8 PAGE INSERT ELEMENTARY MATH

ONTARIO ASSOCIATION FOR MATHEMATICS EDUCATION

Abacus

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REPRESENTING AND COMPARING FRACTIONS

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ABACUS CO-EDITORS



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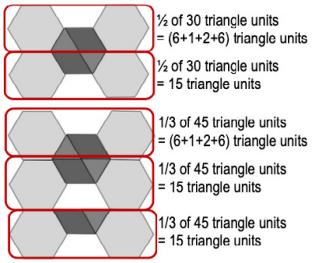
KATHY KUBOTA-ZARIVNIJ currently focuses on K to 6 mathematics professional learning at the provincial level. She is a long-time OAME board member and OAME past president. In relation to her work in mathematics education, she uses complexity thinking to interpret mathematics teaching and learning for students and teachers. **kathkubo@rogers.com**

This issue includes practice tasks for representing and comparing fractions using equivalence and interpreting fractions as a multiplier, using different contexts, models and methods. Depending on the context, the fraction representations could include any of these models: set of objects (bar model, array, objects of different shapes and sizes), linear measure (number line), area of a rectangle and numeric/algebraic expressions/equations. Solve this fraction brainteaser using different methods.

True or False? 1/2 of 30 > 1/3 of 45

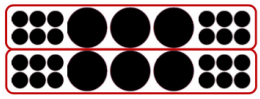
- What mathematics could be used to solve this brainteaser?
- Why might Solution 1 be discussed first, followed by Solution 2 and Solution 3?

Solution 1: Pattern Blocks with Triangle as 1 Unit of Different Wholes

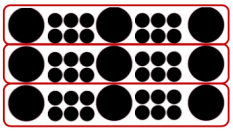


False: ½ x 30 triangles
= 30/2 triangles
= 15 triangles
= 45/3 triangles
= 1/3 x 45 triangles

Solution 2: Set of Different Size Objects of Different Wholes



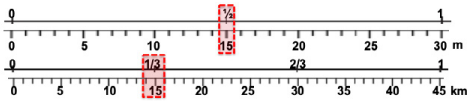
½ x 30 circles = 30/2 = 15 circles



1/3 x 45 circles = 15 circles

False:
½ x 30 circles = 30/2 circles = 45/3 circles
= 1/3 x 45 circles = 15 circles

Solution 3: Number Line with Different Size Linear Units of Different Wholes



True: the linear unit, 1 km is greater than 1 m, so 15 km > 15 m

This December 2020 issue includes a variety of fraction representation and comparison practice tasks, which can be included in your classroom mathematics program as: (a) five-minute quick practice; (b) fifteen-minute mini-lesson; (c) lesson problem for practice lessons; (d) at-home practice; and/or (e) family math activities.

DESIGNING PRACTICE TASKS – MATHEMATICS CONTENT FOCUS

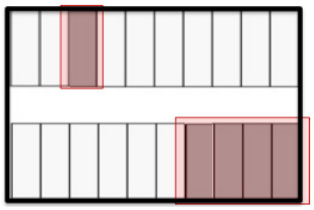
What do students need to know and do mathematically to be proficient with representing and comparing fraction multipliers of a whole, set and measure? When designing fraction practice tasks, consider these mathematical details.

Conceptual Understanding

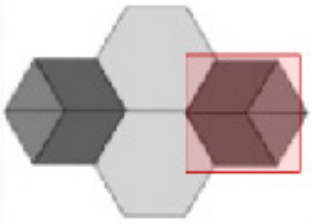
Relate the representation of fractions as a multiplier of a whole, set and measure in different real-life contexts to the models of representation listed below.

a) *Fraction as Multiplier of a Whole*

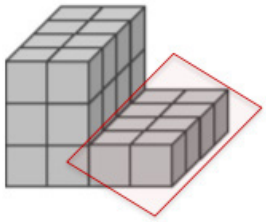
The fraction multiplier of a whole determines the total number congruent parts for which a whole is partitioned. The fraction as a multiplier also specifies the number of parts of that whole. For example, the whole could be the area of a rectangular parking spaces, the number of cubes in a complex rectangular prism and a 2D shape made up of pattern blocks, where 1 triangle is 1 unit.



¼ x 20 parking spaces
= 0.25 x 20 parking spaces
= (25 x 0.01) x 20 parking spaces
= (25 x 20) x 0.01 parking spaces
= 500 x 0.01 parking spaces
= 5 parking spaces for 1 part



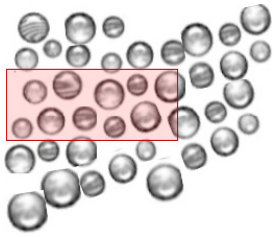
¼ x 24 triangles
= 0.25 x 24 triangles
= (25 x 0.01) x 24 triangles
= (25 x 24) x 0.01 triangles
= [(25 x 20) + (25 x 4)] x 0.01 triangles
= 6 triangles for 1 part



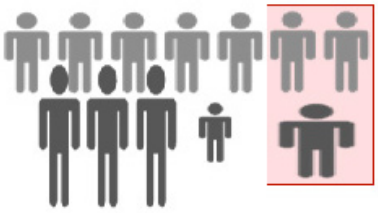
¼ x 32 cubes
= 0.25 x 32 cubes
= (25 x 0.01) x 32 cubes
= (25 x 32) x 0.01 cubes
= [(25 x 30) + (25 x 2)] x 0.01 cubes
= 8 cubes for 1 part

b) *Fraction as a Multiplier of a Set of Objects*

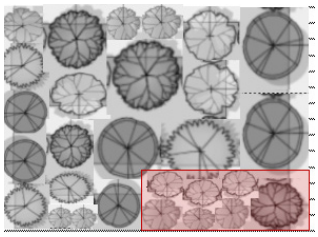
The fraction multiplier of a set of discrete objects of same and different shapes and sizes determines the total number objects for which the set is partitioned. The fraction as a multiplier also specifies the number of objects in that set. For example, a set of objects could be marbles on the floor, people waiting to pay for groceries and trees in a forest.



¼ x 40 marbles
= 0.25 x 40 marbles
= (25 x 0.01) x (4 x 10) marbles
= (25 x 4) x (0.01 x 10) marbles
= [100 x 0.1] marbles
= 10 marbles for ¼ of 1 set



¼ x 12 people
= 0.25 x 12 people
= (25 x 0.01) x 12 people
= (25 x 12) x 0.01 people
= [(25 x 10) + (25 x 2)] x 0.01 people
= 3 people

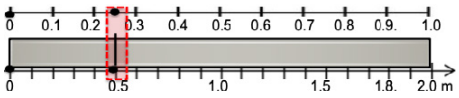


= ¼ x 28 trees
= 0.25 x 28 trees
= (25 x 0.01) x 28 trees
= (25 x 28) x 0.01 trees
= [500 + 200)] x 0.01 trees
= 7 trees

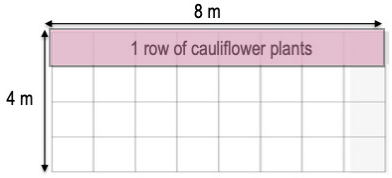
c) *Fraction as a Multiplier of a Measure*

The fraction multiplier for a measure determines the total number of equal-size units for which a measure is partitioned, such as length units (cm), square units (cm2) and cubic units (cm3). The fraction as a multiplier also specifies the number of units for the measurement. For example, lengths of ribbon strips, area of cauliflowers growing in a rectangular garden and the volume of cubes in a rectangular prism made up of cubes.

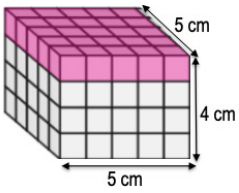
¼ x 2 m ribbon strips
= 0.25 x 2 m ribbon = (25 x 0.01) x 2 m ribbon
= [(25 x 2) x 0.01] m ribbon = 50 x 0.01 m ribbon
= 0.5 m ribbon



¼ x 32 m2 rectangular garden is cauliflower
= 0.25 x 32 m^2 garden = (25 x 0.01) x 32 m^2 garden
= (25 x 32) x 0.01 m2 garden
= [(25 x 30) + (25 x 2)] x 0.01 m^2 garden
= [(750 + 50) x 0.01] m^2 garden
= 800 x 0.01 m^2 garden = 8 m^2 garden



¼ x 100 cm^3 cubes in a rectangular prism
= 0.25 x 100 cm^3 cubes
= (25 x 0.01) x (10 x 10) cm^3 cubes
= 25 x (0.01 x 10 x 10) cm^3 cubes
= 25 cm^3 cubes



1 layer of (5 x 5) 1 cm^3 cubes
= 1 layer of 25 1 cm^3 cubes

Procedural Fluency

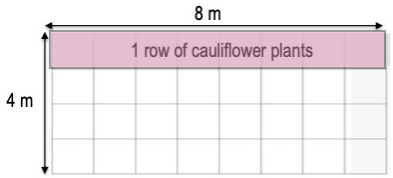
Equivalent fractions represent the same part of the whole. Though equivalent fractions have different numerators and denominators, they represent the same value or proportion of the whole; that is their fractional values are the same.

Two different fractions that are equivalent have numerators and denominators that are multiples of each other. Equivalent fractions are produced by multiplying or dividing the numerator and denominator of a fraction by the same number.

- Multiplying the numerator and denominator by the same number increases the number of parts and total parts proportionally, resulting in a greater number of smaller parts

  ¼ x 32 m^2 garden = ¼ x 8/8 x 32m^2 garden
  = 8/32 x 32m^2 garden = 8 m^2 garden

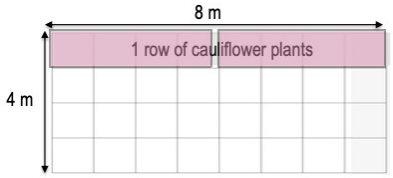
  For ¼, the garden was partitioned into 4 congruent parts. For the equivalent fraction of 1/4 = 8/32, the garden was partitioned into 32 congruent parts.



- Dividing the numerator and denominator by the same number decreases the number of parts and total parts proportionally, resulting in a smaller number of larger parts.

  ¼ x 32 m^2 garden = 8/32 x 32m^2 garden
  = (8/32 ÷ 4/4) m^2 garden = 2/8 m^2 garden

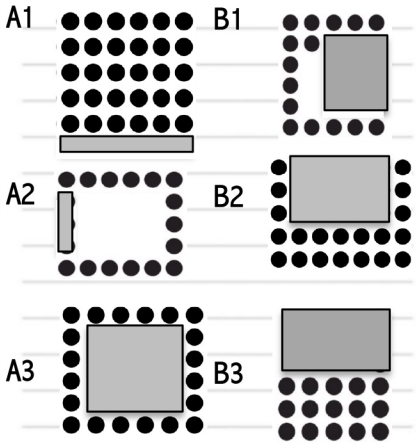
  For the equivalent fraction of ¼ = 8/32 = 2/8, the garden was partitioned into 8 congruent parts.



"QUICK IMAGES, MATH PUZZLES AND GAMES MAT" - Fraction Equivalence and Multiplier

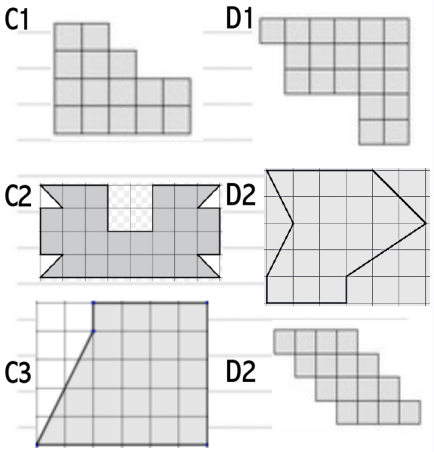
QUICK IMAGE "DOTS"

The dots shown are 2/5 of the whole. How many dots make up the whole?



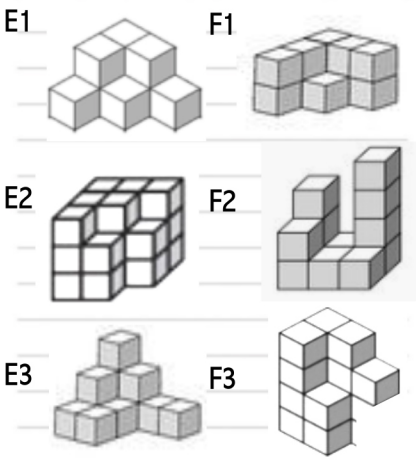
QUICK IMAGE "SQUARES"

The square units shown are 1/3 of the whole. How much is the whole?



QUICK IMAGE "CUBES"

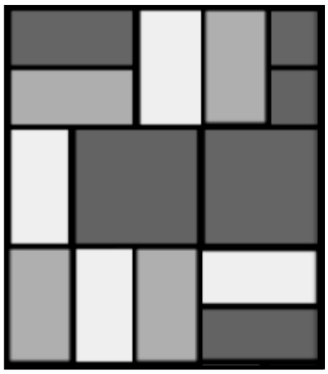
The cubes shown are 4/6 of the whole. What does the whole look like?

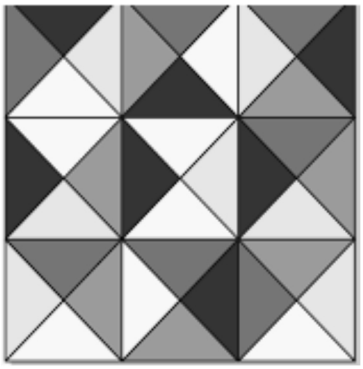


RECTANGLE BRAIN TEASER

Draw sets of equivalent fractions when:

- 1/8 whole =2 small squares
- 1/3 whole =2 rectangles
- 1 whole = 2 medium size squares
- 1 whole = 5 rectangles





TRIANGLE BRAIN TEASER

Draw sets of equivalent fractions when:

- 1 whole = 12 triangles
- 1 whole = 16 triangles

FRACTIONS CONCENTRATION MATH GAME

Materials -1 set of playing cards with no face cards, 1 dice, pattern blocks, meter stick, square tiles

Game Goal - Collect the greatest number of points.

How to Play: (2 players or 2 teams)

- Spread out playing cards, face down. Players take turns revealing 4 playing cards to create equivalent fractions.
- If a pair of equivalent fractions cannot be made, the player 1 turns over face down 2 or 4 cards.
- Player 2 turns over 4 cards to make a pair of equivalent fractions and can include the existing cards left turned up.
- When a player makes a pair of equivalent fractions, the player keeps the 4 cards and turns over 4 new cards to make pairs of equivalent fractions.
- When there are no more cards to turn over, players count the number of cards they have collected. Each card is 1 point.

FRACTIONS BATTLE MATH GAME

Materials - 2 decks of playing cards

Game Goal – Collect the greatest number of points.

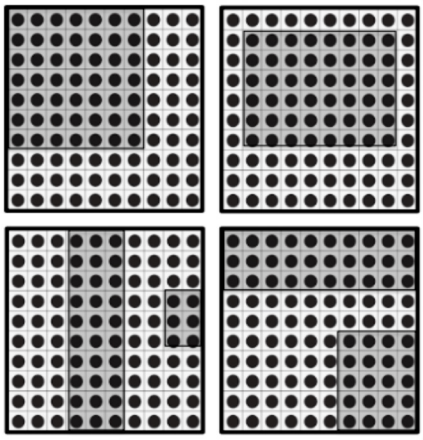
How to Play: (2 players or 2 teams)

- Each player gets a deck of playing cards using only numbers 2 to 10
- Players simultaneously flip over 1 card, which is used to make a fraction. (e.g., 1 card is the numerator and the 2nd card is the denominator)
- Players battle by simultaneously flip over 2 cards to make a fraction. First player to identify equivalent fractions collects the equivalent fraction cards.
- When there are no more cards to flip over, the game ends and players count the number of cards they have.
- Cards with numbers 2 to 6 are worth 1 point. Cards with numbers, 7 to 9 are worth 2 points.

"QUICK IMAGES, MATH PUZZLES AND GAMES" MAT – Fraction Equivalence and Multiplier

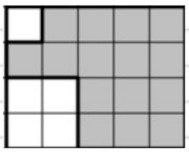
QUICK IMAGE "DOTS"

Dark dots are ½ of the whole. Lighter dots are 1/3 of the whole. Consider each case. Which dots have the greater whole?



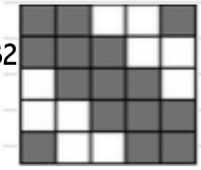
QUICK IMAGE "SQUARES"

B1



Each design is made up of square tiles.

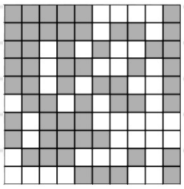
B2



The darker tiles are 1/5 of whole for each design.

How many tiles are in each design?

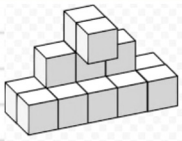
B3



What does the whole look like for each design?

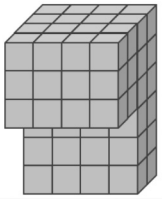
QUICK IMAGE "CUBES"

C1



The cubes shown are 2/6 of 3 different wholes.

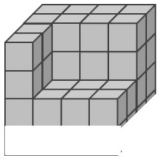
C2



How many cubes are in each prism?

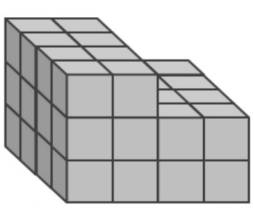
How many cubes are in each whole?

C3

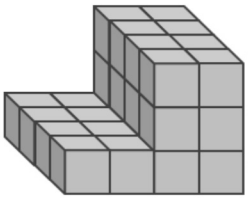


What does the whole look like for each complex rectangular prism?

WHICH IS GREATER? BRAIN TEASER

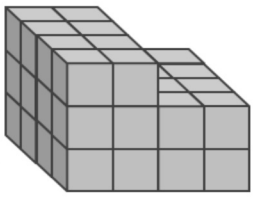


2/5 of 3D figure A

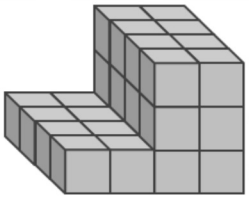


2/8 of 3D figure B

WHAT IS THE WHOLE? BRAIN TEASER



A is 2/5 of the whole



B is 2/8 of the whole

FRACTION OF THE AREA MATH GAME

Materials – 4 dice, 10 x 10 square grid, markers

Game Goal – Cover the greatest area on the grid.

How to Play:

- Players take turns rolling 4 dice and take turns selecting 2 dice to represent the fraction multiplier and the 2 dice to represent factors to determine the number of square units for the whole.
- Players shade in the fractional quantity of the whole on their square grid as a rectangle.
- Game ends when a player cannot record a rectangle within the empty spaces remaining on the 10x10 square grid.
- Example: for dice rolls 3, 5, 3, 4
  3/3 x (5x4) = 1 x 20 = 20 square units
  Marking a 2 x 10 square unit rectangle on the 10x10 square game board grid

FRACTION EQUIVALENCE MATH GAME

Materials - 1 set of playing cards with no face cards

Game Goal – Make 2 sets of equivalent fractions

How to Play: (2 people or 2 teams)

- Players each receive 8 cards; not to be shown. The remaining cards are in a pile, face down.
- Players take turns picking up and then discarding 1 card from their set of 8 cards, to collect sets of cards that can be organized as equivalent fractions.
- When a player has 2 sets of equivalent fractions, then "EQUIVALENCE" is called and the game ends.
- Example: for playing cards, 5, 5, 2, 2, 3, 4, 6, 8
  Equivalent fraction pairs are:
  5/5 = 2/2 and ¾ = 6/8

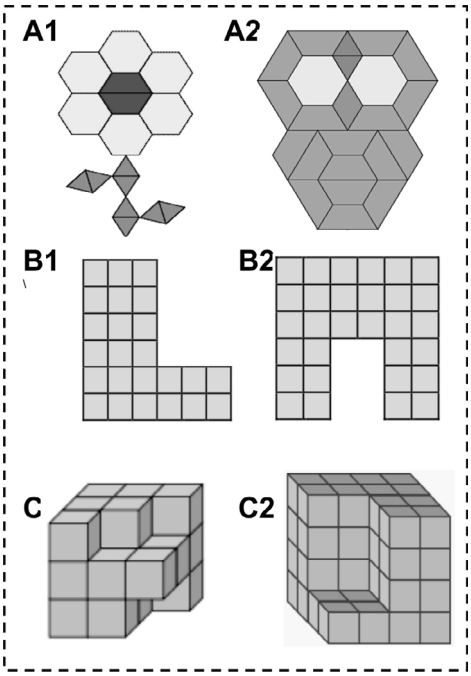
DESIGNING PRACTICE TASKS – MATHEMATICS CONTENT FOCUS

QUICK IMAGES - Squares, Pattern Blocks and Cubes

Using quick image routines provide predictable practise of seeing and explaining relationships between spatial visualization, mental math calculations, mathematical reasoning and communication.

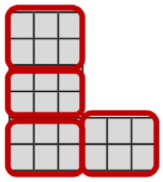
Quick Image Routine – Fraction of a Whole

- Show one image for 3 seconds.
- Students build with materials (e.g., pattern blocks, square tiles, cubes) or sketch each image using pattern block paper, square grid paper and/or isometric drawing paper.
- Show the same quick image for 3 seconds again.
- Students make revisions to their structure or drawing.
- Show the same quick image for 3 seconds for the last time.
- Class Discussion - Identify the area.



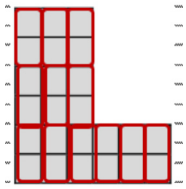
True or False? Equivalent fraction multipliers change the fractional quantity?

¼ x 24 squares = 0.25 x 24 squares
= (25 x 0.01) x 24 squares
= (25 x 24) x 0.01 squares
= [(25 x 20 + (25 x 4)] x 0.01 squares
= (500 + 100) x 0.01 squares
= 600 x 0.01 squares
= 6 squares for ¼ of 24 squares



An equivalent fraction for ¼ of 24 can be determined by multiplying the numerator and denominator by the same number, which increases the number of parts and total parts proportionally, resulting in a greater number of smaller parts.

(¼ x 3/3) x 24 squares = 3/12 x 24 squares
For 3/12, the shape of 24 squares was partitioned into 12 congruent parts.
For the equivalent fraction of 1/4 = 3/12, the shape was partitioned into 12 congruent parts. The greater the denominator, the greater the number of smaller total parts.



(¼ x 6/6) x 24 squares = 6/24 x 24 squares
For 6/24, the shape of 24 squares was partitioned into 24 congruent parts.
For the equivalent fraction of 1/4 = 6/24, the shape was partitioned into 24 congruent parts.
The greater the denominator, the greater the number of smaller total parts.

Therefore, the equivalent fraction multipliers, ¼ = 3/12 = 6/24 do not change the fractional quantity of the whole, as ¼ x 24 = 3/12 x 24 = 6/24 x 24 = 6 squares. But the number of partitions for the 24 squares changed from 4 partitions, to 12 partitions and then 24 partitions.

Viewing the images for a brief period of time (3 seconds) motivates students to create and use efficient methods to determine multiplicative relationships of the fraction multiplier and their equivalent fractions for each image.

MATH PUZZLES – Fraction Number Path Puzzles

How are these two fraction path puzzles similar and different? How does the choice and organization of the numbers influence the choice of possible fraction multipliers and the difficulty of the game? Make variations to these games to accommodate the fraction comparison practice for your students' needs.

Fractional Quantity Path Puzzle

Which fraction number path ends with a calculation result closest to the last number?

8	3	2	7	1
5	7	3	8	7
2	4	4	9	5
9	1	5	9	3
6	1	6	5	4

Fractional Quantity Path Puzzle

Which fraction number path ends with a calculation result closest to the last number?

4	2	4	2	4
2	4	4	4	2
4	4	2	2	4
2	4	2	4	2
2	4	2	2	2

Fraction Quantity Path Puzzle

Materials - 5 x 5 square number grid for each player

Goal – To calculate fraction quantities of 6 different wholes that related through any operation(s) to result in a number closest to the last number.

Game-Playing

1. Start at a corner of the grid.
2. For the 1st set of 4 numbers, draw a rectangle around a 2x2, 1x4 **OR** 4x1 array, which contains 4 numbers.
3. Select 2 numbers to be the numerator and denominator of the fraction multiplier and the other 2 numbers to be related by any operation(s) to result in a number value of a whole. Calculate the fraction quantity of that whole.
   (e.g., 8, 5, 3, 7; 3/5 of 15 = 9)
4. Repeat (2) and (3) for each subsequent set of 4 numbers until the last number is reached.
5. Use arrows to track of the fraction quantity path.

Scoring:

- 1st set, 8,5,3,7; 3/5 x (8 + 7) = 9
- 2nd set, 2,4,9,1; 2/4 x (9 – 1) = 4
- 3rd set, 6,1,6,5; 6/6 x (5 – 1) = 4
- 4th set, 4,9,9,5; 9/9 x (5 – 4) = 1
- 5th set, 2,3,7,8; 2/8 x (7 –– 3) = 1
- 6th set, 1,7,5,3; 1/3 x (7 + 5) = 4



Calculation of scores to compare to the final number, 4
[(9 – 4 – 4) x 1] x (1x 4) = (1x1) x (1x4) = 4. This final calculation is equivalent to the last number. This player wins.

Equivalent Path Puzzle

Materials - 5 x 5 number square grid for each player

Goal – To calculate fraction quantities of 6 different wholes that related through any operation(s) to result in a number closest to the last number.

Game-Playing

1. Start at a corner of the grid.
2. For the 1st set of 4 numbers, draw a rectangle around a 2x2, 1x4 or 4x1 array, which contains 4 numbers.
3. Select 2 numbers to be the numerator and denominator of the fraction multiplier and the other 2 numbers to be related by any operation(s) to result in a number value of a whole. Calculate the fraction quantity of that whole. (e.g., 4,2,4,2; 2/4 x (4 x 2) = 4)
4. Repeat (2) and (3) for each subsequent set of 4 numbers until the last number is reached.
5. Use arrows to track of the fraction quantity path.

Scoring:

- 1st set, 4,2,4,2; 2/4 x (4 x 2) = 4
- 2nd set, 2,4,4,4; 4/4 x (2 x 4) = 8
- 3rd set, 4,4,2,2; 2/2 x (4 – 4) = 0
- 4th set, 2,4,2,4; 4/2 x (4 x 2) = 8
- 5th set, 2,4,2,2; 2/2 x (4 x 2) = 8
- 6th set, 2,2,4,2; 4/2 x (2 x 2) = 8



Calculation of scores to compare to the final number, 4.
4 + 8 + 0 – 8 + 8 – 8 = 12 – 8 + 0 = 4

This final calculation is equivalent to the last number, 4. This player wins.

MATH GAMES – CARDS, DICE, INTERLOCKING CUBES AND NUMBER LINE BALANCE

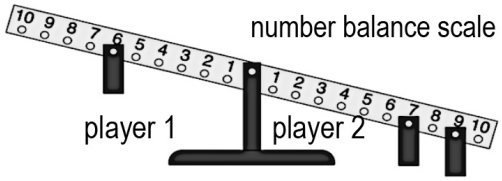
Use similar game boards and game-playing rules to practise new concepts, like representing and comparing fraction multipliers of a whole, set and measure. Such game adaptations support students in understanding similarities and differences between different mathematical concepts.

Developing Students' Metacognition for Learning Mathematics

After playing a game, have students orally describe or write a reflection on the game-playing details in math game journal. Consider these prompts.

- What fraction multipliers did you use most often as you played the game?
- Explain the methods you used to calculate fraction multipliers of a whole, set and measure, such as calculations, visualizing area of a rectangle in square units or a number line?
- When playing a game again, describe which fraction multiplier you selected most often. Why?
- Change one game detail, which makes the game simpler or more difficulty to play.

Keep track of points on a number balance scale. If the goal of the game is to achieve 10 points, then use the number balance scale to show what fraction of 10 points each player has achieved. Use 1 hanging weight, starting at 0.



Concentration Fraction Game

Materials: 1 set of playing cards (aces are 1; face cards are 10)



Game Goal:
Reach 10 points.

How to Play: (2 players or 2 teams)

- Lay cards face down in an array.
- Both players flip over 2 cards at a same time and each select the fraction multiplier and whole that results in a least fraction quantity.
- Player with the least fraction quantity collects the 4 cards.
- Game ends when there are no more cards left to turn over.
- Players count up the number of cards collected.
- Player with the greatest number of points gets 1 point. If a player has ½ the number of cards as the other player, that player gets 2 points.

Fractional Quantities Game

Materials:
2 colour counters, 6 dice, game board

22	47	16	41	10	35	4
5	23	48	17	42	11	29
30	6	24	49	18	36	12
13	31	7	25	43	19	37
38	14	32	1	26	44	20
21	39	8	33	2	27	45
46	15	40	9	34	3	28

Game Goal:
Reach 10 points.

How to Play:

- Players take turns rolling 2 dice to determine the quantity of the whole.
- Players roll 3 dice, to select two numbers that can represent a fraction multiplier.
- Players calculate the fractional quantity of the selected whole and cover that number on the game board, using 1 colour counter, if that number is on the game board.
- The game ends when a player has covered 7 numbers.

Fraction Battle Game II

Materials: 8 dice



Game Goal:
Reach 10 points.

How to Play:

- Each player rolls 4 dice at the same time.
- Players call out a fraction multiplier using any 2 dice. The remaining dice may be combined to make the least whole.
- The greatest fractional quantity is worth 1 point.
- The game ends when a player has collected 10 points.

Variations

- Each player rolls 8 dice and calls out the least fraction multiplier and combines the least whole with the remaining 6 dice.
- Player with the least fraction quantity collects 1 point.

CLOSING

Our next issue (Vol 59, Issue 3) will focus on practice tasks (quick images, math puzzles and games) for representing and comparing fractions, decimals and percent of a whole, set and measurement. These practice tasks can be used for mini lessons and practice lessons.

CALL FOR *ABACUS* SUBMISSIONS

Send your teaching strategy ideas, problems and student solutions to the *Abacus* Co-Editors via email kathkubo@rogers.com or maryloukestell@gmail.com