8 PAGE INSERT ELEMENTARY MATH

ONTARIO ASSOCIATION FOR MATHEMATICS EDUCATION

Abacus

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WHOLE AND DECIMAL NUMBER RELATIONSHIPS FOR DIVISION

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This issue includes practice tasks for decomposing and composing numbers using different division contexts, models and methods. Depending on the context, the division representations could include any of these models: set (bar model), linear (number line), area (rectangle), ratio tables, table of values and numeric and algebraic expressions/equations.

Solve this math problem using different methods.





A Singapore Betta fish farm is sending out 3 672 Betta fish. For every 7 female Bettas, there are 11 male Bettas. How many males? How many females?

- What mathematics could be used to solve this brainteaser?
- Why might Solution 1 be discussed first, followed by Solution 2 and 3?

Solution 1: Subtracting Parts

3672 fish – 700 female – 1100 male = 1872 fish

1872 fish – 700 female – 1100 male = 72 fish

72 fish – 28 female – 44 male
= 0 fish

700 + 700 + 28 = 1428 female
1100 + 1100 + 44
= 2244 male

Solution 2: Applying Multiplication Relationship:

2 x 18 fish = 36 fish
4 x 18 fish = 2 x 36 fish
= 72 fish
100 x 18 fish = 1800 fish
200 x 18 fish = 3600 fish
204 x 18 fish = 72 + 3600 fish
= 3672 fish

204 x 7 fish = 1428 female fish
204 x 11 fish = 2040 + 204 male
= 2244 male fish

Solution 3: Organizing in a Table of Values

Term Number	Female Fish	Male Fish	Total Fish	Number Equation
1	7	11	18	(1 x 7) + (1 x 11) = 18
2	14	22	36	(2 x 7) + (2 x 11) = 36
4	**28**	**44**	**72**	**(4 x 7) + (4 x 11) = 72**
10	70	110	180	(10x7) + (10x11) = 180
100	700	1100	1800	(100x7) + (100 x 11) = 1800
200	**1 400**	**2200**	**3600**	**(200 x 7) + (200 x 11) = 3600**
204	1428	2244	3672	**(204 x 7) + (204 x 11) = 3672**

1428 female Bettas and 2244 male Bettas.

This June 2020 issue includes a variety of division practice tasks, which can be included in your classroom mathematics program as: (a) five-minute quick practice; (b) fifteen-minute mini lesson; (c) lesson problem for practice lessons; (d) at home practice; and/or (e) family math activities.

DESIGNING PRACTICE TASKS – MATHEMATICS CONTENT FOCUS

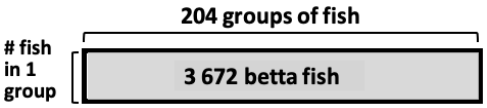
What do students need to know and do mathematically to be proficient with whole and decimal number division? When designing division practice tasks, consider these details.

Conceptual Understanding

Relate division of whole and decimal numbers in different problem-solving contexts to these concepts of division: (e.g., objects, area of rectangle, volume of cubes).

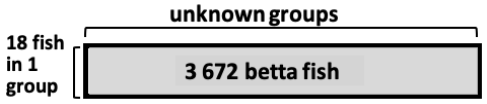
a) Division as Equal Sharing of a Whole (Partitive)

- known - dividend, divisor (number of units or groups)
- unknown - quotient (unit size, size of 1 group)



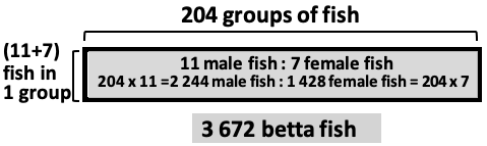
b) Division as Equal Grouping of a Whole (Quotative)

- known - dividend, divisor (unit size, size of 1 group)
- unknown - quotient (number of units or groups)



c) Division as a Ratio

Ratio is the quantitative, absolute or relative comparison between two numbers showing the number of times one value contains or is contained within the other number.



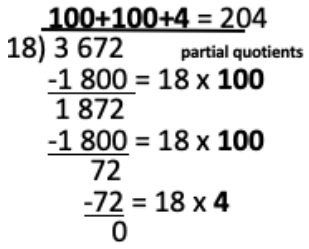
Procedural Fluency

Divide whole and decimal numbers using these methods:

a) Partial Quotient Division

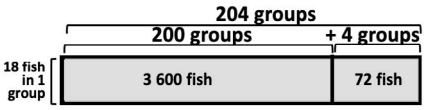
What factors (partial quotients) are multiplied by the divisor to get a product, which is close to the dividend?

- Multiplication is the inverse operation of division.
  divisor x (sum of partial quotient) = dividend
  (100+100+4) groups x 18 fish/1 group = 204 groups x 18 fish
  = 204 groups x (7 females + 11 males)
  = (204 x 7 females) + (204 x 11 males)
  = 1 428 females + 2 244 males = 3 672 fish



b) Decompose Dividend into Addends

The dividend is decomposed into 2 or more addends, which can be divided evenly by the divisor.

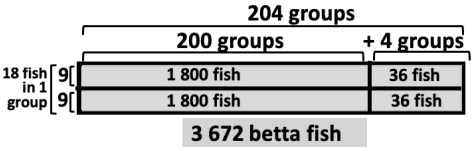


Use multiplicative relationships: 18 x 2 = 36; 18 x 3=54;

- 3672 fish ÷ 18 fish/1 group
  = (3600 fish+ 72 fish) ÷ 18 fish/1 group
  = (3600 fish ÷ 18 fish/1 group) + (72 fish ÷ 18 fish/1 group)
  = (200 + 4) groups = 204 groups of fish

c) Decompose Divisor into Factors

The divisor is decomposed into 2 or more factors, which can be divided evenly into the original dividend and then the resulting dividend.



- 3672 fish ÷ 18 fish/1 group = 3 672 ÷ 2 ÷ 9 fish/1 group
  = (3672 ÷ 2) fish ÷ 9 fish/1 group
  = (1836) fish ÷ 9 fish/1 group = (200 + 4) groups
  = 204 groups of fish

DESIGNING PRACTICE TASKS – MATHEMATICS CONTENT FOCUS

Strategic Competence and Productive Disposition

a) Analyses, selects and explains the multiplication and division relationships and methods, which are most suitable to dividend and divisor size (whole and decimal numbers)
3672 m x 0.1 = 3 672 m ÷ 10 3672 m x 10 = 3 672 m ÷ 0.1

a) Perseveres to solve the brainteaser using division methods suitable to the dividend and divisor, systematically.

3672 m of ribbon divided into 0.7 m and 1.1 m lengths. How many ribbons?

3672 m ÷ 1.8 m/1 ribbon
= 3672 ÷ 18 m/1 ribbon ÷ 0.1
= (3672 m ÷ 18 m/1 ribbon) ÷ 0.1
= 204 ribbons ÷ 0.1
= 204 ribbons x 10
= 2040 ribbons.

Divisibility Rules

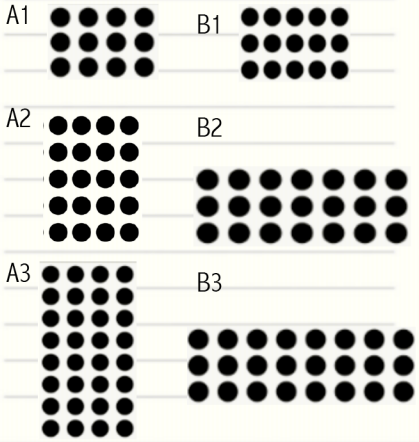
A **divisibility rule** is a heuristic for determining whether a whole number can be evenly divided by another whole number; that is, there is no remainder. For example, if the last digit of a number is 2, 4, 6, 8 or 0, the number is even. Multiple divisibility rules can be applied to the same number to help determine its prime factors. Use these rules to practise division calculations.

Divisible By	A number is divisible by … if …
2	• Last digit even (= 0,2,4,6 or 8)
3	• The sum of the digits is divisible by 3
4	• Last two digits divisible by 4 • Tens digit is even and ones digit is 0, 4 or 8 • Tens digit is odd and ones digit is 2 or 6
5	• Last digit = 5 or 0
6	• Divisible by 2 (even) and 3
7	• Double the last digit of the number and subtract the result from the rest of the number. If the resulting number is evenly divisible by 7, so is the original number
8	• Hundreds digit even: last 2 digits divisible by 8 • Hundreds digit odd: add 4 to last 2 digits and sum divisible by 8 • Last 3 digits divisible by 8
9	• The sum of the digits are evenly divisible by 9.
10	• Last digit is 0
11	• Subtract then add the digits from left to right; the result divisible by 11 Subtract last digit from the rest
12	• Divisible by 3 and 4
13	• Iteratively add 4 times the last digit to the rest
14	• Divisible by 2 and 7
15	• The number must end with 0 or 5 and the sum of its digits must be divisible by 3.
16	• The thousands digit is even and the last three digits form a number that is **divisible** by **16**. • A number is also **divisible** by **16** if the thousands digit is odd and the number formed by the last three digits plus 8 is **divisible** by **16**.
17	• Multiply the last digit by 5 and subtract that from the remaining digit number. If the result is divisible by 17 then the original number is divisible by 17.
18	• Divisible by 2 and 9
19	• Add twice the last digit to the remainder of the number. If the result is divisible by 19 then so was the original number.

"QUICK IMAGES, MATH PUZZLES AND GAMES MAT" – WHOLE NUMBER DIVISION

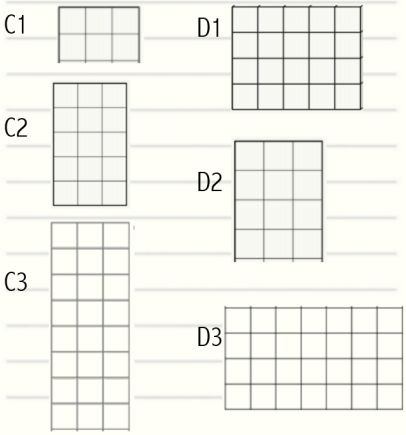
QUICK IMAGE **Dots**

A1-3 Number of groups is 4
B1-3 Size of 1 group is 3 dots
Show as a division equation.



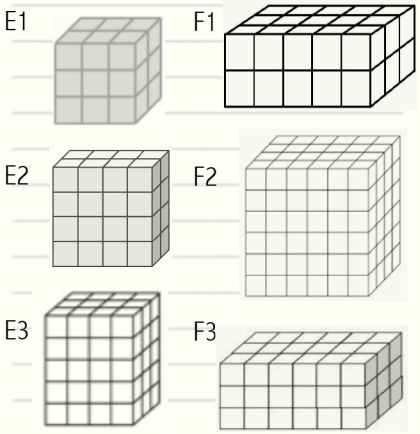
QUICK IMAGE **Squares**

C1-3 Number of groups is 3
D1-3 Size of 1 group is 4 squares
Show as a division equation.



QUICK IMAGE **Cubes**

E1-3 **2 <** number of layers **< 6**
F1-3 **12 <** size of 1 layer < **25** cubes
Show as a division equation.



How Many Ways? BRAIN TEASER

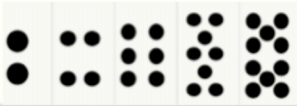
120 ÷ 8

- Decompose the dividend into addends in different ways.
- Write equivalent division expressions.

120 ÷ 8 = (80 + 40) ÷ 8

\Division BRAIN TEASER

What different **division expressions** can be made with any set of dots below? The quotient must be a whole number.



Dicey Division BRAIN TEASER

Roll 5 number cubes to make division expressions with whole number quotients.

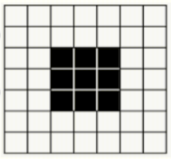
The sum of 4 dice is the dividend and the 5th dice is the divisor.



How Many? BRAIN TEASER

Decompose 80 square units into 4 or more rectangles.

Write the division equation for each rectangle.



Quotient Difference MATH GAME

Materials: 1 set of playing cards (face cards -10 pts..; aces -1 pt.)



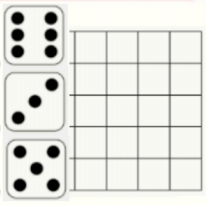
Game Goal:
Create division expressions with whole number quotients.

How to Play:

- From cards spread out and face down, players draw 5 cards, in order to combine 4 cards to make a whole number, dividend; the 5th card is a whole number divisor. Quotient must be a whole number. If not, the player loses her/his turn.
- Players compare, multiplicatively, their quotients and determine the ratio of the 2 quotients. Player with the greatest quotient records the ratio points.
- Play as many rounds as needed to get 10 ratio points.
- Scoring Example: Player 1 (3+2+5+8) ÷2 = 9
  Player 1 (9+8+7+6)÷5 = 40÷ 5 = 8; 9 : 8 ~ 1:0 so 1 pt.

Divide It Up MATH GAME

Materials: 40 x 40 1 cm square grid, 2 colour markers, 3 dice, 20



Game Goal:

- To accumulate greatest number of points from the value of the quotients.

How to Play:

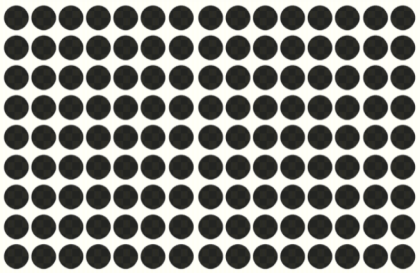
- Players take turns rolling 3 dice, to represent three 1-digit whole numbers. 2 whole numbers are added to be the dividend and the 3rd number is the divisor
- Draw the rectangle that shows the dividend (sum of 2 dice numbers) and the # columns to be the divisor; the quotient is # of rows (partitive division).
- Record rectangle with dividend, divisor and quotient labelled on the grid. Points = number of the quotient.
- Game ends when there is no more space rectangle to record a rectangle, with no overlaps.

"QUICK IMAGES, MATH PUZZLES AND GAMES" MAT – DECIMAL NUMBER DIVISION

QUICK IMAGE **Dots**

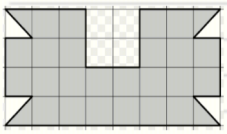
Divide this dot array into smaller rectangular arrays, which only have quotients of 4 dots.

Record the division equations for each array.

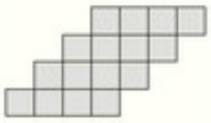


QUICK IMAGE **Squares**

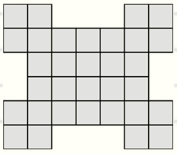
B1



B2



B3



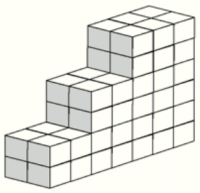
10 squares = 1 ones, then use the unit, tenths.

What is the total number of squares?

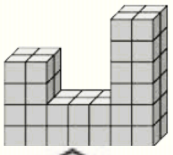
How many parts are in each whole when 1 part is 4 tenths?

QUICK IMAGE **Cubes**

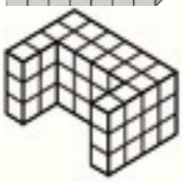
C1



C2



C3



0 cubes = 1 ones, then use the unit, tenths.

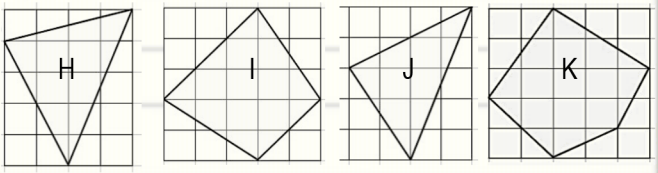
Divide each 3D figure into 4 equal size parts.

What is the total number of cubes (dividend)?

What is the size of 1 part?

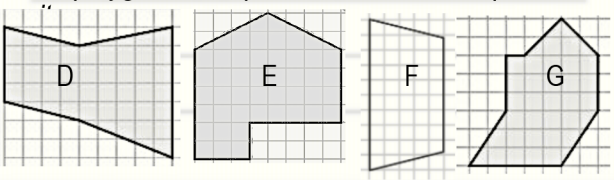
Divide Polygons into 3 Equal Size Parts
BRAIN TEASER

What does each polygon look like when divided into 3 equal size parts? *1 square = 1 hundredth of 1 square unit.*



Divide Polygons into 6 Square Units Parts
BRAIN TEASER

How many parts of 6 square units are there for each polygon? *1 square = 1 tenth of 1 square*



What's the Divisor?
MATH GAME

Materials: 6x6 circle array, plastic page protector, 2 colour dry erase markers

12.3	90.82	1230	1.23	0.123	0.98
36.82	8.37	3.68	86.32	26.83	6.52
50.09	950	59.4	42.90	500.9	9.42
1.6	63.01	1.64	3.00	16.0	1.06
23.19	123.9	43.17	3.91	63.15	0.75
163.1	1.36	274	72.04	309.5	7.53

Game Goal:
- To accumulate lowest (or highest) points.

How to Play:
Players take turns marking pairs of numbers as the dividend and quotient. Label each pair of numbers by drawing a letter over the numbers (e.g., letter A, drawn in green is written over 12.3 (divisor) and 1 230 (quotient).
- Write the division equation and show the divisor.
- 12.3 ÷ __ = 1 230; 12.3 ÷ 0.01 = 1 230
- Game ends when 2 numbers cannot be circled

Quotient Ratio
MATH GAME

Materials: 2 sets of playing cards (face cards are 10 pts);



Game Goal:
- Get points that sum to 50 points.

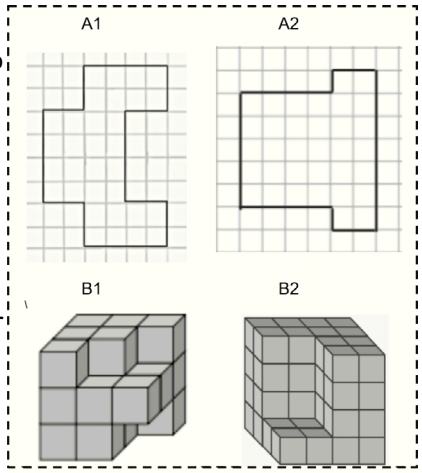
How to Play:
- Black cards are tenths units; red cards are ones unit
- Place cards face down in an array.
- Players simultaneously flip over 2 cards at a time and calculate the quotient.
- Compare quotients as an approximate ratio: 1:1, 1:2, 1:3,
- Player with greatest number, gets points from the ratio.
- Game ends when there are no more cards to flip over.

Division Example:
- king ÷ ace = 10 ÷ 1 = 10
- jack ÷ queen = 10÷ 1.0 = 10
- ratio 10:10 = 1:1, each get 1 point

QUICK IMAGES – DOTS, SQUARES, AND CUBES

Using quick image routines provide predictable practise of seeing and explaining relationships between spatial visualization, mental math calculations, mathematical reasoning and communication.

Quick Image Routine - Division of Wholes



Divide each composite rectangles and rectangular prisms into equal size groups: ÷ 2, ÷ 3, ÷ 4 or ÷ b5, so that there is either:

a) a whole number quotient (grade 1 to grade 5)

b) a decimal number quotient (grade 6 to grade 8)

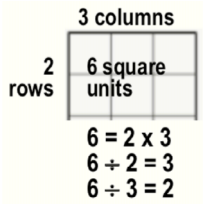
Using number to describe the decomposition of 2D shapes and 3D figures is practised using quick images.

- Show one quick image for 3 seconds.
- Students build with materials (e.g., square tiles, cubes) or sketch each image using, square grid paper and/or isometric drawing paper.
- Show the same quick image for 3 seconds again.
- Students make revisions to their structure or drawing.
- Show the same quick image for 3 seconds for the last time.
- *Class Discussion* – Describe the visualization strategy used to divide the shapes and figures into 2, 3, 4 or 5 equal size parts.

Students will notice that multiplication and division equations are related:

(a) product (multiplication) and dividend (division) are the total or whole

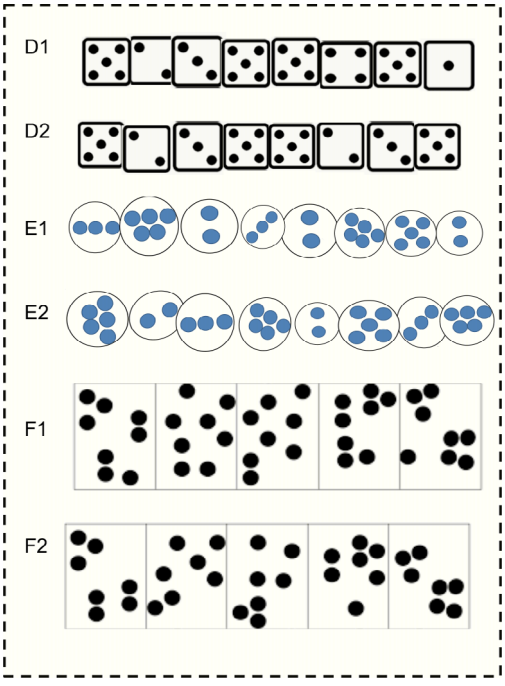
(b) factors (multiplication) are either the divisor or quotient (division).



Quick Image Routine - Multiplicative Comparisons of Quotients

a) How many dots? Write a division equation for each row.

b) Compare multiplicatively the quotient for pairs of rows, D, E and F. Decide whether the multiplicative comparisons of the quotients is 1:1, 1:2 or 1:3.



Number quick images could be composed of dot units (same and/or different size of dots.

- Show one dot image for 3 seconds.
- Tell students to record the total number of dot units and call it the dividend.
- Show the same dot image for 3 seconds again.
- Tell students to record the number of groups (divisor) that the dividend will be divided, so that the quotient is a whole number (e.g., "I divided 8 into 1 group with a quotient, 8, 2 groups with a quotient, 4; 4 groups with a quotient, 2.")
- Show the same dot image for 3 seconds for the last time.
- Tell students to record the division equations to represent their division calculations.
- *Class Discussion* - Describe and compare the division equations used.

MATH PUZZLES – BRAIN TEASERS

Division Equation Search

Draw lines (horizontal, vertical, diagonal) to show division equations. Record the division equations. For example, 20 ÷ 2 = 10 is on this game board.

10	3	4	7	2	14	2	3	2	10
10	2	7	15	8	7	7	10	5	9
8	11	20	2	10	9	9	2	5	15
80	5	5	8	2	6	6	35	4	45
15	10	5	6	8	1	5	1	8	6
12	6	2	12	6	7	6	56	32	9
9	3	3	7	8	1	10	10	8	3
6	7	4	3	3	1	5	9	5	7
7	6	9	12	10	2	72	10	7	11
9	27	18	36	3	30	2	1	2	2

Division Challenge 3 x 3 Square Grid

Dividends are shown on the square grid. Select divisor and quotients so all division equations are true. (e.g.,16 ÷ 8 = 2; 60÷ 5 = 12)

2	12	24
8	5	40
16	60	÷

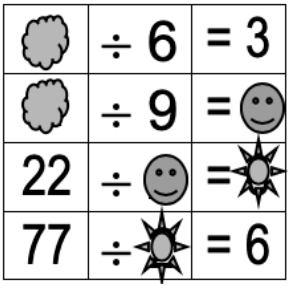
		15
		32
12	40	÷

		21
		36
42	18	÷

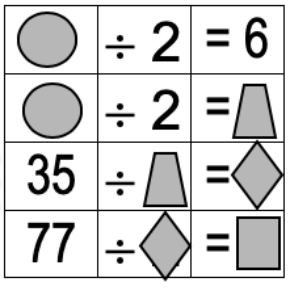
		6
		45
10	27	÷

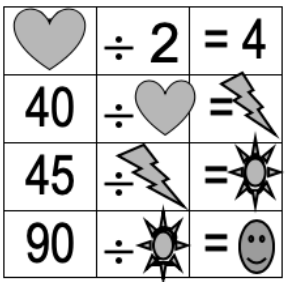
Logic Division Puzzles

Use multiplicative and relational thinking to determine the unknown numbers in division equations.



A	÷ 2	= 6
A	÷ 4	= B
33	÷ B	= C
66	÷ C	= D





Division with Unknown Numbers

Problem 1:

When N is divided by P, the quotient Q and the remainder is T. P, Q, T are 1-digit numbers.

What are possible values of the dividend, divisor, quotient and remainder.



Problem 2:

The Bike Shop has 225 bicycles and tricycles for rent. There are no training wheels. How many bicycles were for rent? How many tricycles were for rent?



MATH GAMES – CARDS, DICE, INTERLOCKING CUBES AND NUMBER LINE BALANCE

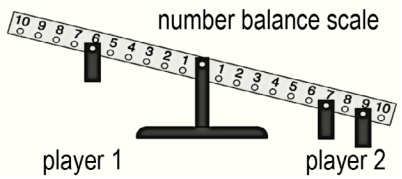
Playing number math games develops flexibility in strategic thinking, problem solving and deepens student's conceptual understanding of number relationships. Math games engage students in applying their learning in dynamic contexts, which are defined only by the game rules, rather than a specific set of questions. As soon as the rules of a game are understood, students often require little teacher input and exchange their questions and ideas with their class mates. Student's general interest to improve their number knowledge becomes a shared experience with their classmates.

Developing Students' Metacognition for Learning Mathematics

After playing a game, have students orally describe or write a reflection on the game-playing details in a math notebook or math journal. Consider these prompts.

- What division equations did you use as you played the game?
- Explain which divisors were used more often? Why?
- When you play the game again, describe different strategies you will try to be more successful.
- Change one game detail, which makes the game simpler or more difficult to play.

Players	Dice 1	Dice 2	Dice 3	Dice 4	Division Equation
1	6	2	4	2	(6 x 2 x 2) ÷ 4 = 6
2	5	2	5	5	(5 x (2 +5) ÷ 5 = 7
1					
2	4	5	3	3	3 x (4 + 5) ÷ 3 = 9



Concentration Division Game



Materials: *1 set of playing cards (aces are 1; face cards are 10)*

Game Goal: *Reach 20 points (sum of quotients for each turn).*

How to Play:

- *Players flip over 3 cards at a same time and calculate the quotient for dividend divided by the divisor.*
- *Dividend could be the sum or the product of the 2 number cards. The 3rd card is the divisor.*
- *Discard flipped cards into a pile.*
- *Game ends when there are no more cards left to turn over.*
- *Example for Cards 2, 8, 10*
  (10 + 8) ÷ 2 = 18 ÷ 2 = 9
  (10 x 8) ÷ 2 = 80 ÷ 2 = 40

Cover-Up Division Game

22	47	16	41	10	35	4
5	23	48	17	42	11	29
30	6	24	49	18	36	12
13	31	7	25	43	19	37
38	14	32	1	26	44	20
21	39	8	33	2	27	45
46	15	40	9	34	3	28

Materials: *2 colour counters, 4 dice, game board*

Game Goal: *Reach 100 points.*

How to Play:

- *Players take turns rolling 4 dice to make a dividend and divisor through the addition, subtraction and/or multiplication of 2 dice to be the dividend or divisor.*
- *The quotient must be a number on the gameboard. Players cover the quotient number on the game board with a colour counter. Use gameboard numbers once.*
- *One point is allotted for each covered number.*
- *The game ends when all numbers are covered.*

Division Battle Game



Materials: *4 dice*

Game Goal: *Reach 40 points.*

How to Play:

- *Each player rolls 4 dice at the same time.*
- *Players add, subtract and/or multiply 2 or 3 numbers on the dice to be the dividend or divisor, so that the quotient is a whole number.*
- *Players record the division equation; The quotient is the point allocation.*
  18 ÷ 2 = 9, so 9 points.
- *The game ends when a player reaches exactly 40 points.*
- *If points collected exceed 40 points, then the next quotient can be subtracted.*

CLOSING

Our next issue (Vol 59, Issue 1) will focus on practice tasks (quick images, math puzzles and games) for fractions. These practice tasks can be used for mini lessons and practice lessons.

CALL FOR *ABACUS* SUBMISSIONS

Send your teaching strategy ideas, problems and student solutions to the *Abacus* Co-Editors via email kathkubo@rogers.com or maryloukestell@gmail.com