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TEACHING AND MULTIPLYING

TABLE OF CONTENTS

1. Verbing Multiplications with "Sets of, Rows of, Jumps of" (by Lisa Lunny Borden)
2. Doing the Math (Lesson Plan insert for 5 x 6): Everything any teacher needs to discuss 5 x 6 in their classroom
3. Evan Throop Robinson shares how to use an Indigenous story, *Aliet Learns to Make Lu'sknikn,* to elicit ideas about multiplication.
4. Moving Forward: Readings for continuing professional learning

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IN THIS ISSUE

We will be continuing our work on mental math, with a focus on multiplication. Multiplication is a hot topic for many educators and parents because of their past experiences learning facts and rules. The reality is, however, that multiplication is quite complex—it has multiple meanings, which can be thought about, using different contexts and representations (for examples, see Davis & Renert, 2009).

In light of September 30, 2021 being the first National Day for Truth and Reconciliation, we will be dedicating some of this issue to Indigenous ways of knowing/learning multiplication. We have two special guest authors: Lisa Lunny Borden and Evan Throop Robinson, both of whom have been working with Mi'kmaq schools in their work at St. Francis Xavier University in Nova Scotia. Their work on multiplication with Mi'kmaq students speaks to the value of making mental actions, performed in your mind, visible for everyone to see.

The focus of this issue is on the movement (whether physical or mental) involved with learning multiplication. We invite readers to draw specific attention to words ending in "ing" that students may use to describe and do multiplication, or what we interpret our guest authors have coined as "verbing."

VERBING MULTIPLICATIONS WITH "SETS OF, ROWS OF, JUMPS OF"

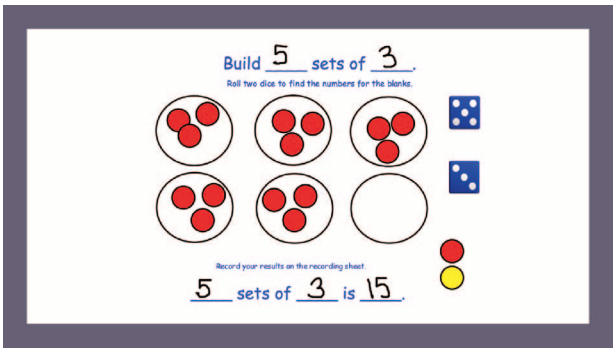
By Lisa Lunney Borden

For almost two decades now, I have been considering the ways in which we might teach math, using more verbs, focusing on action and process, rather than things. This notion emerged in my research with Mi'kmaq communities in Nova Scotia. Mi'kmaq, like all Indigenous languages in what we now call Canada, is a verb-based language that attends to flux and flow and movement, rather than static objects. To work multiplicatively with verbing, we work at building concepts concretely and paying attention to how they form.

One of my favourite tasks for introducing multiplication is a series of centres focused on building multiplication facts, without ever saying the word "multiplication." In this task, we invite students to use dice and concrete materials to build equal sets, equal lengths, and equal rows. As students build, we ask them to count the total. How they count reveals a great deal about their thinking. This task is rooted in playful engagement with quantity that allows students to build equal sets with and without a ten frame—each offering a different way to see quantity—as equal lengths on a number line, and as an area model. Linear, set, and area models are important ways in which to see multiplication, and each needs to be explored.

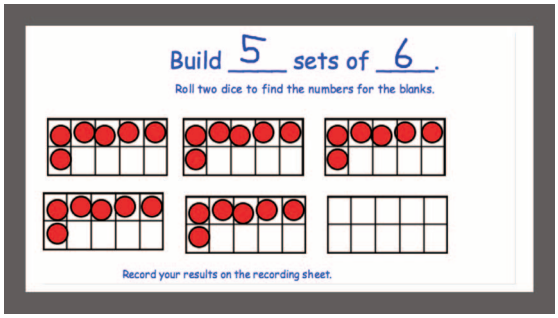
Below I describe each centre and share some ways in which I invite students to think about what they are building and how they are counting it. You can download the following slides: https://docs.google.com/presentation/d/1BiTPmkJsjHq4hc6kzSdOeXDU3hXAov8E/edit?usp=sharing&ouid=108204641278681174745&rtpof=true&sd=true.

Sets of



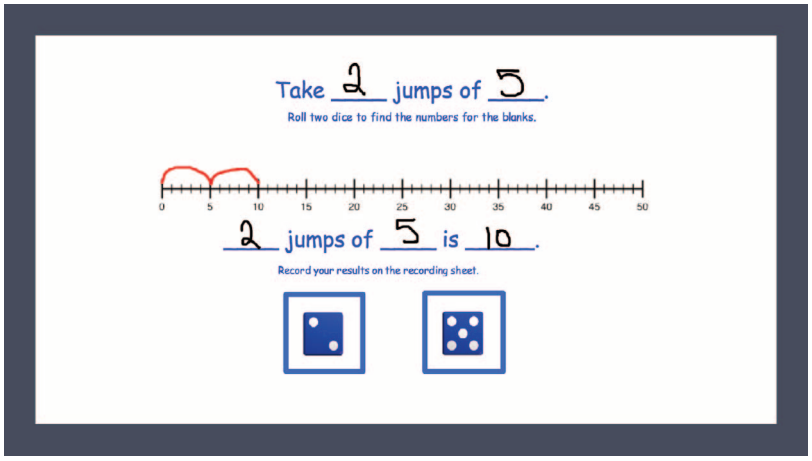
Students roll two dice and use counters to build equal sets. For example, here a student has rolled a 5 and a 3 and has chosen to build 5 sets of 3. They could have chosen to build 3 sets of 5. It is an interesting question to ask if these would give the same solution. While we as teachers know that multiplication is commutative (order doesn't matter), young children just learning multiplication would not. This is an interesting question to explore.

As a teacher, I am very much interested in watching to see how students count to get the total. This reveals rich information about their mathematical thinking. If students count items one at a time, I might prompt with the question: "Is there an easier way to count that?" You might see students switch to a skip-counting strategy or some other form of grouping for the total.



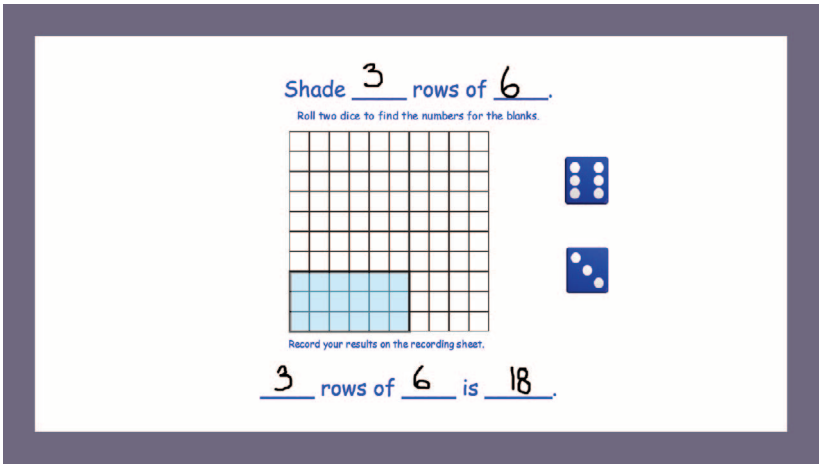
"Sets of" with ten frames evokes a different way of seeing sets. In the example, a student has built 5 sets of 6. As a teacher, I want to see if students might use the structure of a ten frame to support counting. For example, here it would be interesting to see if they count by 5's and then add on the 1's. If they do, they are seeing a very valuable way of partitioning numbers to make multiplication easier, which we will later refer to as the "distributive property." This property allows us to break the sets into parts, multiply each of the parts, and then put them back together again.

Jumps of



This centre invites students to think about equal lengths as jumps on a number line. Again, exploring the ways in which students get to the total can help reveal information about their mathematical thinking. For example, do students need to count each individual tick mark on the number line, or can they jump a given amount without counting?

Rows of



For “rows of,” students are given square colour tiles to build rows, which are then recorded as an area model on grid paper. The area model for multiplication becomes invaluable later on in working with decimals, fractions, and algebra, so it is an important representation to explore in early multiplication work. Students can explore if order matters with this model as well, and hopefully will see that the difference between 3 rows of 6 and 6 rows of 3 is just a rotation of the rectangle. In watching students count the total, I am always interested in how they counted. For example, one student who built a 4 by 6 rectangle told me that the total was 24. When I asked how he counted it, he declared that he had done 6 and 6 is 12 and then 12 more. Here is an instance of a double double strategy being used for multiplication by 4.

These centres can be done over several days, each day inviting students to share something they learned that helped them to count more easily. For example, a student might suggest that by using the ten frame, they were able to count by 5’s and add the 1’s when building sets of 6. This then becomes a strategy others can try. Or, as was the case above, a student might share a double double strategy for making 4 sets of something. Students can be invited to see if that always works. One day of centre exploration could be dedicated to the question, “Does order matter?”

Multiplication Lesson Plan for a Number Talk about 5 x 6

Getting Started

- Step 1: Make sure to VERBALLY (no writing) prompt students. For example, you can say: "Today's number talk is about 5 x 6." **Follow-up prompts:** What are you picturing In your head when you think about 5 x 6? Which number are you seeing in your mind?
- Step 2: Provide students with a couple of minutes of think time. During think time, display an image of 5 x 6, using manuipulatives (see Figure 1 for an example). You can further prompt at this point with: Are you seeing or thinking about groups? Area? Sets? How would you explain what you did to a friend?
- Step 3: Have students explain their strategy to a partner.

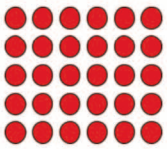


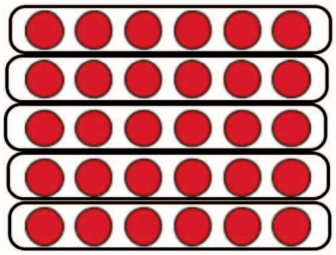
Figure 1: Image for 5 x 6 to display
Put the dots out after a couple of minutes of think time, and make sure all the dots are spaced equally apart.

AFTER (Highlights/Summary)

- The teacher revisits the different solutions, the key ideas, strategies, and models of representation that are related to the lesson-learning goal.
- The teacher records key ideas, strategies, and models of representation separately, so that students see the explicit focus of learning from the lesson.

Anticipate Solution Strategies for 5 x 6

Solution 1: Making 5 groups of 6



Quick Tips (Making 5 groups of 6)

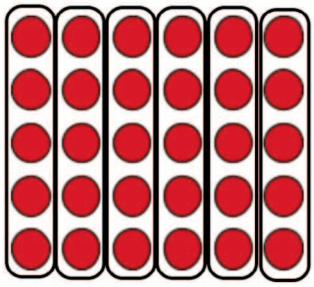
- *Ask students how they counted all the dots or figured out how many dots there are in total.*
- *Possible ways:*
  - *Skip-counting: 6, 12, 18, 24, 30*
  - *6 + 6 = 12, 12 + 12 = 24, 24 + 6 = 30*
  - *Decompose every group of 6 into 5 and 1:*

    5 x 5 + 5 = 25 + 5 = 30

AFTER (Practice)

- The teacher chooses two or three problems, similar to the lesson problem for students to solve individually (or in pairs).
- Problems are different by number (choice, size), problem contexts, or variation of problem needing to be solved.
- Students are asked to use a strategy different from the one they used in the number talk to solve the practice problems.

Solution 2: Making 6 groups of 5



Quick Tips
Ask clarifying questions when students are sharing their ideas:

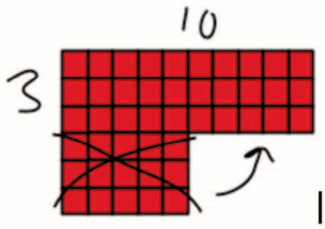
- *What did you do first?*
- *Can you say that again?*
- *Does everyone see the groups of 5? Show me.*
- *Can you tell me more?*
- *How does that make sense?*
- *How do you know that?*
- *Do you really like 5? Why?*

Video resources: These anticipated solutions are static representations of what children may offer. Please watch the video (link below) to see how students may share their solution strategies. The attached "open-go" number talk video can be easily used in your classroom. NO PREP or manipulatives required! www.vimeo.com/639373976/725de441ca

- Curriculum Expectations (MOE, 2020) (not to put on the board with students)

Prior Grade	Current Grade	After Grade
Grade 3: B2.2 recall and demonstrate multiplication facts of 2, 5, and 10, and related division facts	**Grade 4:** B2.2 recall and demonstrate multiplication facts for 1 × 1 to 10 × 10, and related division facts	**Grade 5:** B2.2 recall and demonstrate multiplication facts from 0 × 0 to 12 × 12, and related division facts

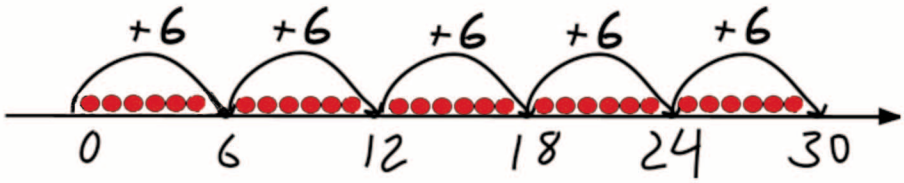
Solution 3: Changing the question to 3 x 10 by **halving and doubling**



Quick Tips (manipulatives)
- *Actually have a paper grid of 5 x 6 ready, and use scissors to cut and then move to make a grid of 3 x 10.*
- *Have magnetic dots handy to use on the whiteboards, or use a projector so that students can see the dots moving.*

Quick Tip
- *Placing the dots on a number line, as students count, can help students make connections between their images of dots (quantity) and the number line.*
- *Limit the time of a number talk. Number talks should typically be 10 minutes or less.*

Solution 4: Adding 6's (repeated addition)



DURING
- 5–10 minutes
- "Can someone come up and tell us what they did to figure out 5 x 6?"; "What did you see in your mind?"
- As the student explains, the teacher records by moving manipulatives and make annotations as needed.
- Invite another student to share. "Is there someone who did something different?"
- The teacher again records by moving manipulatives and making annotations as needed.

Quick Tip
- *Mental math is much more than mere computations.*

Ending (Consolidation)
- Have students analyze and compare solution strategies.
- "How are these solution strategies different or the same?"
- "If your solution strategy was not talked about, which of the solution strategies was most similar to yours? Which one was it most like? OR Which one was most different from your solution strategy?"

AFTER (Next steps and practice)
- If appropriate, the teacher can challenge students to use a different strategy next time (e.g., "If you used your fingers to keep track of the 6 groups of 5 , for our next talk, I challenge you to make use of the number 10."
- Problems could vary by number (choice, size), problem contexts, or what is unknown or needs to be solved.
- Students make their own written document of the strategies shared.
- Students can make their own word problems for the question.

Quick Tip
Mathematical Annotations:
Includes mathematical vocabulary, symbols, elaborations of mathematical details from solutions, and labels describing approach/strategy. All annotations are records of students' mathematical discussions.

Note: Template adapted from *OAME ABACUS* Template, created by Kathy Kubota-Zarivnij and Mary Lou Kestell.

CHILDREN'S LITERATURE AND MATH LEARNING

By Evan Throop-Robinson



ALIET LEARNS TO MAKE LU'SKNIKN

https://youtu.be/xOoyUowuygI

Teaching with Treaty Education in mind means embracing a way of life enjoyed by Indigenous peoples today and throughout history. It means more than papers signed by nations hundreds of years ago (Meader, 2020). In Nova Scotia, elementary students learn about Mi'kmaq culture and ways of knowing through curriculum developed by Mi'kmaq Kina'matnewey (MK). The curriculum uses the character of Aliet, a loveable bear, to further students' holistic language development and engagement in mathematical thinking.

Children's Literature



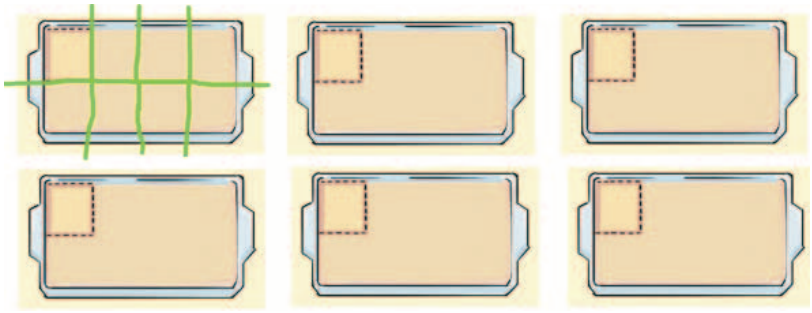
Good stories make great starting points for establishing connections with children's experiences and mathematical thinking (Trakulphadetkrai et al., 2019). The Aliet curriculum showcases Indigenous culture and language through storytelling. Mi'kmaq children can participate in this oral tradition by listening and relating to stories about familiar activities, such as learning to skate, participating in Treaty Day, and baking with grandparents. Non-Indigenous children can experience who the Mi'kmaq are through the stories, and come to know Mi'kmaq teachings and ways of being in cultural context. In *Aliet Learns to Make Lu'sknikn*, Aliet enjoys time in the kitchen with Nukumi (Grandmother), baking a delicious biscuit to share with Niskamij (Grandfather). Nukumi shows Aliet how to score the biscuit dough to make sharing even easier to do.

Connecting Mathematics



Aliet's story creates a wonderful context for mathematics learning and play, while connecting her experience to Mi'kmaq culture and language. The experience comes to life in the classroom, with students preparing to make lu'sknikn to share. Nukumi shows Aliet a way to mark the top of the dough, and creates rows and columns for the biscuit. Students can mark the dough similarly on their pan of lu'sknikn, and count the groups of biscuits by rows or by columns. The area model helps students to visualize the pan of lu'sknikn in a new way and to develop their spatial reasoning skills.

Increasing the number of pans encourages students to investigate finding the total quantity and efficient methods for sharing the lu'sknikn equitably. Teachers may introduce an idea for students to double the recipe for community sharing, and prompt further inquiry into quantities and products. Verbing the activities (Lunney Borden et al., 2021) by talking about processes and actions associated with Aliet's baking lesson, and focusing specifically on verbs like doubling, increasing, grouping, and forming (rows and columns), as in these mathematical contexts, engages students more fully as they participate actively in learning. Even before students hear about "multiplication," they experience multiplicative concepts by doing, and demonstrate conceptual understanding by making equal sets, skip-counting, and grouping objects.



Children's literature provides context and purpose for problem solving. *Aliet Learns to Make Lu'sknikn* brings Mi'kmaq culture and tradition together for students, and invites all students to engage in mathematical thinking. When students participate in hands-on activities in the kitchen involving measuring, counting, and sharing, learning comes to life. And, as we all know, learning always tastes better with a nice hot snack out of the oven!

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Lunney Borden, L., Throop Robinson, E., Carter, E., & Prosper, S. (2021). Sets of, rows of, jumps of: Verbing multiplication. *For the Learning of Mathematics, 41*(3), 2–8.

Meader, J. (2020, February 26). *N.S. elementary students starting to learn Mi'kmaw culture, history. CBC News*. www.bit.ly/3BuzrJU

Trakulphadetkrai, N.V., Aerila, J.A., & Yrjänäinen, S. (2019). Bringing mathematics alive through stories. In K. Kerry-Moran & J.A. Aerila (Eds.), *Story in children's lives: Contributions of the narrative mode to early childhood development, literacy, and learning* (pp. 199–225), Educating the Young Child (Advances in Theory and Research, Implications for Practice), 16. Springer, Cham. doi.org/10.1007/978-3-030-19266-2_11

CONCLUDING REMARKS

Our biggest take-away from this special issue on Indigenous perspectives for learning about multiplication is how the activities and suggestions highlight movement. There is much to be had by focusing on students' count*ing*, doubl*ing*, increas*ing*, group*ing*, chang*ing* the question (i.e., 5 x 6 to 10 x 3) and see*ing*/show*ing* 3 rows of 6 and 6 rows of 3 simultaneously in an area model. In fact, the word "multiplication" does not exist in the Mi'kmaq language. It is important to note that neither of our guest authors makes mention of memorized multiplication times tables or learning to regurgitate facts. Perhaps, this is because multiplication times tables lean more toward static ideas. The Mi'kmaq language and ways of knowing have been great reminders for us to be privileging the *'ing'/*(verbing) involved in the teaching of mathematics.

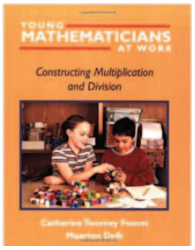


MOVING FORWARD: READINGS FOR CONTINUING PROFESSIONAL LEARNING

If you're interested in the different ways multiplication can be taken up or looking for professional learning experiences, we highly recommend the following readings:



Davis, B. (2008). Is 1 a prime number? Developing teacher knowledge through concept study. *Mathematics Teaching in the Middle School, 14*(2), 86–91.



Fosnot, C.T., & Dolk, M. (2001). *Young mathematicians at work: Constructing multiplication and division.* Heinemann, 88 Post Road West, PO Box 5007, Westport, CT 06881.

Other references cited in this issue:

Davis, B., & Renert, M. (2009). Mathematics-for-teaching as shared dynamic participation. *For the Learning of Mathematics, 29*(3), 37–43.



We Want to Hear from You

We are still very interested in how mental math translates into practice for you! We are hoping you'll share these ideas, as well as what you would like us to focus on in our next issue, by completing this short survey.

https://forms.gle/26mK2r1Y5nkj4rzp8

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