15 min + 5 min + 25 min + 5 min + 35 min = 85 min

Between 7:30 a.m. to 9:00 a.m. there is 1 h and 30 min or 90 min
->7:30 a.m. to 8:30 a.m. is 1 h
->8:30 a.m. to 9:00 a.m. is ½ h
–> so 1-1/2 h time passed
-> 60 min + 30 min = 90 min

I should get to school with 5 min before 9:00 a.m. (90 – 85 = 5).

Solution 2:

Wake Up	*7:30 a.m.*	*7 h 30 min*
Breakfast	*15 min*	*+ 85 min*
Brush teeth	*5 min*	*7 h 30 min*
Wash and dress	*25 min*	*+ 60 min*
Backpack	*5 min*	*+ 25 min*
Walk to school	*35 min*	*7 h 30 min*
Total	*85 min*	*1 h 25 min*
Arrive	*8:55 a.m.*	*8 h 55 min*
		school

I will get to school at 8:55 a.m.

Compare Time Passed
-Start to School Time
-Start to Events Time

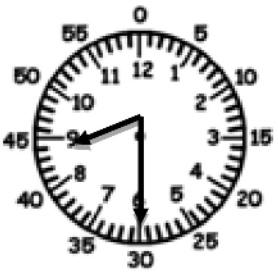
Add Events Time
To Start Time
(min, then h, min)

Note: Mathematical annotations include mathematical vocabulary, symbols, elaborations of mathematical details from solutions, labels describing the method/strategy, and questions to further thinking. All annotations are records of students' mathematical discussion.

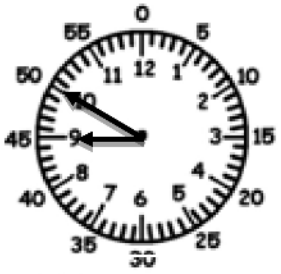
Grade 3	Grade 4	Grade 5
- Read time using analogue clocks, to the nearest five minutes, and using digital clocks, and represent time in 12-hour notation	- Estimate and measure (i.e., using an analogue clock), and represent time intervals to the nearest minute) - Estimate, and determine elapsed time, with and without using a time line, given the durations of events expressed in five-minute intervals, hours, days, weeks, months, or years	- Estimate and determine elapsed time, with and without using a time line, given the duration of events expressed in minutes, hours, days, weeks, months, or years



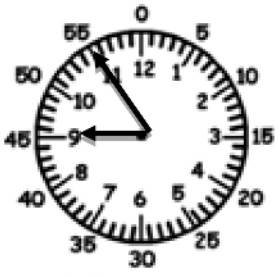
7:30 p.m.



8:30 p.m.



8:50 p.m. *(or 10 min before 9:00 p.m.)*



8:55 p.m. *(5 min before 9:00 p.m.)*

Passage of Time** or **Elapsed Time
How much time passes from start to end

Time ***Benchmarks***
¼ hour -> 15 minutes
½ hour – 30 minutes
1 hour – 60 minutes

Solution 3:

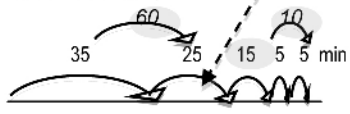
Wake Up at 7:30 a.m.	
Breakfast	15 min
Brush teeth	5 min
Wash and dress	25 min
Backpack	5 min
Walk to school	35 min

Minutes
15 + 5 + 25 + 5 + 35
= 30 + 30 + 5 + 15 + 5
= 60 + 25 => 1 h + 25 min
Start at7:30 a.m.
-> 1 h later is 8:30 a.m.
-> 25 min later is 8:55 a.m.

ReGrouping to Time Benchmarks
->minutes to 30 minutes
->minutes to 60 minutes
->60 minutes to 1 hour

Time Line

Solution 4:



Add Events Time To Start Time (Regroup 30 min; h,min)

Add On Events Time To Start Time (Regroup h, min)

AFTER (Highlights/Summary) – 5 minutes
- Teacher and students revisit the student solutions for key ideas, strategies, and models of representation that are related to the lesson learning goal.
- Teacher lists key ideas, strategies, and models of representation separately, so the students can see how the mathematical details from their solutions relate explicitly to the lesson learning goal.

Highlights/Summary:

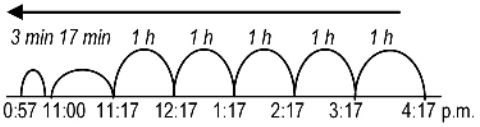
- *Time Benchmarks:*
  *-> **1 hour** (h) or 60 minutes (min)*
  *-> ½ **hour**, half hour (h) or 30 minutes (min)*
  *-> ¼ **hour**, quarter hour, or 15 minutes (min)*
- *Every time the hour hand passes the 12, you add one hour on and start counting by minutes*
- *Different combinations of times that make 1 hour:*
  -> 30 min + 30 min = 1 h
  -> 30 min + 25 min + 5 min = 60 min = 1 h
- *Time line: A number line with time labels shows "**elapsed time**" (time that passes)*

AFTER (Practice) – 5 to 10 minutes
- Teacher chooses 2 or 3 problems, similar to the lesson problem for student to solve in pairs as a scaffold and individually.
- Problems could vary by number (choice, size), problem contexts, or what is unknown or needs to be solved
- Students are asked to solve these problems using a strategy different from the one they used for the lesson problem.

On Time Problem C:

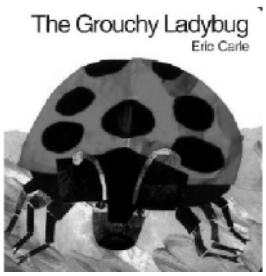
Navid and his friends were picked up at the amusement park at 4:17 pm. The boys had been there for 5 h and 20 min. What time did they arrive at the park?

Solution 2: *Count Back on Time Line*



Solution 1: *Count Back Hours and Minutes*
4:17 p.m. Count Back
->5 h - start 4, then 3, 2, 1, 12 p.m., 11 a.m.
->17 min - count back 17 min to end up at 0 min
-> 11:00 a.m. count back 3 min to end at 10:57 a.m.

Note: A classroom board is longer proportionally than these 2 pages. Due to the space constraints on these pages, the mathematical annotations are recorded above the solutions with arrows, rather than on and around the solutions.



LINKS TO LITERATURE: *THE GROUCHY LADY BUG* BY ERIC CARLE

The Grouchy Ladybug by Eric Carle, is a classic picture book that can be used to reinforce the patterns in the passage of time through the miniature clocks in the upper corner of each page which show each passing hour as the story progresses. The illustrations also show the changing position of the sun as the hours pass reinforcing the passage of time.

- Create a concrete time line of the events in the story. Sequence the events of the story by using clothes pins or paper clips and string to hang pictures of the animals that the ladybug meets each hour. This creates a time line for the hours in a day.
- Clocks can also be used to develop a time line for the story.
- What if the grouchy ladybug sleeps in for one-half hour? How will the times on our timeline change?

LET'S DO MATH – PASSAGE OF TIME



Before (Getting Started)

On Time Problem A:

See the hour hand on the clock face to the left.

About what time might it be?

Solution 1

The hour hand is past half way between 5 and 6 so it could be 4:45 or even 25 to five.



Solution 2

It might be any time after half past 4, like 4:35.

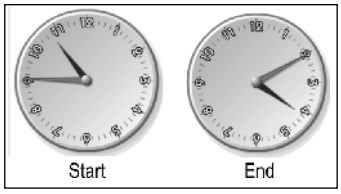


During (Working On It)

On Time Problem B:

At a soccer tournament, Yusuf's first game started at the time on the first clock and he was done at the time shown on the second clock.

How long was Yusuf at the soccer tournament?

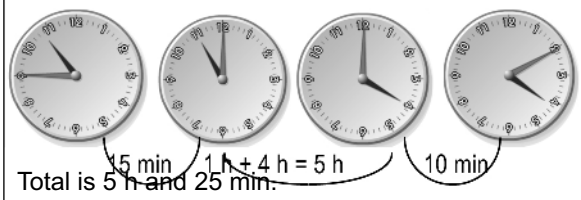


Scaffolding Questions

- How many minutes between the time on the first clock and the next hour?
- How many hours between that hour and the hour on the end clock?
- How many more minutes to end time?

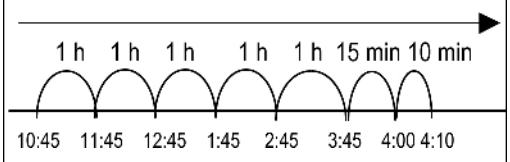
After (Consolidation)

Solution 1



Total is 5 h and 25 min.

Solution 2



Total is 5 h and 25 min.

Anticipating Student Responses:

Coordinating Discussion for Student Learning:

Why might solution 1 be chosen first for student discussion followed by solution 2?

- solution 1 -> counting on using analog representations 15 minutes to 11:00 a.m. then 5 hours to 4:00 p.m. and then 10 minutes to 4:10 p.m.
- solution 2 -> represented time on a line rather than on a clock face, by counting up first by hours and then by minutes.

LET'S DO MATH – TRACK AND FIELD

Before (Getting Started)

Race Problem A:

Keon walks 0.5 km to school and takes 8 min to get there. Kat walks 2 km to school and takes 30 min to get there. Who walks at a faster pace?

Solution 1

Keon 8 min for 1/2 km
and – 16 min for 1 km
and – 32 min for 2 km.
Kat 30 min for 2 km.
Kat gets there faster.

Solution 2

Kat takes 30 min to go 2 km

This is the same as 15 min for 1 km and 7.5 min for 0.5 km.

Kat walks a little faster.

During (Working On It)

Sprints Problem B:

Usain Bolt, a Jamaican Olympic sprinter, has a best time of 45 s for a 400 m sprint.

If he could run at this rate for 1 km, what would his speed be in kilometres per hour?

After (Consolidation)

Anticipating Student Responses:

Solution 1

Distance	400 m	800 m	200 m	1000 m = 1 km	32 km
Time	45 s	90 s	22.5 s	90 + 22.5 = 112.5 s	3600 s = 1 h

He can run 32 km/h.

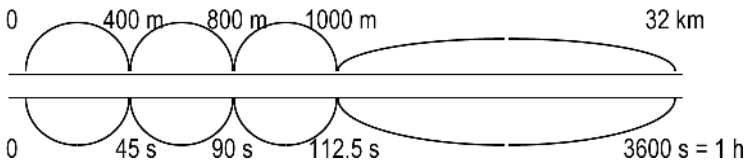
Solution 2

I need to know how many groups of 45 s are in 3600 s (1 h).

3600 / 45 = 80

400 m x 80 = 32 000 m or 32 km.

Solution 3



Coordinating Discussion for Student Learning:

Why might solution 2 be chosen first for student discussion, followed by solution 3, and then solution 1?

- solution 2 -> calculating the number of 45 s there are in 3600 s or 1 h and then multiplied the distance in metres by 80 to get 32 000 m or 32 km. This is a comparison focusing on time.
- solution 3 -> compares on double number lines, lining up distances and corresponding times.
- solution 1 -> shows the proportional relationships in a ratio table making the distance 1 km first.

NEXT STEPS FOR YOUR PROFESSIONAL LEARNING

Application to Your Classroom

- Try these time and distance/time ratio problems yourself first and then with your students.
- To suit your students' learning needs, you might adjust the numbers (e.g., for time, use 12-hour and 24-hour digital readings, use whole and decimal number distance/time ratios, such as 12 km/h or 33.5 m/s) to help them develop flexibility in understanding time, time units relationship, elapsed time, and determining distance/time rate or speed in order to solve problems.
- Think about how the context of the problem evokes visual images and serves as a tool for understanding and determining time and distance/time rate (e.g., clock hands moving, picture schedule, time lines with minute, half hour, hour units of time, double number lines showing distance and elapsed time).

- Provide students with non-examples of telling time, elapsed time, and distance/time ratios (e.g., A microwave clock shows 10:10 then 11:10. Marc says that 100 min have passed, while Olivia says that the 60 min has passed. Who is right? How do you know?)
- Practise noticing the different strategies and the breadth of the mathematics your students use in their solutions. Take note of the number and kinds of solutions that show strategies for telling time (e.g., using non-standard units (e.g., counting numbers or clapping hands, personal referents, pendulum or metronome swings) determining elapsed time (e.g., counting hour to hour and minute to minute, using time line marked with hours and minutes, adding up or subtraction hours and minutes, knowing that 1:00 represents 60 min), and calculating distance/time rates. Which strategy do students use more often? (e.g., double number line comparison), determine the distance for 1 unit hour (requiring division; choose common whole (hour) and compare distances (requiring multiplying and/or dividing).
- In preparation for your class discussion of student solutions, decide which solution you will discuss first, second, and third by referring to the developmental learning sequence for determining time, elapsed time, and distance/time rates.
- Think ahead of the kinds of mathematical annotations you will record on and around the student solutions to make explicit key mathematical ideas, strategies for determining time and distance/time ratio measurements when the students discuss and analyze each other's solutions.
- Choose 4 student work samples to analyse further in terms of their mathematical details.

Suggested Readings

Some readings used in the Research Summary and in the development of the problems and solutions are listed below:

Barnes, M.K. (2006). "How many days 'til my birthday? Helping kindergarten students understand calendar connections and concepts. *Teaching Children Mathematics*, pp. 290-295.

Dixon, J. K. (2008). Tracking time: Representing elapsed time on an open timeline. *Teaching Children Mathematics*, pp. 18-24.

Jones, D., & Arbaugh, F. (2004). What do students know about time? *Mathematics Teaching in the Middle School, 10*(2), pp. 82-84.

Kamii, C., & Long, K. (2003). The measurement of time: Transitivity, unit iteration, and conservation of speed. In D. Clements & G. Bright. (Eds.), *Learning and teaching measurement (2003) Yearbook* (pp. 169-180). Reston, VA: National Council for Teachers of Mathematics.

McMillen, S., & Hernandez, B. O. (2008). Taking time to understand telling time. *Teaching Children Mathematics*, pp. 248-256.

Piaget, J. (1970). *The child's conception of movement and speed.* New York: Basic Books.

Randall, C. (2005). Big ideas and understandings as the foundation for elementary and middle school mathematics. *Journal of Mathematics Education Leadership, 7*(3), 9-24.

Small, M. (2008). *Making math meaningful to Canadian students, K-8*. Toronto: Nelson Education.

Small, M. (2010). *Big ideas from Dr. Small, Grades K-3*. Toronto: Nelson Education.

Zhou, Z., Peverly, S. T., Boehm, A. E., & Chongde, L. (2000). American and Chinese children's understanding of distance, time, and speed interrelations. *Cognitive Development, 15*, pp. 215-240.

Closing

Our next issue will focus on key notions of 2-D geometry, 3-D geometry, and transformations, how teachers need to know these notions for teaching, as well as, ways that teachers can use these mathematical ideas and pedagogical strategies through problem solving-based teaching and learning strategies.

CALL FOR *ABACUS* SUBMISSIONS

Send your teaching strategy ideas, problems and student solutions to the *Abacus* Co-Editors via email