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HOW CAN TEACHERS MAKE VISIBLE THE THINKING OF STUDENTS AS THEY LEARN?

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IN THIS ISSUE

In this issue, we will focus on teaching addition and include a discussion on making visible the thinking of students. We see many affordances to be had through examining and discussing the place and role of pedagogical documentation in the mathematics classroom. We think of pedagogical documentation broadly to be a "process for making pedagogical (or other) work visible and subject to dialogue, interpretation, contestation, and transformation" (Dahlberg, 2012, p. 225).

Aligning with this conceptualization of pedagogical documentation, we share an activity/process we have come to call "Whose thinking is it, anyways?" This activity can be played like a game and be paired with a number talk to create what some might call a supercharged number talk. The goals of the game are to: 1) offer written forms of mental-math strategies that students have shared during a number talk, and 2) provoke students to analyze and compare their peers' strategies that were used to solve a mental-math question. A Grade 2 example for 14 + 13 of the game "Whose thinking is it, anyways?" is provided in this issue; however, it can easily be adapted for mental-math lessons at all grade levels.

Our special guest author, Dr. Allison Tucker, from St. Francis Xavier University, whose research expertise is pedagogical documentation, continues the discussion by sharing her thoughts on pedagogical documentation, what it looks like in a mathematics classroom, and how it can be extended for teachers, students, and families.

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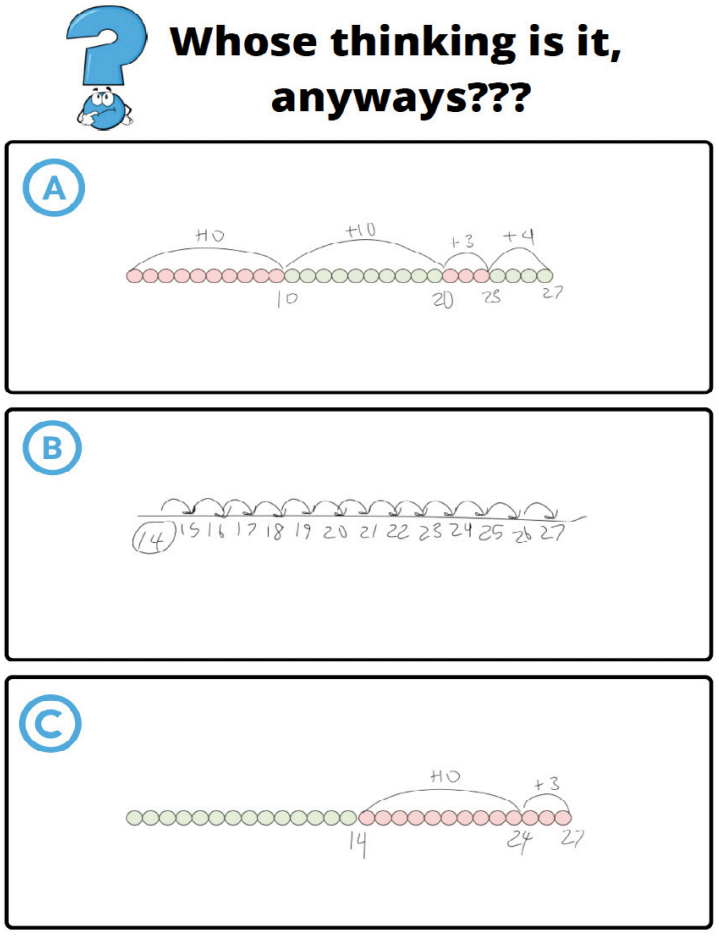
“WHOSE THINKING IS IT, ANYWAYS?”

“Whose thinking is it, anyways?” is a practical game that teachers can use in their classrooms at all grade levels to focus on students’ mathematics ideas about addition, subtraction, multiplication, or division. The game was developed as a way to offer written forms of ideas that students share in class, and to provide students with experiences in comparing and analyzing their peers’ mathematical solution strategies. The game can take place over the course of two days or even within a week.

Here’s how you can play:

Day	Teacher	Student
One	Number talk for 14 + 13 • Ask (verbally) students to solve an arithmetic question (e.g., 14 + 13). • Ask one student to explain the strategy they used to solve the problem. • Ask if any other student used a different strategy. • Asking students to share a different strategy continues until the students have no other strategies to share.	• Explain the strategy, and discuss how it is different from the already-shared strategy.
Two	Game: “Whose thinking is it, anyways?” for 14 + 13 • Hand out a document that contains three student strategies, represented in written form. We suggest that the written forms use a mathematical tool (e.g., number line). • Ask students to identify which classmate shared the strategy and which strategy they think is most efficient.	• Students are tasked with identifying which classmate shared the strategy, and identifying which strategy they think is most efficient.

To the right is an example of a worksheet created after day one. It depicts three different student strategies to compute 14 + 13. This is an example of a worksheet that was used to play “Whose thinking is it, anyways?” for 14 +13.



ALL TOGETHER: NURTURING EARLY MATHEMATICAL THINKING THROUGH PEDAGOGICAL DOCUMENTATION

by Allison Tucker

What is pedagogical documentation?

Pedagogical documentation is a process through which educators make visible the thinking of children as they learn (Knauf, 2020; de Sousa, 2019; Tarr, 2010; Wien, 2013). As curious observers of children, educators watch for and record through watching, listening, photographing, and reflecting on the learning explorations of the children (Ontario Ministry of Education [OME], 2012). They capture the learning happening in the moment, and also come to understand the learning trajectory of the child through emerging unfolding and ongoing learning stories (Knauf, 2020; Steele, Hives, & Scott, 2016).

Teachers as Curious Researchers

A surface understanding of pedagogical documentation might equate pedagogical documentation with simply photographing children as they are engaged in learning explorations. However, there is a significant difference: the educator is not looking for cute photo opportunities; rather, they are observing the children as they play, through the lens of a curious researcher (Tarr, 2010). The researcher observes intently, using their knowledge of child development to guide their eye. Their knowledge of children's learning, and the ways in which learning deepens and grows with experience, allows them to notice changes, understand the child's thinking, move into the play to scaffold learning through anticipated questioning, and offer language that supports the child to connect and construct their understanding.

What does pedagogical documentation look like in a classroom?

The following is an example of how pedagogical documentation of mathematical thinking about addition might be taken up in a Kindergarten classroom. It describes an observed play episode, in which Sam and Jadyn, who had initially been playing in the house area, decided to plan a birthday party for baby. Two of the children determined they would make cookies for the party. Their conversation unfolded as follows:



Jadyn: I am making cookies for the party, star cookies because that is the kind the baby likes.
Sam: I'll help you. We need to make some for everyone! Teacher, we are making cookies for everyone!
Teacher: Cookies for everyone? How many do you think you will need?
Sam: One for everyone. One for Jadyn, ore for Nila, one for Cody, one for Maggie...lots! Maybe a hundred!
Teacher: A hundred? Who are all the cookies for?





Sam and Jadyn begin to look at the cookies and name children again.

Teacher: Hmm, I wonder how many cookies you will need in all?

Jadyn steps back watching Sam continue to name his classmates, then goes to a list of children's names and begins counting the names of all the children. When he is finished counting, he calls back to Sam and their teacher.

Jadyn: We need 14 cookies in all!
Sam: 14? Okay! That's a lot! We need to make lots of cookies!

What can the educator learn from the pedagogical documentation of this episode?

While this is not an unusual scene in a Kindergarten classroom, the educator has noticed it and documented it because it shows mathematical understanding of both Sam and Jadyn. *The Kindergarten Program* (OME, 2016) states that children will have opportunities to "investigate addition and subtraction in everyday experiences" (p. 225), and that they will "demonstrate an ability to use problem-solving skills in a variety of contexts including social contexts" (p. 255). Thinking in terms of addition, Jadyn and Sam approach addition (finding a total number) from different places of learning through their problem-solving, classroom experience. The educator's research, looking back on the cookie experience to notice the learning that occurred, provides insight into the children's mathematical thinking:

> Sam is thinking about the total as "a lot," demonstrating his growing understanding of quantity, but to determine how many exactly they will need to produce, he associates each item (cookie) with a student, showing he is using one-to-one correspondence to ensure there are enough.
>
> Jadyn, in his adoption of the term "all together," and his approach to count the number of students in the class to inform them how many to make, is thinking of the total as a quantity and equating the number of students with the number of cookies. He uses the total number of students' names to find the total number of cookies required.

How might the educator use what they have noticed?

As researchers of their own teaching practices, educators use pedagogical documentation as a means through which to consider the learning journey of the child and to intentionally respond to that learning in ways that might nurture deeper investigation and understanding on the part of the child. The educator might plan to support Sam and Jadyn in the following ways:

Sam

- Anticipate opportunities to question and scaffold an understanding of "all together."
- Look for opportunities for Sam to encounter "how many" problems, such as the one with the cookies, and question how he might find out how many items there are "*in total*" or "*all together*."

Jadyn

- Anticipate situations for Jadyn to apply his new understanding.
- Look for opportunities to question and scaffold his learning by asking him to articulate his thinking (*How did you figure out how many cookies you would need*? *What did you do to find out how many you needed?*), to strengthen his understanding.

Extending the Invitation to Notice

Using pedagogical documentation invites the extension of learning in numerous ways beyond capturing moments. It offers educators opportunities to position themselves as curious researchers of children's learning (Tarr, 2010; Wien, 2013), to invite children to reflect on and deepen their thinking of what they are learning, and to open the conversation to families to share in the learning occurring in the classroom.

Educators might view the documentation in that it:

- captures what children are learning, how they are thinking mathematically, and applying their thinking in a variety of situations
- feeds the curiosity of the educator—researching what children are learning and how one might respond
- enables them to generously observe and notice, in a multitude of ways, how children approach challenges, and how they as educators might respond (de Sousa, 2019)
- highlights how the community of learners supports each other and honours the role children play in each other's learning journeys

Children, when included in pedagogical documentation, might have opportunities to:

- talk about what was noticed, to revisit and articulate what was happening as a full participant in their learning (Picchio, Di Giandomenico, & Mussatti, 2014)

- share their thinking with educators and other children, possibly extending their thinking through the explored memory
- feel honoured that their learning is of value in the classroom

Families, when invited to interact with pedagogical documentation, might:

- share in experiences of the classroom at home, as it provides a lens into the children's lives lived in the classroom
- be offered an opportunity to more deeply understand the complexity of the learning processes of young children (Barbili & Tzioga, 2014)
- continue conversations when they see similar investigations happening at home, connecting school and home, as they understand the ways children mathematically interact with and understand the world

Pedagogical Documentation: All-Together Thinking

As children come into classrooms, they bring with them their own curiosities, their wonderings, their stories of learning, to which they continually contribute. Pedagogical documentation, when approached as a way to capture moments of learning that deepen and grow over time and to nurture that learning, creates an interwoven understanding of learning on the parts of educators, children, and families. For educators, it contributes to their understanding of the narrative arc of the child's learning story. For the child, it can help them come to understand themselves as the authors of their learning stories. For families, it can invite them into the space of pedagogical relationship and, with educators, to see their children as the capable and competent beings they are, and are becoming. Pedagogical documentation is a way of noticing and nurturing the child as a learner, all together.

CLOSING REMARKS

Although pedagogical documentation is described as a process, we are very interested in the documents/artifacts that are "left after" the "going-ons" and their potential in mathematics education. In the case of "Whose thinking is it, anyways?," the document that remains is the worksheet. In some ways, the "left after" of the pedagogical documentation paper is the paper itself. We see both documents relating to the prompts: "Explain," "Justify." For the "Whose thinking is it anyways?" worksheet, there is an explanation of how to solve the question. The pedagogical documentation paper provides justification of the learning that occurred.

Actually, the prompts "Explain" and "Justify" are deeply rooted in the work of students and teachers:

a) On tests and assignments, students are constantly being asked to communicate in written forms by being asked to "explain" or "justify."

b) Teachers are expected to assign grades to students and then "justify," "explain," or convince others that the grade assigned represents students' understandings. This is done by gathering evidence/documents, and typically includes test and assignment documents, where students were prompted to "explain" and "justify" their answers.

This relationship—teachers and students being expected to perform similar processes—makes us think of ways to improve the process so that what is "left after" (i.e., the artifact) clearly documents what took place. In our own practice, we have experimented with the following two prompts with students, which has served to amplify the work in production:

a) How can you record your ideas so that someone, who was not here, can know what we did and how you were thinking?

b) I am going to pretend you're not here and try to make sense of your work. I'll talk out loud, but you can't talk. That way, you can see what sense people might be able to make of what you wrote.

We, as teachers, also practise how we could write to fellow teachers and our students to show the strategies students shared. We think there is a lot of learning for us as teachers and students to extend the "Whose thinking is it, anyways?" by involving students in creating the worksheet and getting feedback from one another.

Addition **Lesson Plan for a Number Talk about 14 + 13**

Getting Started

- Step 1: Make sure to VERBALLY (no writing) prompt students. For example, you can say: "Today's number talk is about 14 + 13."
  Follow-up prompts: What are you picturing in your head when you think about 14 + 13? Which number are you seeing in your mind?
- Step 2: Provide students with a couple of minutes of think time. During think time, display an image of 14 + 13, using manipulatives (see Figure 1 for an example). You can further prompt at this point with: Are you seeing or thinking about groups? Area? Sets? How would you explain what you did and how you are thinking to a friend?
- Step 3: Have students explain their strategy to a partner.

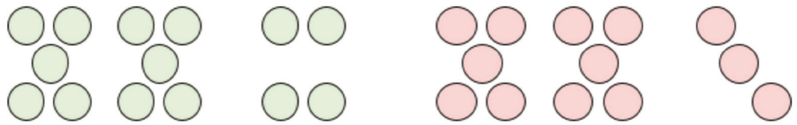
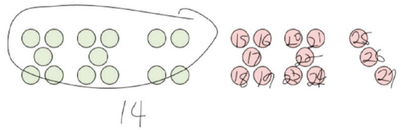


Figure 1: Images of 14 + 13 for the teacher to display
*Teacher notes: It is important to leave a gap between the 10 and the 4 and the 10 and the 3.

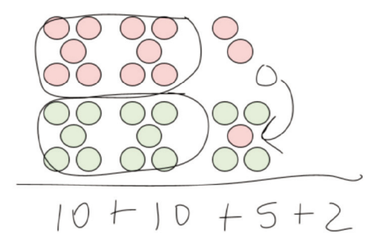
Anticipate Solution Strategies for 14 + 13

Solution 1: Counting on from 14



Quick Tips
- *Ask students how they counted all the dots or figured out how many dots there are in total.*
- *Possible ways:*
  - *Counting three times: first counting to 14, then counting to 13, then starting from the beginning all the way to 27*
  - *Counting on by 1's from 14*
  - *Counting by 5's or 10's and 1's*

Solution 2: Decomposing 14 and 13 into 10's and 1's, then adding the 10's (10 + 10 = 20) and then 1's (4 + 3 = 7).



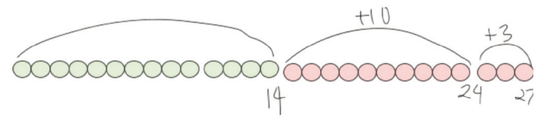
Quick Tip
Ask clarifying questions when students are sharing their ideas:
- *What did you do first?*
- *Can you say that again?*
- *Does everyone see the groups of 5? Show me.*
- *Tell me more?*
- *How does that make sense?*
- *How do you know that?*
- *Do you really like 5? Why?*

Video resources: These anticipated solutions are static representations of what children may offer. Please watch the videos (link below) to see how students may share their solution strategies. The attached "open-go" number-talk video can easily be used in your classroom. NO PREP or manipulatives required!
https://vimeo.com/669661309/0ae1ee2c74

- Curriculum Expectations (MOE, 2020) (not to put on the board with students)

Prior Grade	Current Grade	After Grade
Grade 1: use mental-math strategies, including estimation, to add and subtract whole numbers that add up to no more than 20, and explain the strategies used	**Grade 2:** use mental-math strategies, including estimation, to add and subtract whole numbers that add up to no more than 50, and explain the strategies used	**Grade 3:** use mental-math strategies, including estimation, to add and subtract whole numbers that add up to no more than 1000, and explain the strategies used

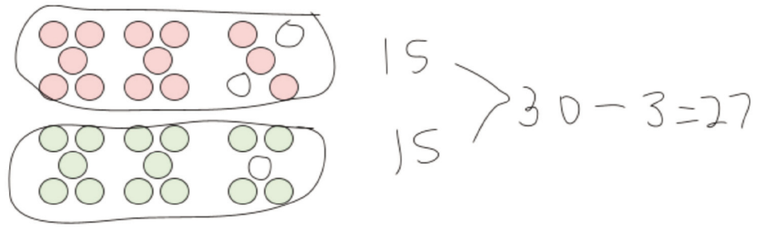
Solution 3: Counting on by 10's and 1's



Quick Tips (manipulatives)
- *Use a beaded number line and pull a group of 14 to show that this number goes "in your mind."*
- *Actually have magnetic dots to use on the whiteboards, or use a projector so that students can see the dots moving.*

Quick Tip
- *Placing the dots on a number line, as students count, can help students make connections between their images of dots (quantity) and the number line.*

Solution 4: Changing the question to 15 + 15 (compensation)



DURING
- 5–10 minutes
- "Can someone come up and tell us what they did to figure out 14 + 13?," "What did you see in your mind?"
- As the student explains, the teacher records by moving manipulatives and making annotations as needed.
- Invite another student to share. "Is there someone who did something different?"
- The teacher again records by moving manipulatives and making annotations as needed.

Quick Tips
Mathematical annotations:
- *Mental math is much more than mere computations.*
- Include mathematical vocabulary, symbols, elaborations of mathematical details from solutions, and labels describing approach/strategy. All annotations are records of students' mathematical discussion.

ENDING (Consolidation)
- Have students analyze and compare solutions.
- "How are these solutions different or the same?"
- "If your solution was not talked about, which of the solutions was most similar to yours? Which one was it most like?"
  OR "Which one was most different from your solution?"

AFTER (Next steps and practice)
- If appropriate, the teacher can challenge students to use a different strategy next time (e.g., "If you used your fingers to keep track of 13, for our next talk, I challenge you to make use of the number 10."
- Problems could vary by number (choice, size), problem contexts, or what is unknown or needs to be solved.
- Students make their own written document of the strategies shared.
- Students can make their own word problems for the question.



Note: Template adapted from OAME *ABACUS* Template, created by Kathy Kubota-Zarivnij and MaryLou Kestell.

In the end, in mathematics education, the importance of math talk has been established (e.g., Campbell & Boylard, 2018), and there are many recommendations for conducting/orchestrating math discussions (Smith & Stein, 2018). We think the time is ripe to now turn attention to the "left after" artifacts/documents from number talks—the affordances to students and teachers are abundant. Specifically, because tests are riddled with "explain" or "justify," we predict that making thinking visible processes can greatly impact test scores and ultimately societal discussion about testing.

MOVING FORWARD: CONTINUING PROFESSIONAL LEARNING

Interested in reading more about the importance of math talk? Making thinking visible? We recommend checking out the following readings/resources:

- *Why Students Need More 'Math Talk'* (Campbell & Boylard, 2018)
- Harvard Graduate School of Education's Project Zero
- *5 Practices for Orchestrating Productive Mathematical Discussion* (Smith & Stein, 2018)

We are very still very interested in how mental math translates into practice for you! We are hoping you'll share these ideas, as well as what you would like us to focus on in our next issue, by completing the short survey at https://forms.gle/26mK2r1Y5nkj4rzp8.

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