8 PAGE INSERT ELEMENTARY MATH

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PROPORTIONAL REASONING – RATE

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ABACUS CO-EDITORS



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The development of proportional reasoning is fostered by intentional learning experiences where students explore, discuss, and experiment with situations that require proportional reasoning. These situations include real-world and every-day situations, such as: determining the better buy when 10 bags costs $9.50 and 24 bags costs $22, adjusting calculations for distance travel time when the speed fluctuates, or determining if 6 cents per litre is better than a 5% discount on your fuel charges. Imagine the impact of underdeveloped proportional reasoning in real-world situations; it could have life-threatening or disastrous consequences, such as incorrect doses of medicine being administered, miscalculation of time when expecting the arrival of a tornado or tsunami given anticipated speed in relation to time needed for safe evacuation!

In this issue, the Research Summary and problems (with multiple solutions) focus on strategies for representing proportional reasoning situations within measurement contexts. The problems we provide are designed for use within a three-part, problem-solving lesson structure.

LINKS TO LITERATURE AND MANIPULATIVES



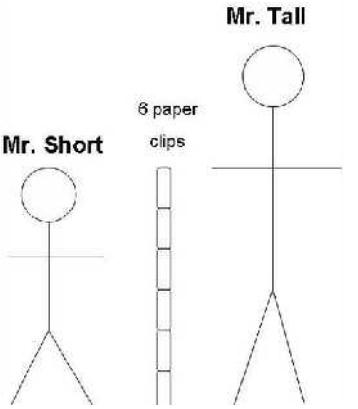
PAT MARGERM is an independent literacy and mathematics consultant with more that 30+ years of K to 8 teaching experience with students, in-service and pre-service teachers. She is a long-time OAME board member and works in various aspects of mathematics education.

It is interesting to consider how many rate problems involve time and money – two areas of math that are often identified as challenging to both teach and learn. The passage of time (duration of an event) is difficult to experience in the classroom, since it cannot be felt, seen, or heard. The picture book for this issue, *Just a Second* provides many examples of rates by comparing units of time to a variety of quantities such as distance, mass, volume, heartbeats, population changes, to name a few. This book provides interesting ways to think about the idea of speed by making comparisons in real life contexts. What visuals enable students to model relationships in rate problems? For this issue, the double number line is used as a tool for representing details in rate problems.

RESEARCH SUMMARY – PROPORTIONAL REASONING – FROM RATIO TO RATE

From Additive to Multiplicative Thinking

According to Piaget's theory of intellectual development, a child acquires proportional reasoning when progressing from the stage of concrete operations to the stage of formal operations. Karplus and Peterson (1970) who are pioneers in the research of proportional and rational number reasoning designed "The Mr. Tall/Mr. Short" problem to assess and enhance students' levels of proportional thinking. Since their study, variations in using this problem have been used by other researchers.



Mr. Short is six paper clips in height. If he is measured in large buttons, he is four large buttons in height. Mr. Tall is similar to Mr. Short, but is six large buttons in height. Predict the height of Mr. Tall in paper clips. Explain.

Reasoning	Explanation
Multiplicative Reasoning 1	He is nine paper clips tall. Each button is equal to one and a half paper clips. If he is six buttons tall you multiply six times one and a half to get nine paper clips.
Multiplicative Reasoning 2	Mr. Tall is 1 ½ times as high as Mr. Short. Since Mr. Short is 6 clips high, Mr. Tall must be 6 * 1 ½ = 9 clips high.
Multiplicative Reasoning using addition	For every two buttons there are three paper clips. Mr. Tall is 2 buttons taller than Mr. Short so he must be 3 paper clips taller. 6 + 3 = 9 paper clips.
Additive Reasoning 1	Mr. Tall is 8 paper clips high. Mr. Short is 4 large buttons high and 6 paper clips high. So the buttons are 2 less than the paper clips. Since Mr. Tall and Mr. Short are similar, and Mr. Tall is 6 buttons high, he must be 8 paper clips high.
Additive Reasoning 2	Mr. Tall is two more buttons taller than Mr. Short so he will also be two more paper clips taller than Mr. Short resulting in 8 paper clips.
Estimate	Nine, I figured he would be a bit taller.
Haphazard	Since Mr. Tall is 2 more buttons than Mr. Short, I took the 6 paper clips and multiplied by 2 to get 12 paper clips.

According to Lamon (2006), being able to make comparisons in additive and multiplicative terms is also referred to as absolute and relative thinking respectively. Being able to see these various equivalent relationships is a key to proportional reasoning.

Tree Growth Problem: In a schoolyard, a pine tree was 4 m high and the maple tree was 5 m high. Five years later, the height of both trees was measured. The pine tree is now 7 m high, while the maple tree is 8 m high. Which tree has grown the most?

- From an additive (absolute) perspective, both trees have grown taller by the same amount of 3 m. From a multiplicative (relative) perspective, the pine tree has grown 3⁄4 of his original height and the maple tree has grown 3/5 of his original height. The pine tree has grown more than the maple tree because 3/4 is greater than 3/5.
- What would this relative change look like on a number line?
- If the trees both doubled, the pine tree would be 8 m high and the maple tree would be 10 m high.
- The pine tree had a 75% increase, which means it is 1- 3/4 or 175% of its original size. While the maple tree had a 60% increase, which means it is 1- 3/5 or 160% of its original size.

Rate

- The growth of a pine tree and maple tree over 5 years is compared.
- A car travels 540 km and uses 60 L of fuel, so 540 km / 60 L = 9 km / 1 L.
- A student works 5 hours and is paid $60, so $60 / 5 h = $12 / 1 h.
- 1 dozen eggs costs $3.00, so $3.00 / 12 = $0.25 /1 egg.

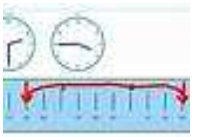
These examples show the comparison of two quantities (or 2 terms) that have different measuring units using division, which is called a rate. A rate is unlike a ratio where both quantities have same units.

- A rate can be represented as a fraction that includes 2 different units. For example, a puppy grows 2 kg per 1 week (2 kg / 1 week) or 8 kg / 4 weeks.
- A **unit rate** is a rate where the second quantity or term is a unit of 1. For example, 60 km/hour really means 60 km is travelled in 1 hour.
- If two rates are different but equivalent, they are proportional. For example, 2 km / 3 h = 6 km / 9 h. These 2 rates are proportional by a factor of 3.

$$\frac{2\text{ km}}{3\text{ h}} \overset{\times 3}{=} \frac{6\text{ km}}{9\text{ h}}$$

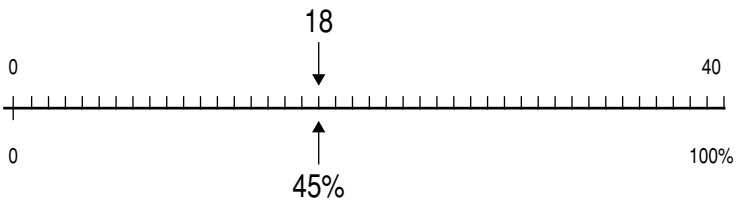
LINKS TO MANIPULATIVES:

The number line is a powerful tool for visualizing number relationships. The scale on a number line can be adapted for different mathematical contexts, such as fractions, money and elapsed time.

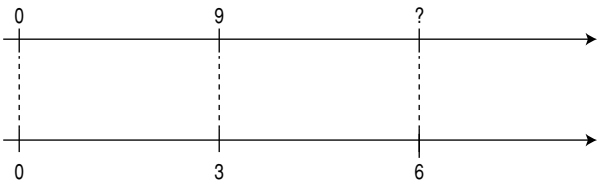


A double number line models ratio comparisons like percent.

What is 18 out of 40 as a percent? In this representation students see the “stretching” of 40 to 100, so that 18/40 is equivalent to 45%.



The double number line also models rates. For example, Jim can walk 9 km in 3 hours. If he maintained the same speed, how far can he walk in 6 hours? In this representation students can visualize the relationship between the two units in the rate.



LESSON LEARNING GOALS

Identify the curriculum expectations (MOE, 2005) grade-before and grade-after the grade specific learning goal, in order to:
- see the development of mathematical knowledge, skills, and strategies across grade levels
- determine the mathematical focus of the problems chosen for Before (activation), During (solving problem), and After (Practice) parts of the lesson

BEFORE (Getting Started) – 5 to 10 minutes

- Activating students' mathematical knowledge and experience using a prompt or problem, related to the math in the lesson problem.
- Includes student responses to highlight ideas and strategies.

Hats Problem:

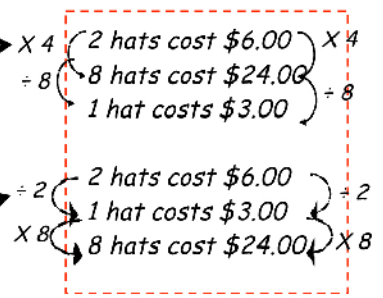
If 2 hats cost $6.00, then how much do 8 hats cost?

Solution 1

2 hats x 4 = 8 hats
$6 x 4 = $24

Solution 2

If 2 hats cost $6, then
half of 2 hats is 1 hat, so
half of $6 is $3
1 hat costs $3
8 hats cost $3 x 8 = $24



"half of" is the same as "÷ 2"

DURING (Working on It) – 15 to 20 minutes

- Understanding the problem – Teacher asks, "What information from the problem are we using to make a plan to solve it? Explain." Teacher records in a list below the problem, the information that students identify.
- Students solve the problem on chart paper (landscape) with markers (visible for class discussion) in pairs or small groups.
- Teacher circulates to record different student solutions, in addition to the ones the teacher anticipated.

Iced Tea Problem 1:

Two stores have a sale on ice tea:
- Price Chopper 12 cans for $15.00
- Food Basics 20 cans for $23.00

Which is the better buy?

What information are we going to use to solve the problem?
- *PC – 12 cans cost $15.00*
- *FB – 20 cans cost $23.00*
- *Cans have same amount of ice tea (same capacity)*
- *Which is cheaper?*

AFTER (Consolidation) – 20 to 25 minutes

- Teacher selects 2 or more solutions for class analysis and discussion in a sequence (1st, 2nd, 3rd, etc.) based on mathematical relationships between the solutions and the lesson learning goal.
- Students (authors) explain and discuss their solutions with their classmates; co-construction of success criteria towards the lesson learning goal
- During whole class discussion, solutions are organized (often re-organized) to show mathematical elaboration from one solution to the next and towards lesson learning goal.
- Teacher mathematically annotates (math terms, math symbols, labelled diagrams, concise explanations) on and around solutions to make mathematical ideas, strategies, and models of representation explicit.

***Rate** is a special kind of ratio; a multiplicative comparison of two or more quantities of different units (e.g., price of 12 cans, number of cans)*

- *ratio as fraction*
- *doesn't matter which quantity is numerator or denominator*

What is the cost of 1 can if divide or fair share:
- *$15.00 equally among 12 cans?*
- *$23.00 equally among 12 cans?*

Solution 1

PC FB
12/15 < 20/23
12 ÷ 15 = 0.80 < 0.87 = 20 ÷ 23
FB get more of a can per $1

15/12 > 23/20
1 - 3/12 > 1 - 3/20
3/12 > 3/20
At FB 1 can cost less

Solution 2

Price Chopper ->
$15.00 ÷ 12 cans
= $1.25 per 1 can

Food Basics ->
$23.00 ÷ 20cans
= $1.15 per 1 can

Food Basics price is cheaper per can $1.25 > $1.15 per 1 can

Fraction Comparison
Ratio of 2 Quantities

Division - Unit Rate
$ Cost for 1 Can

Note: Mathematical annotations include mathematical vocabulary, symbols, elaborations of mathematical details from solutions, labels describing the method/strategy, and questions to further thinking. All annotations are records of students' mathematical discussion.

Grade 5	Grade 6	Grade 7
- demonstrate an understanding of simple multiplicative relationships involving whole-number rates	- represent ratios found in real-life contexts using concrete materials, drawings and standard fractional notation	- demonstrate an understanding of rate as a comparison or ratio of two measurements with different units; solve problems involving the calculation of unit rates

Unit Rate *- describes how many units of the first type of quantity corresponds to one unit of the second type of quantity (order implied)*

quantity 1 (how many units)	*quantity 2 (one unit)*
$ cost	*of 1 can*
how much of can	*for $1.00*

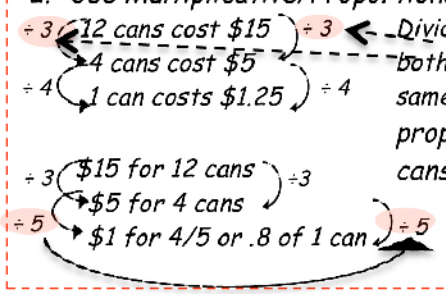
Different Ways to Calculate Unit Rate

1. Divide (or fair share) 1st quantity among 2nd quantity = number of units of 1st quantity for 1 unit of 2nd quantity

$15.00 ÷ 12 cans = $1.25 per 1 can (1 unit)

12 cans ÷ $15.00 = .80 of 1 can for $1.00 (1 unit)

2. Use Multiplicative/Proportional Relationships



Divide (or multiply) both quantities by the same amount keeps proportional ratio of cans to cost

Solution 3

Price Chopper
12 cans cost $15
4 cans cost $15÷3 = $5.00
1 can costs $5÷4 = $1.25

Food Basics
20 cans cost $23
4 cans cost $23÷5 = $4.60
1 can costs $4.60÷4 = $1.15

Food Basic is cheaper

Solution 4

Price Chopper ->
12 cans ÷ $15.00
= 0.80 of 1 can for $1

Food Basics ->
20 cans ÷ $23.00
= 0.87 of 1 can for $1

Food Basics price is better - get more of 1 can for $1

Division - Proportionality
Unit Rate - $ Cost for 1 Can

Division - Unit Rate
Amount of Can for $1

AFTER (Highlights/Summary) – 5 minutes

- Teacher and students revisit the student solutions for key ideas, strategies, and models of representation that are related to the lesson learning goal (anticipated success criteria)
- Teacher lists key ideas, strategies, and models of representation so the students can see how the mathematical details from their solutions relate explicitly to the lesson learning goal; summary of co-constructed

Highlights/Summary:

*A **rate** is a multiplicative comparison or ratio between two quantities with different units (e.g., money, amount of iced tea), in a particular order (e.g., 20 cans for $23.00 is not the same as 23 cans for $20.00)*

Compare rates using multiplicative relationships by:
*a) **fraction comparison** (15/12 › 23/20 or 12/15 ‹ 20/23)*
*b) **unit rate** (number of units to one unit*
ice tea - total price divided by number of cans
(e.g., $15.00 ÷12 cans = $1.25 per can; 23.00 ÷20cans = $1.15 per can)
cost ($1.00) - total cans divided by total cost
(e.g., 12 cans ÷ $15.00= 0.80 of 1 can for $1; 20 cans ÷ $23.00 = 0.87 of 1 can for $1)

AFTER (Practice) – 5 to 10 minutes

- Teacher chooses 2 or 3 problems, similar to the lesson problem for student to solve in pairs as a scaffold and individually.
- Problems could vary by number (choice, size), problem contexts, or what is unknown or needs to be solved
- Students are asked to solve these problems using a strategy different from the one they used for the lesson problem.

Ice Tea Problem 2:

Which is the better buy for ice tea cans?
- T & T - 24 cans for $30.00
- No Frills - 40 cans for $46.00

Compare using fraction comparison or unit rate.

Solution 1

T&T NF
30/24 > 46/40
15/12 > 23/20

No Frills cost less (proportional to the price of Food Basics – doubled the price and # of cans)

Solution 2

24 cans for $30 is 0.80 of 1 can for $1 (24÷30= 0.8)

40 cans for $46 is 0.87 of 1 can for $1 (40÷46 = 0.87)

Get more of 1 can for $1 at No Frills

Note: A classroom board is longer proportionally than these 2 pages. Due to the space constraints on these pages, the mathematical annotations are recorded above the solutions with arrows, rather than on and around the solutions.

LET'S DO MATH – DETERMINING RATES IN DIFFERENT WAYS

Before (Getting Started)

Rate of Growth Problem 1:

Examine each picture and each rate of growth. Match a picture to a rate of growth. Explain the rate of growth represented.

			
0.5 cm / 1 month	10 m / 1 pedal cycle	4000 drips / 1 L	3 mm / 1 week

Solution 1:

The plant might grow at 3 mm / 1 week. The plant would grow (3, 6, 9. 12) 12 cm in a month because there are about 4 weeks in a month.

If the boy is growing 0.5 cm / month then he will grow 6 cm in a year. 12 x 0.5 = 6)

Solution 2:

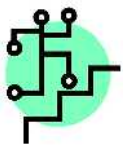
Drips from a tap might add up to 4000 drips / 1 L. If a tap drips 3 drops / min then it would be 180 drips / h (3 drops x 60 min or 1 h) or 4320 drips / day (180 x 24 h or 1 day) which means more than 1 L of water a day is lost.

A girl going 10 m / 1 pedal cycle would have to pedal 100 times to go 1000 m or 1 km. (10 m / 1 cycle = 100 m / 10 cycles = 1000 m / 100 cycles)

During (Working On It)

Climbing Rates Problem 2:

There are 1776 stairs in the CN Tower. For last year's United Way Climb, the median rate of participants; climb was 1 h 20 min to make the climb. What was the climbing rate?





After (Consolidation)

Anticipating Student Responses:

Solution 1:

1776 stairs takes
1 h 20 min or 80 min
-> 177.6 stairs in 8 min
-> 22.2 stairs in 1 min.

This person climbed about 22 stairs /min.

Solution 2:

Stairs	1776	888	444	1332	222	22.2
Time (min)	80	40	20	60 min or 1 h	10	1

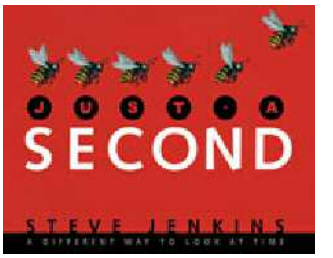
This person climbed at a rate of 1332 stairs / 1 h.

Coordinating Discussion for Student Learning:

Why might solution 2 be chosen first for student discussion followed by solution1?

- *Solution 2* – shows a ratio table being used to multiplicatively compare the number of stairs and the time taken to climb them; showing a rate for 1 h and a rate for 1 min. This is like a double number line.
- *Solution 1* – shows a rate calculation using a unit rate or a rate for 1 min; comparing number of stairs to time, reducing the time to one unit or 1 minute.

LINKS TO LITERATURE:



"A second goes by pretty quickly. In fact, several seconds have passed since you started reading this sentence. And they just keep going by. There are 60 seconds in every minute, 3 600 second in every hour, and on and on. But a lot can happen in a second. And that's what this book is about," writes author, Steven Jenkins. "Just a Second: A Different Way to Look at Time," is a picture book that has a range of facts that can be used as contexts for developing rate problems

> *In one second . . . a black mamba slithers a frightening 7 m. How far can the snake slither in 4 s?*

Students might solve this problem by counting, using repeated addition (7+ 7+ 7+ 7), doubling (7+ 7 = 14; 14 + 14 = 28) or multiplying (4 x 7 = 28).

> *In one minute . . . A giant tortoise lumbers about 4 ½ m. This fact from the book can be adapted to pose a problem in which students identify the unit rate. (e.g., a giant tortoise can lumber about 18 m in 4 min. How far can it travel in one minute?*

Students might solve this problem using strategies such as halving (9 m in 2 min, 4 ½ m in 1 min), using a known fact to estimate (4 x 4 = 16, 16 is close to 18 so 2/4 left over so 4 2/4).

LETS DO MATH –



Before (Getting Started)

Lemonade Problem:

One can of lemonade concentrate requires 4 cans of water to make juice. For a community picnic, 100 cans of juice are required. How many cans of lemonade concentrate are needed?



Solution 1:

1 can lemonade + 4 cans water
= 5 cans juice

$$\frac{\text{1 can concentrated}}{\text{5 cans juice}} = \frac{\text{? cans}}{\text{100 cans juice}}$$

$$\frac{\text{1 can concentrated}}{\text{5 cans juice}} = \frac{\text{20 cans}}{\text{100 cans juice}}$$

Solution 2:

Need to compare combined parts to whole

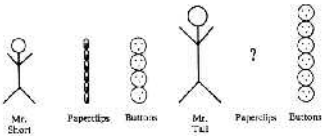
Concentrate	1	2	4	20
Water	4	8	16	80
Juice	5	10	20	100

Need 20 cans of concentrate for 100 cans of juice.

During (Working On It)

Comparing Heights Problem:

How many paperclips tall is Mr. Tall?



After (Consolidation)

Solution 1:

6 buttons for 4 buttons
so 6 : 4 = 3 : 2
So Mr. Tall is 3/2 the height of Mr. Short.

Mr. Tall is 3/2 the height of Mr. Short whose height measures 4 buttons or 6 paperclips.

3/2 x 4 buttons = 1.5 groups of 4 buttons
= 6 buttons
3/2 x 6 paperclips
= 1.5 groups of 6 paperclips

Solution 2:

Mr. Short → 6 paperclips to 4 buttons to Mr. Short. This is the same as 3 : 2
Mr. Tall → paperclips : buttons = ? : 6
3 groups of half of Mr. Short will match Mr. Tall.
We need 3 groups of half of 6, which is 9.



Mr. Short is 6 paper clips tall. Three groups of 3 is 9. Mr. Tall is 9 paperclips tall.

Coordinating Discussion for Student Learning:

Why might solution 1 be chosen first for student discussion followed by solutions 2 and 3?

- *Solution 1* – These are part-to-part relationships. Mr. Tall is 3/2 the height of Mr. Small. 3/2 x 4 = 6 buttons and 3/2 x 6 = 9 paperclips. The ratio of buttons (Mr Small) and buttons (Mr Tall) must be compared or the rates (a kind of ratio) of buttons to paperclips (Mr Small) must equal buttons to paperclips (Mr Tall).
- *Solution 2* – The parts in this question are broken up and students count 3 paperclips as 1/2 of Mr. Small. Because there are 3/2 as many buttons, we need 3/2 paperclips – that is 3 groups of 3 or 9.

NEXT STEPS FOR YOUR PROFESSIONAL LEARNING

Application to Your Classroom

- Try the proportional reasoning problems in this issue yourself first so that you can anticipate possible mathematical responses and then offer them to your students. Compare your anticipated responses to the students' responses. How are the solutions similar? How are they different?
- To suit your students' learning needs, vary the ways that you have students engage with noticing and using multiplicative (proportional) relationships inherent in multiplication and division operations, in representing and comparing fractions and rates. Use different models of representation, such as sets of objects and concrete materials and visuals, such as counters, base ten blocks, square tiles and fraction bars to model proportionalities.
- Practise noticing the different strategies and the breadth of the mathematics your students use in order to describe, show and make sense of multiplicative thinking (proportional reasoning) in relation to solving rate problems. Take note of the kinds of proportional reasoning language students' use in the junior years (e.g., number of units of one quantity in relation to a unit of 1 for the second quantity; a fraction can represent a rate (a ratio between two quantities of different units)).
- Think ahead of the kinds of mathematical annotations you will record on and around the student solutions to make explicit key mathematical ideas, strategies for representing and determining rate (e.g., fractional comparison, unit 1 (quantity 1 compared to quantity 2; quantity 2 compared to quantity 1) as the students discuss and analyze each other's solutions to co-construct success criteria for the lesson learning goal.

SUGGESTED READINGS

Some readings used in the Research Summary and in the development of the problems and solutions are listed below:

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CLOSING

Our next issue will focus on proportional reasoning within the measurement, geometry and spatial sense strand. Also, we will outline the ways that teachers need to know these notions for teaching, as well as, the ways that teachers can use these mathematical ideas and pedagogical strategies within a problem solving-based teaching and learning lesson framework.

CALL FOR *ABACUS* SUBMISSIONS

Send your teaching strategy ideas, problems and student solutions to the *Abacus* Co-Editors via email