8 PAGE INSERT ELEMENTARY MATH

ONTARIO ASSOCIATION FOR MATHEMATICS EDUCATION

Abacus

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REPESENTING AND COMPARING FRACTIONS AS DIVISION OF A WHOLE, SET AND MEASURE

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This issue includes practice tasks for representing and comparing fractions of a whole, set and measure using different contexts, models and methods. Depending on the context, the fraction representations could include any of these models: set of objects (bar model, array, objects of different shapes and sizes), linear measure (number line), area of a rectangle and numeric/algebraic expressions/equations. Solve this fraction brainteaser using different methods.

True or False? 1/2 < 1/3

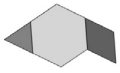
- What mathematics could be used to solve this brainteaser?
- Why might Solution 1 be discussed first, followed by Solution 2 and Solution 3?

Solution 1: Pattern Blocks as Parts of the Same Whole

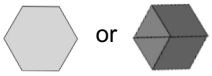
1 whole



1/2 of 1 whole



1/3 of 1 whole



False: 1/2 > 1/3 of the same whole because the area of ½ of the whole is greater than 1/3 of the whole

Solution 2: Pattern Blocks as Parts of a Set of Discrete Objects

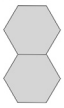
1 set of 6 pattern blocks



1/2 of 1 set of 6 pattern blocks
= 6 pattern blocks ÷ 2 parts of 1 set
= 3 pattern blocks



1/3 of 1 set of 6 pattern blocks
= 6 pattern blocks ÷ 3 parts of 1 set
= 2 pattern blocks

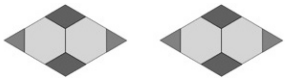


False: 1/2 > 1/3 of the same set of pattern blocks because 3 pattern blocks > 2 pattern blocks, but the size of 1/3 of the set of pattern has a greater area.

Solution 3: Pattern Blocks as Parts of Different Wholes

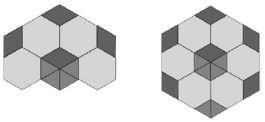
1/2 of whole A

1/3 of whole B



Whole A

Whole B



False: 1/2 of whole A = 1/3 of whole B compared by area because: area (whole A) < area (whole B)

This September 2020 issue includes a variety of fraction representation and comparison practice tasks, which can be included in your classroom mathematics program as: (a) five-minute quick practice; (b) fifteen-minute mini-lesson; (c) lesson problem for practice lessons; (d) at-home practice; and/or (e) family math activities.

DESIGNING PRACTICE TASKS – MATHEMATICS CONTENT FOCUS

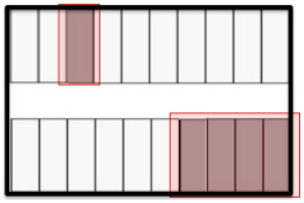
What do students need to know and do mathematically to be proficient with representing and comparing fractions of a whole, set and measure? When designing fraction practice tasks, consider these mathematical details.

Conceptual Understanding

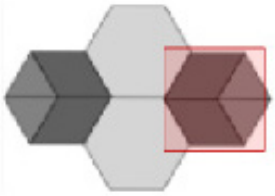
Relate the representation of fractions of a whole, set and measure in different real-life contexts to the models of representation listed below.

a) *Fraction of a Whole*

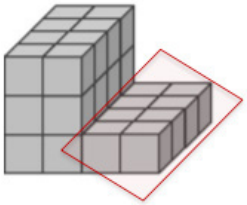
A whole is divided or partitioned into parts that are the same size and shape. For example, the whole could be the area of a rectangular parking lot, number of cubes in a complex rectangular prism and a 2D shape made up of square tiles or pattern blocks.



¼ of the parking lot is used
¼ of 20 parking spaces
= 20 parking spaces ÷ 4 parts
= 5 parking spaces



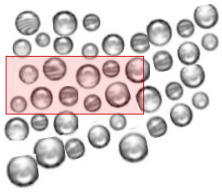
¼ of the polygon is a hexagon
¼ of 4 hexagons
= 4 hexagons ÷ 4 parts
= 1 hexagon



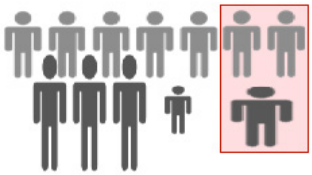
¼ of complex rectangular prism is 8 cubes
¼ of 32 cubes
= 32 cubes ÷ 4 parts
= 8 cubes

b) *Fraction of a Set of Objects*

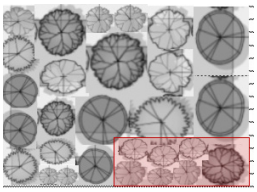
A set of discrete objects of same and different shapes and sizes are divided into parts, which have an equal number of objects. For example, a set of objects could be marbles in a bag, people at an amusement park and trees in a forest.



¼ of 1 set of marbles
¼ of 20 marbles
= 40 marbles ÷ 4 parts
= 10 marbles in 1 part



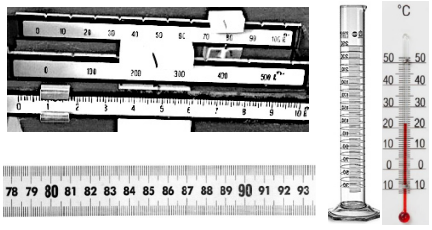
¼ of 1 set of park visitors
¼ of 12 people
= 12 people ÷ 4 parts
= 3 people



¼ of 1 set of trees in a forest
¼ of 28 trees
= 28 cubes ÷ 4 parts
= 7 trees

c) *Fraction of a Measure*

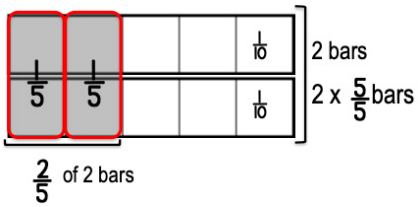
A number line is used represent the measure of length (meters), mass (kilograms), time (hours), temperature (˚C) and capacity (litres). Fractional units provide greater measurement precision. Measurement tools include: meter stick (length), triple beam balance (mass), analog clock (time), graduated cylinder (capacity) and thermometer (temperature).



DESIGNING PRACTICE TASKS – MATHEMATICS CONTENT FOCUS

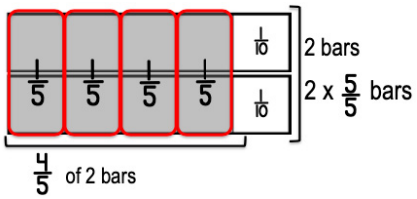
Procedural Fluency

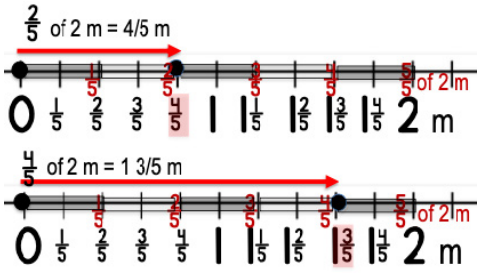
Fractions can be directly compared only if the whole is known. To represent fractions of same and different wholes, specific visual models represent directly real-life contexts.

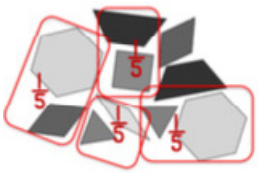


a) *Same Denominator, Compare Numerator of the Same Whole*
When comparing fractions of the same whole with the same denominator (total number of parts), the fraction with the greater numerator is the greater fraction, or the fraction with the least numerator is the least fraction.

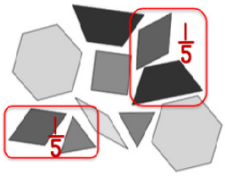
- a/b > c/b when a > c
- If the whole is 2 bars, meters or sets of blocks for both fractions, then 4/5 > 2/5. Both fractions have the same whole; that is, the whole is divided into 5 equal size parts, fifths (1/5, unit fraction). The fraction, 4/5 has a greater numerator, 4, than the fraction, 2/5, which has 2 parts.
- Dividing or partitioning a whole into 5 equal parts means that each 1/5 of 2 bars, meters and sets of blocks has the fractional quantity of 2/5 bars, meters or set of blocks.







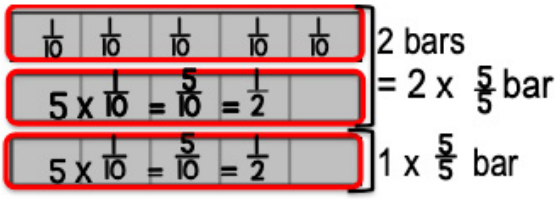
2 sets of 5 blocks = 10 blocks
4/5 X 10 blocks = 4 x 1/5 x 10 blocks
4 x (10÷5) blocks = 4 x 2 blocks
= 8 blocks



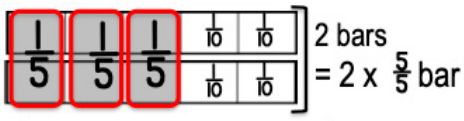
2 sets of 5 blocks = 10 blocks
2/5 x 10 blocks = (2 x 1/5) x 10 blocks
= 2 x (10÷5) blocks = 2 x 2 blocks
= 4 blocks

b) *Same Numerator, Compare Denominator of Same Whole*
When comparing fractions of the same whole have the same numerator, the fraction with the greater denominator is the least fraction, or the fraction with the least denominator is the greater fraction. If the same whole is divided into an increasing number of parts, then the size of the parts decrease.

- a/b > a/c when b < c
- If the whole is 2 bars, meters or sets of blocks for both fractions, then 3/5 < 3/2 because with the same numerator, 4, the denominators, 5 > 2. Both fractions have the same number of parts, but the size of each unit fraction is different; that is, the unit fraction, ½ is greater than the unit fraction, 1/5.
- Dividing (partitioning) a whole into 5 equal parts means that each 1/5 of 1 whole, has the fractional quantity of 2/5 bars, meters or set of blocks for 2 m. Dividing a whole into 2 equal parts, so that each ½ of 1 whole has the fractional quantity of 1 bar of 2 bars, 1 m of 2 m and 5 blocks of a set of 10 blocks.



3/2 of a whole = 3/2 x 2 bars = (1+1/2) x 2 bars
= (1 x 2 bars) + (1/2 x 2 bars) = 2 bars + 1 bar
= 3 bars = 3 x 5/5 bar = 3 x 5 x 1/5 bar = 15/5 bars

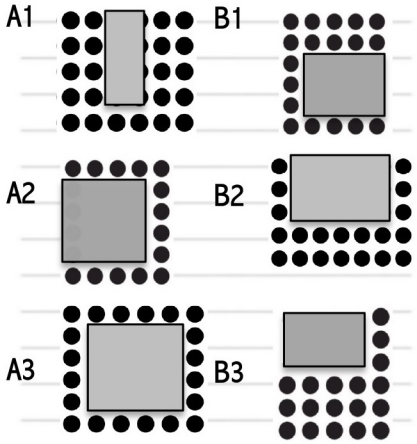


3/5 of a whole = 3/5 x 2 bar = (3 x 1/5) x 2 bars
= 3 x (2 ÷ 5 parts) = 3 x 2/5 bars = 3 x 2 x 1/5 bar
= 6/5 bars = 1 bar + 1/5 bar = 1 1/5 bars

"QUICK IMAGES, MATH PUZZLES AND GAMES MAT" - Fractions as Division

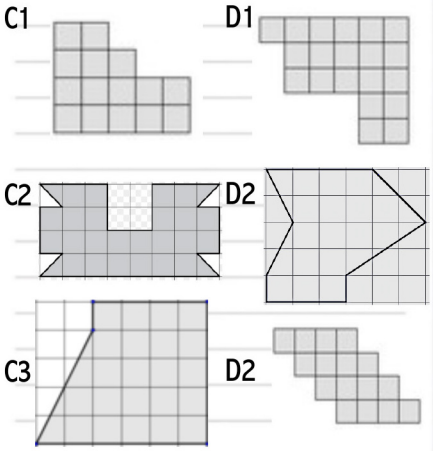
QUICK IMAGE "DOTS"

About How Many Dots Are Covered by 1/3,1/4 or 1/5 of Each Rectangle?



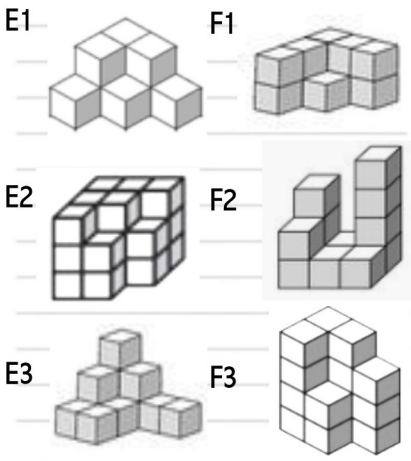
QUICK IMAGE "SQUARES"

What Fraction of Each Polygon Is About 8 Square Units?



QUICK IMAGE "CUBES"

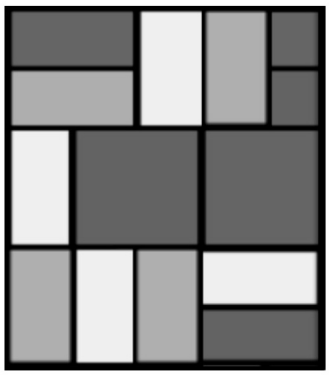
About How Many Cubes are 1/3 of Each Complex Rectangular Prism?

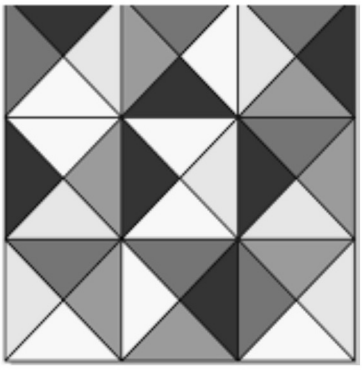


"RECTANGLE" BRAIN TEASER

What fraction of this design has the greatest number of congruent rectangles?

Solve as:
- a fraction of a whole
- a fraction of a set.





"TRIANGLE" BRAIN TEASER

What fraction of this design have triangles with an area of 2 square units?

Solve as:
- a fraction of a whole
- a fraction of a set.

"FRACTION REPRESENTATION" MATH GAME

Materials -1 set of playing cards with no face cards, 1 dice, pattern blocks, meter stick, square tiles

Game Goal - Collect the greatest number of points.

How to Play: (2 players or 2 teams)
- Spread out playing cards, face down. Players take turns revealing 2 playing cards to determine the fraction (greater number is the denominator.)
- Both players represent the fraction using a different set of manipulatives and gain 1 point for an accurate representation. Together, both players determine the accuracy of representations.
- The player who revealed the playing cards rolls the dice to determine additional points to collect.
- The game ends when there are no more playing cards to reveal. Players add the points collected.

"FRACTIONAL QUANTITY" MATH GAME

Materials -3 dice; 40 x 40 1 cm square grid

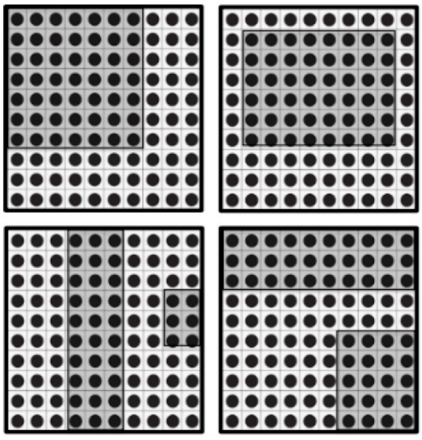
Game Goal – Collect the greatest number of points.

How to Play: 2 players or 2 teams)
- Players take turns rolling the 2 dice to determine the fraction (greater number is the denominator); Roll 3 dice to determine the quantity of the whole (sum the dice numbers).
- Players represent the fraction of the whole on the square grid, with no overlap.
- Players draw the rectangle on the square grid, which represents the quantity of the whole and shade in the fraction of that whole. The fractional quantity of the whole is the number of points that the player collects. Players lose their turn if the fraction of the whole cannot be represented.
- The game ends when there is no space of the grid.

"QUICK IMAGES, MATH PUZZLES AND GAMES" MAT – Fractions as Division

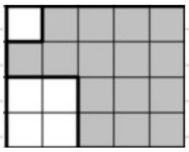
QUICK IMAGE "DOTS"

Which fraction (shaded rectangle) of these 4 sets is greater? Why can these fractions be compared?



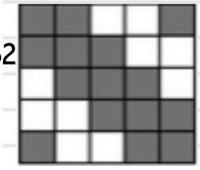
QUICK IMAGE "SQUARES"

B1



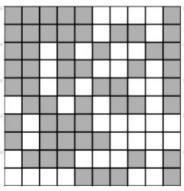
Which fraction (shaded squares) of a set is greater? Which fractions cannot be compared?

B2



Why? Which fraction model are you using: area or set of objects?

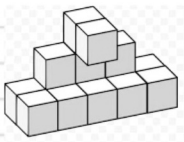
B3



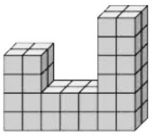
How would you change the diagram so that the fractions could be compared?

QUICK IMAGE "CUBES"

C1



C2



C3

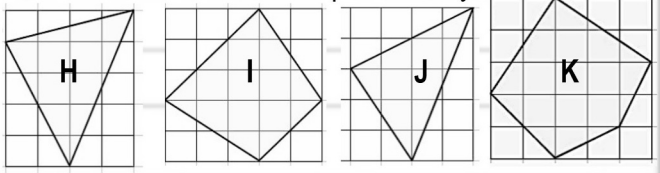


About how many cubes show each fraction of the whole for each complex rectangular prism?

- 1/2 of the whole
- 4/6 of the whole
- 3/2 of the whole
- 20/10 of the whole

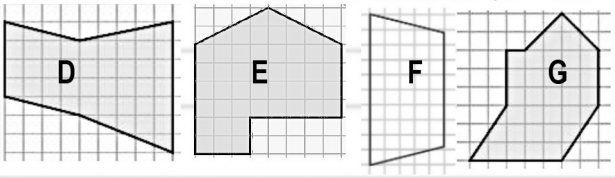
"COMPARING FRACTIONS" BRAIN TEASER

What fraction of the square grid is each polygon? Which fractions can be compared? Why?



"ORDERING FRACTIONS" BRAINTEASER

What fraction of the square grid is each polygon? Order the fractions of the same whole. Explain.



"COMPARING FRACTIONS of a WHOLE" MATH GAME

Materials - 6 dice, fraction strip (1/1 to 10/10)

Game Goal - Collect the greatest number of points.

How to Play: (2 people or 2 teams)

- Players take turns rolling 6 dice and take turns selecting 2 dice (one at a time) to make a fraction that is less than 1 (e.g., greater number is the denominator)
- Player with the greater fraction of the same whole, 1, roll the remaining 2 dice to determine a 3rd fraction.
- Player with the fraction that is closest to the 3rd fraction collects 3 points.
- Player with the fraction that is greater than the other players' fraction collects 2 points.
- The game ends when a player collects 50 or more points.

"COMPARING FRACTION QUANTITIES" MATH GAME

Materials - 1 set of playing cards with no face cards

Game Goal – Collect the least number of points

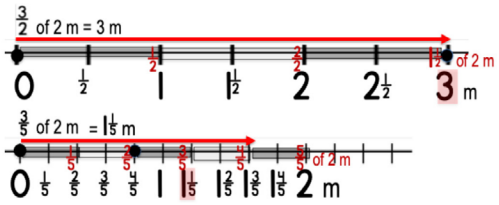
How to Play: (2 people or 2 teams)

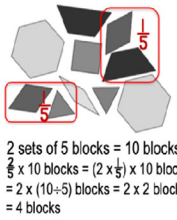
- Black cards are the numerator; red cards are the numerator.
- Place cards face down in two separate arrays: red, black cards
- Players both flip over 2 cards at a time, simultaneously and then identify the fractions represented by the red/black cards.
- Compare the fractions using benchmarks of less than ½ and greater than ½. Player with least fraction number, gets 10 points. Put cards in a discard pile.
- Game ends when there are no more cards to flip over.

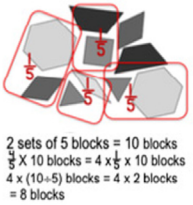
DESIGNING PRACTICE TASKS – MATHEMATICS CONTENT FOCUS

Procedural Fluency

a) *Same Numerator, Compare Denominator of Same Whole*







b) *Benchmark Fractions*

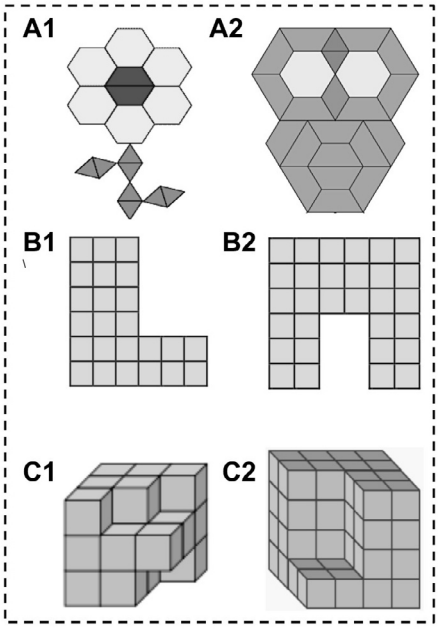
Benchmark fraction numbers include 0, 1 and ½. Use benchmark fractions for comparison when:

- one fraction is greater than ½ and the other fraction is greater than1 of the same whole, set or measure (e.g., 3/2 of 2 sets > 3/5 of 2 sets)
- one fraction is less than ½ and the other fraction is greater than ½ of the same whole, set or measure (e.g., 2/5 < ½ < 4/6, so 2/5 < 4/6)
- both fractions are 1 unit fraction away from benchmark, 1, so that the unit fraction with the greater denominator is the least fraction, which is closer to the benchmark 1 (e.g., 1> 5/6 > 4/5)

QUICK IMAGES - SQUARES, PATTERN BLOCKS AND CUBES

Using quick images routines, provide predictable practice for seeing and explaining relationships between spatial visualization, mental math calculations, mathematical reasoning and communication.

- There are 4 groups of 6 square tiles.
- The whole or shape B1 is composed of 24 square tiles.
- When the whole is divided into groups of 6 square tiles, then there are 4 parts, which means that 6 square tiles of the whole, 24 square tiles is the unit fraction, 1⁄4.

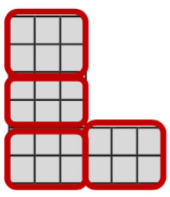


Quick Image Routine – Fraction of a Whole

- Show one image for 3 seconds.
- Students build with materials (e.g., pattern blocks, square tiles, cubes) or sketch each image using pattern block paper, square grid paper and/or isometric drawing paper.
- Show the same quick image for 3 seconds again.
- Students make revisions to their structure or drawing.
- Show the same quick image for 3 seconds for the last time.
- *Class Discussion* – Identify the unit fraction for each shape.

Viewing the images for a brief period of time (3 seconds) motivates students to create and use efficient methods to determine multiplicative relationships of the unit fraction for the shape

If 6 square tiles represent the unit fraction, then how many units of 6 square tiles is Figure B1?



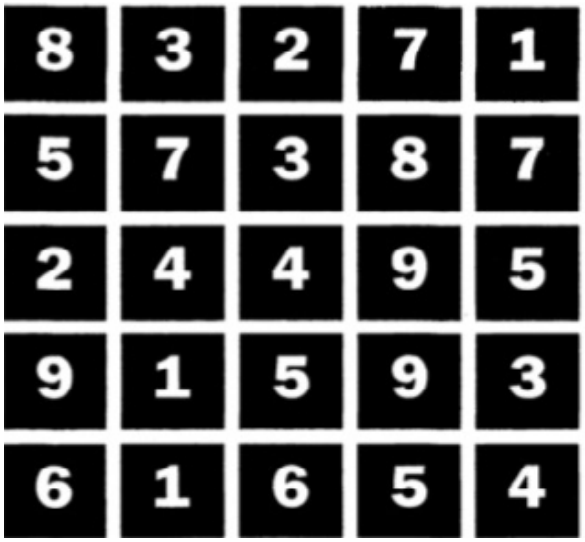
There are 4 groups of 6 square tiles.
The whole or shape B1 is composed of 24 square tiles. When the whole is divided into groups of 6 square tiles, then there are 4 parts, which means that 6 square tiles of the whole, 24 square tiles is the unit fraction, ¼.

MATH PUZZLES – FRACTION NUMBER PATH PUZZLES

How are these two fraction path puzzles similar and different? How does the choice and organization of the numbers influence the choice of possible fractions and the difficulty of the game? Make variations to these games to accommodate the fraction comparison practice for your students' needs.

Least to Greatest Fractions Path Puzzle

Which five-fraction number path ends with the greatest fraction?



Equivalent Fraction Path Puzzle

Which five-equivalent-fraction number path starts from one side to a different side?



Least to Greatest Fractions Path Puzzle

A five-fraction path is made from one side to a different side, by connecting numbers, horizontally, vertically and diagonally.

- Use arrows to show the direction of the fraction path.
- Start the path at the numerator of the first fraction, then its denominator. then the numerator of the second fraction, then its denominator.
- No number overlaps and no unused numbers between the numerator and denominator.

An Example

The five-fraction number path is:

- 1/6 (start)
- 1/5
- 4/7
- 3/2
- 7/1 (ends) on the opposite side



Equivalent Fraction Path Puzzle

A five-equivalent-fraction path is made from one side to a different side, by connecting numbers, horizontally, vertically and diagonally.

- Use arrows to show the direction of the fraction path and the connections between the numbers.
- Start the path at the numerator of the first fraction, then its denominator, then the numerator of the second fraction, then its denominator,
- No number overlaps and no unused numbers between the numerator and denominator

An Example

The five-equivalent fraction number path is:

- 4/4 (start)
- 4/4
- 2/2
- 2/2
- 4/4 (ends) on the opposite side



MATH GAMES - CARDS, DICE, INTERLOCKING CUBES AND NUMBER LINE BALANCE

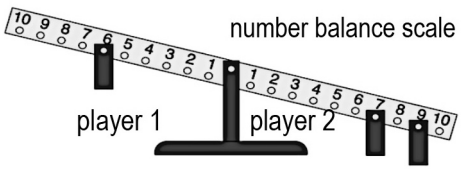
Using similar game playing rules and game boards, practise new concepts, like representing and comparing fractions Such game adaptations support students in understanding similarities and differences between different mathematical concepts.

Developing Students' Metacognition for Learning Mathematics

After playing a game, have students orally describe or write a reflection on the game-playing details in a math notebook or math journal. Consider these prompts.

- What fractions did you use as you played the game?
- Explain the fraction comparison methods you used while playing the game.
- When you play the game again, describe the fraction comparison method you will use to be more successful.
- Change one game detail, which makes the game simpler or more difficult to play.

Keep track of points on a number balance scale. If the goal of the game is to achieve 10 points, then use the number balance scale to show what fraction of 10 points each player has achieved. Use 1 hanging weight, starting at 0.



Concentration Fraction Game



Materials: 1 set of playing cards (aces are 1; face cards are 10)

Game Goal: Reach 10 points.

How to Play: (2 players or 2 teams)

- Lay out cards in an array face down.
- Both players flip over 2 cards at a same time and identify the fraction (greater number is the denominator).
- Player with the greater fraction collects the 4 playing cards.
- Game ends when there are no more cards left to turn over.
- Players count up the groups of 4 cards they have collected. Player with the greater number of cards gets 1 point. If a player has double the number of groups of 4 cards, then the player gets 2 points.

Fractional Quantities Game

Materials: 2 colour counters, 6 dice, game board

22	47	16	41	10	35	4
5	23	48	17	42	11	29
30	6	24	49	18	36	12
13	31	7	25	43	19	37
38	14	32	1	26	44	20
21	39	8	33	2	27	45
46	15	40	9	34	3	28

Game Goal: Reach 10 points.

How to Play:

- Players roll 2 dice to determine which number (the whole) to cover on the gameboard, with colour counters.
- Players roll 3 dice, in order to select two numbers that can represent a fraction less than 1.
- Players calculate the fractional quantity of the selected whole on the game board.
- Players compare their fractional quantities. Player with the greater quantity collects 1 point. If the fractional quantity is double the other player's fractional quantity, then the player collects 2 points.

Fraction Battle Game

Materials: 6 dice



Game Goal: Reach 10 points.

How to Play:



- Each player rolls 3 dice at the same time.
- Players call out the least fraction using any 2 of the 6 dice.
- The player with the greatest fraction collects 1 point.
- The game ends when a player has collected 10 points.

Variations

- Each player rolls 6 dice and calls out the least fraction in the set of dice.
- Players compare their choice of least fraction.
- Player with the least fraction collects 1 point.

CLOSING

Our next issue (Vol 59, Issue 2) will focus on practice tasks (quick images, math puzzles and games) for representing and comparing fractions, using equivalence and interpreting fractions as a multiplier. These practice tasks can be used for mini lessons and practice lessons.

CALL FOR *ABACUS* SUBMISSIONS

Send your teaching strategy ideas, problems and student solutions to the *Abacus* Co-Editors via email kathkubo@rogers.com or maryloukestell@gmail.com