8 PAGE INSERT ELEMENTARY MATH

ONTARIO ASSOCIATION FOR MATHEMATICS EDUCATION

Abacus

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WHOLE AND DECIMAL NUMBER RELATIONSHIPS FOR MULTIPLICATION

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This issue includes practice tasks for decomposing and composing numbers using different multiplication contexts, models and methods. Depending on the context, the multiplication representations could include any of these models: set (bar model), linear (number line), area (rectangle), graphical (line graph) and numeric and algebraic expressions/equations.

Solve this math brainteaser using different methods.



A 100 kg rice bag takes up an area of 122 cm x 71 cm. Andy said, "Please put 8% of the 25 rice bags in the store." Hilary chuckled, "Sure, I will put out 25% of 8 rice bags."

Are they referring to the same amount of rice?

- What mathematics could be used to solve this brainteaser?
- Why might Solution 1 be discussed first, followed by Solution 2 and Solution 3?

Solution 1: Mass of Rice

100 kg/1 bag x 25 bags x 8 x 1/100
= [25 x 8 x (100 ÷ 100)] kg
= (25 x 8) kg
= 200 kg of rice

100 kg/1 bag x 8 bags x 25 x 1/100
= 8 x 25 kg
= 200 kg of rice

They are referring to the same measure of rice, 200 kg.

Solution 2: Area of Rice Bags

Area (1 rice bag) = 122 cm x 71 cm
= (100 x71) + (20x71) + (2x71)] cm2
= (7100 + 1420 + 142) cm2 = 8662 cm^2

8 x 0.01 x 25 bags x 8662 cm^2/1 bag
= [(8x25) x (8662 x 0.01)] cm^2 = (200 x 86.62) cm^2
25 x 0.01 x 8 bags x 8662 cm^2/1 bag
= [(25x8) x (8662 x 0.01)] cm^2 = (200 x 86.62) cm^2

Same area of the bags, laid out side by side
(2x100) x 86.62 = 2 x(100 x 86.62) = (8662 x 2) cm^2

Solution 3: Number of Rice Bags

8% of 25 bags = 25% of 8 bags
0.8 x 25 bags = 0.25 x 8 bags (8 x 0.10) x 25 bags
= (25 x 0.01) x 8 bags

(8 x 0.10 x 25) bags
= (8 x 25) x 0.10 = 200 x 0.10
= 2.00 bags = 2 bags

Andy and Hilary are referring to 2 bags of rice with 100 kg in 1 bag.

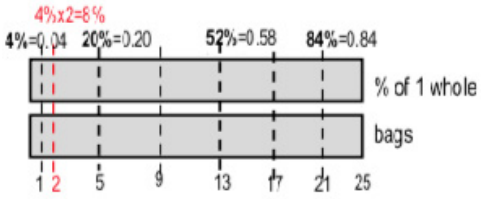
This March 2020 issue includes a variety of multiplication practice tasks, which can be included in your classroom mathematics program as: (a) five minute quick practice; (b) fifteen minute mini lesson; (c) lesson problem for practice lessons; (d) at home practice; and/or (e) family math activities.

DESIGNING PRACTICE TASKS – MATHEMATICS CONTENT FOCUS

What do students need to know and do mathematically to be proficient with whole and decimal number multiplication? When designing multiplication practice tasks, consider these details.

Conceptual Understanding

Relate multiplication of whole and decimal numbers in different real life contexts to models of representation, as listed below.

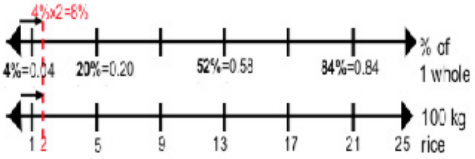


comparative bar model (for # bags)

a) *Multiples of Sets of Discrete Objects*

A set of objects can be grouped by specific number units size, such as 25 rice bags

- 25 bags x 800 = 25 bags x (8 x 100)
  = (200 x 100) bags = **20 000 bags**
- 8% of 25 bags = 25 bags x 8/100 = 25 bags x 8 ÷ 100
  = 25 bags x 8 x 0.01 = (200 ÷ 100) bags = (200 x 0.01) bags = **2 bags**
- 800% of 25 bags = 25 bags x 800/100 = (25 x 800 ÷ 100) bags
  = (25 x 800 x 0.01) = (20 000 ÷100) bags = (20 000 x 0.01) bags
  = (200 x 100 x 0.01) = (200 x 1) bags = **200 bags**

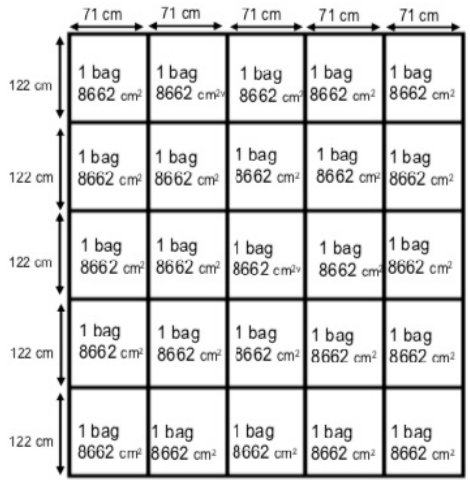


linear model (for kg of rice)

b) *Multiples of a Unit Length*

A unit length could be a measure of length (metres), mass (kilograms), temperature (˚C) or capacity (litres)

- 800 x 100 kg/1 bag x 25 bags
  = 100 kg/1 bag x 25 bags x 800
  = (100 x 25 x 800) kg = [(25 x 8) x 100 x 100] kg of rice
  = (200 x 10 000) kg of rice = **2 000 000 kg of rice**
- 8% of 100 kg/1 bag x 25 bags = 100 kg/1 bag x 25 bags x 8 x 1/100
  = [25 x 8 x (100 ÷ 100)] kg = (25 x 8 x 1) kg = [(25 x 4) x 2)} kg
  = (100 x 2) kg = **200 kg of rice**
- 800% of 100kg/1 bag x 25 bags = (100 kg/1 bag x 25 bags) x (800 x 0.01)
  = [25 x 800) x (100 x 0.01)] kg = [(200 x 100) x 1] kg = **20 000 kg of rice**



area (rectangle) model

c) *Area (Rectangle)*

- Area (1 rice bag) = 122 cm x 71 cm = (100 x71) + (20x71) + (2x71)] cm^2
  = (7100 + 1420 + 142) cm2 = 8662 cm^2
- 8% of Area (25 rice bags) = 8 x 0.01 x 25 bags x 8662 cm^2/1 bag
  = [(8x25) x (8662 x 0.01)] cm^2 = (200 x 8662 x 0.01) cm^2
  = (2 x 100) x (8662 x 0.01) cm^2 = (2 x 8662) x (100 x 0.01)] cm^2
  = {[(2 x 8600) + (2 x 62)] x (100 x 0.01)} cm^2 = [(17 200 + 124) x 1] cm^2
  = 17 324 cm^2

d) *Combinations of Discrete Objects*

Determining the number of possible arrangements of a collection of objects with 2+ attributes; the selection order doesn't matter.

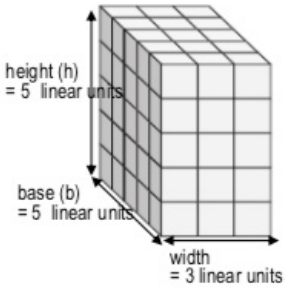
- number of combinations for 8 types of rice and 25 sizes of bags
  8 rice types x 25 bag sizes = (8 x 25) combinations = **200 combinations**
- number of combinations for 80 types of rice and 25 bag sizes
  80 rice types x 25 bag sizes = [(8 x 10) rice types x 25 bag sizes = [(8 x 25) x 10] combinations = (200 x 10) combinations = **2 000 combinations**
- number of combinations for 800 types of rice and 25 sizes of bags
  = 800 rice types x 25 bag sizes = (8 x 100) rice types x 25 bag sizes
  = [(8 x 25) x 100] combinations = 200 x 100 combinations
  = **20 000 combinations**

e) *Volume (Rectangular Prism)*

Amount of 3-dimensional space an object occupies, measured in cubic units.

Volume (rectangular prism)

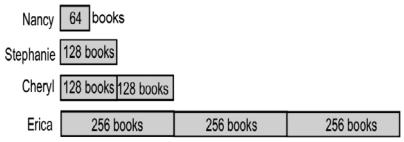
= 5 units x 5 units x 3 units
= 25 units^2 x 3 units
= 75 cubic units



f) *Multiplicative Comparisons*

A comparison in which one quantity can be described as a multiple of the other quantity.

Erica has read 3 times as many pages in a book as Cheryl who read 2 times as many pages as Nancy who read ½ of a book which is 128 pages. Stephanie read 2 times as many pages as Nancy.

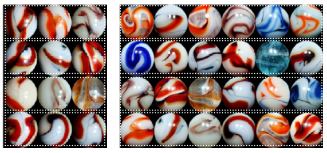


g) *Equivalent Ratios*

Two or more quantities with same units are compared multiplicatively.

Organize 32 marbles into 2 groups as a 5:3 ratio.

Term Number	Group A (3)	Group B (5)	Total Marbles	Number Expression
1	3	5	8	(3x1) + (5x1)
2	6	10	16	(3x2) + (5x2)
3	9	15	24	(3x3) + (5x3)
4	12	20	32	(3x4) + (5x4)



Procedural Fluency

Multiply whole and decimal numbers using these multiplication number properties:

a) *Commutative Property for Multiplication*
Multiplying same numbers in any order results in the same product:

- *a* x *b* x *c* = *b* x *c* x *a* = *a* x *c* x *b*
- 8% of 25 bags = 25% of 8 bags
  8 x 0.01 x 25 bags = 25 x 0.01 x 8 bags = (8 x 25 x 0.01) bags
  = 200 x 0.01 = 2.00 bags = **2 bags of rice**

b) *Associative Property for Multiplication*
Multiplying same factors by associating different pairs of factors results in the same product

- (*a* x *b)* x *c* = *a* x (b x *c)*
- 8% of 25 bags = (8 x 0.01) x 25 bags = (25 x 0.01) x 8 bags = (8 x 25) x 0.01 = 200 x 0.01
  = 2.00 bags = **2 bags of rice**

c) *Distributive Property for Multiplication (Over Addition)*
Multiplying the sum of two or more addends by a number will give the same result as multiplying each addend individually by the number and then adding the partial products together.

- **1 Factor Decomposed:** *a* x (*c* + *d*) = (*a* x *c*) + (*a* x d) and
  2 Factors Decomposed: (*a* + *b*) x (*c* + *d*) = *ac* + *ad* + *bc* + *bd*
- 8% of 25 bags = 0.08 x (20 + 5) bags = (16 x 0.01 x 10) bags + (40 x 0.01 x 1) bags
  = 1.6 +0.4 = 2 bags
- 25% of 8 bags = (0.20 + 0.05) x (3 + 5) bags
  = [(0.20 x 3) + (0.20 x 5) + (0.05 x 3) + (0.05 x 5)] bags
  = (0.6 + 1.0 + 0.15 + 0.25) bags = 2.00 bags = 2 bags of rice

Strategic Competence and Productive Disposition

a) Analyses, selects and explains the multiplication methods and models, which are most suitable to the given real life context and the size of the factors being multiplied.

b) Perseveres to solve the brainteaser using different multiplication methods and methods systematically.

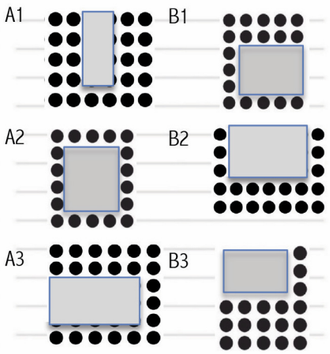
For 8% of 25 bags = 25% of 8 bags, the most efficient calculation method uses the associative property for multiplication. Associating pairs of factors to emphasize the quantity and number unit, 0.01 (hundredths).

(8 x 0.01) x 25 = (25 x 0.01) x 98
(8 x 25) x 0.01 = (25 x 8) x 0.01

"QUICK IMAGES, MATH PUZZLES AND GAMES MAT" - WHOLE NUMBER MULTIPLICATION

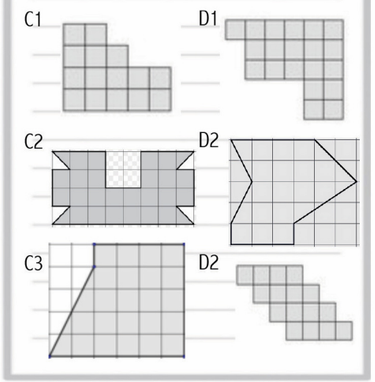
QUICK IMAGE Dots

How Many Dots Are Covered?
Show as a multiplication equation.



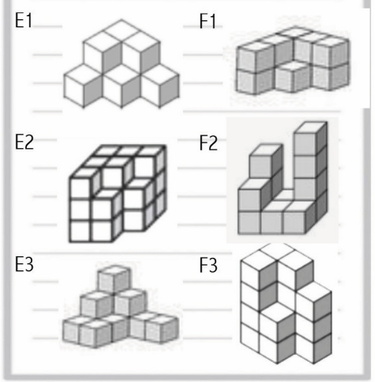
QUICK IMAGE Squares

Ratio for Area for C and D
Show as a multiplication equation.



QUICK IMAGE Cubes

Ratio for Volume for E and F
Show as a multiplication equation.



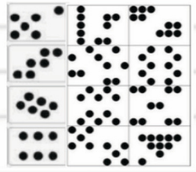
How Many Ways? BRAIN TEASER

How many ways can these be arranged? What is the multiplication equation?

- 2 different letters be arranged?
- 3 different digits be arranged?
- 2 different digits and 2 different letters

Close to 250? BRAIN TEASER

___ x ____ x ___ ~ 250



Combinations? BRAIN TEASER

What different combinations can be rolled for:

- 2 six-sided dice
- 3 six-sided dice
- 4 six-sided dice
- 5 six-sided dice?



How Many? BRAIN TEASER

How many prime factors of 2, 3 and 5 are equivalent to these composite numbers?

- 45, 156, 345
- Show a multiplication equation.

Product Difference MATH GAME

Materials: 1 set of playing cards (face cards -10 pts..; aces -1 pt.)



Game Goal:

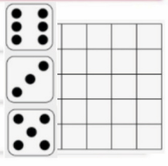
- To have the least product difference.

How to Play:

- Players draw 5 cards and calculate the product of the 5 number values of the cards. (e.g., 10 x 10 x 8 x 8 x 8)
- Players compare their products and determine the ratio of the 2 products. Player with the greatest product records points from the ratio
- Play as many rounds as needed to accumulate10 ratio points.
- Ratio Points Example:
  Player 1 product is 51200 : Player 2 product (

Building Towers MATH GAME

Materials: 40x40 1 cm square grid, 1 cm interlocking cubes, 3 dice



Game Goal:

- To accumulate greatest number of points (in cubic units)

How to Play:

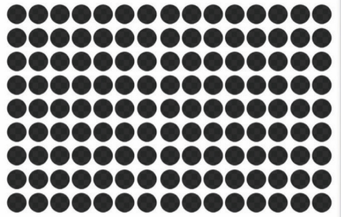
- Players take turns rolling 3 dice, to represent 1-digit whole numbers.
- Players record the dice roll (3, 5, 6) as multiplication equation with 3 factors. (e.g., 3 x 5 x 6 = 90), which is 90 points.
- Build the structure frame of a rectangular prism on the grid.
- Game ends when there is no more space to build a rectangular prism

"QUICK IMAGES, MATH PUZZLES AND GAMES MAT" - DECIMAL NUMBER MULTIPLICATION

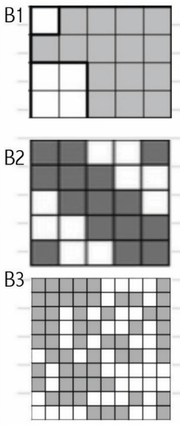
QUICK IMAGE Dots

Circle 2 sets of dots, with no overlaps using colour markers, at these ratios:

a) 1.25 : 1 (blue)
b) 1.5 : 1 (green)
c) 2.0 : 1 (orange)
d) 2.25 : 1 (red)
e) 2.5 : 1 (black)



QUICK IMAGE Squares

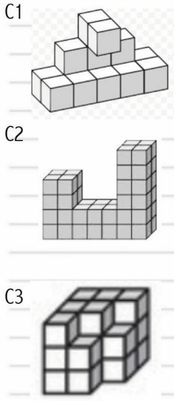


What is the ratio between the area of white squares and area of grey squares for B1, B2 and B3?

a) If 10 squares = 1 ones, then use the unit, tenths.

b) If 100 squares = 1 ones, then use the units, tenths and hundredths.

QUICK IMAGE Cubes



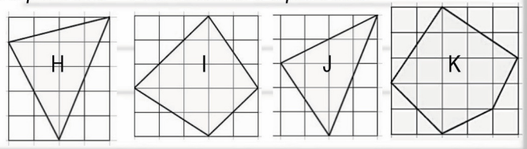
What are the least number of multiplication expressions used to measure the volume of C1, C2 and C3?

a) If 10 cubes = 1 ones, then use the unit, tenths.

b) If 100 cubes = 1 ones, then use the units, tenths and/or hundredths.

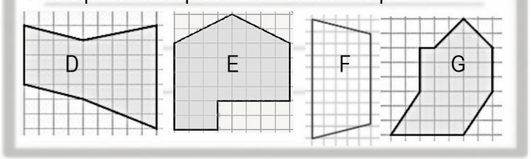
Least Area Ratio BRAIN TEASER

Which 2 polygons have the least area ratio?
1 square unit = 1 hundredth of 1 square unit.



Greatest Area Ratio BRAIN TEASER

Which 2 polygons have the greatest area ratio?
One square unit represents 1 tenth of 1 square unit.



Decimal Number Ratio MATH GAME

Materials: 6x6 circle array, plastic page protector, 2 colour dry erase markers

12.3	90.82	1230	1.23	0.123	0.98
36.82	8.37	3.68	86.32	26.83	6.52
50.09	950	59.4	42.90	500.9	9.42
1.6	63.01	1.64	3.00	16.0	1.06
23.19	123.9	43.17	3.91	63.15	0.75
163.1	1.36	274	72.04	309.5	7.53

Game Goal:

- To accumulate lowest (or highest) points.

How to Play:
Players take turns marking pairs of numbers that have an ratio of 2:1, 3:1, 4:1, approximately, with no overlaps.

- Label each pair of numbers by drawing a letter over the numbers (e.g., letter A, drawn in green is written over 12.3 and 36.82, to show an approximate 3:1 ratio, and the pair of numbers are worth 3 points.
- Game ends when 2 numbers cannot be circled

Product Ratio MATH GAME

Materials: 2 sets of playing cards (face cards are 10 pts); paper, pencil



Game Goal:

- To accumulate points that sum to 40 points.

How to Play:

- Black cards are tenths units; red cards are hundredth units
- Place cards face down in an array.
- Players both flipping over 2 cards at a time and calculate the product. Determine the approximate ratio of the 2 cards.
- Player with greatest number, gets points from the ratio.
- Game ends when there are no more cards to flip over.

Points Example:
Player 1 - 10 tenths + 10 hundredths = 1.0 + 0.10 = 1.1
Player 2 - (10 +1) hundredths = 0.11
1.1 : 0.11 = 10 : 1 Player 1 gets 10 points

QUICK IMAGES – DOTS, SQUARES, PATTERN BLOCKS AND CUBES

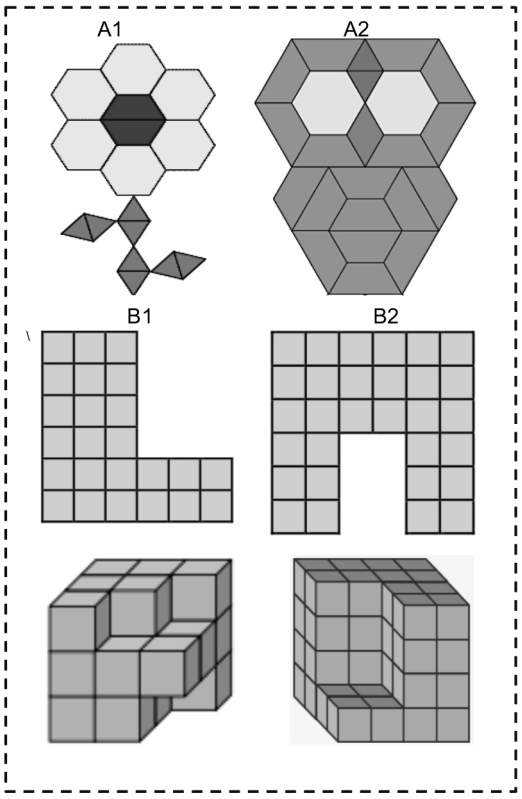
Using quick image routines provide predictable practise of seeing and explaining relationships between spatial visualization, mental math calculations, mathematical reasoning and communication.

Quick Image Routine – Multiplicative Comparisons

a) Calculate the area of the composite rectangles (B1, B2) and the volume of the composite rectangular prisms (C1, C2), using the least number of multiplication expressions.

b) What would a ratio of 2:1 look like for polygon (A1, A2, B1, B2) and composite rectangular prisms (C1, C2)?

- Which attribute could be used to show a 2:1 ratio using either 2D or 3D shapes: colour, type, number, size?
- How many pattern blocks (of each type), square tiles or cubes would you need?



- Show one quick image for 3 seconds.
- Students build with materials (e.g., pattern blocks, square tiles, cubes) or sketch each image using pattern block paper, square grid paper and/or isometric drawing paper.
- Show the same quick image for 3 seconds again.
- Students make revisions to their structure or drawing.
- Show the same quick image for 3 seconds for the last time.
- *Class Discussion* - Identify the attribute they would use to show a ratio of 2:1.

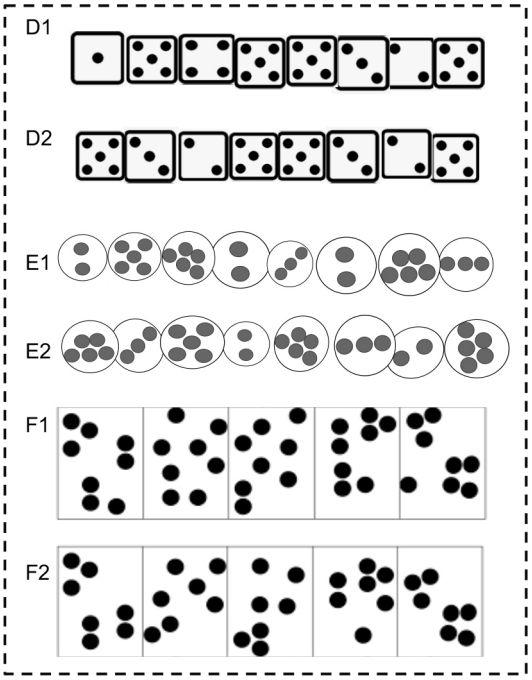
Quick Image Routine – Multiplicative Comparisons

a) How many dots? Create the least multiplication expressions for each unit size. (e.g., E1 (5 x 5) + (1 x 2) = 27

b) Compare multiplicatively each set of dots and identify whether the comparison is at a ratio of 1:1, 1:2 or 1:3.

Number quick images could be composed of dot units (same and/or different size of dots.

- Show one dot image for 3 seconds.
- Tell students to record the total number of dot units.
- Show the same dot image for 3 seconds again.
- Tell students to record the unit sizes used. (e.g., " I used units of 1, units of 5 and units of 10.")
- Show the same dot image for 3 seconds for the last time.
- Tell students to record the multiplication expressions used in their calculations.
- *Class Discussion* - Describe and compare the multiplication expressions used.



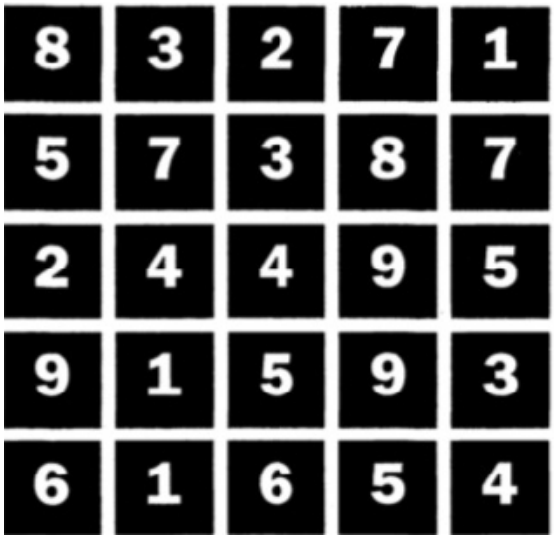
MATH PUZZLES – BRAIN TEASERS

Number Path Puzzles

How are these two number path puzzles similar and different? How does the choice and organization of the numbers influence the use of multiplication methods and the difficulty of the game? Make variations to these games to accommodate the multiplication practice, which your students need.

Product Path Puzzle

Which five whole number path ends with the least product?



Greatest Product Path Puzzle

Which set and combination of nine whole numbers will have the greatest product?



Least Difference Path Puzzle

A five number path is made from one side to a different side.

- **Select 5 numbers**, horizontally, vertically and/or diagonally. From the 1st to the 5th number, carry out these operations:
- ***Sum A*** - add the 1st two numbers to get sum A (9 + 2 = 11)
- ***Product B*** - multiply the 3rd number by sum A (11 x 7 = 77)
- ***Sum C*** - add the 4th number to the product B (77 + 3 = 80)
- ***Product D*** - multiply 5th number from sum C
- (80 x 7 = 560)

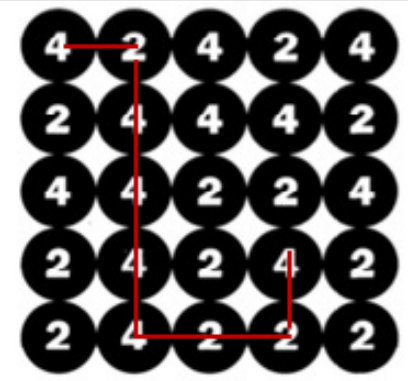
The five number path, 9, 2, 7, 3, 7 results in this equation.

{[(9 + 2) x 7] + 3} x 7
= (11 x 7 + 3) x 7
= 80 x 7
= 560



Greatest Difference Path Puzzle

- Select nine adjacent whole numbers, horizontally and vertically only. (4, 2, 4, 4, 4, 4, 2, 2, 4)
- Group the nine whole numbers to make two whole numbers or 2 sums (4 + 2 + 4 + 4 + 4 + 4 + 2 + 4) = 28 and (2)
- Multiply the two sums to get a final product. (4 + 2 + 4 + 4 + 4 + 4 + 2 + 4) x (2) = 28 x 2 = 56
- Decide if 56 is the greatest product, by trying other combinations.



MATH GAMES - CARDS, DICE, INTERLOCKING CUBES AND NUMBER LINE BALANCE

Math games are highly motivating because the student is actively participating in mathematical thinking and is control of decision-making throughout the game. For example, the choices that students make in selecting numbers that will have the greatest product involves immediate feedback from the other game player. With game rules, there are well-defined limits and directions, within which students can develop strategies. Math games are productive learning experiences as they can be used to consolidate class work, which motivates and enables students to extend their conceptual understanding and develop their proficiency in procedural fluency.

Developing Students' Metacognition for Learning Mathematics

After playing a game, have students orally describe or write a reflection on the game-playing details in a math notebook or math journal. Consider these prompts.

- What multiplication equations did you use as you played the game?
- Explain the number properties for multiplication you used while playing the game.
- When you play the game again, describe different strategies you will try to be more successful.
- Change one game detail, which makes the game simpler or more difficulty to play.

Keep track of points on a number balance scale.

In this example, player 1 has 6 points and player 2 has 16 points. The inequality statement is $6 < (7 + 9)$ or $6 < [6 + (1 + 9)]$

Concentration X Game

Materials: 1 set of playing cards (aces are 1; face cards are 10)

Game Goal: Reach 20 points.

How to Play:

- Players flip over 2 cards at a same time and calculate the product of the numbers on the cards.
- Compare the products as a ratio. Player with greater ratio receives either 2 or 3 points for a ratio of 2:1 or 3:1 respectively.
- Discard flipped cards into a pile.
- Game ends when there are no more cards left to turn over.

Point Example:

Player 1 : Player 2
is 90:30 = 3:1

Player 1 gets 3 points for one round.

Cover-Up Multiplication Game

Materials: 2 colour counters, 3 dice, game board

Game Goal: Reach 100 points.

How to Play:

- Players take turns rolling 3 dice to make 2 or 3 numbers, either 1-digit and/or 2-digit whole numbers.
- On the gameboard, cover a number (48) with a counter, which is the product of the 2 or 3 numbers. (e.g., 2 x 4 x 6 = 48)
- Use gameboard numbers once.
- The game ends when all numbers are covered.
- One point for each covered number.

Multiplication Battle Game

Materials: 6 dice

Game Goal: Reach 40 points.

How to Play:

- Each player rolls 3 dice at the same time.
- Players multiply the three numbers on the dice and record the multiplication equation.
- The player with the greatest product wins points by ratio.

Point Example:

Player 1 : Player 2
2 x 1 x 4 : 2 x 5 x 6
8 : 60
1 : 7 approximately

Player 2 gets 7 points for one round.

CLOSING

Our next issue (Vol 58, Issue 4) will focus on practice tasks (quick images, math puzzles and games) for decomposing and dividing whole and decimal numbers using the area of a rectangle model. These practice tasks can be used for mini lessons and practice lessons.

CALL FOR *ABACUS* SUBMISSIONS

Send your teaching strategy ideas, problems and student solutions to the *Abacus* Co-Editors via email

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