8 PAGE INSERT ELEMENTARY MATH

ONTARIO ASSOCIATION FOR MATHEMATICS EDUCATION

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READING AND COMPARING GRAPHS TO MAKE INFERENCES

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ABACUS CO-EDITORS



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Graphs are primarily used for communication and for analysis. For the purpose of communication, the data is organized in a graph to convey particular meanings explicitly and clearly. However, for the purpose of analysis, data is read for more than what it presents directly. Sometimes, when comparing data sets, analysis requires consideration of the details and contexts surrounding the data set in order to "read between the lines." Reading data and seeing it as clues in order to make other possible conclusions offers deeper understandings of the phenomenon that the data is representing. In fact, going beyond the surface details of data in order to see other meanings that data and sets of data can suggest or imply requires inference.

In this issue, the Research Summary and problems (with multiple solutions) focus on strategies for reading quantitative and qualitative data to make inferences. The problems we provide are designed for use within a three-part, problem-solving lesson structure.

LINKS TO LITERATURE AND MANIPULATIVES



PAT MARGERM is an independent literacy and mathematics consultant with more that 30+ years of K to 8 teaching experience with students, in-service and pre-service teachers. She is a long-time OAME board member and works in various aspects of mathematics education.

On this Visual Literacy website,**http://k-8visual.info/**, Steve Moline,author of *I See What You Mean,* identifies graphs as a visual text that measures, ranks and/or compares. He maintains that presenting information visually makes information more memorable. Kelly Gallagher poses these questions in his book, *Deeper Reading; Comprehending Challenging Texts 4-12* to students as they "read" graphs: "What does it say?" (literal comprehension) and "What Does it Mean?" (inferential comprehension). In an information and technologically-based society, it is important for students to develop critical literacy skills, such as reading, interpreting and drawing conclusions from data for a purpose and audience. In this issue, we will look at a picture book that uses both words and graphs to tell the story.

RESEARCH SUMMARY

Small (2009) outlines a developmental sequence towards inferring and drawing conclusions from graphs. First, students need to understand how a graph is constructed in order to read the graph for simple information. Then, they are able to make comparisons and consider multiple pieces of data simultaneously and describe trends; that is, making inferences about data that will support their drawing of conclusions. Part of the process includes justifying any inferences or conclusions that they make from the data (i.e., table, graphs). Interpolation can be made when the reader determines possible data values between two given or known data values on a graph or in a table. Extrapolation is about predicting data values beyond known data values using trends in the data.

Friel, Curcio and Bright (2001) explain that there are critical factors that appear to influence the comprehension of graphs, which in turn suggest instructional approaches. For example, based on the work of Jolliffe (1991) and Wood (1968) who identify three sorts of behaviours that are related to graph comprehension: translation, interpretation and extrapolation / interpolation.

- A beginning level focuses on locating and extracting data from a graph; such translation entails a change in the form of communication, like between graphs and tables where the contents of a table of data is described, while a graph is interpreted at a descriptive level by commenting on specific structure of a graph.
- An intermediate level, *integration*, *interpolation* and *interpretation*, requires the rearrangement and sorting of the less important from the important details, such as looking for relationships among specifiers (visual features representing data) in a graph or between a specifier and a labeled axis.
- The third level, *extrapolation* from the data includes analyzing relationships implicit in a graph, which are embedded within the deep structure of the data presented in the graph.

This is akin to Curcio's (1987) framework of reading the data, reading between the data and reading beyond the data. Rubenstein and Thompson (2012, p. 548) elaborate on the three levels of reading information (Curcio, 1987; Friel, Curcio, & Bright, 2001): (a) reading information directly requires answering explicit questions for which an answer is "right there;" (b) reading between the data includes analyzing interpolations, making comparisons, finding relationships or applying operations, like determining a median; (c) reading beyond the data includes analyzing interpolations, producing generalizations, making predictions, identifying trends and making inferences (e.g., What might be another data value if we surveyed another person?).

Instructional Strategies

Small (2009) remarks that graphs can be misleading, intentionally or accidentally, through the misuse of intervals and/or scale. Interval graphs should have intervals of equal size. Unequal intervals are misleading in graphs. The appropriate choice of scale is necessary, especially when comparing graphs. The use of a different scale and an axis break can make a graph misleading. Consider using misleading graphs as an instructional approach to provoking students to make inferences about the meaning of data in relation to its graphical structure.

Consider using graphs without labels. Provide a horizontal bar graph, a broken-line graph, a Venn Diagram or a line plot without labels. You might or might not provide a context, depending on how open you want to make the problem. Then, ask students to make inferences about the data trends in the graph and hypothesize possible labels for the axes.

Kitchen (1999) uses the data representations in a local community newspaper over an extended period of time as a source for students to examine how a wide variety of statistics are displayed. Rather than making inferences about the data, the students are examining the ways that statistics are used and misused. Some key prompts that Kitchen (p. 318) uses include:

- How often were charts and tables used to illustrate statistical data?
- How often were tables used to represent statistical data?
- What types of graphs were used to depict the data and how often was each type of graph used in the articles?
- Look for general patterns during the time period in which the articles including statistics appeared. Are the articles connected in any way? In what types of articles do statistics most frequently occur?

- Summarize observations that your group made about your investigations about how statistics were used in the print media over the course of two months.
- How were statistics used in the articles? What does the use of statistics do to enhance the articles?
- Discuss the use or misuse of statistical representations chosen for three articles in particular.

Consider that Kitchen's instructional sequence could be adapted for the use of any data, rather than statistical data. Making inferences about the ways that data and data representations are used within and across a series of articles over a period of time and within particular newspaper features (e.g., front page, business, sports, local news) are certainly ways to develop students' media literacy through data analysis and sense making.

LINKS TO MANIPULATIVES:

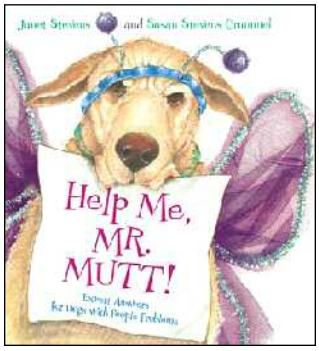
Students can use a variety of materials to critically investigate the representation of data.

In this example, students used wrappers to create the following concrete graph to identify their purchasing power to lobby for fair trade for chocolate. Ask students what strategies could be used to make the graph easier to read and interpret? Alternatively, provide different sizes of square grid paper (e.g., ½ cm, 1 cm, or 2 cm) and compare the impact on the representation and interpretation of the data. Ask students how the size and number of squares on their grid paper influenced their choice of scale.



LINKS TO LITERATURE:

Mr. Mutt offers sage advice to dogs everywhere. The book is written in a letter format. Dogs write Mr. Mutt with their problem(s), and he writes back with his answer(s). Each answer includes a graph to support the explanation. Although the book is humorous, students can write their own advice column and add their own research in charts and graphs. They can write about issues, such as, "Should Rottweilers and pit bulls be banned?"



There are books and poems with growing and shrinking patterns, such as *Ten Black Dots* by Donald Crews and *Five Green and Speckled Frogs.* The patterns in the stories can be represented graphically so that students can investigate the idea that graphs can tell stories.

LESSON LEARNING GOALS

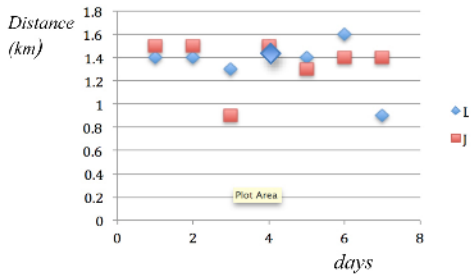
Identify the curriculum expectations (MOE, 2005) grade-before and grade-after the grade specific learning goal:
- to see the development of mathematical knowledge, skills, and strategies across grade levels
- to determine the mathematical focus of the problems chosen for Before (activation), During (lesson), and After (Practice) parts of the lesson

BEFORE (Getting Started) – 5 to 10 minutes

- Activating students' mathematical knowledge and experience using a prompt or problem, related to the math in the lesson problem.
- Includes student responses to highlight ideas and strategies.

Race Problem

This scatterplot shows the distance (km) ran by Lesley (L) and Jan (J) over 7 days.



a) Label the scatterplot.
b) Record the data in a table

day	1	2	3	4	5	6	7
L	1.4	1.4	1.3	1.5	1.4	1.6	0.9
J	1.5	1.5	0.9	1.5	1.3	1.4	1.4

DURING (Working on It) – 15 to 20 minutes

- Understanding the problem – Teacher asks, "What information from the problem are we using to make a plan to solve it? Explain." Teacher records in a list below the problem, the information that students identify.
- Students solve the problem on chart paper (landscape) with markers (visible for class discussion) in pairs or small groups.
- Teacher circulates to record different student solutions, in addition to the ones the teacher anticipated.

AFTER (Consolidation) – 20 to 25 minutes

- Teacher selects 2 or more solutions for class analysis and discussion in a sequence (1st, 2nd, 3rd, etc.) based on mathematical relationships between the solutions and the lesson learning goal (co-construction of success criteria)
- Students (authors) explain and discuss their solutions with their classmates.
- During whole class discussion, solutions are organized (often re-organized) to show mathematical elaboration from one solution to the next and towards lesson learning goal.
- Teacher mathematically annotates (math terms, math symbols, labelled diagrams, concise explanations) on and around solutions to make mathematical ideas, strategies, and models of representation explicit to students.

Constructing a Broken-Line Graph

- label horizontal axis with months
- label vertical axis with precipitation data
- choose scale range according highest and lowest precipitation values (0 to 330 mm)
- scale units - 1 unit is 50 mm on vertical axis; precipitation (mm) and 1 unit for each month on horizontal axis
- plot data values as points on vertical axis to month on horizontal axis
- use line segments to connect the data points in order

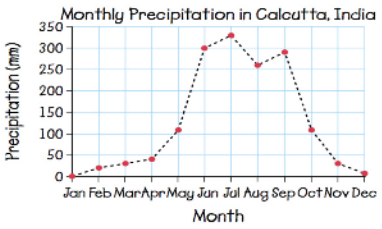
Make conclusions about data and graphs by:
- *reading the data - identify data values and whether data increases or decreases in value*
- *reading between the data - seeing relationships within 1 set of data and between 2 sets of data*

Climate Problem

Karin is studying climate to compare countries.
a) She made a broken-line graph. How did she do it?

Monthly Precipitation in Calcutta, India (mm)

Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
0	20	30	40	110	300	330	260	290	110	30	10



Adapted from Nelson Mathematics, Grade 5

b) How does the Prince Rupert compare to Calcutta?

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Calcutta, India	0	20	30	40	110	300	330	260	290	110	30	10
Prince Rupert, B.C.	240	190	210	160	130	100	120	130	190	310	310	280

Solution 1
The precipitation is opposite in Prince Rupert to Calcutta (January is high in Prince Rupert and low in Calcutta).

Solution 2
Prince Rupert precipitation decreases, increases, then decreases Calcutta precipitation increases, stays the same and decreases.

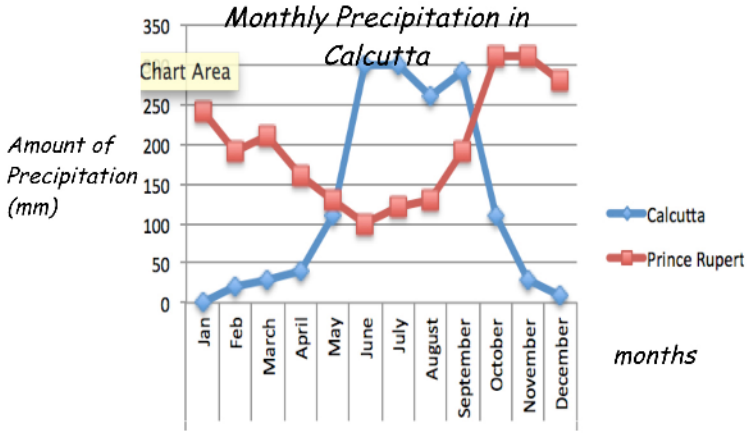
Compare Data Values at Same Month

Describe Each Trend Individually

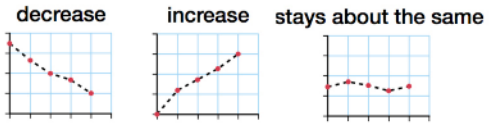
Note: Mathematical annotations include mathematical vocabulary, symbols, elaborations of mathematical details from solutions, labels describing the method/strategy, and questions to further thinking. All annotations are records of students' mathematical discussion.

Grade 4	Grade 5	Grade 6
-read, interpret and draw conclusions from secondary data presented in charts, tables and graphs	- Read, interpret and draw conclusions from secondary data presented in charts, tables and graphs (including broken-line graphs)	-demonstrate an understanding of how data from charts, tables and graphs can be used to make inferences and convincing arguments

Secondary data *- data that someone else collected, like "Monthly Precipitation in Calcutta and Prince Rupert*



Trends *- general direction of data in graph*



Make conclusions about data and graphs by:

- *reading beyond the data – looking for additional details through research and including the contexts to "read between the lines"*

Solution 3

Calcutta precipitation increases from Jan to June (opposite in Prince Rupert), stays about the same from June to Sept (same as Prince Rupert), then decreases from Sept to Dec (opposite in Prince Rupert).

Solution 4

I looked up climate and seasons for both cities. The summer weather during June to August, it is different in Prince Rupert than Calcutta. Prince Rupert has little rainfall, while Calcutta is hot, humid and has the most precipitation in monsoon season.

Compare Trends Relationally

Connecting to Other Data and Info

AFTER (Highlights/Summary) – 5 minutes

- Teacher and students revisit the student solutions for key ideas, strategies, and models of representation, related to the lesson learning goal (summary of co-constructed success criteria)
- Teacher lists key ideas, strategies, and models of representation separately, so the students can see how the mathematical details from their solutions relate explicitly to the lesson learning goal.

Highlights/Summary (Summarized Success Criteria):

Read, Interpret and Draw Conclusions by:

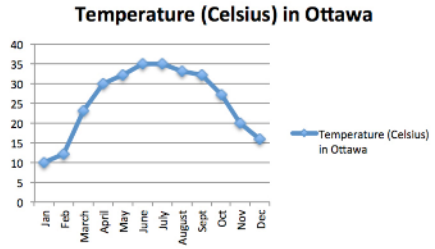
- *reading the data – identify data values and whether data increases or decreases in value (see Solution 1 and 2)*
- *reading between the data – seeing relationships within 1 data set and/or between 2 sets of data (see Solution 3)*
- *reading beyond the data – using additional details, such as context for the data collection, additional data through research to "read between the lines" (see Solution 4)*

AFTER (Practice) – 5 to 10 minutes

- Teacher chooses 2 or 3 problems, similar to the lesson problem for student to solve in pairs as a scaffold and individually.
- Problems could vary by number (choice, size), problem contexts, or what is unknown or needs to be solved
- Students are asked to solve these problems using a strategy different from the one they used for the lesson problem.

Tourism in Ottawa Problem

Travel and Tourism Ottawa advertises that any time of the year is a great time to visit Ottawa. How does this graph help to support their claim?



Solution 1

The seasons have a range of temperature so that different activities can be experienced throughout the year, like skating, swimming. But you can visit the Parliament buildings anytime during the year.

Solution 2

You can choose which time of the year you prefer according to the activities available in Ottawa. In the winter months, I found out skating on the Rideau Canal is popular in Dec to Feb, but boating is popular May to August.

Note: A classroom board is longer proportionally than these 2 pages. Due to the space constraints on these pages, the mathematical annotations are recorded above the solutions with arrows, rather than on and around the solutions.

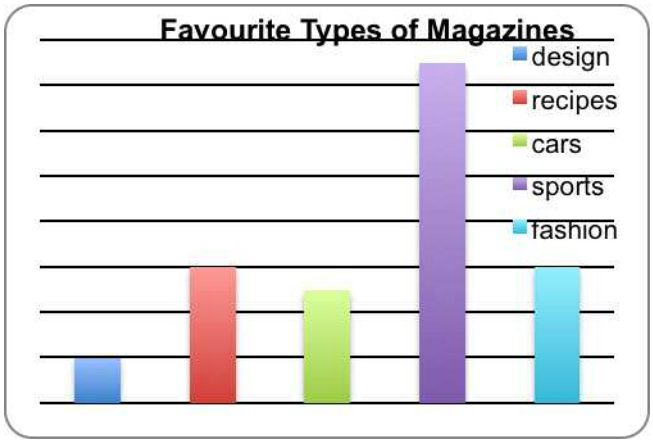
LETS DO MATH – MAKING INFERENCES FROM GRAPHS

Before (Getting Started)

Favourite Magazines Problem:

Here is some data collected.

What can you infer from it? Explain.



Solution 1

- Comparing responses for sports magazines - most people like sports magazines; 15/34 or almost half the people voted for sports; 2X votes for sports than recipes; design was less than half sports votes
- I am inferring that the data collected was a biased sample because there are more votes for sports and that isn't true because there is such a variety of magazines being sold

Solution 2

- Looking at the gender of readers: cars and sports magazines more often read by males; design, recipes and fashion magazines more often read by females
- Cars and sports -> 20; design, fashion, recipes -> 14
- I am inferring that more males were sampled than females, possibly a 20:14 male to female ratio based on the choice of favourite magazines

During (Working On It)

Magazine Sales Problem:

Colin's school is collecting data about magazine sales at his school. The stem-and-leaf plot shows the number of sales after 4 weeks. What can you infer from the graph?

Magazine Sales

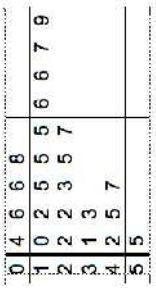
0	4 6 6 8
1	0 2 5 5 5 6 6 7 9
2	2 2 3 5 7
3	1 3
4	2 5 7
5	5

After (Consolidation)

Solution 1

- 24 students sold magazines (add number of data values)
- mode -> 15 magazines; median -> (17+19) /2 = 18 magazines sold (11 less than 17 and 11 more than 19).
- most sales were in the 10-20 range.

Solution 2



- stem-and-leaf plot is like a horizontal bar graph just make a quarter CCW turn.
- reading data - most sales were between 10 and 20; 3 sales in the 40s and one in the 50s
- sales are in the 40s and 50s were because people bought many different kinds of magazines, while those who had sales in 10s and 20s bought one magazine only.

Coordinating Discussion for Student Learning:

Why might solution 1 be chosen first for student discussion followed by solution 2?

- Solution 1 - shows the median and the mode identified from the stem-and-leaf plot and shows the majority of sales
- Solution 2 - identifies the similarity between a stem-and-leaf and a bar graph and reads the data from the graph; the graph should have a label on its horizontal axis that reads ones digit for the Number of Sales on the vertical axis the tens digit for the Number of Sales

LET'S DO MATH – DECIPHERING INFORMATION

Before (Getting Started)

TV Viewing Problem:

The data in this table suggests the average TV watching per week is 5:11 h. How did they get such an answer and where does the 9 years come from?

Total Use of Television	Data
Average time spent watching television in 1 week	5:11 h
Children birth to 7 years	3:14
Children 8-14	4:35
Adolescents 15-24	7:12
Young people 24+	5:02
Years the average person spent watching TV	9 yrs

Solution 1

If I read the numbers as decimals I get

Average = (5:02+7:12+4:35+3:14) / 4 = 19.63 / 4
= 4.9 h. It doesn't match 5:11 h

Then I tried adding hours and minutes separately. 19 h and 63 min is the same as 20.05 h. (3/60 = 0.05) 20.05 / 4 – 5.01 h which does not match 5:11 h. It is a mystery.

Solution 2

The range is about 4 h (7:12 – 3:14) so 15-24 year olds watch about 4 h more TV in a week than children under 7.

Adolescents watch about 2 more hours of TV than those over 24.

Solution 3

I'm trying to figure out where 9 years of total TV time comes from. 9 x 365 x 24 = 78 840 h.

If I watch 5:11 h per week that means 78 840 / 5.11 = 15 428.57 wks.
So that means 15 428.57 / 52 = 296.7 I think it's saying I live 296 years or so – I don't get it.

During (Working On It)

TV Watching Problem:

Some parents are worried about the amount of time their children are watching TV each week. The principal asked the grade 5s to analyse the data they collected and represent it, so parents could have an informed discussion about the issue.

3.5	23	4.5	21	10	18.5	11	12.5	5	5
0	24	16	15	6.5	7	8	13.5	15	12
14.5	17	6	18	8	21	10	11	21.5	11

After (Consolidation)

Anticipating Student Responses:

Solution 1

I will determine the mean as it represents the average of all the hours.

Sum of the data 369/30 = 12.3 h.

The average watching time is 12.3 h per week per student.

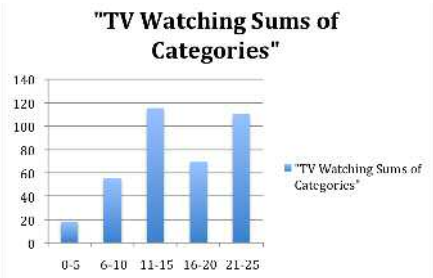
But that doesn't represent the lows and the highs.

Solution 2

I totaled the number of hours in each category.

I do not think this helps the discussion.

Number of Hours TV Viewing	Total Number of Hours
0-5	18
6-10	55.5
11-15	115.5
16-20	69.5
21-25	110.5



Coordinating Discussion for Student Learning:

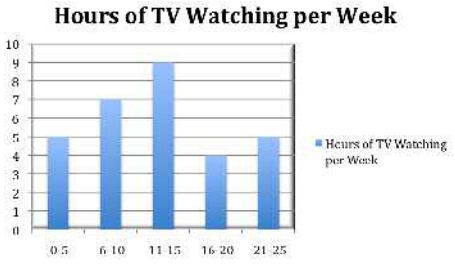
Why might solution 1 be chosen first for student discussion followed by solutions 2 and 3?

- ***Solution 1*** - shows the average of the 30 responses. This does not show the spread nor the frequency of students in each category
- ***Solution 2*** - shows the sum of the numbers in each category; this representation of data is not useful because showing the distribution of hours in each category does not provide a relation to the number of students across the categories; for example if most of the students were in the category of 0 to 5, then it is less problematic than if more students were in category 21 to 25
- ***Solution 3*** - offers parents an image of the responses represented in an organized way.

Solution 3

This shows the spread of the data and the parents could use this to discuss appropriate times.

Number of Hours TV Viewing	Total Number of Hours
0-5	5
6-10	7
11-15	9
16-20	4
21-25	5



NEXT STEPS FOR YOUR PROFESSIONAL LEARNING

Application to Your Classroom

- Try these data problems yourself first and then offer them to your students.
- To suit your students' learning needs, vary the ways that you have students engage with reading and comparing graphs of different forms, like bar graphs, circle graphs, Venn Diagrams, and broken line graphs. Data may differentiate by its form (discrete, continuous) and the number of data sets being compared.
- Consider the purposes for comparing graphs and its practical uses in local classroom and school community contexts. Some interesting contexts for comparing sets of data include analysis of global warming impact, in terms of snow accumulation across the years.
- Practise noticing the different strategies and the breadth of the mathematics your students use to read graphs to tell a story or compare graphs to make inferences and conclusions. Take note of the range of solutions that show strategies in making inferences, like telling a story from a graph (e.g., when the graph shows an increase it shows that the bath tub is filling up with water and when the graph shows a decrease, maybe the water is draining from the bathtub) and making inferences from graphs (e.g., relating the context and data from the graphs as evidence for an argument).
- Think ahead of the kinds of mathematical annotations you will record on and around the student solutions to make explicit key mathematical ideas, strategies for reading data as the students discuss and analyze each other's solutions.

SUGGESTED READINGS

Some readings used in the Research Summary and in the development of the problems and solutions are listed below:

Chapin, S., Koziol, A., MacPherson, J., & Rezba, C. (2002). *Navigating through data analysis and probability in grades* 3-5. Reston, VA: NCTM.

Curcio, F. (1987). Comprehension of mathematical relationships expressed in graphs. *Journal for Research in Mathematics Education, 18*, 38-393.

Graham, A. (2006). *Developing thinking in statistics.* Thousand Oaks, CA: Sage Publications.

Kitchen, R. (1999). Analyzing and making sense. *Mathematics Teacher,* 318-322.

Rubenstein, R. & Thompson, D. (2012). Reading visual representations. *Mathematics Teaching in the Middle School,* 17(9), 544-550.

Small, M. (2010). *Big ideas from Dr. Small, Grades K-3.* Toronto: Nelson Education.

Small, M. (2010). *Big ideas from Dr. Small, Grades 5-8.* Toronto: Nelson Education.

CLOSING

Our next issue will focus on key concepts of probability, such as comparing theoretical and experimental probability and using whole number and fractions to represent probably. As well, the next issue will outline the ways that teachers need to know these notions for teaching, as well as, the ways that teachers can use these mathematical ideas and pedagogical strategies within a problem solving-based teaching and learning lesson framework.

CALL FOR *ABACUS* SUBMISSIONS

Send your teaching strategy ideas, problems and student solutions to the *Abacus* Co-Editors via email