8 PAGE INSERT ELEMENTARY MATH

ONTARIO ASSOCIATION FOR MATHEMATICS EDUCATION

Abacus

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WHOLE AND DECIMAL NUMBER RELATIONSHIPS FOR SUBTRACTION

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ABACUS CO-EDITORS



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This issue includes practice tasks for decomposing and comparing numbers to see part-whole number relationships and equivalent number expressions within the context of the operation, subtraction. Number representations include set (bar model), linear (number line), area (rectangle), graphical, numeric and/or algebraic models. Solve this puzzle using two different methods.

Thinking Mathematically and Pedagogically

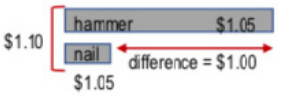
- What mathematics are you using to solve the Puzzle?
- What could Puzzle Solution 2 and Solution 3 be? How are they similar and different to Puzzle Solution 1? Which puzzle solution should be discussed 1st, 2nd and 3rd?

"Hammer and Nail" Puzzle

A hammer and a nail cost $1.10, and the hammer costs $1.00 more than the nail.

- How much does the nail cost?
- Solve this puzzle using two different models and methods.

Solution 1: *Comparison of 2 Wholes Bar model*



Solution 2: *Systematic Comparisons Chart*

hammer	$1.00	$1.10	$1.05
nail	$0.10	$0.10	$0.05
difference $1	$0.90	$1.00	$1.00
sum $1.10	$1.10	$1.20	$1.10

Solution 3: *Algebraic Equation*

If x = cost of the nail and $x + \$1$ = cost of hammer, then

$x + x + \$1.00 = \$1.10$

$2x + \$1.00 = \$1.10$

$2x = \$1.10 - \$1.00 = \$0.10$

$x = 1/2 \times \$0.10$;

$x = \$0.05$

To solve this problem, think about modelling this subtraction situation, as a comparison of two wholes (hammer cost compared to nail cost), as in solutions 1 and 2. If you think of subtraction as the separation of one whole (hammer cost + nail cost) into 2 parts (hammer cost, nail cost), what would the bar model look like?

This December 2019 issue includes a variety of practice tasks, which can be integrated into your classroom mathematics program as: (a) five minute quick practice; (b) fifteen minute mini lesson; (c) lesson problem for practice lessons; (d) at home practice; and/or (e) family math activities. These practice tasks require students to decompose and compare whole and decimal numbers in relation to the number operation, subtraction.

DESIGNING PRACTICE TASKS – Criteria for Task Design

Mathematics Content Focus

What do students need to know and do specifically to be mathematically proficient with whole and decimal number subtraction? Consider these mathematics details as a focus for designing subtraction practice tasks.

Conceptual Understanding

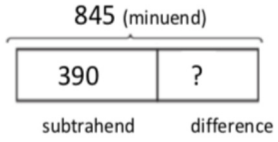
- recognizes subtraction is either: (a) separation of a whole (minuend) into 2 or more parts, where 1 part is the subtrahend and the other part is the difference and (b) comparison of 2 wholes to determine the difference (result of subtraction) using models, such as the bar model, number line, area of rectangle, number equation, algebraic equation
- explains the relationship with decomposition and composition of whole and decimal numbers to the subtraction of whole and decimal numbers in relation to different subtraction methods

Procedural Fluency

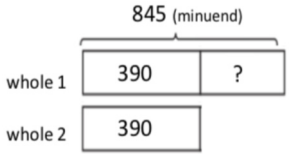
- subtracts 2 or more whole and decimal number subtrahends (1 to 7 digits) using different methods: (a) subtract quantities of same number units; (b) decompose the minuend into 2 parts; (c) decompose the subtrahend into tens; (d) constant difference (add or subtract same number to minuend and subtrahend to make the subtrahend tens and hundreds or parts of the minuend to be equivalent to the subtrahend)

845 – 390 = ___

a) separation of 1 whole into 2 or more parts



b) comparison of 2 wholes to determine what is the same and what is the difference



Strategic Competence, Adaptive Reasoning and Productive Disposition

- selects subtraction methods that are most suitable for the number and size of the minuend and subtrahend
- analyses and explains the selection of the most appropriate subtraction method and model of representation for the given problem context
- perseveres to solve the puzzle using different subtraction methods

Variations of a Math Puzzle

Different variations of a math puzzle are based on varying specific aspects of the math puzzle. First change one aspect at a time, then two aspects:

- *contexts* only (e.g., hammer and nail to pencil and eraser)
- *contexts and its related model of representation* (e.g., set model -hammer and nail are objects represented in a bar model)
- *unknowns* (e.g., cost of 1 hammer; cost of1 eraser; total cost of hammer and nail; cost difference between hammer and nail)
- *knowns* (e.g., different number of hammers and nails; different cost of hammers and nails; different difference between cost of hammers and nails)

"Hammer and Nail Puzzle" Variations:

a) A hammer and nail cost $1.10. The hammer costs $1.00 more than the nail. How much did the hammer cost?

b) A pencil and eraser cost $1.10. The pen costs $1.00 more than the eraser. How much did the eraser cost?

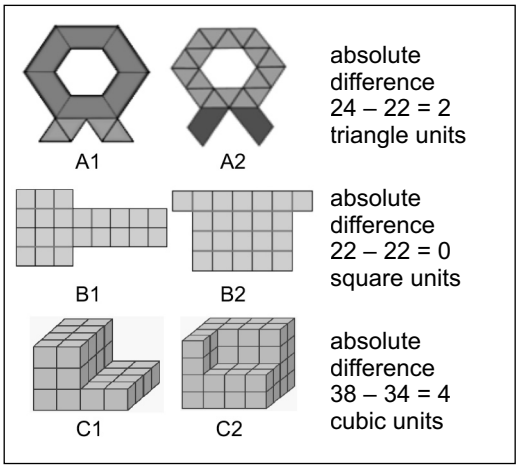
c) A hammer and nail cost $1.20. The hammer costs $1.10 more than the nail. How much did the nail cost?

d) Two hammers and 10 kg of nails cost $15.50. One hammer and 2 kg of nails cost $7.35. How much did 1 nail cost? How much did 1 hammer cost?

e) Two hammers and 10 kg of nails cost $15.50. 1 hammer and 3 kg of nails cost $7.35. How much did 1 nail cost? How much did 1 hammer cost?

QUICK IMAGES – DOTS, SQUARES, PATTERN BLOCKS AND CUBES

Using quick image routines provides predictability to the students to practise seeing and explaining relationships between spatial visualization, mental math calculations, mathematical reasoning and mathematical communication.



Quick Image Routine -Comparing 2 Composite 2D Shapes and 2 Composite 3D Figures

a) Compare two geometric images using same geometric units. (e.g., triangle units, square units, cube units)

b) What are the absolute and relative differences?

- Show two quick images for 3 seconds.
- Students build with materials (e.g., pattern blocks, square tiles, cubes) or sketch the "difference" between the 2 images, using pattern block paper, square grid paper and/or isometric drawing paper.
- Show two quick images for 3 seconds again.
- Students make revisions to their structure or drawing.
- Show two quick images for 3 seconds for the last time.
- *Class Discussion* - Describe the comparison methods used and identify the same shape unit.

Viewing the images for a brief period of time (3 seconds) motivates students to create and use efficient methods to compare and determine absolute and multiplicative differences.

Quick Image Routine – Comparing Two Dot Images

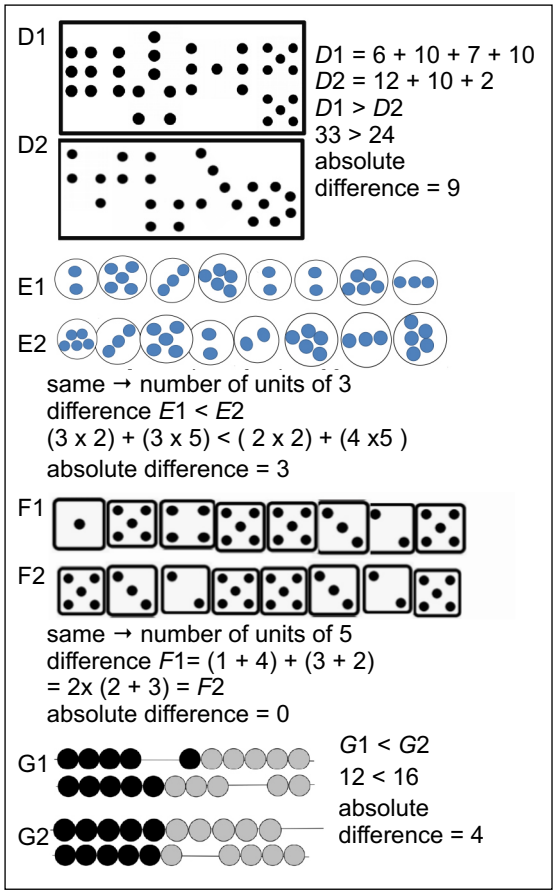
a) Compare the two dot images using same number unit (e.g., tens, ones, tens, hundredths).

b) What are the absolute (exact difference) and relative differences (multiplicative difference)?

Number quick images could be composed of dot units (same and/or different size of dots.

- Show two dot images for 3 seconds.
- Tell students to record the total number of dot units.
- Show two dot images for 3 seconds again.
- Tell students to record the unit sizes used. (e.g., " I used units of 1, units of 5 and units of 10.")
- Show two dot images for 3 seconds for the last time.
- Tell students to record the subtraction equation and the calculations used (choice of subtraction method).
- *Class Discussion* - Describe and compare the subtraction equations and subtraction methods used.

In all of the examples provided, absolute difference rather than relative difference better represented the differences between the number of 2D shapes, 3D figures and dots. Why?



"QUICK IMAGES, MATH PUZZLES AND GAMES MAT" - WHOLE NUMBER SUBTRACTION

QUICK IMAGE Dots

Difference in # Dots for A and B
Show as a subtraction equation.

A1 B1

A2 B2

A3 B3

QUICK IMAGE Squares

Difference in # Squares for C and D
Show as a subtraction equation.

C1 D1

C2 D2

C3 D2

QUICK IMAGE Cubes

Difference in # Cubes for E and F
Show as a subtraction equation.

E1 F1

E2 F2

E3 F3

Who Got There? PUZZLE

31 students walk to the skating rink. 7 stopped to throw snow balls. 11 students go back to school and 13 of them reached the skating rink.

What happened to the rest of the students?

How Many ? PUZZLE

A line of 20 people waited to sample mini muffins. The first person took one muffin. Each person thereafter took more muffins than the person before, until no muffin was left. Did the 20 people each get a muffin?

Stacked Dice PUZZLE

What is the least difference between numbers shown on 4 staked dice?

6-2=4; 6-4=2
4-2 = 2; least difference is 2



How Many ? PUZZLE

There are 4 500 apples, and the boy scouts take away 300 apples.

How many apples do the boy scouts have?

'Race to Zero MATH GAME

Materials: 1 set of playing cards (face worth 10; aces are worth 1)



Game Goal:

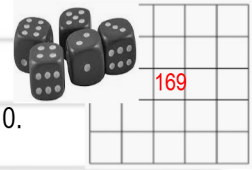
- First player to reach score of 0 points

How to Play:

- Each player begins with 100 points.
- Players face up 1 card (1-digit) or 2 cards (2-digit) at the same time. Player with the larger number identifies the difference between the 2 numbers and claims it as points to subtract from the starting points, 100.
- If players show the same number card, those cards are placed in a common pile. When a player has no cards left, the cards in the common pile are shuffled and divided equally among all the players.
- Play as many rounds as needed to get to 0 points.

5 Squares in a Row MATH GAME

Materials: 5x5 blank square grid, 2 colour markers, 5 dice



Game Goal:

- To accumulate points closest to 0.

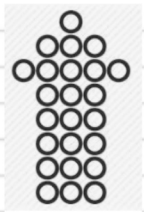
How to Play:

- Players take turns rolling 5 dice, to make a 1-digit, 2-digit and/or 3-digit whole numbers.
- Players record the dice roll (2, 3, 4, 5, 6) as subtraction equation with 2 or 3 addends. (e.g., 234 - 65 = 169) and subtract it from the starting points, 1 000. (e.g., 1 000 - 169 = 831)
- Record the difference on any blank square on the square grid, using different colour marker.
- Players who complete 5 squares in a row (horizontal, diagonal, vertical) get an additional turn.

"QUICK IMAGES, MATH PUZZLES AND GAMES MAT" - DECIMAL NUMBER SUBTRACTION

QUICK IMAGE Dots

A1



A2



A3



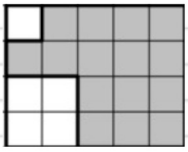
What is the difference between the number of white dots for each arrow, A1, A2 and A3?

a) If 10 dots = 1 ones, then use the unit, tenths.

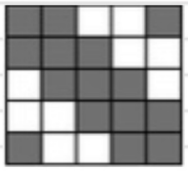
b) If 100 dots = 1 ones, then use the unit, tenths and hundredths.

QUICK IMAGE Squares

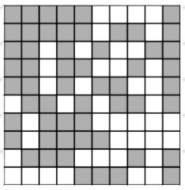
B1



B2



B3



What is the difference between the number of white squares and grey squares for B1, B2 and B3?

a) If 10 squares = 1 ones, then use the unit, tenths.

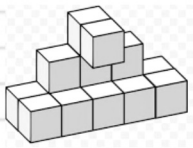
b) If 100 squares = 1 ones, then use the units, tenths and hundredths.

QUICK IMAGE Cubes

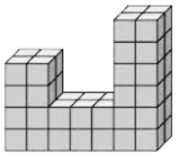
C1



C2



C3



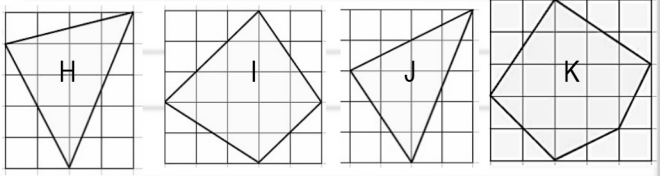
What is the difference between the number of cubes for structures, C1, C2 and C3?

a) If 10 cubes = 1 ones, then use the unit, tenths.

b) If 100 cubes = 1 ones, then use the units, tenths and hundredths.

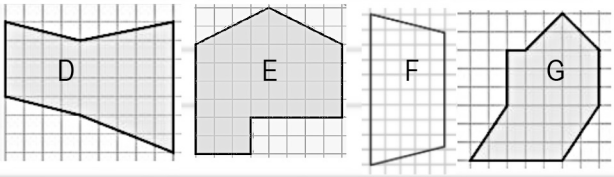
The GREATEST Difference TEASER

Which 2 polygons have the greatest difference?
One square unit represents 1 hundredth of 1 square unit.



The LEAST Difference TEASER

Which 2 polygons have the least difference?
One square unit represents 1 tenth of 1 square unit.



Connect 2 MATH GAME

Materials: 6x6 circle array, plastic page protector, colour dry erase markers

12.3	90.82	1230	1.23	0.123	0.98
36.82	8.37	3.68	86.32	26.83	6.52
50.09	950	59.4	42.90	500.9	9.42
1.6	63.01	1.64	3.00	16.0	1.06
23.19	123.9	43.17	3.91	63.15	0.75
163.1	1.36	274	72.04	309.5	7.53

Game Goal:

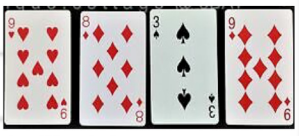
- To accumulate lowest (or highest) points.

How to Play:

- Join 2 numbers, vertically and horizontally to calculate the difference.
- Each number may be used once, and connecting lines may not intersect.
- Keep track of the points by recording the difference and subtraction equation for each set of 2 numbers connected.
- Game ends when 2 numbers cannot be joined.

Race to 40 MATH GAME

Materials: 2 sets of playing cards (no jacks, queens, kings, aces are 1); paper, pencil



Game Goal:

- To accumulate points that sum to 40 points.

How to Play:

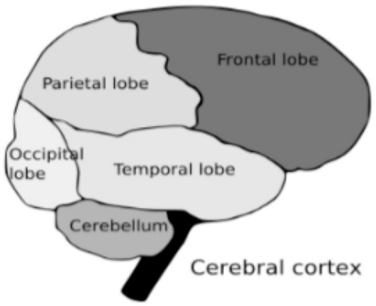
- Black cards are tenths units (1 pt); red cards are hundredth units (2 pts).
- Place cards face down in an array.
- Players take turns flipping over 4 cards at a time and calculate the difference. Set aside cards revealed.
- Record points as a difference of the subtraction equation.
- Game ends when there are no more cards to flip over.

Points Example:

3 tenths- (9+8+9) hundredths= (30 -26) x 0.01
= 4 x 0.01 → 4 x 2 points = 8 points

MATH PUZZLES – PATTERNS, LOGIC RIDDLES

Math puzzles include brain teasers, logic puzzles and math riddles, which have specific rules and conditions and requires mathematics to solve them. Because math puzzles often do not involve competition between two or more players, students can take as many opportunities as possible to try different ways to solve the puzzle. Marilyn vos Savant advises that the most important part about staying sharp is doing novel things. It is much better to solve 20 different problems every day than doing the same thing.



What happens in these different parts of the brain?
How will doing math puzzles improve our mathematical thinking, learning, memory and mathematical communication?
The brain has two hemispheres, each divided into four lobes. Each lobe is responsible for different functions:

- **frontal cortex** is responsible for executive functions, such as decision making, planning, and some memory for pattern recognition, abstract reasoning, hypothesis testing and implicit assumptions
- **temporal lobe** is responsible for language and memory. This area is stimulated when recalling number properties and number operation relationships, so that mental energy is not wasted on performing these calculations
- **parietal lobe** is responsible for spatial skills
- **occipital lob**e is responsible for vision

What is the 100th number in this pattern? 83, 99, 107, 123, 131

Add one thousand twenty and one thousand twenty. Subtract twenty. Subtract twenty again. Subtract ten. Subtract ten again. What is the final result?

Number puzzles have a specific set of rules. You first have to figure out the pattern rule, then solve the puzzle using that pattern rule. Think about the mathematics, which is used to solve these different math puzzles.

Puzzle 1 - Unknown Numbers

29	15	14	?
88	32	56	82
51	29	?	45
62	47	15	56

Hint: 88-32=56; 88-6=82

Variations:

- Change the number unit for the numbers (e.g., 290, 150, 140, ?)
- Change the location of the unknown numbers.
- Change the size of the square grid.
- Change the pattern rule.

Puzzle 2 - 5 Clues

- It is a two-digit whole number.
- It is less than 50.
- The sum of its digits is 12.
- The absolute difference of the digits is 4.
- If the digits are reversed, it is 6 less than the next greater ten.

Hint: 58 has an absolute difference of 3.

Variations:

- It is a two-digit odd whole number. One of its digits is half the other. The number is greater than 50.
- The number is greater than the number nickels in a loonie. The number is less than the number of pennies in five quarters. The number is an odd number. When you count by 5s, you say the number. The sum of the digits is 8.

Puzzle 3 – Why Not?

Why will an odd number not be the unknown number?

3	4	1	5	2	9
8	2	2	3	7	5
4		0	8	4	6

Hint: 75 - 29 = 46; 75 – 46 = 29

Variations:

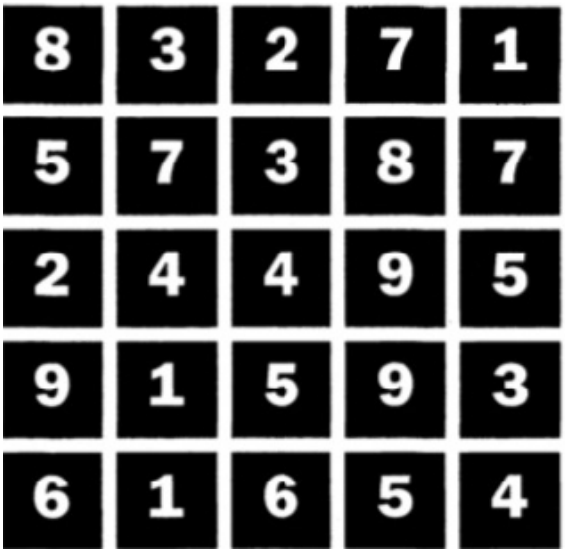
- Change the number unit for the numbers (e.g., 290, 150, 140, ?)
- Change the location of the unknown numbers.
- Change the size of the square grid.
- Change the number relationship.

Number Path Puzzles

How are these two number path puzzles similar and different? How does the choice and organization of the numbers influence the use of subtraction and difficulty of the game? Make your own variation of these games to suit the kinds of practice your students need.

Least Difference Path Puzzle

Which path five whole number path ends with the least difference?



Greatest Difference Path Puzzle

Which set and combination of nine whole numbers will have the greatest difference?



Least Difference Path Puzzle

A five number path is made from one side to a different side. Select 5 numbers, horizontally, vertically and/or diagonally. From the 1st to the 5th number, carry out these operations:

- Sum A - add the 1st two numbers (9 + 2 = 11)
- Difference B - subtract 3rd number from sum A (11 – 7 = 4)
- Sum C - add 4th number to the difference B (4 + 3 = 7)
- Difference D - subtract 5th number from sum C (7 – 7 = 0)
- The five number path, 9, 2, 7, 3, 7 results in the difference of 0, the least difference.



Greatest Difference Path Puzzle

- Select nine adjacent whole numbers, horizontally and vertically only.
  (4, 2, 4, 4, 4, 4, 2, 2, 4)
- Group the nine whole numbers to make two whole numbers (2 sums)
  (4 + 2 + 4 + 4 + 4 + 4 + 2 + 4) and (2)
- Choose combinations of numbers in each of the 2 groups, to make the greatest difference between the two numbers.
  (4 + 2 + 4 + 4 + 4 + 4 + 2 + 4) – (2)
  = 28 – 2 = 26
- The greatest difference is 26.



MATH GAMES - CARDS, DICE, INTERLOCKING CUBES AND NUMBER LINE BALANCE

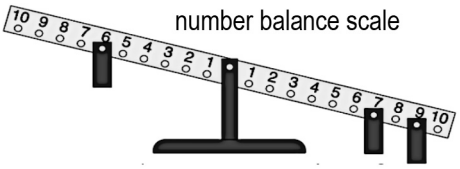
Math games provide problem solving situations where students apply fundamental number concepts, such as number combinations and number computation methods through reasoning. As students play the math games, they notice new patterns, number relationships and strategies, which contributes to improved precision in their computation fluency. Because math games provide students with practice, the teacher can observe students' development of adaptive and strategic mathematical thinking, individually, in pairs and in small groups.

Developing Students' Metacognition for Learning Mathematics

After playing a game, have students orally describe or write a reflection on the game-playing details in a math notebook or math journal. Consider these prompts:

- Describe the mathematics you used as you played the game.
- Explain the calculation methods you used while playing the game.
- When you play the game again, describe different strategies you will try to be more successful.
- Change an aspect of the game to make the game more challenging.

Keep track of points on a number balance scale.
Individual player's points: left side (6); right side (7+9=16).
Difference between the left side (6) and right side (16)
6 < (7+ 9) or 6 < [6 + (1+9)]



Subtraction Concentration Game

Materials: 1 set of playing cards (aces worth 1; jacks, queens, kings worth 10)



Game Goal:
Get least points.

How to Play:

- Players take turns flipping over 4 cards at a time, and calculate the difference between the sum of the red cards and black cards.
- The absolute difference is the points earned each round.
- Discard flipped cards into a pile.
- Game ends when there are no more cards left to turn over.

Point Example: absolute difference = 0 because
10 +10 = 10 + 10 or 20 – 20 = 0

Comparing Towers Game

Materials:
2 colour counters, 2 dice, game board

22	47	16	41	10	35	4
5	23	48	17	42	11	29
30	6	24	49	18	36	12
13	31	7	25	43	19	37
38	14	32	1	26	44	20
21	39	8	33	2	27	45
46	15	40	9	34	3	28

Game Goal:
Reach 100 points.

How to Play:

- Players take turns rolling 2 dice to make a 1 or 2 digit whole number.
- On the gameboard, use a counter to cover 2 numbers, which have a difference equal to the 1 or 2 digit whole number.
- Use gameboard numbers once.
- The game ends when all numbers are covered.
- Players add their points.

Subtraction Battle Game

Materials:
12 dice



Game Goal:
Reach 20 points.

How to Play:

- Each player rolls 6 dice at the same time to create two 3-digit whole or decimal number.
- Players calculate the difference between the two 3-digit numbers.
- The player with the least difference wins the round point.

Variations:

- Use 2 different colour dice to represent whole and decimal numbers.

CLOSING

Our next issue (Vol 58 Issue 3) will focus on practice tasks (quick images, math puzzles and games) for decomposing and multiplying whole and decimal numbers using the area of a rectangle model. These practice tasks can be used for mini lessons and practice lessons.

CALL FOR *ABACUS* SUBMISSIONS

Send your teaching strategy ideas, problems and student solutions to the *Abacus* Co-Editors via email kathkubo@rogers.com or maryloukestell@gmail.com