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MAKING SENSE OF FRACTIONS – RATIO, RATE, AND PROPORTION

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ABACUS CO-EDITORS



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What are some ways that teachers prepare students to develop proportional reasoning? Figuring out the ingredients needed to make chocolate chip cookies or rice crispie squares for 40 people from a recipe that makes enough for 12 people is an effective approach. Predicting how many runs a baseball team can make by the end of the season if by game 4, they had scored 22 runs is another context for learning about proportions. Also, figuring out the cost of an item for 5 people, 10 people, 100 people, given a particular rate develops students' understanding and use of proportional reasoning. Proportional reasoning can be developed within the context of other mathematical concepts, such as one-to-many correspondence for creating a scale for a bar graph or legend for a concrete graph (1 book represents 5 books) and place value (e.g. flats, roads, and unit cubes represents 100, 10, or 1 for whole numbers or 100 hundredths, 10 hundredths (1 tenth), and 1 hundredth for decimal numbers).

In this issue, the Research Summary and problems (with multiple solutions) will focus on strategies for determining proportional relationships. Consider solving these problems yourself first before examining the solutions we offer you. Also, think about how these problems can be used within the framework of a three-part, problem-solving lesson. It's been our experience that when we do the mathematics ourselves, we are deepening our conceptual understanding of the mathematics that we are teaching, and we are compelled to rethink our choices of learning materials and teaching strategies to further our students' learning.

LINKS TO LITERATURE AND MANIPULATIVES



PAT MARGERM is an independent literacy and mathematics consultant with more that 30+ years of K to 8 teaching experience with students, in-service and pre-service teachers. She is a long-time OAME board member and works in various aspects of mathematics education.

At this time of the year, teachers are often reflecting on what worked and what they would like to change for next year. The two picture books featured in Links to Literature are focused on social justice themes. Doing the math in these books could increase students' understanding of a social justice issue described in the story. The manipulative, Pattern Blocks is featured to show how it can be used to make mathematical connections across the mathematics curriculum strands, such as Patterning and Algebra and Number Sense and Numeration. I hope that these ideas will be helpful as you begin to plan for the next school year.

RESEARCH SUMMARY – RATIO, RATE, AND PROPORTION

Proportional Thinking

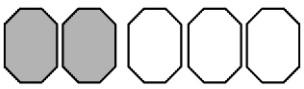
A proportion is a relationship between quantities; that is, how two quantities are related multiplicatively (Small, 2010). Thinking of 8 as two 4s, instead of as 2+6 or 4+4 highlights the difference between multiplicative thinking and additive thinking. In fact, proportional thinking involves the ability to generate a missing component through multiplicative reasoning, regardless of the numerical aspects of the problem situation. A student who recognizes that a person who runs 3 km in 4 minutes finishes a 15 km run faster than one who runs 5 km in 8 minutes is thinking proportionally. Students need to look at a relationship between numbers instead of looking at one number in isolation. Subsequently, proportional reasoning requires the mental storage and processing of several bits of information as it involves a sense of co-variation and multiple interpretation and comparisons of both the meaning of two individual ratios and the comparison of ratios (Lesh, Post, & Behr, 1988).

According to Baroody (1999) and Randall (2005), if two quantities vary proportionally, the quantities are either directly related (as one increases, the other increases) or inversely related (as one increases, the other decreases). So, a big idea for proportionality is that "if two quantities vary proportionally, that relationship can be represented as a linear function" (Randall, 2005, p. 14). In fact, when two quantities vary proportionally, then the ratio of the corresponding terms is constant. When you graph the terms or equal ratios as ordered pairs (first term, second term) or (x, y) and connect the points, the graph is a straight line; that is, a linear function.

Developing proportional reasoning is one of the goals of the grades 4 to 8 mathematics curriculum. Proportional thinking involves the use of multiplicative relationships in the form of rates, ratios, and percents to solve problems. Ratios, rates, and percents are like fractions and decimals as they are comparisons of quantities, whereas:

- a *ratio* compares quantities with the same unit (3 boys to 2 girls, the unit being children)
- a *rate* compares quantities with different units (e.g., distance/time or price per number of items)
- a *percent* always compares a quantity to 100.

Ratio



Ratio is a comparison between two quantities (or numbers). Whenever a situation can be described by one ratio, it can be described by several ratios (Randall, 2005; Small, 2009). In this example, for every 2 grey counters, there are 3 white counters. The 2 is called the first term of the ratio and the 3 is the second term. 2:3 is called *part-to-part* ratios because they compare 2 parts of something. 2 to 5 (2:5) or 3 to 5 (3:5) are called *part-to-whole* ratios because they compare a part to the whole.

Smith (2002) adds that a ratio is a relational number that has two properties: (1) it relates two quantities in one situation, and (2) it projects that relationship onto a second situation in which the relative amounts of the two quantities remain the same. Suppose a soccer team scores 12 goals in 4 games. Think of the ratio, 12:4 in order to estimate how many goals the team would score in a 16 game season (e.g., 12:4 is equivalent to 48:16). The first situation is goals in 4 games and the second situation is projected goals by the end of the season (16 games). Also, ratios can be expressed as units, by finding an equivalent ratio where the second term is 1 (e.g., 12:4 is equivalent to (12 ÷ 4):(4 ÷ 4) or 3:1). As well, ratios can be expressed as a unit (unit ratio), in terms of a equivalent ratio that has a second term being 1.

(According to Cai and Sun 2002), students need to begin with the knowledge they have learned (division 12 ÷ 4 = 3) and use this knowledge to describe the new situation (ratio of 3:1). Because students have studied the division of integers and understand that a fraction also represents division, the connections among ratios, division and fractions are easily made. The division a ÷ b is the result of the ratio operation where a/b is the value of the ratio of a:b. The value of the ratio is represented by the fraction a/b, so a:b, a ÷ b, and a/b or 12:4 = 12 ÷ 4 = 3. Like Cai and Sun (2002), Smith (2002) names ratios as fractions using quotients; that is, numerals of the form a/b (e.g., 4/5). For the grey/white counter example, expressing part-to-part ratio of 3 white counters to 2 grey counters means three halves (3/2) as many white counters as grey counters.

Small (2009) explains that though ratios are introduced explicitly in the later junior grades, kindergarten and primary students consider them in informal ways, like when we say we have 5 fingers for each hand and 2 feet for every person. As multiplication is introduced in grades 2 and 3, the concept of ratio is implicit when students solve problems like how many wheels are on 4

bicycles. Students use the ratio 2:1 (2 wheels to 1 bicycle) to figure out that there are 8 wheels on 4 bicycles.

From Additive to Multiplicative Reasoning

Ratios are multiplicative not additive relationships. Students need to think about how quantities in situations relate and change. According to Kent, Arnosky, and McMonagle (2002), the transition from additive to multiplicative thinking is a difficult one for elementary students. Doubling, tripling, and counting by tens are some of the strategies used by elementary grades students to multiply numbers in proportional reasoning situations. However, these numbers (2, 3, 10) have limited use in other problem contexts. Consequently, students tend to persevere with additive strategies rather than multiplicative strategies.

Bottles Problem:

How many bottles for different number of cases?

Cases					5	6						12
Bottles					125	150						

For this problem, initially students concentrate on the additive nature of the table. Students only see that as number of cases increases or decreases by 1 the number of bottles increases or decreases by 25. The number of bottles for one case can be determined by subtracting 25 bottles for each case from 5 cases until the number of cases equals 1. Eventually, a student uses division to calculate the unit rate; that is, one case is 125 bottles divided by 5 cases to get 25 bottles per 1 case.

Rate

A ratio is understood initially in terms of the separate physical measures. A ratio becomes a rate when the student no longer relies on the physical situation and considers the potential generalized applicability of the multiplicative relationship. Small (2009) says that a rate is like a ratio only with different units; for example, a rate could be $13.50 per person to represent the cost of a movie. See the Printing problem as another example.

Printing Problem:

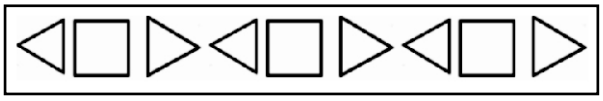
Printer can print 20 pages in two and one-half minutes. How many pages does it print per minute?

Page	20	40	8
Minute	2.5	5	

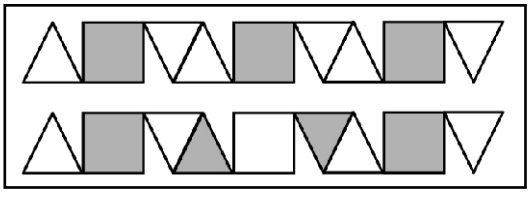
LINKS TO MANIPULATIVES: PATTERN BLOCKS

Students can use ratios to analyze multiplicative relationships between the number of pattern blocks in a repeating pattern.

- How many triangles are there for every square in the pattern?
  - *2 triangles to one square; 2:1; 2/1*
  - *1 square to 2 triangles; 1:2; 1/2*



- How many triangles will you have used if your pattern has been extended to use 26 squares?
  - *2 triangles:1 square → (2 x 26) triangles:(1 x 26) squares → 52 triangles:26 squares*
  - *1 square:2 triangles → (1 x 26) squares:(2 x 26) triangles → 26:52 or 26/52*
  - (1:2) x 26 = (1 x 26):(2 x 26) = 26:52
- What other repeating patterns can you make using the same ratio of triangles to squares?
  - *What is a patterning rule for these 2 patterns?*
  - *How are these 2 patterns similar and different?*



LESSON LEARNING GOALS

Identify the curriculum expectations (MOE, 2005) grade-before and grade-after the grade specific learning goal:
- to see the development of mathematical knowledge, skills, and strategies across grade levels
- to determine the mathematical focus of the problems chosen for Before (activation), During (lesson), and After (Practice) parts of the lesson

BEFORE (Getting Started)

- 5 to 10 minutes
- Activating students' mathematical knowledge and experience using a prompt or problem that is directly relates to the mathematics in the lesson problem.
- Includes student responses to prompt/problem to highlight key ideas/strategies.

What is the ratio of:
a) hockey pucks to sticks?
b) hockey tape to sticks?
c) Which ratio is greater?



Solution 1
6 pucks to 3 sticks is 6/3
1 tape to 3 sticks is 1/3
6/3 >1/3
There is a higher ratio of pucks to sticks than tape to sticks.

ratio as a fraction
1st term is 6
2nd term is 3

ratio symbol
6 to 3 is 6:3
1 to 3 is 1:3

Solution 2
6 to 3 or 2 to 1
1 to 3 or 1/3 to 1
More pucks to sticks.

unit ratio
2nd term is 1

DURING (Working on It)

- 15 to 20 minutes
- Understanding the problem – Teacher asks, "What information from the problem are we using to make a plan to solve it? Explain." Teacher records in a list below the problem, the information that students identify.
- Students solve the problem on chart paper (landscape) with markers (visible for whole class discussion) in pairs or in small groups.
- Teacher circulates to record different student solutions, in addition to the ones the teacher anticipated.

Guitar Hero Problem:

Which teacher is best at Guitar Hero? Show your work.
Ms. Delores passed 5 out of 10 songs.
Mr. Kay passed 3 out of 5 songs.
Mr. Pape passes 7 out of 15 songs.

What information will we use to solve this problem?
- *Ms. Delores – 5 out of 10 songs*
- *Mr. Kay – 3 out of 5 songs*
- *Mr. Pape – 8 out 15 songs*
- *Compare ratios of songs passed to total songs*
- *Show your work – calculations, labelled diagrams*
-

AFTER (Consolidation)

- 20 to 25 minutes
- Teacher selects 2 or more solutions for class analysis and discussion in a sequence (1st, 2nd, 3rd, etc.) based on mathematical relationships between the solutions and the lesson learning goal.
- Students (authors) explain and discuss their solutions with their classmates.
- During whole class discussion, solutions are organized (often re-organized) to show mathematical elaboration from one solution to the next and towards lesson learning goal.
- Teacher mathematically annotates (math terms, math symbols, labelled diagrams, concise explanations) on and around solutions to make mathematical ideas, strategies, and models of representation explicit to students.

part *whole*

benchmark fraction of 1/2

Solution 1

Passed	Total	Comparison
5	10	Equal to 1/2
3	5	More than 1/2 as 2.5 is half of 5
7	15	Less than ½ as 7.5 is half of 15

Mr. Kay scored the highest because he passed a greater fraction of the songs.

Using division of parts to total parts to convert fraction to decimal

Like a unit rate …
5 per 10
-> 5÷10 per 10÷10
-> 0.5 per 1)

3 per 5
-> 3÷5 per 5÷5
-> .6 per 1

7 per 15
-> 7÷15 per 15÷15
-> .47 per 1

Solution 2
5/10 = 5÷10 = 0.5
3/5 = 3÷5 = 0.6
7/15 ~ 0.47

The greatest decimal is 0.6 or 3/5, so Mr Kay is best at Guitar Hero. .

Comparing to Benchmark Fractions

Converting Fractions and Decimals

← Comparing Ratios as Numbers in Isolation →

Note: Mathematical annotations include mathematical vocabulary, symbols, elaborations of mathematical details from solutions, labels describing the method/strategy, and questions to further thinking. All annotations are records of students' mathematical discussion.

Grade 1	Grade 5	Grade 6
- describe relationships that involve simple whole-number multiplication - demonstrate an understanding of simple multiplicative relationships involving unit rates through investigation using concrete materials and drawings	- describe multiplicative relationships between quantities by using simple fractions and decimals - demonstrate an understanding of simple multiplicative relationships involving whole-number rates, through investigation	-represent ratios found in real-life contexts, using concrete materials, drawings, and standard fractional notation - represent relationships using unit rates

equivalent ratios
5:10 = 30:60
3:5 = 36:60
7:15 = 28:60

common whole of 60

part-whole ratio
- comparing part to whole
5:10, 30:60; 3:5; 36:60; 7:15; 28:60

Solution 4
Ms Delores:Mr Kay:Mr Pape
5:10 = (5x6):(10x6) = 30:60
3:5 = (3x12):(5x12) = 36:60
7:15 = (7x4):(15x4) = 28:60

If they had all played 60 songs and had the same results, Mr Kay passes most songs (30:36:28)

part-part ratio
- comparing part to part

multiplicative relationships
5:10, 3:5, 7:15
5x3:10x3 = 15:30
3x6: 5x6 = 18:30
7x2:15x2 = 14:30

ratio
- a comparison of two quantities.

common whole of 30

Solution 3
5/10 x 3/3 = 15/30
3/5 x 6/6 = 18/30
7/15 x 2/2 = 14/30

If they played 30 songs, Mr. Kay passed the most songs (18/30 > 15/30 > 14/30).

Solution 5

Delores		Kay		Pape	
5	10	3	5	7	15
2.5	5	6	10	2.3	5
10	20	9	15	**14**	**30**
15	**30**	12	20	21	45
30	**60**	**18**	**30**	**28**	**60**
		36	**60**		

If they had all played 30 or 60 songs Mr Kay has the best score – Guitar Hero.

Making Common Wholes ***Using a Ratio Table***

Relating Quantities Multiplicatively

AFTER (Highlights/Summary)
- 5 minutes
- Teacher and students revisit the student solutions for key ideas, strategies, and models of representation that are related to the lesson learning goal.
- Teacher lists key ideas, strategies, and models of representation separately, so the students can see how the mathematical details from their solutions relate explicitly to the lesson learning goal.

Highlights/Summary:
- *ratio is a comparison of two quantities that are related multiplicatively*
- *part-whole ratio – comparison of a part to the whole 5:10 is a comparison of 5 passed songs to 10 total songs*
- *part-to-part ratio – comparison of part to part 30:36:28 is a comparison of 30 to 36 to 28 passed songs out of a common whole of 60 songs*
- *ratio table – shows a set of multiplicative relationships between ratios that can be compared at common wholes*

AFTER (Practice)
- 5 to 10 minutes
- Teacher chooses 2 or 3 problems, similar to the lesson problem for student to solve in pairs as a scaffold and individually.
- Problems could vary by number (choice, size), problem contexts, or what is unknown or needs to be solved
- Students are asked to solve these problems using a strategy different from the one they used for the lesson problem.

Pen Problem:
Riley, Hunter, and Ella together bought a box of 100 pens at a garage sale. They asked, "How many pens should we each get?" Ella said, "I should get the most because I put in the most, $7. Hunter offered $2 and Riley $1."
How should they fairly split up the pens?

Solution 1
Riley:Hunter:Ella = 1:2:7
Riley paid for 1/10 of the pens
Hunter paid for 2/10 of the pens
Ella paid for 7/10 of the pens.

For $10 altogether.
They want to share 100 pens in same ratios.
1/10 = 10/100 2/10 = 20/100
7/10 = 70/100
Riley gets 10, Hunter gets 20 and Ella gets 70. (1:2:7 = 10:20:70)

Solution 2

1	10	10	100
2	10	20	100
7	10	70	100

Ella gets 70 pens.
Hunter gets 20 pens.
Riley gets 10 pens.

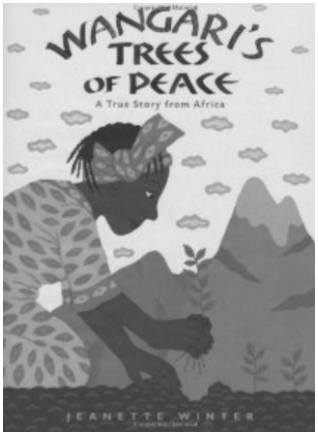
Note: A classroom board is longer proportionally than these 2 pages. Due to the space constraints on these pages, the mathematical annotations are recorded above the solutions with arrows, rather than on and around the solutions.



LINKS TO LITERATURE:

In the book, *The Carpet Boy's Gift*, the main character, Nadeem, is a bonded laborer, a child who works in a carpet factory to pay off a loan his family accepted from the factory owner. While reading the book there are opportunities for students to use mathematics to gain a deeper understanding of the problems and choices that Nadeem faces. For example, Nadeem is fined 50 rupees for falling asleep at the loom.

- If the exchange rate is $0.02 Canadian for 1 rupee, how much is Nadeem's fine in Canadian money?
- How much is Nadeem's fine of 100 rupees in Canadian money?
- It takes Nadeem six months to pay off the 50 rupee fine. At this rate how long will it take him to pay off a fine of 500 rupees?



The book, *Wangari's Trees of Peace*, is based on the true story of Wangari, who left Kenya to study and then returned home six years later to be shocked to see whole forests being cut down. She decides to do something and starts by planting nine seedlings in her own backyard.

- If Wangari planted 9 seedlings a day, how many days will it take for her to plant 425 trees?
- Wangari asks other women to help her plant seedlings. If each woman plants 9 seedlings, how many trees will 72 women plant?

LET'S DO MATH – LEMONADE RECIPE PROBLEM

Before (Getting Started)

Lemonade Recipe Problem A:

Write ratios to describe different combinations of water and lemonade concentrate: (1 cup, 2 cups, 3 cups, 4 cups).

How do you know that you have listed all possible combinations?

CUPS	1	2	3	4
1	1:1	1:2	1:3	1:4
2	2:1	2:2	2:3	2:4
3	3:1	3:2	3:3	3:4
4	4:1	4:2	4:3	4:4

During (Working on It)

Lemonade Recipe Problem B:

Which lemonade recipe has a stronger lemonade taste?

Recipe 1: 1 cups lemonade concentrate, 2 cups water
Recipe 2: 2 cups lemonade concentrate, 3 cups water
Recipe 3: 3 cups lemonade concentrate, 5 cups water

After (Consolidation)

Anticipating Student Responses:

Solution 1

Ratios of lemonade to water

Recipe 1: 1:2 = 0.50:1

Recipe 2: 2:3 = 0.67:1

Recipe 3: 3:5 = 0.60:1

Solution 2

Recipe 1: lemonade to water = 1/2 = 30/60

Recipe 2: lemonade to water = 2/3 = 40/60

Recipe 3: lemonade to water = 3/5= 36/60

If all recipes were made with 60 cups of water, there would be the most lemonade in recipe 2.

Solution 3

1	2	2	3	3	5
3	6	4	6	9	15
15	30	20	30	18	30

Recipe 2 has the greater ratio of lemonade.

Coordinating Discussion for Student Learning:

Why might solution 1 be chosen first for student discussion, followed by solutions 2 and 3?

- solution 1 –> calculates unit rate of number of units of lemonade to 1 unit of water; recipe 2 has more lemonade for the same amount (1 unit) of water
- solution 2 –> common whole is represented here in the denominator (60) so it says, like the others, that recipe 2 has more lemonade in it (40 units) than the other 2 (36 and 30 units)
- solution 3 –> uses the ratio table to re-present calculations used to get the same whole; recipe 2 has more lemonade (20 units) in 30 cups of water than the others (15 and 18 units)

LET'S DO MATH – PENCIL PROBLEM

Before (Getting Started)

Pencil Problem A:

10 pencils cost $1.80.

- How much do 5 pencils cost?
- How much do 20 pencils cost?
- How much does 1 pencil cost?

10 pencils	5 pencils	20 pencils	1 pencil
$1.80	$1.80÷2 = $0.90	$1.80 x 2 = $3.60	$1.80÷10 = $0.18

During (Working On It)

Pencil Problem B:

A school needs to buy 22 400 pencils. Here are the prices for pencils at 3 different stores

Store A sells 60 pencils for $1.80
Store B sells 30 pencils for $0.99
Store C sells 15 pencils for $0.55

- Which store has the lowest price?
- What is the cost of the pencils at the lowest price store?

After (Consolidation)

Anticipating Student Responses:

Solution 1

180¢ / 60 = 3

99¢ / 30 = 3.3

55¢/15 = 3.667

Store A sells 1 pencil for less than the other stores

$0.03 rather than $0.033 or $0.03667

$0.03 x 22 400 = $$672.00.

Solution 2

60	180¢	30	99¢	15	55¢
1	3¢	10	33¢	3	11¢
11	33¢	10	33¢	9	33¢

Store A gives you more pens for $0.33 (11 pens) than the other 2 stores (10 and 9 pens).

Solution 3

60	180¢	30	99¢	15	55¢
22 400	**$672.00**	**22 400**	**$739.20**	**22 400**	**$880.00**

Store A's price would be $672.00 less than the other 2 ($739.20 and $880.00).

Coordinating Discussion for Student Learning:

Why might solution 1 be chosen first for student discussion, followed by solutions 2 and 3?

- solution 1 - unit rate cost; that is the cost of 1 pen in cents
- solution 2 - common whole of 33 cents store A gives 11 pens for $0.33, whereas the others only give 10 or 9 pencils for $0.33
- solution 3 - ratio table comparatively shows the cost of 22 400 pens.

For Professional Discussion:

What if the Pencil Problem was written this way.

Would the anticipated solutions shown about still be appropriate for this version of the problem? How do you know?

Pencil Problem C:

A school needs to buy 22 400 pencils. Here are the prices for 3 different stores. Which store has the lowest price? What is the cost of the pencils at the lowest price store?

Store A sells 30 pencils for $0.90
Store B sells 60 pencils for $1.98
Store C sells 45 pencils for $1.65

NEXT STEPS FOR YOUR PROFESSIONAL LEARNING

Application to Your Classroom

- Try these ratio and proportion problems yourself first and then with your students.
- To suit your students' learning needs, you might adjust the numbers (e.g., 2/3 or 2:3; 4/5 or 4:5; 6/4 or 6:4)
- Think about how the context of the problem evokes visual images and serves as a tool for understanding and determining proportional relationships between numbers (e.g., speed limit; recipes, table seating)
- Provide students with non-examples of proportional relationships (e.g., additive rather than multiplicative relationships) and have students explain why the number relationships are not proportional. For example, Jayne says that 3/5 is equivalent to 9/15. Didi disagrees with Arlene. What do you think? Explain using a concrete model.
- Practise noticing the different strategies and the breadth of the mathematics your students use in their solutions. Take note of the number and kinds of solutions that show strategies for determining proportional relationships. Which strategy do students use more often: additive/subtractive, unit rate/division, common whole, ratio table?
- In preparation for your class discussion of student solutions, decide which solution you will discuss first, second, and third by referring to the developmental learning sequence for determining proportional relationships to solve problems
- Think ahead of the kinds of mathematical annotations you will record on and around the student solutions to make explicit key mathematical ideas, strategies for determining proportional relationships when the students discuss and analyze each other's solutions.
- Choose 4 student work samples to analyse further in terms of their mathematical details.

Suggested Readings

Some readings used in the Research Summary and in the development of the problems and solutions are listed below:

- Baroody, A. (1999). *Fostering children's mathematical power: An investigative approach to K-8 mathematics instruction.* NJ: Lawrence Erlbaum Associates.
- Cai, J., & Sun, W. (2002). Developing students' proportional reasoning: A Chinese perspective. In B. Litwiller & G. Bright (Eds.), *Making sense of fractions, ratios, and proportions:* 2002 NCTM Yearbook. Reston, VA: National Council of Teachers of Mathematics.
- Fosnot, C., & Dolk, M. (2002). *Young mathematicians at work: Constructing fractions, decimals, and percents.* Portsmouth, NH: Heinemann.
- Kent, Arnosky, & McMonagle. (2002). Using representational contexts to support students' multiplicative reasoning. In B. Litwiller (ed.), *Making sense of fractions, ratios, and proportions:* 2002 NCTM Yearbook. Reston, VA: National Council of Teachers of Mathematics.
- Lesh, R., Post, T., & Behr, M. (1988). Proportional reasoning. In J. Hiebert & M. Behr (Eds.) *Number concepts and operations in the middle grades* (pp. 93-118). Reston, VA: Lawrence Erlbaum & National Council of Teachers of Mathematics.
- Randall, C. (2005). Big Ideas and Understandings as the Foundation for Elementary and Middle School Mathematics. *Journal of Mathematics Education Leadership* (7)3, 9-24.
- Small, M. (2009). *Big ideas from Dr. Small: Creating a comfort zone for teaching mathematics, Grades 4 - 8.* Toronto, ON: Nelson Educational Ltd.
- Smith, J. P. (2002). The development of students' knowledge of fractions and ratios. In B. Litwiller & G. Bright (Eds.), *Making sense of fractions, ratios, and proportions:* 2002 NCTM Yearbook. Reston, VA: National Council of Teachers of Mathematics.
- Van de Walle, J., & Lovin, L. (2006). *Teaching student-centered mathematics, Grades 3-5.* New York: Pearson Education, Inc.

CLOSING

Our next issue will focus on key notions of measurement (i.e., linear, area, volume/capacity, time), how teachers need to know these notions for teaching, as well as, ways that teachers can use these mathematical ideas and pedagogical strategies through problem solving-based teaching and learning strategies.

CALL FOR *ABACUS* SUBMISSIONS

Send your teaching strategy ideas, problems and student solutions to the *Abacus* Co-Editors via email