8 PAGE INSERT ELEMENTARY MATH

ONTARIO ASSOCIATION FOR MATHEMATICS EDUCATION

Abacus

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COLLECTING AND ORGANIZING DATA

TABLE OF CONTENTS

Abacus Editor Greetings 1
RESEARCH SUMMARY 2
- Collecting and Organizing Data
LINKS TO MANIPULATIVES . . . 3
- Using Every Day Objects and Photos
LET'S DO MATH (Bansho (Board-Writing) Plan) 4-5
- Conserving Energy Problems A, B, C
LINKS TO LITERATURE 6
Not My Fault – by Leif Kristiansson
LET'S DO MATH 6
- Venn and Vegetables
- Venn and Languages Problem
LET'S DO MATH 7
- Counting Birds Problem
- Causes of Bullying Problem
NEXT STEPS FOR YOUR PROFESSIONAL LEARNING. 8
- Application to Your Classroom
- Suggested Readings

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ABACUS CO-EDITORS



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The main purpose for collecting and organizing data is to gather information in order to answer questions. Such questions are derived often from curiosities about a phenomenon, dilemmas about a situation, or observations that require further investigation. When students collect and organize data, they have an opportunity to learn more about themselves, their environment, issues in their school or community, topics in various subject areas, and so on. Learning activities should help students understand the processes that are involved in formulating questions, seeking relevant information, and organizing that information in meaningful ways.

In this issue, the Research Summary and problems (with multiple solutions) focus on strategies for collecting and organizing quantitative and qualitative data and determining mean, median, and mode where appropriate. The problems we provide are designed for use within a three-part, problem-solving lesson.

LINKS TO LITERATURE AND MANIPULATIVES



PAT MARGERM is an independent literacy and mathematics consultant with more that 30+ years of K to 8 teaching experience with students, in-service and pre-service teachers. She is a long-time OAME board member and works in various aspects of mathematics education.

Welcome to a new school year and a new math focus for the ABACUS. According to the curriculum expectations, students in grades 2 to 8 collect data through surveys. How does a grade 8 survey compare to a grade 2 survey? In this issue, we will use a picture book to try to collect more meaningful survey results than "what is your favourite ice cream?" Also, we will look at a few ways to collect data using manipulatives.

RESEARCH SUMMARY – COLLECTING AND ORGANIZING DATA

Collecting and Organizing Data

According to Small (2010), "people collect data in order to solve problems. School activities should support this idea – that data collection is a tool that can help solve problems, rather than an end in itself. There should always be a purpose for collecting data, and the method of data collection should suit that purpose" (p. 169).

When students collect and organize primary data, they are compelled to make decisions that are required at different steps of the data management process. Randall (2005) describes a big idea about data collection; that is, "some questions can be answered by collecting and analyzing data, and the question to be answered determines the data that needs to be collected and how best to collect it" (p. 20). Some questions associated with collecting and organizing data are as follows:

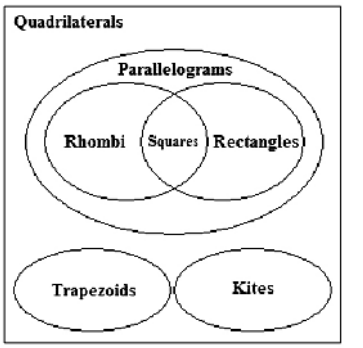
- What do we need to find out? What question needs to be answered? (e.g., How can the school improve on reducing school waste?)
- What kind of data needs to be gathered in order to answer the question?
- Where and from whom should data be collected? (e.g., from direct observations of a situation; asking a set of focused questions to grade 3 students, teachers, or parents)
- How will data be collected? (e.g., carrying out an experiment, taking a poll, paper-pencil or electronic survey)
- How will data be recorded? (e.g., responses directly on a paper-pencil or electronic survey, tallies)
- What ways could data be organized so that it provides answers to a question? (e.g., similar items in categories, by order of data value (least to greatest)) How can we sort and classify objects using multiple attributes (colour, size, texture, thickness)?

Sorting and Classifying Data

According to Small (2010), sorting and classifying data is a prerequisite to many forms of data collection and display.

Venn Diagrams are tools for sorting by multiple attributes in different parts of one of the circles. The example of a Venn Diagram shows the inclusions and intersections of the various types of quadrilaterals.

Carroll Diagrams are cross-classification charts that enable sorting of two or more attributes with a focus on one attribute at a time.



Prime	Not prime
2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53	1, 4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20, 21, 22, 24, 25

	Prime	Not prime
Even	2	4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26
Not even	3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41	1, 9, 15, 21, 25, 27, 33, 35, 39, 41, 45, 49

Mean, Median, and Mode

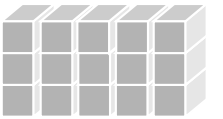
Randall (2005) describes the big idea about data distribution; that is, "there are special numerical measures that describe the center and spread of numerical data sets." Small (2010) explains that there are three measures of central tendency or averages: mean, median, and mode." The best descriptor of the centre of a numerical data set is determined by the nature of the data and the question to be answered. Outliers affect the mean, median, and mode in different ways" (p. 21) and the size or the range of data.

- ***Range*** of a set of numbers is the difference between the highest and lowest numbers in the set. The range measures the spread of a set of data. However, range is frequently not a good measure of distribution because it includes the extremes and it can give us a distorted picture of the data.

- ***Mean or arithmetic mean*** describes a set of data by indicating the value that would result from putting all the values together and distributing them evenly. It is calculated by dividing the sum of the numbers by the number of members in a set. It is conceptualized as making equal groups or as a balance point where there is an equal number of data above and below the mean. Students need to use different sets of numbers and data and apply the algorithm to calculate the mean and see the relationship between the range of data and the value of the mean.
- ***Mode*** of a set of numbers is that number that occurs most frequently in the set. If there is not a more frequent number then there is no mode; if there are two numbers with the highest frequency, then the set of data is bi-modal. Because an extremely high number or an extremely low number can distort the picture of the tendency shown by the mean, the mode is frequently used.
- ***Median*** of a set of numbers is the middle number of the set when the set is ordered. To determine the median, order the numbers from least to greatest, and choose the position of the middle score. For example, if the numbers are 14, 6, 8, 8, 5, 12, 34, 23, and 2, then arrange them as: 2, 5, 6, 8, 8, 12, 14, 23, and 34. Then locate the middle position; in this case the fifth position is the number eight. If the total number of numbers is even, add the two numbers in the middle position and divide them by 2. That will be the median for that set of numbers.



Distribute the cubes evenly to get the mean of 3.



Common Errors, Misconceptions, and Instructional Strategies

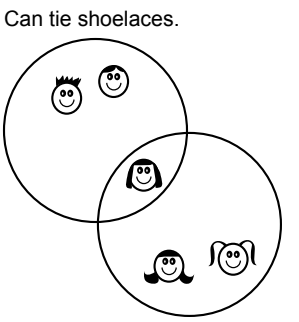
Batanero, Godino, Vallecillos, Green and Holmes (2006) report on Russell and Mokros' categories for classifying students' misconceptions about averages as being: the most frequent value or mode; the reasonable value, the midpoint and an algorithmic relationship. They suggest these strategies to support students in understanding the meaning and appropriate use of mean:

- Given different data sets, choose a central value (i.e., mean, median, mode) that best represents or summarizes the data.
- Build a data set that has a given mean, mode, and/or median. Have the students notice when and what sorts of data sets have the mean, mode, and median are close yet different in value.
- Examine what happens to central values (i.e., mean, median, and mode) when you add extreme data values to the sets.

LINKS TO MANIPULATIVES: USING EVERY DAY OBJECTS AND PHOTOS

A. How can we help each other? (Using student photographs in early primary).

- Organize the pictures of the students in response to the following questions on a Venn diagram: Can you tie shoelaces? Can you zip zippers?
- Tape the photographs on the Venn diagrams and post it near the coats and shoes.
- Students can refer to the Venn diagram to see who to ask for help when it is time to get dressed.



B. What are the students in our school doing to the environment?

- Young students investigate litter in the schoolyard by collecting litter and categorizing it in the classroom.
- Older students may investigate issues with litter in the schoolyard, by mapping or counting the litter in various locations and/or at various times of the day.

LESSON LEARNING GOALS

Identify the curriculum expectations (MOE, 2005) grade-before and grade-after the grade specific learning goal:
- to see the development of mathematical knowledge, skills, and strategies across grade levels
- to determine the mathematical focus of the problems chosen for Before (activation), During (lesson), and After (Practice) parts of the lesson

BEFORE (Getting Started) – 5 to 10 minutes

- Activating students' mathematical knowledge and experience using a prompt or problem, related to the math in the lesson problem.
- Includes student responses to highlight ideas and strategies.

AFTER (Consolidation) – 20 to 25 minutes

- Teacher selects 2 or more solutions for class analysis and discussion in a sequence (1st, 2nd, 3rd, etc.) based on mathematical relationships between the solutions and the lesson learning goal.
- Students (authors) explain and discuss their solutions with their classmates.
- During whole class discussion, solutions are organized (often re-organized) to show mathematical elaboration from one solution to the next and towards lesson learning goal.
- Teacher mathematically annotates (math terms, math symbols, labelled diagrams, concise explanations) on and around solutions to make mathematical ideas, strategies, and models of representation explicit to students.

Comparing Central Measures Problem A:

Which set of data has a mean, median and mode that are equal? Explain.

Group 1: 5, 5, 5, 5, 5, 20 Group 2: 6, 2, 5, 8, 5, 4

Group 3: 1, 8, 1, 8, 6, 8

Solution 1:

Group 2 -> the range is the smallest and the numbers are pretty evenly spaced; 4 data points above 5 and 4 below 5; mean is 5; median and mode are 5

Solution 2:

Not Group 1 -> 20 is an outlier that throws off the mean by making it bigger than median and mode; Not Group 3 - median, mode, median are different

Group 2:

Range->8-2=6
Mean->30/6=5
Median is 5
2,4,5,5,6,8
Mode->5

Group 1:

***Range**->20-5=15*
***Mean**->45/6=7.5*
***Median** is 5*
5,5,5,5,5,20
***Mode**->5*

How did you calculate the median value?
- order the data from least to greatest value
- count in from lowest and highest value towards the middle until locate the middle data value

***Mode** - the data value that appears most often in a set of data*
Australia – seven 1000s
Canada – six 1000s

***Range** - difference between the highest and the lowest values in a set*

DURING (Working on It) – 15 to 20 minutes

- Understanding the problem – Teacher asks, "What information from the problem are we using to make a plan to solve it? Explain." Teacher records in a list below the problem, the information that students identify.
- Students solve the problem on chart paper (landscape) with markers (visible for class discussion) in pairs or small groups.
- Teacher circulates to record different student solutions, in addition to the ones the teacher anticipated.

Conserving Energy Problem B:

Students were asked to score, from 0 to 1000 about the importance of conserving energy? Who cares more?

BrthPlce	Sex	Conserving Energy
Victoria	Female	101
South Australi	Male	101
New South Wa	Female	208
South Australi	Female	302
South Australi	Male	362
New South Wa	Female	391
Queensland	Male	391
Western Austr	Male	444
New South Wa	Male	503
Victoria	Male	521
New South Wa	Female	740
Montenegro	Female	769
Victoria	Female	781
Queensland	Female	1000
New South Wa	Male	1000
New South Wa	Female	1000
Victoria	Female	1000
New South Wa	Female	1000
Victoria	Female	1000
New Zealand	Female	1000

Province	Gender	Conserving Energy
Ont.	Male	5
Ont.	Male	5
Man.	Male	241
Ont.	Male	316
Man.	Male	380
Ont.	Female	422
Ont.	Male	439
Ont.	Female	481
Ont.	Male	529
N.S.	Female	679
Ont.	Female	711
Ont.	Male	738
Ont.	Female	797
Ont.	Female	872
Sask.	Male	1000
Ont.	Male	1000
Alta.	Female	1000
Ont.	Female	1000
Alta.	Female	1000
Alta.	Male	1000

Solution 1:

Range
- *Australia: 1000 – 101 = 899*
- *Canada: 1000 – 5 = 995*

Australian's spread of data 899 is smaller than the Canadian's at 995.

Both have a mode of 1000 so mean and median will be close. So students in both countries care about conserving energy.

Solution 2:

Mode
- *Australia: 1000*
- *Canada: 1000*

Median
- *Australia: (521 + 740) / 2 = 630.5*
- *Canada: (679 + 711) / 2 = 695*

695 > 630.5 so Canadians care more about conserving energy.

Range and Mode Comparisons

Mode and Median HowComparisons

Note: Mathematical annotations include mathematical vocabulary, symbols, elaborations of mathematical details from solutions, labels describing the method/strategy, and questions to further thinking. All annotations are records of students' mathematical discussion.

Grade 5	Grade 6	Grade 7
compare similarities and differences between two related sets of data by determining measures of central tendency (i.e., mean, median, and mode)	demonstrate an understanding of mean and use the mean to compare two sets of related data	determine, through investigation, the effect on a measure of central tendency (i.e., mean, median, and mode) by adding or removing a value or values

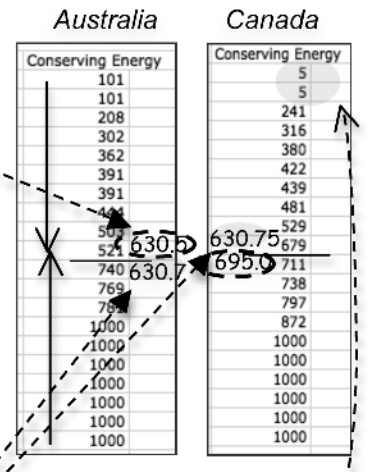
How are the median and mean different?

Median *- data value separating the upper half of the data from the lower half of the data*

- *Middle value between 521 and 740 -> (521+740)/2=630.5*
- *Middle value between 679 and 711 -> (679+711)/2=695*

Arithmetic Mean *- sum of the numbers divided by the size of the collection*

- *(202+510+753+835+1024 +2290+7000)/20 = 12614/20=630.7*
- *(10+557+802+920+1208+ 1449+1669+6000)/2 = 12615/20 = 630.75*



Outliers- *data values much lower (or higher) than rest of data*

- *5, 5*
- *creates greater range of data (1000-5=995)*

AFTER (Highlights/Summary) – 5 minutes

- Teacher and students revisit the student solutions for key ideas, strategies, and models of representation that are related to the lesson learning goal.
- Teacher lists key ideas, strategies, and models of representation separately, so the students can see how the mathematical details from their solutions relate explicitly to the lesson learning goal.

Highlights/Summary:

Compare "*Averages*" *(Measures of Central Tendency):*

- ***mode*** *- data value that occurs most often (e.g., both data sets, 1000 occurs 6 or 7 times; mode is 1000)*
- ***median*** *- data value separating the upper half of the data from the lower half of the data; middle data value in a set of ordered data from least to greatest (e.g., 630.5 and 695)*
- ***(arithmetic) mean*** *- sum of the data values divided by the number of data (e.g., 630.7; 630.75)*
- ***outliers*** *- data values much lower (or higher) than the rest of the data values (e.g., 5, 5 in Canada data)*

AFTER (Practice) – 5 to 10 minutes

- Teacher chooses 2 or 3 problems, similar to the lesson problem for student to solve in pairs as a scaffold and individually.
- Problems could vary by number (choice, size), problem contexts, or what is unknown or needs to be solved
- Students are asked to solve these problems using a strategy different from the one they used for the lesson problem.

Conserving Energy Problem C:

Compare Canadian males' and females' concerns about conserving energy.

Province	Gender	Conserving Energy
Ont.	Female	422
Ont.	Female	481
N.S.	Female	679
Ont.	Female	711
Ont.	Female	797
Ont.	Female	872
Alta.	Female	1000
Ont.	Female	1000
Alta.	Female	1000
Ont.	Male	5
Ont.	Male	5
Man.	Male	241
Ont.	Male	316
Man.	Male	380
Ont.	Male	439
Ont.	Male	529
Ont.	Male	738
Sask.	Male	1000
Ont.	Male	1000
Alta.	Male	1000

"Averages"

- *Arithmetic mean*
- *Mode*
- *Median*

Solution 4:

Country	Range	Mean
Australia	899	630.7
Canada	995	630.75
Country	**Median**	**Mode**
Australia	630.5	1000
Canada	695	1000

Students in both countries care about conserving energy. The numbers suggest that their concern about conserving energy is above average (600 compared to 500).

Solution 1:

Median

- *Females: 797*
- *Males: 439*

797>439; so females care more about conserving energy than males.

Solution 2:

Arithmetic Mean

- *Females: 6962/9 = 773.6*
- *Males: 5653/11 = 513.9*

773.6>513.9; so females care more about conserving energy than males.

Solution 3:

Calculating Mean:
Australia ->12614/20= 630.7
Canada ->12615/20= 630.75

Students in both countries care equally about conserving energy.

Mean Comparisons

Mode, Median and Mean Comparisons

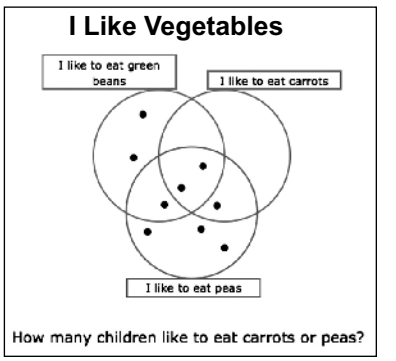
Note: A classroom board is longer proportionally than these 2 pages. Due to the space constraints on these pages, the mathematical annotations are recorded above the solutions with arrows, rather than on and around the solutions.

LET'S DO MATH – VENN DIAGRAMS

Before (Getting Started)

Venn and Vegetables Problem:

How many children like to eat green vegetables?



Solution 1	*Solution 2*
Nine people like green vegetables as all the dots are in the green bean circle or the peas circle.	Six of the dots are in the green (beans and peas) circles and three like carrots

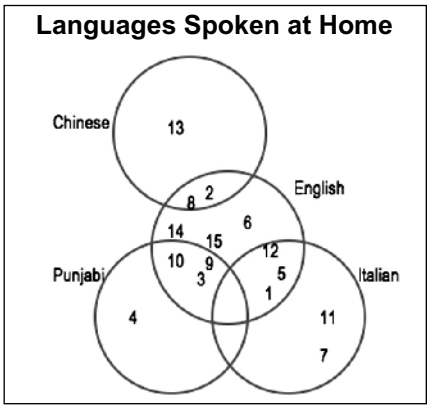
During (Working On It)

Venn and Languages Problem:

Fifteen people were asked the question, "What languages do you speak at home?" Their responses are shown by their number in the Venn diagram. What does the Venn diagram tell you about the survey results?

After (Consolidation)

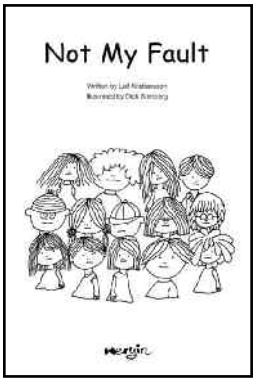
Solution 1	*Solution 2*
11 out of 15 of the children speak English at home. 3 speak two languages – 3 English and Italian and 3 English and Punjabi.	Most speak English and one more language at home. One speaks only Punjabi and one speaks only Chinese. Two speak only Italian. Three speak only English.



Coordinating Discussion for Student Learning:

Why might solution 1 be chosen first for student discussion followed by solution 2?

- *Solution 1* presents information about those that speak English and those that speak 2 languages at home.
- *Solution 2* reports on different information – the number of people who speak a single language at home and that most speak English at home.



LINKS TO LITERATURE

Not My Fault by Leif Kristiansson is a book that is very powerful in its deceptively simple illustrations and text. Each child in the group offers an excuse for not taking responsibility for stopping the bullying of a child. The end of the book has powerful images of global strife.

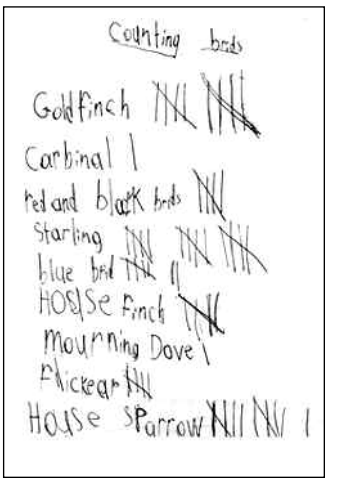
- Create a page of the excuses and ask the students to organize them into categories (e.g., blaming the bullied child, blaming others). Compare the categories used to sort the excuses.
- Use the page of excuses and collect data about which excuses students have heard being used. Which excuses are most common?
- Do we have a bullying problem in our class? Do we have a bullying problem in our school? How could we conduct a survey to find out? Would responses vary by gender, grade, age (adults and students)?

LET'S DO MATH – ORGANIZING AND TALLYING

Before (Getting Started)

Counting Birds Problem:

Marcus created the tally sheet when he went bird watching. What does is tally chart say about his sightings?



Solution 1

Marcus saw 9 different kinds of birds. He saw the most Starlings and only 1 Morning Dove and 1 Cardinal.

Solution 2

Birds	Numbers	Birds	Numbers
Goldfinch	10	House Finch	5
Cardinal	1	Morning Dove	1
Red and Black bird	5	Flicker	5
Starling	15	House Sparrow	11
Blue Bird	7		

During (Working On It)

Causes of Bullying Problem:

Students were asked, "Why do you think some kids bully others?"

Here are some responses:

trying to be cool, insecure, having family problems, trying to be cool, trying to be cool, having family problems, jealous, looking for fun, insecure, angry, trying to be cool, having family problems, trying to be cool, angry, looking for fun, trying to be cool, having family problems, jealous, trying to be cool, insecure, trying to be cool, insecure, jealous, trying to be cool, trying to be cool, insecure, having family problems, trying to be cool

What are the issues that cause students to bully others?

After (Consolidation)

Solution 1

	Tally	Numbers
Trying to be cool	𝍸 𝍸 |	11
Insecure	𝍸	5
Having family problems	𝍸 ||	7
Jealous	|||	3
Looking for fun	||	2
Angry	||	2

I read the reasons one at a time and made a tally mark for each. Then I can see that trying to be cool and having family problems were the two biggest reasons people thought that others bullied.

Solution 2

There are 30 answers. Trying to be cool is the most then insecure and having family problems. The last two are sad but no reason to bully and the first idea that it is cool is just wrong!

Coordinating Discussion for Student Learning:

Why might solution 2 be chosen first for student discussion followed by solution 1?

- *Solution 2* presents the general information about the excuses that occur most often.
- *Solution 1* presents the data in an organized tally chart so all excuses can be seen and their frequency identified.

NEXT STEPS FOR YOUR PROFESSIONAL LEARNING APPLICATION TO YOUR CLASSROOM

- Try these data collection and organization problems yourself first and then offer them to your students.
- To suit your students' learning needs, vary the ways that you have students engage with data collection and organization, as well as determining mean, median, and mode (e.g., use concrete materials, such as centicubes or interlocking cubes, counters; organize data using TinkerplotsTM (dynamic data software) or ExcelTM spreadsheets).
- Think about how the context of the problem evokes the use of data collection and organization (e.g., who cares about conserving energy by country and then by gender).
- Practise noticing the different strategies and the breadth of the mathematics your students use in their solutions. Take note of the number and kinds of solutions that show strategies for analysing data (e.g., central measures, graphs)
- Think ahead of the kinds of mathematical annotations you will record on and around the student solutions to make explicit key mathematical ideas, strategies for collecting and organizing data as the students discuss and analyze each other's solutions.

SUGGESTED READINGS

Some readings used in the Research Summary and in the development of the problems and solutions are listed below:

- Batanero, C., Godino, J. D., Vallecillos, A., Green, D. and Holmes, P. (2006). Errors and difficulties in understanding elementary statistical concepts. *International Journal of Mathematics Education in Science and Technology, 25*(4), 527-547
- Chapin, s., Koziol, Al, MacPherson, J., & Rezba, C. (2002). *Navigating through data analysis and probability in grades 3-5.* Reston, VA: NCTM.
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- Small, M. (2010). *Big ideas from Dr. Small, Grades 5-8*. Toronto: Nelson Education.

CLOSING

Our next issue will focus on key notions of organizing data to construct and read graphs, how teachers need to know these notions for teaching, as well as, the ways that teachers can use these mathematical ideas and pedagogical strategies within a problem solving-based teaching and learning lesson framework.

CALL FOR *ABACUS* SUBMISSIONS

Send your teaching strategy ideas, problems and student solutions to the *Abacus* Co-Editors via email