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8 PAGE INSERT ELEMENTARY MATH

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MAKING SENSE OF FRACTIONS – EQUIVALENCE FOR FRACTIONS, DECIMALS, AND PERCENTS

TABLE OF CONTENTS

ABACUS EDITOR GREETINGS 1
RESEARCH SUMMARY
- Fractions and Equivalence 2
LINKS TO MANIPULATIVES
-Paper Folding. 3
LET'S DO MATH:
Pancake Problem (with bansho plan) 4-5
LINKS TO LITERATURE 6
- The Lion's Share:: A Tale of Halving Cake and Eating It, Too
LET'S DO MATH:
Garden Problem. 6
LET'S DO MATH:
Fun Fair Crispy Quads Problem 7
NEXT STEPS FOR YOUR PROFESSIONAL LEARNING
- Application to Your Classroom
- Suggested Readings

A Publication of the OAME/AOEM

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Fosnot and Dolk (2002) remind us that "children need to be allowed to grapple with big ideas and progressively refine their strategies. Part of what it means to do mathematics is to explore relationships and find ways to make your procedures, proofs, and strategies elegant. Support the human drive to create, to know, and to make efficient – drive for understanding, efficiency, and elegance is at the heart of what it means to be a young mathematician at work" (pp. 70-71). So, what are the ways that students can construct robust understandings about and strategies for determining equivalence between fractions, decimals, and percents through experience, rather than being told to follow rules like "make the denominators the same" and "whatever you do to the bottom you do to the top?" Kamii (1995) argues that by telling or showing students that certain fractions are equivalent deprives them of the possibility of thinking and inventing notions of equivalence from their own direct experience. So, teaching needs to include the use of realistic problems that provoke students to invent solutions, so that understandings of fractional equivalence can grow out of their own mathematical thinking. For example, rather than providing students with ready-made pictures and fraction circles, students record their own figurative knowledge on paper.

In this issue, the Research Summary and problems (with multiple solutions) focus on the underlying notions of equivalence through conceptual modeling and strategic calculations. As suggested in our previous Abacus issues, consider solving these problems yourself first before examining the solutions we offer you. Also, think about how these problems can be used within the framework of a three-part, problem-solving lesson. It's been our experience that when we do the mathematics ourselves, we are deepening our conceptual understanding of the mathematics that we are teaching, and we are compelled to rethink our choices of learning materials and teaching strategies to further our students' learning.

LINKS TO LITERATURE AND MANIPULATIVES



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The literature connection for this issue focuses on the picture book, *"The Lion's Share: A Tale of Halving Cake and Eating It, Too"* by Matthew McElligott. This book can also be used to discuss character traits such as respect, sharing, and fairness. Paper, a manipulative that is often overlooked, can be used to model fractions, and through paper folding, constructing equivalent fractions is particularly effective.

RESEARCH SUMMARY – EQUIVALENCE FOR FRACTIONS, DECIMALS, AND PERCENTS

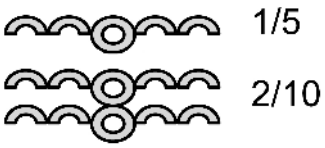
Randall (2005) identifies a big idea for equivalence: "Every fraction/ratio can be represented by an infinite number of different but equivalent fractions/ratios" and "Decimal numbers can be named in an infinite number of different but equivalent forms" (p. 14). This notion of equivalence is often used in comparing fractions. Fosnot and Dolk (2002) remind us that in order to compare two fractions, the whole must be the same, yet the fractional pieces of the whole do not have to be congruent, but do have to be equivalent. So, for students to understand this underlying idea of equivalence, they need concrete experiences in exploring how a fractional quantity stays the same even though its parts look different. These big ideas are a developmental leap that often requires a shift in the learner's perspective about representations of fractions.

Further, Small (2009) describes fractional equivalence in terms of fractions having more than one name. For example, 1/5 is another name for 2/10, since the same part of the same whole is represented by both fractions.



$$\frac{1}{5} \rightarrow \frac{1 \times 2}{5 \times 2} \rightarrow \frac{2}{10}$$

The area and set models show that the number of parts of the rectangle or set is doubled with smaller size parts for the area model) and same proportion of parts to the whole. So 1 part out of 5 becomes 2 parts out of 10.



Equivalent Improper Fractions and Mixed Numbers

The improper fraction, 8/3 can be renamed as a mixed number, 2 and 2/3. In this case, the improper fraction is conceived as division to show whole number and fractional parts. For example, 8/3 → 8 ÷ 3 = 2 2/3, since 8/3 makes 2 wholes (6/3 =2) and 2 thirds or 8 ÷ 3 = 2 R2 where the remainder is 2 out of 3 parts. The remainder is the fraction part of the mixed number.

Equivalent Fractions, Decimals, and Percents

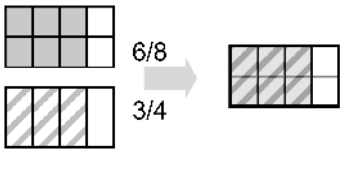
According to Fosnot and Dolk (2002), decimals and percents are really just specific equivalents of fractional relations. Decimals are fractional base-ten equivalents making use of place value, and percents are relationship based on a one-hundred-part whole. With decimals and percents, just as with fractions the whole matters. Forty hundredths means 40/100, and forty percent means 40/100. So, 40/100 = 0.40 = 40%. In these cases, the whole is the same, 100. Decimals and percents are additional instances of fractional equivalences and bring up new insights about the relationship between place value and quantity.

Strategies for Determining Equivalence

How do you know that 6/8 = 3/4?

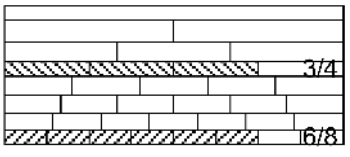
Rectangular Grid Overlay

- Using two same size rectangles (same size whole), divide one rectangle into 8 equal-size parts (shade in 6 of the 8 parts) and the other rectangle into 4 equal-size parts (shade in 3 of 4 parts).
- Overlay one rectangle over the other and compare the area of the shaded parts to check for equivalence (and/or to see which fraction is smaller (or larger) in comparison to the other fraction).



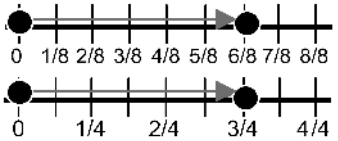
Fraction/Percent Strip

- Highlight 6/8 on the fraction/percent strip and 3/4 on the strip.
- Compare the lengths for equivalence.

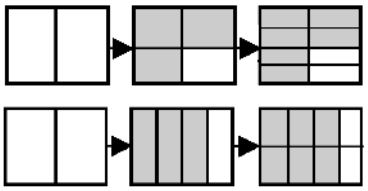


Double Number Line

- Draw two number lines with same size whole, marking 6/8 and 3/4.
- Compare their lengths for equivalence.

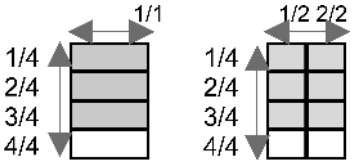


Paper Folding



- Fold a sheet of paper in fourths (or fold in half, then in half again) widthwise (or lengthwise)
- Shade in three of the fourths.
- Fold the paper again in half lengthwise (or widthwise).
- The part that was 3/4 is now 6/8, so 3/4 is equivalent to 6/8

Linear Dimensions and Area of a Rectangle



- On a rectangle, mark off one length as 3/4 and the other length as 2/2, to get eighths.
- Do you still have 3/4 of 1 whole?
  3/4 x 1/1 → (3x1)/(4x1) → 3/4
  and 3/4 x 2/2 → (3x2)/(4x2) → 6/8.

Ratio Table

A table can be used to record multiplying or dividing a numerator and denominator by the same amount. Doubling and halving are useful strategies.

3	(x1) **3**	(x2) **6**	(x3) **9**	(x4) **12**
4	(x1) **4**	(x2) **8**	(x3) **12**	(x4) **16**

Separating into Parts

- Separating a larger part into equivalent smaller parts has the same effect as multiplying original fraction (numerator and denominator) by fractional equivalents of 1 (e.g., 5/5, 2/2)
- For example, 2/2 can be separated into 10 parts by multiplying the numerator and denominator by 5 → 2/2 x 5/5 => (2x5)/(2x5) =10/10; 2/2 can be separated into 4 parts by multiplying the numerator and denominator by 2 → 2/2 x 2/2 = (2x2)/(2x2) = 4/4.

Combining Parts

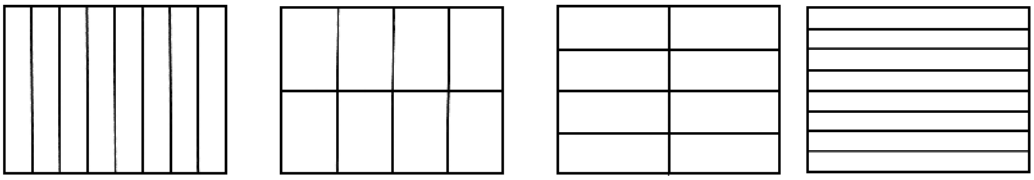
- Combining smaller parts to make an equivalent larger part; it has the same effect as dividing the numerator and denominator fractional equivalents of 1 (e.g., 2/2)
- For example 6/8 can make equivalent larger parts by dividing by 2/2 → 6/8 ÷ 2/2 → (6÷2)/8÷2) = 3/4;
- 24/12 can make equivalent larger parts by dividing 24/12 by 6/6 → 24/12 ÷ 6/6 → (24÷6)/(12÷6) = 4/2

LINKS TO MANIPULATIVES: PAPER FOLDING

Paper folding a rectangular piece of paper involves visualizing, reasoning, and making connections between conceptual representations of multiplication, division, and fractions. Here are three types of paper folding tasks:

Predicting the Outcome of a Sequence of Folds.

- If you fold this paper in half 3 times, how many equal parts will you create?
- Students who are using additive rather than multiplicative thinking will predict that there is one more space after each fold.
- Students can compare solutions in which the paper was folded in various ways. (e.g.,
- How does each of these solutions show that 1/2 is the same as 4/8?



Producing a Target Number of Equal Parts.

- Fold a piece of paper so that you create 12 equal parts.
- What different ways (or sequence of folds) can the paper be folded to create 12 equal parts?

Predicting the Final Fold to Product a Target Number of Equal Parts

- Camilla folded a piece of paper into 3 equal parts, then 8 equal parts. Derrick folded his piece of paper into 6 equal parts. If he wants to make exactly as many parts as Camilla, how many times should he fold his paper?
- If Camilla folded a piece of paper into 8 equal parts then 3 equal parts, does that change how Derrick folds his paper after he makes 6 equal parts? Explain.

LESSON LEARNING GOALS

- Curriculum Expectations (MOE, 2005) (not to put on the board with students; for board (bansho) plan only)
- Show relevant curriculum expectations the grade before and after the grade-specific learning goal, to see development of mathematical thinking across grade levels. Use the curriculum expectations to determine the appropriateness of the problems chosen for Before (activation), During (lesson), and After (practice)

BEFORE (Getting Started)

- 5 to 10 minutes
- Activating students' mathematical knowledge and experience that is directly related to the mathematics in the lesson problem
- Use a smaller problem similar to previous lesson problem
- Use student work responses for class analysis and discussion to highlight key ideas and/or strategies

AFTER (Consolidation)

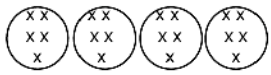
- 20 to 25 minutes
- Teacher selects 2 or more solutions for class analysis and discussion.
- Teacher decides which solution to share 1st, 2nd, 3rd based on mathematical relationships between the solutions.
- Students (authors) explain and discuss their solutions with their friends.
- During discussion, solutions are organized to show mathematical elaboration from one solution to the next and towards lesson learning goal.
- Teacher mathematically annotates on and around solutions to make mathematical ideas, strategies, and models of representation explicit to students, using students' ideas, relating students' informal vocabulary with formal vocabulary.

Pancake Problem A:

Four pancakes are shared among 5 children.
How much pancake does each child get? Explain your thinking.

- *divide 1 pancake into 5 pieces (for 5 children), so each child gets 1/5 of 1 pancake; so for 4 pancakes, one child gets 1/5 of 1 pancake x 4 pancakes or 4/5 pancake*

Solution 1



Solution 2



1/5 four times
4/5 of 1 pancake

DURING (Working On It)

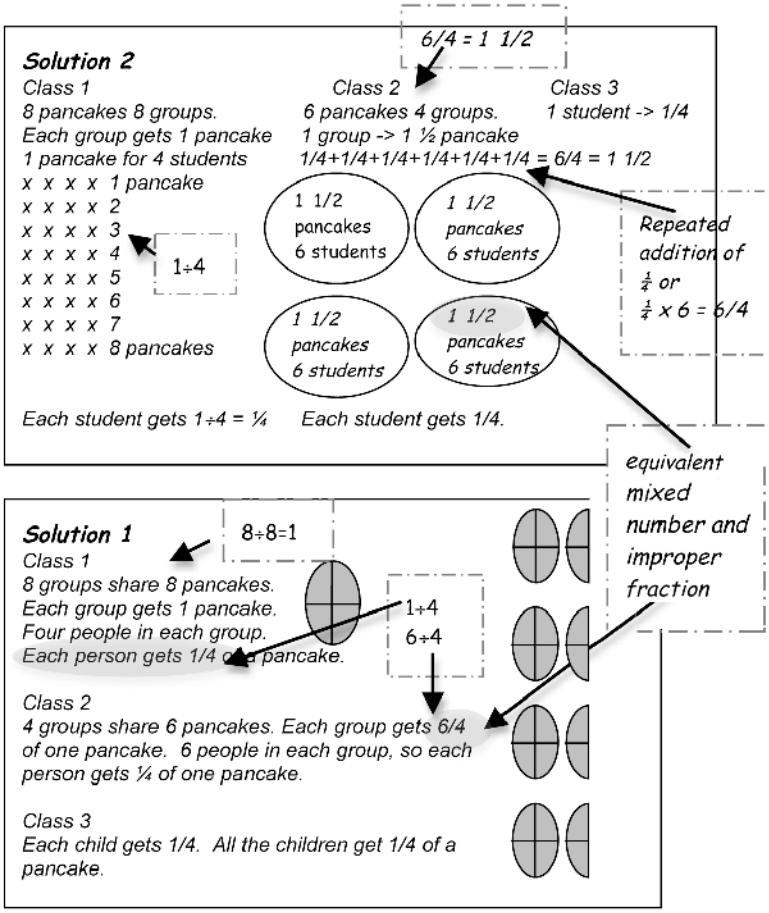
- 20 minutes
- Understand the problem. Make a Plan. Carry Out the Plan.
- Understanding the problem – Ask, "What information from the problem will we use to make a plan to solve it? Explain."
- Teacher records in a list, below the problem, the information that students identify (We don't want different solutions to different problems; we want different solutions to the same problem. Students are learning to discern important information (i.e., problem definition)).
- Students solve the problem on chart paper with markers (visible for later discussion) in pairs or in small groups
- Teacher circulates to record different student solutions, in addition to the ones the teacher has anticipated prior to the lesson.

Pancake Problem B:

In one classroom, 8 groups of 4 children share 8 huge pancakes each one gets the same amount. In another classroom, 4 groups of 6 children equally share 6 huge pancakes. A third classroom reports that each child got 1/4 of one pancake.
Did all the children from the three classrooms get the same amount? Explain your thinking.

What information will we use to solve this problem?

- *Classroom 1 - 8 groups of 4 children share 8 pancakes*
- *Classroom 2 - 4 groups of 6 children share 6 pancakes*
- *Classroom 3 - $\frac{1}{4}$ of 1 pancake per child*
- *Is the amount of pancakes for each child equivalent?*



Fractions - Fair Sharing (Division)
Comparing Unit Fractions

Note: Mathematical annotations includes mathematical vocabulary, symbols, elaborations of mathematical details from solutions, labels describing approach/strategy, and questions to further thinking and practice. All annotations are records of students' mathematical discussion.

Grade 4	Grade 5	Grade 6
Demonstrate and explain the relationship between equivalent fractions using concrete materials	Demonstrate and explain the concept of equivalent fractions using concrete materials	Determine the relationship among fractions, decimal numbers, and percents

AFTER (Highlights/Summary)

- 5 minutes
- Teacher revisits in the different student solutions the key ideas, strategies, and models of representation that are related to the lesson learning goal.
- Teacher records key ideas, strategies, and models of representation separately, so the students can see the explicit focus of learning from their solutions, as it relates to the lesson learning goal.

Highlights/Summary:

- *For equivalent fractions, the whole must be the same, yet the fractional pieces of the whole do not have to be congruent, but do have to be equivalent (1/4= 6/24=8/32).*
- *The improper fraction (e.g., 3/2) can be renamed as a mixed number, 1 1/2 showing whole number and fractional parts*
- *Calculation Strategies for Equivalent Fractions*
  - *Division – Divide to make equivalent fraction with fewer number of (larger size) parts in the whole*
  - *Multiplication – Multiply to make equivalent fraction with greater number of (smaller size) parts in the whole*

AFTER (Practice)

- 10 minutes
- Teacher chooses 2 or 3 problems, similar to the lesson problem for students to solve individually or in pairs as a scaffold).
- Problems could vary by number (choice, size), problem contexts, or what is unknown or needs to be solved.
- Students are asked to use a strategy different from the one they used for the lesson problem to solve practice problems.

Pancake Problem C:

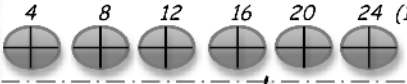
The classroom teachers want each child to get at least 1/3 of 1 large pancake. In Mr. T's classroom, 32 students got 8 pancakes. In Mrs. K's classroom, 6 pancakes for 24 students. The third classroom, 30 students got 10 pancakes. Did the teachers make enough pancakes for the classes?

Divide 8 pancakes by 8 groups of 4 students (or 32)

4 8 12 16 20 24 28 32 (1/4 of 1) pcs

Divide 6 pancakes by 4 groups of 6 students (or 24)

4 8 12 16 20 24 (1/4 of 1) pcs



8/8 = 6/6 = 1/1 = 1
Same whole, different number of parts

Division Strategy for Equivalent fractions
8/32 ÷ 8/8 = 1/4
6/24 ÷ 6/6 = 1/4
8/32 = 6/24 = 1/4

Multiplication Strategy for Equivalent fractions
1/4 x 6/6 = 6/24
1/4 x 8/8 = 8/32
1/4 = 6/24 = 8/32

Solution 3

8 groups of 4 students -> 32 students
8 pancakes cut into 32 pieces
Each student gets 8/32 of the collection of pancakes or 1/4 of one pancake.

4 groups of 6 = 24 students
6 pancakes cut into fourths = 24 pieces
Each student gets 6/24 of the collection of pancakes or 1/4 of one pancake.

Each student gets 1/4 of one pancake. (1 pancake for 4 students)
They all get the same amount.

Solution 4

Class 1 - 8 groups of 4
Class 2 - 4 groups of 6
Class 3 – 1/4

1	2	3	4	5	6	7	8	9
4	8	12	16	20	24	28	32	36

They all get 1/4 of a pancake each because they are equivalent fractions.

Solution 1

Class 1: 8 pancakes for 32 students -> (8/32)/(8/8) = 1/4
Class 2: 6 pancakes for 24 students -> (6/24)/(6/6) = 1/4
Class 3: 10 pancakes 30 students -> (10/30) / (10/10) = 1/3

Only class 3 got enough pancakes because 1/4 < 1/3.

Dividing the same size whole into 4 parts rather than 3 means the one divided into 4 will have smaller pieces.

Solution 2

Pancakes	Students	Am't per Student
8	32	8/32
4	16	4/16
2	8	2/8
1	4	1/4
6	24	6/24
3	12	3/12
1	4	1/4
10	30	10/30
1	3	1/3

Only the 3rd class gets enough. 1/4 < 1/3 because 3/12 < 4/12

Fractions - Fair Sharing (Division) Using Equivalent Fractions

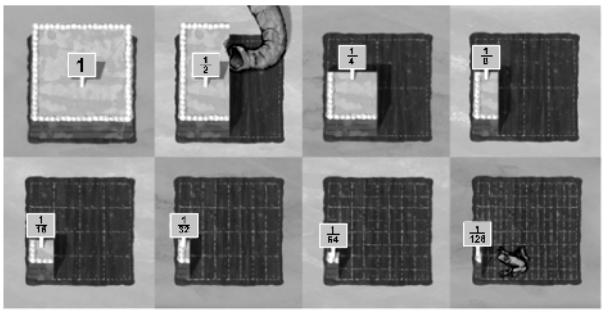
Equivalent Fractions Using Multiplication in a Ratio Table

Note: A classroom board is longer proportionally than these 2 pages. Due to the space constraints on these pages, the mathematical annotations are recorded above the solutions with arrows, rather than on and around the solutions.

LINKS TO LITERATURE:

"The Lion's Share: A Tale of Halving Cake and Eating It, Too"

This story is about a lion that invites a group of animals to dinner. These animals range in size from an elephant to an ant. The animals turn out to be an inconsiderate group of guests at the dinner table. Each of the animals helps themselves to half of the cake before passing it on to the next smallest guest. Provide each student with a piece of paper to represent the cake. As you tell the story, ask the students to show what is happening to the parts of the cake as each animal cuts the remaining piece in half.



Using the illustration on the inside covers or the students' models, challenge the students to identify equivalent fractions:

- How many 1/4s are there in half of the cake?
- How many 1/8s are there in half of the cake?
- How many 1/8s are there in 1/4 of the cake?
- How do you know? Use a paper model to show your thinking.

(from **http://matthwmcelligott.com/liosshare/math.php**)

LET'S DO MATH – GARDEN PROBLEM

Before (Getting Started)

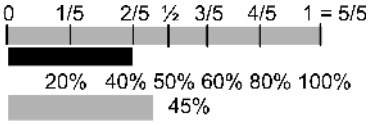
Garden Problem A:

Krysten has 2/5 of her garden planted in peas. Matthew has 45% of his garden planted in peas.

Who has a greater area of peas planted?

Solution 1

Their gardens have to be the same size in total or these comparisons cannot be done.



Solution 2

2/5 = (2/5) x (4/4) = 8/20

45% = (45/100) / (5/5)= 9/20

Matthew has more because 9/20 > 8/20.

During (Working On It)

Garden Problem B:

Aditi's rectangular garden is separated into 3 parts: 20% is tomatoes, 6/8 of it is corn, and 0.05 is herbs. How do these 3 areas of the garden compare? Show your work.

After (Consolidation)

Anticipating Student Responses:

Coordinating Student Discussion for Learning:

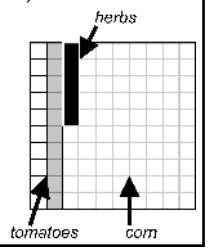
Why might solution 1 be chosen first for student discussion, followed by solutions 2 and 3?

Solution 1

20% = 20/100

6/8 = (6/8) / (2/2) = 3/4 => (3/4) x (25/25) = 75/100

0.05 is 5 hundredths = 5/100



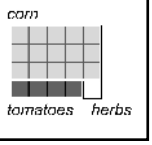
Solution 2

20% = 20/100 = (20/100) / (20/20) = 1/5 => (1/5) x (4/4) = 4/20

6/8 = (6/8) / (2/2) = 3/4 => (3/4) x (5/5) = 15/20

0.05 or 5 hundredths = 5/100 => (5/100) / (5/5) = 1/20

15/20 > 4/20 > 1/20
She has the most corn.



Solution 3

20% = 20/100 = 20 hundredths = 0.20

6/8 = 0.75 or divide 6 by 8

0.05 = 5 hundredths = 0.05

0.75 > 0.20 > 0.05

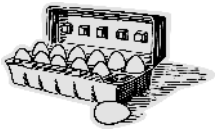
- solution 1 –> compare areas using common denominator of 100. Percent is defined as a special fraction out of 100 and 0.05 is 5 hundredths. 6/8 is divided by 2/2 to get 3/4 which is multiplied by 25/25 to get 75/100. Comparing fractions with same denominator requires a comparison of numerators only.
- solution 2 –> compare areas using a common denominator of 20.
- solution 3 –> compare areas using a decimal form of hundredths which is like solution 1.

LET'S DO MATH – CRISPY CRISPS PROBLEM

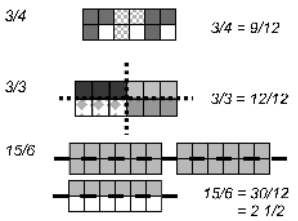
Before (Getting Started)

How much is each fraction of a dozen eggs?

a) 3/4 b) 3/3 c) 15/6



Solution 1



Solution 2

3/4 = (3/4) x (3/3) =
(3x3)/(4x3) = 9/12

3/3 = (3/3) x (4/4) =
(3x4)/(3x4) = 12/12

15/6 = (15/6) x (2/2) =
(15x2)/(6x2) = 30/12 = 2 6/12

During (Working On It)

Crispy Crisps Problem

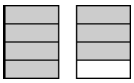
On Day 1 of the Fun Fair, the Grade 5s took 7 pans of Crispy Crisps to sell. Each pan was cut into 4 treats. They sold 5 1/4 pans of these treats. On Day 2, they decided to cut their 6 new pans of Crispy Crisps into 8ths and sell them at 2 for the price of one. They sold 44 of the treats cut into 8ths. The teacher cut the treats remaining from days 1 and 2 into 12ths to share with the students in the class. For a class of 28 students, were there enough so that each student got 1/12th of one pan?

After (Consolidation)

Anticipating Student Responses:

Solution 1

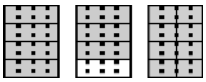
Day 1
7 pans in 4ths or 28 Crisps
5 1/4 pans sold 1 3/4 pans left or
7 Crisps 7/4 of a pan



Day 2
6 pans in 8ths or 48 treats
40 sold so 8 Crisps left or 8/8 of a pan



After the Fair
1 3/4 and 8/8 were left
16/16 + 12/16 + 16/16 = 44/16
44/16 = 22/8 = 11/4 = 33/12



Yes there were 33 pieces for the 28 students.

Solution 2

Day 1
28/4 prepared and 5 1/4 pans is 21/4 sold, 7/4 left

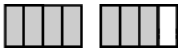
Day 2
48/8 prepared and 40/8 sold, 8/8 left

After the Fair
7/4 = 7/4 x (2/2) = 14/8
14/8 and 8/8 = 22/8

22/8 / (2/2) = 11/4 x (3/3) = 33/12
33 > 28 so there is enough for each student to get 1/12.

Solution 3

Day 1
1 3/4 pans remain



Day 2
1 pan remains



After the Fair
2 3/4 pans remain cut into 12ths – draw folding lines
33/12



Coordinating Student Discussion for Learning:

Why might solution 1 be chosen first for student discussion, followed by solutions 2 and 3?

- solution 1 → using a paper-folding-like drawing to represent the pans of 8ths as pans of 4ths are divided down the middle and the remaining Crisps are split into 16ths and combined. Then dividing by 2/2 twice makes equivalent fractions, 44/16 to 22/8 then to 11/4. By multiplying 11/4 by 3/3, there is 33/12 left over → 33 x 1/12 which is more than needed for 28 students (i.e., 28 x 1/12)
- solution 2 → using ratio table strategies – dividing by 2/2 and then multiplying by 3/3.
- solution 3 → focused on whole pans of Crisps remaining, using mixed fraction 2 3/4 to write as 12ths.

For Professional Discussion:

- What if the Problem was written this way:
- On Day 1 of the Fun Fair, the Grade 5s took 7 pans of Crispy Crisps to sell. Each pan was cut into 4 treats. They sold 5 1/4 pans of these treats. On Day 2, they decided to cut their 6 new pans of Crispy Crisps into 8ths and sell them at 2 for the price of one. They sold 44 of the treats cut into 8ths. Were there enough Crispy Crisps for the class to fill a special order for 3 full pans of treats?
- Would the anticipated solutions shown still be appropriate for this version of the problem? How do you know?

NEXT STEPS FOR YOUR PROFESSIONAL LEARNING

Application to Your Classroom

- Try these fraction representation problems yourself first and then with your students.
- To suit your students' learning needs, you might adjust the numbers (e.g., common fractions, improper fractions, mixed numbers, decimals, and percents) and/or the context of the problem.
- Think about how the context of the problem evokes particular ways to determine equivalence of fractions, decimals, and percents (e.g., pennies for percents,)
- Provide students with non-examples of comparing and ordering fractions and have students explain why the comparison is inaccurate. For example, Arlene says that 3/5 is equivalent to 9/15. Didi disagrees with Arlene. What do you think? Explain using a concrete model.
- Practise noticing the different strategies and the breadth of the mathematics your students use in their solutions. Take note of the number of solutions that show strategies for comparing and ordering fractions. Which model of fraction (i.e., region, set, linear measurement) do students use more often for determining equivalence?
- In preparation for your class discussion of student solutions, decide which solution you will discuss first, second, and third. Refer back to the developmental learning sequence for comparing and ordering fractions.
- Think ahead of what mathematical annotations you will record on and around the student solutions to render explicit key mathematical ideas, strategies, and terms of comparing and ordering fractions as the students discuss and analyze each other's solutions.
- Choose 4 student work samples to analyse further in terms of their mathematical details.

SUGGESTED READINGS

Some readings used in the Research Summary and in the development of the problems and solutions are listed below:

- Baroody, A. (1999). *Fostering children's mathematical power: An investigative approach to K-8 mathematics instruction.* NJ: Lawrence Erlbaum Associates.
- Fosnot, C., & Dolk, M. (2002). *Young mathematicians at work: Constructing fractions, decimals, and percents.* Portsmouth, NH: Heinemann.
- Randall, C. (2005). Big Ideas and Understandings as the Foundation for Elementary and Middle School Mathematics. *Journal of Mathematics Education Leadership* (7)3, 9-24.
- Small, M. (2009). *Big ideas from Dr. Small: Creating a comfort zone for teaching mathematics, Grades 4 - 8.* Toronto, ON: Nelson Educational Ltd.
- Kamii, C., & Clark, F.B. (1995). Equivalent fractions: Their difficulty and educational implications. *Journal of Mathematical Behavior* (14)4, 365-378.
- Van de Walle, J., & Lovin, L. (2006). *Teaching student-centered mathematics, Grades 3-5.* New York: Pearson Education, Inc.

CLOSING

Our next issue will focus on key notions of proportional thinking (i.e., rate, ratio), how teachers need to know these notions for teaching, as well as, ways that teachers can use these mathematical ideas and pedagogical strategies through problem solving-based teaching and learning strategies.

CALL FOR *ABACUS* SUBMISSIONS

Send your teaching strategy ideas, problems and student solutions to the *Abacus* Co-Editors via email

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