8 PAGE INSERT ELEMENTARY MATH

ONTARIO ASSOCIATION FOR MATHEMATICS EDUCATION

Abacus

Volume 50 • Number 2 • December 2011

COMPOSING AND ANALYSING 3-D OBJECTS

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A Publication of the OAME/AOEM

ABACUS CO-EDITORS



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It's a three-dimensional world out there! So, by the time children enter school, they have already developed significant notions about shape and space through everyday experiences in moving about in their environment and in interacting with the objects around them. Classroom experiences can build on what students already understand about 3-D geometry and provoke them to recognize and appreciate 3-D geometry in the world; develop reasoning and problem-solving skills related to geometric thinking; relate geometric ideas to other math strands; and use geometric ideas in other subjects (e.g., creating 3-D art, build tall, stable and strong structures). So, to develop students' understanding of and reasoning about the geometric attributes of 3-D objects, what kinds of tasks and problems should teachers use and in what sequence, from grade 1 to grade 6?

In this issue, the Research Summary and problems (with multiple solutions) focus on strategies for classifying prisms and pyramids by geometric properties (e.g., shape of faces, number of faces, edges, and vertices), constructing nets of prisms and pyramids representing 3-D objects using views and isometric sketches. The problems we provide are designed for use within a three-part, problem-solving lesson. Consider solving these 3-D object problems yourself first, before examining the solutions we provide to deepen your conceptual understanding of the mathematics you teach and develop a sense of the range of mathematical thinking possible.

LINKS TO LITERATURE AND MANIPULATIVES



PAT MARGERM is an independent literacy and mathematics consultant with more that 30+ years of K to 8 teaching experience with students, in-service and pre-service teachers. She is a long-time OAME board member and works in various aspects of mathematics education.

I usually focus on picture books for the links to literature, but there are novels that link to mathematics. If you teach an older grade, you might take a look at *All of the Above* by Shelley Pearsall. The story is inspired by actual events. It is the story of four inner city students and their quest to set a math record by building the world's largest tetrahedron. Students can be challenged to make tetrahedrons from nets and combine them to make larger ones. Check out other ideas in this issue.



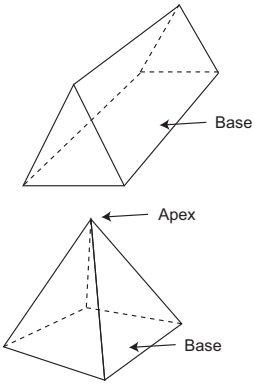
RESEARCH SUMMARY – ANALYSING AND REPRESENTING 3-D OBJECTS

Randall (2005) explains that three-dimensional objects or shapes can be described, classified and analyzed by their attributes. There is more than one way to classify 3-D objects. In fact, all polyhedra can be described completely by their faces, edges, and vertices. More specifically, 3-D geometric attributes include: type of faces, number of faces or edges, number and position of identical (congruent) faces, number of corners/vertices, and round parts. Students develop an understanding of the geometric attributes of 3-D objects by comparing and sorting different 3-D objects (e.g., prisms, pyramids) and by representing 3-D objects in different ways.

Some Key Concepts in Elementary 3-D Geometry

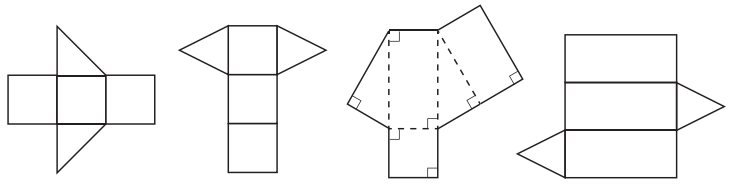
Typical 3-D objects studied include:

- ***prism*** - a solid object whose bases or ends have the same size and shape and are parallel to one another, joined by parallelograms; named by the shape of its base (e.g., triangular prism has 2 opposite facing triangles joined by 3 rectangles)
- ***pyramid*** - a solid object with a polygon-shaped base and triangular faces that meet at a common point (apex or vertex); named by its base (e.g., square-based pyramid has a square base with 4 triangular faces)



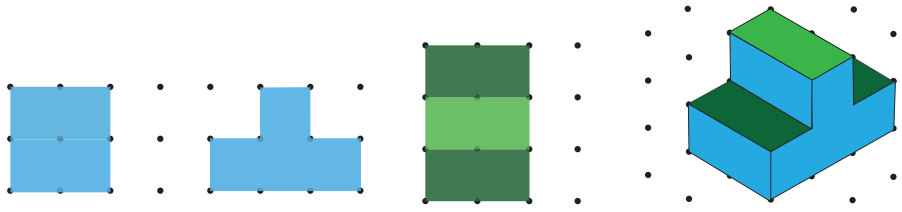
To represent a 3-D object, students need to examine its components, the spatial relationship between its components, and its geometric properties. 3-D objects can be represented in different ways: (Johnston-Wilder & Mason (2005), Small (2007, 2010))

- ***face maps*** – are made by tracing all the faces of one 3-D object. The face map for a triangular prism includes a drawing of 2 congruent triangles and 3 rectangles, with side lengths that correspond to the edges where they meet.
- ***nets*** – are 2-D representations of a 3-D object that can be folded to recreate the 3-D object. There are often many different nets for a single 3-D object; that is, the faces do not change, but they can be connected in different ways.

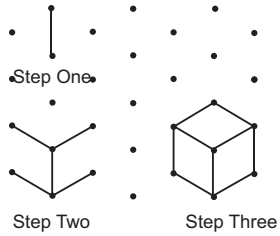


Different Nets for Triangular Prism

- ***orthographic drawings*** – are 2-D representations of objects that shows one side at a time.



- ***isometric drawings*** – are drawn on equilateral triangle dot paper (isometric dot paper, so that the edges that are parallel on the original shape are represented by parallel lines in the drawing.)



Types of Geometric Reasoning

Pittalis and Christou (2010) outline four different types of geometric reasoning:

- ***manipulation of different representations of 3-D objects*** – includes representations of a 3-D objects using 2-D shapes using particular conventions when drawing, such as isometric (2 point perspective), and orthographic (front, side, top views) drawings
- ***identification and construction of nets*** – is directly related to combining and analyzing visual images with regards to the net's component parts and the recombining of these parts to make new images; requires coordination between mental representation of the object as a whole and the decomposition of its component parts; requires translations from 3-D objects to 2-D nets by focusing on the component parts of the objects in both representations
- ***spatial structuring of cube arrays*** – identifying a 3-D object's spatial components, combining them into spatial composites and then establishing interrelations among components and composites (e.g., spatially structuring a cube with an edge of five units means that 25 unit sized cubes are needed to cover its base and that five units/layers fit in the cube)
- ***identifying 3-D objects and their elements*** – means understanding the characteristics parts of 3-D objects, the comparative relations between the same or different structural parts and how the elements of the solid are interrelated (e.g., any polyhedron is composed by the same parts, their size, number, and shape of each face define the special differences of each polyhedron)
- ***calculating of the volume and area of 3-D objects*** – three-dimensional geometric thinking is connected to students' calculation of volume and surface area of a solid; that is, conceptualization of the numerical operations of the formulas with the internal structure of the solid.

Common Errors, Misconceptions, and Instructional Strategies

Students develop an understanding of the geometric attributes of 3-D objects by comparing and sorting different 3-D objects (i.e., prisms, pyramids, polyhedral) and by representing 3-D objects in different ways. 3-D geometric attributes include: type of faces, number of faces or edges, number and position of identical (congruent) faces, number of corners/vertices and round parts. Small (2007) identifies the following common errors. Students:

- confuse vocabulary associated with 2-D shapes and 3-D objects (e.g.,refer to the flat surfaces of a 3-D objects as a "side")
- lose track of counting faces, edges, and vertices on 3-D objects
- have difficulty identifying the base of a prism, especially when the prism is short.

LINKS TO MANIPULATIVES: POLYDRONS™



Polydrons™ are plastic shapes that students snap together in order to make 3-D objects. Polydrons™ can be solid shapes, magnetic frames, and snap together frames.

These materials are very useful in developing the concept of composing and decomposing 3-D objects.

To develop geometric reasoning, prompt students to agree or disagree with statements like:

- You need at least three 2-D shapes to make a 3-D object.
- You need at least three congruent 2-D shapes to make a 3-D object.
- There is only one way to build a triangular based pyramid.

Have students use polydrons™ to build models that satisfy these statements. Provide class discussion to compare their responses before and after investigating the 3-D geometric relationships. Then challenge students to create their own statements.

LESSON LEARNING GOALS

Identify the curriculum expectations (MOE, 2005) grade-before and grade-after the grade specific learning goal:
- to see the development of mathematical knowledge, skills, and strategies across grade levels
- to determine the mathematical focus of the problems chosen for Before (activation), During (lesson), and After (Practice) parts of the lesson

BEFORE (Getting Started) – 5 to 10 minutes

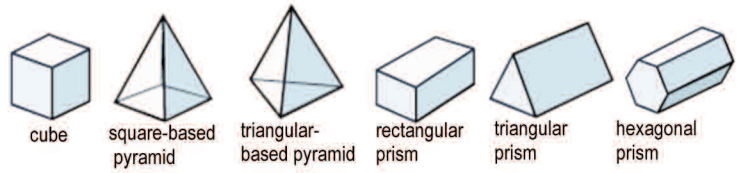
- Activating students' mathematical knowledge and experience using a prompt or problem that is directly relates to the mathematics in the lesson problem.
- Includes student responses to prompt or problem to highlight ideas, strategies, and tools that will provide access to lesson problem

AFTER (Consolidation) – 20 to 25 minutes

- Teacher selects 2 or more solutions for class analysis and discussion in a sequence (1st, 2nd, 3rd, etc.) based on mathematical relationships between the solutions and the lesson learning goal.
- Students (authors) explain and discuss their solutions with their classmates.
- During whole class discussion, solutions are organized (often re-organized) to show mathematical elaboration from one solution to the next and towards lesson learning goal.
- Teacher mathematically annotates (math terms, math symbols, labelled diagrams, concise explanations) on and around solutions to make mathematical ideas, strategies, and models of representation explicit to students.

Polyhedra Problem A:

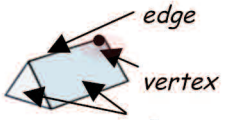
How are these polyhedra similar and different? Explain.



Solution 1
cube and rectangular prism – both have 6 faces and 12 edges; different lengths

Solution 2
pyramids – triangular faces meet at same vertex at top; different bases: square or triangle and 4 or 3 triangular faces

polyhedron
3-D shape whose faces are polygons



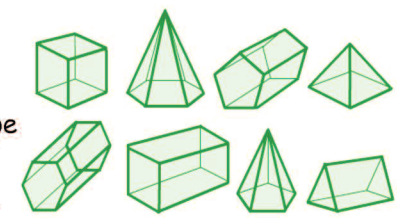
Two Types of Polyhedra
-prism - named by the shape of the base; two end faces or bases are congruent and parallel connected by parallelogram sides
-pyramids – named by the shape of the base; triangle faces that converge at a vertex (apex)

DURING (Working on It) – 15 to 20 minutes

- Understanding the problem – Teacher asks, "What information from the problem are we using to make a plan to solve it? Explain." Teacher records in a list below the problem, the information that students identify.
- Students solve the problem on chart paper (landscape) with markers (visible for whole class discussion) in pairs or in small groups.
- Teacher circulates to record different student solutions, in addition to the ones the teacher anticipated.

Polyhedra Problem B:

Make a shape chain so that each shape shares a face with the same shape as one of the faces on the shape that follows it.

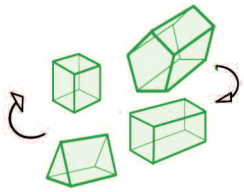


What information will we use to solve this problem?
-Identify faces on two shapes that are the same (e.g., both shapes have a triangle or both have squares)
-Make a line or chain following the rule.

-4 prisms (cube, rectangular, triangular, hexagonal)

-2 prisms (cube, rectangular prism)
-1 pyramid (square-based)

Solution 1



Solution 2



All have square bases.

Prisms with Rectangular Faces

Prisms and Pyramids with Square Bases

Note: Mathematical annotations include mathematical vocabulary, symbols, elaborations of mathematical details from solutions, labels describing the method/strategy and questions to further thinking. All annotations are records of the students' mathematical discussion.

Grade 3	Grade 4	Grade 5
compare and sort prisms and pyramids by geometric properties (i.e., number an shape of faces, number of edges, number of vertices, using concrete materials	identify and describe prisms and pyramids and classify them by their geometric properties (i.e., shape of faces, number of edges, number of vertices) using concrete materials	distinguish among prisms, right prisms, pyramids, and other three-dimensional figures

AFTER (Highlights/Summary) – 5 minutes

- Teacher and students revisit the student solutions for key ideas, strategies, and models of representation that are related to the lesson learning goal.
- Teacher lists key ideas, strategies, and models of representation separately, so the students can see how the mathematical details from their solutions relate explicitly to the lesson learning goal.

Highlights/Summary:

-properties of 3-D shapes - the number of faces, edges, and vertices in each shape.
-bases - two identical parallel faces of a prism
-pyramids always have a vertex where the triangular faces meet → apex
-number of faces (not including the bases) - related to the number of edges on the base.

AFTER (Practice) – 5 to 10 minutes

- Teacher chooses 2 or 3 problems, similar to the lesson problem for student to solve in pairs as a scaffold and individually.
- Problems could vary by number (choice, size), problem contexts, or what is unknown or needs to be solved
- Students are asked to solve these problems using a strategy different from the one they used for the lesson problem.

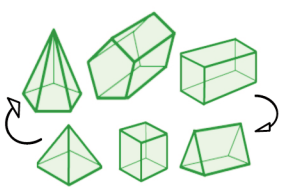
Polyhedra Problem C:

What is the relationship between the number of faces on a prism and a pyramid with the same base?

-4 prisms (cube, rectangular, triangular, hexagonal)
-2 pyramids (square-based, pentagonal-based)

-pentagonal base
-rectangular face
-square base

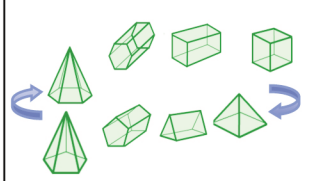
Solution 3



Prisms and Pyramids with Polygonal Bases and Faces

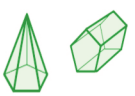
-hexagon base and triangle face
-hexagon base and rectangle face
-square base and rectangle face
-square base and square face
-square base and triangle face
-triangle base and rectangle face
-pentagon base and triangle face
-pentagon base and rectangle face

Solution 4

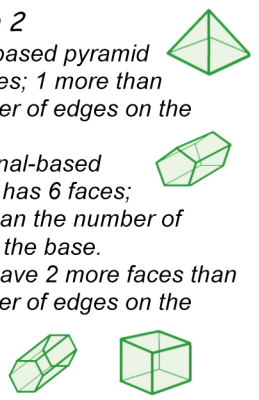


Prisms and Pyramids with Polygonal Bases and Faces

Solution 1
The prism has 7 faces and the pyramid 6.



Solution 2
-Square-based pyramid has 5 faces; 1 more than the number of edges on the base.
-Pentagonal-based pyramids has 6 faces; 1 more than the number of edges on the base.
-Prisms have 2 more faces than the number of edges on the base.

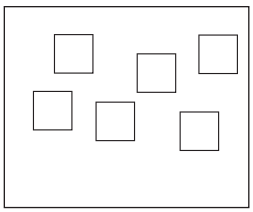


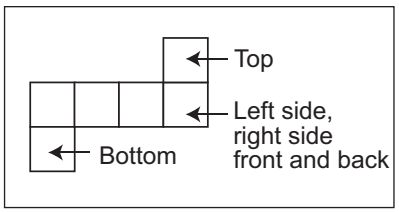
Note: A classroom board is longer proportionally than these 2 pages. Due to the space constraints on these pages, the mathematical annotations are recorded above the solutions with arrows, rather than on and around the solutions.

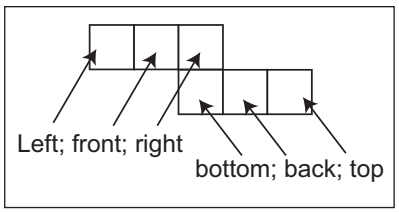
LET'S DO MATH – DESIGNING CUBE NETS

Before (Getting Started)

Cubes Nets Problem A - Join 6 square faces to make a net that folds up to make a cube.

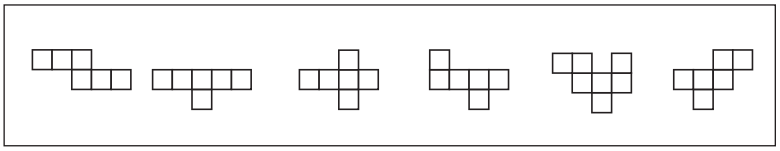






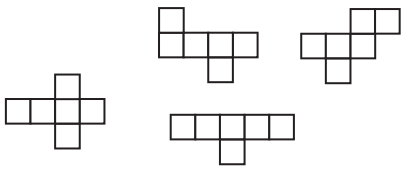
During (Working On It)

Cube Nets Problem B:
Which nets can be folded to make a cube?



After (Consolidation)

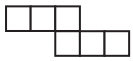
Solution 1

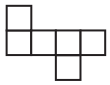


All have 6 faces must make nets.

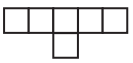
Solution 2

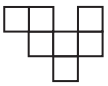
Yes





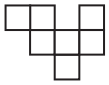
No



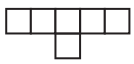


Solution 3

No, has 7 faces.



No, first and last squares on top row overlap.



Coordinating Discussion for Student Learning:

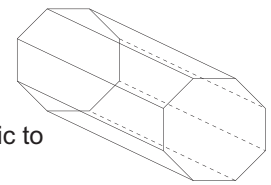
Why might solution 1 be chosen first for student discussion followed by solution 2, and then solution 3?

- solution 1 – all nets have 6 faces, but details about what makes the net fold into a cube requires elaboration
- solution 2 – all nets have 6 faces, with accurate cube nets, with no details about where the nets need to be folded to make a cube
- solution 3 - shows explanations for the two nets that do not work.

LINKS TO LITERATURE: *CHANGES, CHANGES*



Math concepts, such as shape, size, length, weight, and spatial relationships can be explored through block play. At different stages, children create stacks, lines, towers, then bridges, and enclosures. Teachers can choose a variety of materials, such as foam and wood for blocks. Some students may need to use blocks made of transparent plastic to make sense of 3-D object diagrams.



Changes, Changes by Pat Hutchins is a wordless picture book that naturally connects to block play. It shows a couple of people, made of wooden blocks, using building blocks to construct a house. The house catches on fire and the couple uses the blocks to build other things to solve problems. Have the students examine the pictures, ask them questions like:

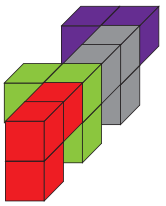
- Which blocks do we see on the bottom of the structure?
- Which blocks do we not see on the bottom of the structure? (e.g., blocks with "pointy tops", cylinders on their sides, spheres that roll).

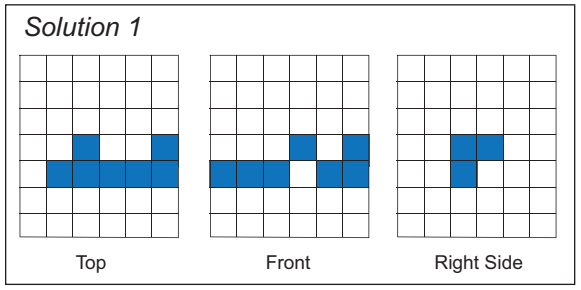
LET'S DO MATH – TAKING PERSPECTIVES

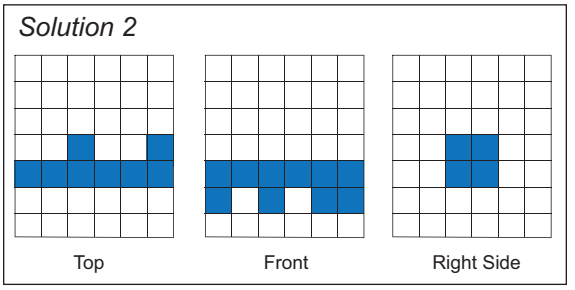
Before (Getting Started)

Perspectives Problem A:

- Examine the 3-D object made from linking cubes.
- Draw the front, top, and right side views.



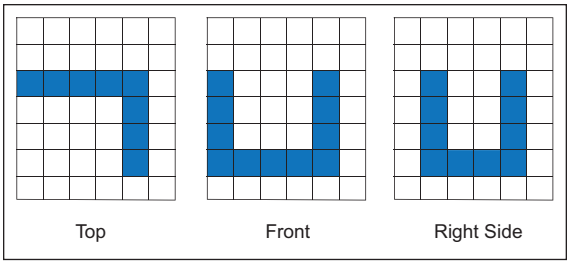




During (Working On It)

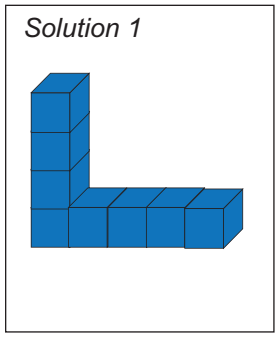
Perspectives Problem B:

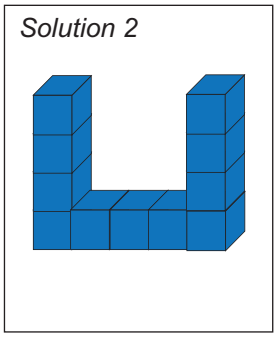
- Three views are given for a cubic design statue that sits outside of city hall in my friend's town.
- Use linking cubes to build a model of the statue.

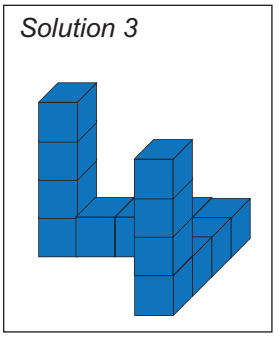


After (Consolidation)

Anticipating Student Responses:







Coordinating Discussion for Student Learning:

Why might solution 1 be chosen first for student discussion, followed by solution 2, and then solution 3?

- solution 1 - structure satisfies the front view only.
- solution 2 - structure could be turned to show a 3-D object that satisfies the top OR the side views but not more than one.
- solution 3 - structure satisfies all three views.

NEXT STEPS FOR YOUR PROFESSIONAL LEARNING

Application to Your Classroom

- Try these 3-D geometry problems yourself first and then with your students.
- To suit your students' learning needs, you might change the type of concrete material (e.g., geoblocks, polydronsTM, pattern blocks), the number or type of geometric attributes used.
- Think about how the context of the problem evokes visual images and serves as a tool for understanding and determining the geometric attributes of a set of 3-D objects, the sorting rule for 3-D objects (e.g., sorting through building toy pieces)
- Provide students with non-examples of the geometric attributes of 3-D objects (e.g., Teri asks Nathan why he thinks that a cube is not a rectangular prism. What would be a convincing argument?)
- Practise noticing the different strategies and the breadth of the mathematics your students use in their solutions. Take note of the number and kinds of solutions that show strategies for sorting and classifying 3-D objects by geometric attributes and geometric properties. Which strategy do students use more often? (e.g., counting the number of faces, comparing the lengths)
- In preparation for your class discussion of student solutions, decide which solution you will discuss first, second, and third by referring to the developmental learning sequence for sorting and classifying 3-D objects, as well as making sketches
- Think ahead of the kinds of mathematical annotations you will record on and around the student solutions to make explicit key mathematical ideas when students discuss and analyze each other's solutions. Think about the strategies students use to identify geometric attributes and properties of 3-D real life objects and concrete materials,

Suggested Readings

Some readings used in the Research Summary and in the development of the problems and solutions are listed below:

- Charles, R. (2005). Big ideas and understandings as the foundation for elementary and middle school mathematics. *Journal of Mathematics Education Leadership, 7*(3), 9-24.
- Johnston-Wiilder, S. & Mason, J. (Eds.). (2005). *Developing thinking in geometry.* London: Paul Chapman Publishing.
- Pittalis, M. Christou, C. (2010). Types of reasoning in 3-D geometry thinking and their relation with spatial ability. *Educational Studies in Mathematics,* 75, 191-212.
- Small, M. (2009). *Big ideas from Dr. Small, Grades 4-8*. Toronto, ON: Nelson Education.
- Small, M. (2008). *Making math meaningful to Canadian students, K-8.* Toronto, ON: Nelson Education.
- Small, M. (2007). *Geometry: Background and strategies.* Toronto, ON: Nelson Education.

CLOSING

Our next issue will focus on key notions of geometric transformations, how teachers need to know these notions for teaching, as well as, the ways that teachers can use these mathematical ideas and pedagogical strategies within a problem solving-based teaching and learning lesson framework.

CALL FOR *ABACUS* SUBMISSIONS

Send your teaching strategy ideas, problems and student solutions to the *Abacus* Co-Editors via email