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ELEMENTARY TEACHERS' CONCEPTIONS OF MENTAL MATH AND TEACHING DIVISION

ABACUS CO-EDITORS



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IN THIS ISSUE

In this issue, we will focus on the basics of teaching division and how teachers describe mental math to other teachers and to students (the topic of the survey that we sent to all subscribers of the *Abacus* last year). Mental math and the way educators are thinking about it is important because: 1) mental math can build important foundation, and 2) here, in Ontario, there is now a specific focus on mental math in the elementary school curriculum—meaning, it is crucial that time is spent on how mental math is and should be occurring in elementary mathematics classrooms. Moreover, we feel that there is much to be gained by discussing how practice and research recommendations about mental math are in line or disjointed.

Our special guest author, Dr. Evan Throop Robinson, from St. Francis Xavier University (St. FX) in Nova Scotia, whose research expertise is elementary mathematics education, shares his thoughts on soft starts. We have heard other teachers talk about soft starts as an alternative to "bell work." Basically, at the beginning of a lesson or the day, just like adults starting a day, students might like to ease into their math lesson or day. We interpret Dr. Throop Robinson's conception of soft starts in the mathematics classroom to include students' prior knowledge and some materials/manipulatives—meaning that soft starts are short activities on topics students typically have much experience in, and that students can do on their own or in small groups. These activities can be done while other students are completing daily routines (e.g., hanging up jackets, finding pencil cases, saying "Good morning").

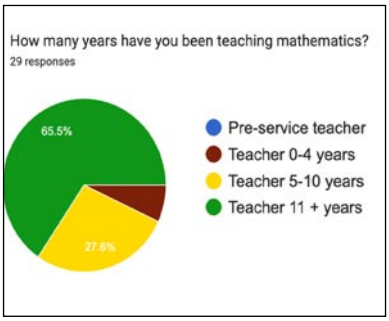
HOW TEACHERS ARE THINKING AND TALKING ABOUT MENTAL MATH IN THEIR CLASSROOMS

We share perspectives of our teaching colleagues in Nova Scotia because uptake of the survey that we sent out to the readership of the *Abacus* was low. The results from our study are based on elementary teachers' conceptions of mental math, who were either current or previous students of St. FX's Certificate of Elementary Mathematics Pedagogy (CEMP) program. About 210 teachers across the province of Nova Scotia received the survey. We received 29 responses. The survey looked at how teachers are currently thinking about mental math—what does it look like, sound like, and feel like in their classroom or classrooms they work in? The goal of the survey was to help us learn how teachers are thinking about and using mental math in classrooms, and to inform future professional learning experiences for teachers.

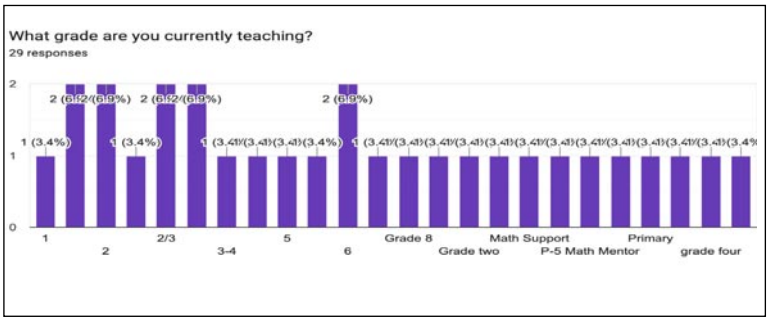
The survey consisted of the following questions:

1. How many years have you been teaching mathematics?
2. What grade are you currently teaching?
3. How would you explain what mental math is to your teacher colleagues?
4. How would you explain what mental math is to your students?
5. What does mental math look like in your classroom? (e.g., describe lessons, activities, worksheets, resources, frequency, assessment)

We share the responses to the questions, using graphics, and conclude with some of our own thoughts on what the data might be indicating. The graphics for questions 3, 4, and 5 were created by wordclouds.com.



QUESTION 1: How many years have you been teaching mathematics?



QUESTION 2: What grade are you currently teaching?

QUESTION 3: How would you explain what mental math is to your teacher colleagues?

Many of the teachers explained mental math to colleagues by describing it as the ability to do calculations/computations in your head, without the use of paper or pencil. They stated that mental math involves:

- student thinking
- strategies for solving number problems
- making sense of numbers



Figure 1

QUESTION 4: How would you explain what mental math is to your students?

Many of the teachers explained mental math to students by describing it as the ability to do calculations/computations in their head, without the use of paper or pencil. They stated that mental math involves:

- thinking
- using strategies
- helping students to "see" math in their heads



Figure 2

QUESTION 5: What does mental math look like in your classroom? (e.g., describe lessons, activities, worksheets, resources, frequency, assessment)

It looks like students engaged on a regular basis (daily) in the following activities:

- Playing Games (e.g., esti-mysteries)
- Working on open-ended tasks (e.g., Show Me All the Ways to Make 59)
- Subitizing
- Talking with peers
- Student explanations
- Using different representations (e.g., ten-frames, dot arrangements)



Figure 3

Our Thoughts on the Data/Discussion

It is interesting to note some similarities and differences among the question responses. There were a lot of similarities between how teachers explained mental math to their colleagues and their students; in fact, one teacher stated “the same way as above” to question #4. However, a few differences have piqued our interest. For example, with teacher colleagues, mental math was associated with sense making, which appeared much less often in explanations to students.

We also noticed the relationship between mental math and making thinking visible. Visualizing, or “seeing,” what’s in our mind (mental) was determined to be valuable, but what was less clear was how teachers do this. We recommend that teachers and researchers dedicate time and effort to investigate “making thinking visible.”

THE BASICS OF TEACHING DIVISION

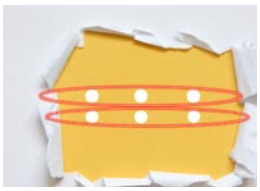
We feel that there are a few things that really need to be focused on when teaching division in elementary and middle schools: 1) students’ descriptions of how they see division in their minds, and 2) how experiences of division involving whole numbers can be extended to division involving fractions.

Students’ Descriptions of Division

We typically hear beginning learners of division describe two ways that they think about division. For example, if we ask students 3 ÷ ½, they reword the question to:

1. How many times does 3 fit into 6? Or how many groups of 3 result in a total of 6?
2. If 6 is split into 3 equal groups, how many are in each group?

When we ask students to draw something to give us insight into how they are thinking about the question, we typically get two different pictures:




These pictures can and *should be* gradually manipulated to become one another, for instance:



The last area of difficulty that arises when teaching division is the involvement of fractions as the divided or the divisor. One way we have successfully remedied this is by reminding students of what they already know about division of whole numbers. For instance, if students don’t know where to start 3 ÷ ½, remind them that there are two ways to think about 6 ÷ 3:

1. How many times does 3 fit into 6?
2. If 6 is split into 3 equal groups, how many are in each group?

Ask students whether they can use the form of questions involving whole numbers to make sense of questions with fractions. Here are the general forms of these questions:

1. How many times does ___ fit into ___?
2. If ___ is split into ___ equal groups, how many are in each group?

For example, 3 ÷ ½ can take the form of this question:

How many times does ½ fit into 3 ?

Division **Lesson Plan for a Number Talk about 3 ÷ ½**

Getting Started

Step 1: Make sure to VERBALLY (no writing) prompt students. For example, you can say: "Today's number talk is about **three divided by a half**."

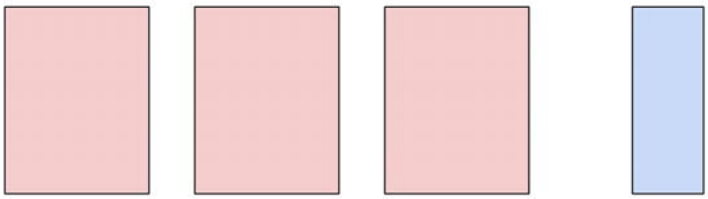
Follow-up prompts: "What are you picturing in your head when you think about **three divided by a half**?" "Which number are you seeing in your mind?"

Step 2: Provide students with a couple of minutes of think time. During think time, display images students could use to solve **three divided by a half** (see Figure 1 for examples). You can further prompt at this point with: "Are you seeing or thinking about groups? area? counting? multiplication?" "How would you explain what you did and how you are thinking to a friend?"

Step 3: Have students explain their strategy to a partner.

Figure 1: Tools for teacher to display

1)



2)



*Teacher notes: Start by: 1) posting/showing three sheets of paper in the same colour and one piece of paper in another colour that is cut into two equal pieces, and 2) post a blank number line with only zero labelled.

DURING (5–10 minutes)

"Can someone come up and tell us what they did to figure out 3 ÷ ½." "What did you see in your mind?"

As the student explains, the teacher records by moving manipulatives and making annotations as needed.

Invite another student to share. "Is there someone who did something different?"

The teacher again records by moving manipulatives and making annotations as needed.

Quick Tip

Ask clarifying questions when students are sharing their ideas:

What did you do first?
Can you say that again?
Does everyone see one-half?
Tell me more.
How does make sense?
How do you know that?

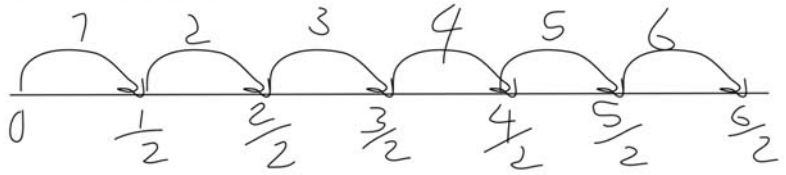
Video resources: These anticipated solutions are static representations of what children may offer. Please watch the videos (link below) to see how students may share their solution strategies. The attached "open-go" number-talk video can easily be used in your classroom. NO PREP or manipulatives required!

www.vimeo.com/669661309/0ae1ee2c74

- Curriculum Expectations (Ontario Ministry of Education, 2020) (not to put on the board with students)

Prior Grade	Current Grade	After Grade
Grade 5: B2.8 multiply and divide one-digit whole numbers by unit fractions, using appropriate tools and drawings	**Grade 6:** B2.10 divide whole numbers by proper fractions, using appropriate tools and strategies	**Grade 7:** B2.8 multiply and divide fractions by fractions, using tools in various contexts

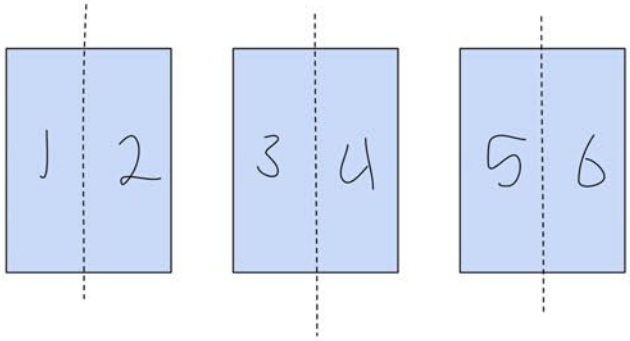
Solution 1: Counting by ½'s



Quick Tips

Ask students how many halves will fit into three OR how many groups of halves will make three wholes.
Count by halves: one one-half, two one-halves, three one-halves, four one-halves, five one-halves, six one-halves.
Fit six halves onto the three pieces of paper.

Solution 1: Fitting ½'s into 3 wholes



Quick Tips (manipulatives)

Have full pieces of paper, and papers cut into two equal halves, on the whiteboards, or use a projector so that students can move the papers. The half pieces of paper should be a different colour from the full pieces of paper.

Quick Tip

Mathematical annotations:

Mental math is much more than mere computations. Include mathematical vocabulary, symbols, elaborations of mathematical details from solutions, and labels describing approach/strategy. All annotations are records of students' mathematical discussion.

ENDING (Consolidation)

Have students analyze and compare solutions:

"How are these solutions different or the same?"

"If your solution was not talked about, which of the solutions was most similar to yours?" "Which one was it most like?" OR "Which one was most different from your solution?"

AFTER (Next steps and practice)

If appropriate, the teacher can challenge students to pose their own questions to classmates who would use the same strategy to solve a problem.

Problems could vary by number (choice, size), contexts, by the unknown, or what needs to be solved.

Students make their own written document of the strategies shared.

Students can make their own word problems for the question.



Note: Template adapted from OAME *Abacus* Template, created by Kathy Kubota-Zarivnij and MaryLou Kestell.

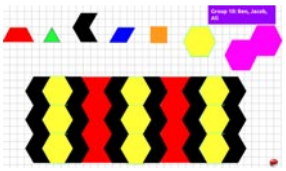
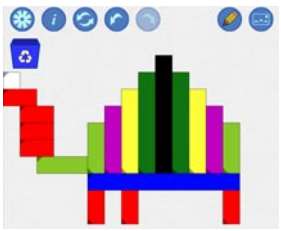
USING SOFT STARTS TO LAUNCH LESSONS

As a purposeful transition into a lesson or even the day, soft starts provide students with playful, exploratory activity to stimulate interest and curiosity about a concept. San Giovanni (2019) suggests teachers adopt strong daily routines that "have potential to ignite engagement and take advantage of students' brains being ready to learn" (p. 3). Soft starts gently invite students to take up mathematical challenges and share what they know with their peers.

Starting Softly

As students arrive in my classroom, there is always a bustle of activity and many conversations happening at once. To build upon this energy (rather than trying to rise above it!), my lessons start softly with a hands-on task. My favourite soft starts use familiar manipulatives (see Table 1). Students find blocks, rods, and/or base-ten materials on their table along with simple challenges:

Table 1

Manipulative	Challenge	Product (Concrete)	Product (Virtual)
Pattern Blocks	Picture This: Create a picture, using blocks to show symmetry, patterning, or groupings (assign values to each pattern block and then find the total value)		
Cuisenaire Rods	Cuisenaire Creatures: Design a creature, using rods (find the creature's age by adding the lengths of all the rods)		
Base-ten Materials	Planet Decima: Build a citizen, using base-ten blocks (roll dice to collect units, tenths, hundredths, and thousandths, and find the total decimal value)		

Why start softly?

Zevenbergen (2005) described an "orientation phase," or opening, to a mathematics lesson, which introduces a topic, motivates students, and provides time for teachers to assess students' prior knowledge before moving into a lesson. Using soft starts promotes the following orienting activities:

- Observing students working in small groups, respond to questions from individuals, and provide extensions
- Eliciting what learners already know
- Developing new mathematical ideas
- Talking, doing, and sharing emerging ideas with peers

Soft starts are invaluable for community building as I join conversations, cross-pollinate ideas from one group to another, and provide one-on-one support for students, without the whole class looking on. Often there is laughter, joy, and playful engagement around the room as a result of the soft start.

Introducing Dividing with a Soft Start

I place piles of manipulatives (square tiles, counters, multilink cubes, base-ten blocks) on tables around the room and display a problem for the class to solve.

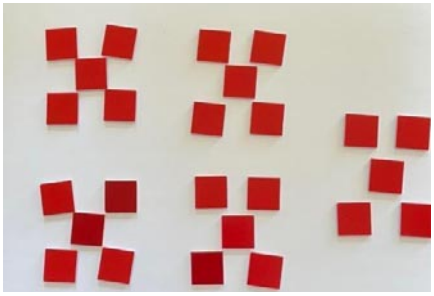
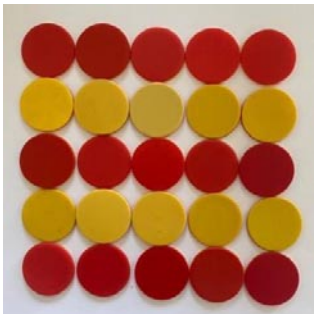
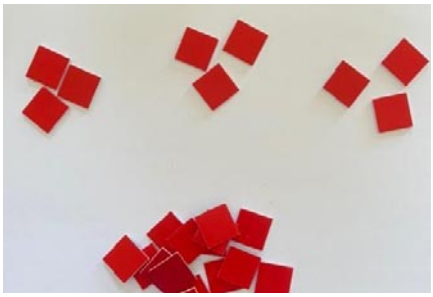
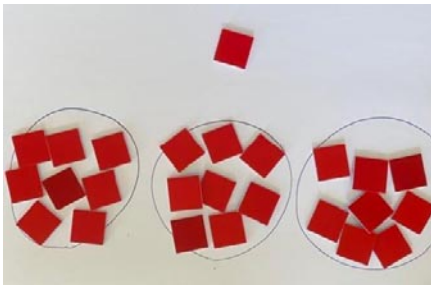
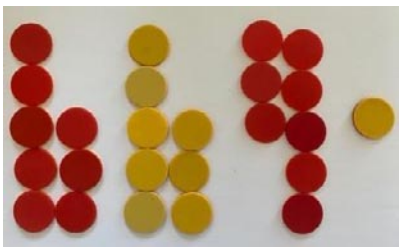


Rafaele wants to share her treats equally among herself and two friends. She has 25 treats.

I walk around the room, but don't say too much until I hear students beginning to talk within their groups. Some students will ask me questions, and I usually answer with more questions: What do you think? What have you tried? What does your group say about that?

Students will start to organize the concrete blocks they have chosen to represent the quantities in the problem in various ways (Table 2).

Table 2

Square Tiles	Counters	Multilink Cubes
 Making familiar "5's" arrangements	 Forming a 5 by 5 array	 Making "10's" towers
 Sharing one by one into 3 groups	 Showing groupings of 10	 Splitting 10's into 5's
 Grouping equally	 Building from "5"	 Levelling 3 towers

Conversations intensify and, depending on what I see and hear, I begin probing students' thinking with a variety of questions as I visit each group:

- Why did you make this grouping?
- How did you start to make the groups?
- What were you thinking about?
- How did you know that?
- What do these groups represent?
- What will you do with the leftover pieces?
- What do these blocks represent?
- How does that make sense to you?
- How would you document the process you chose?

Students will show their problem solving, using the blocks and any notes they might have that document their thinking. During these conversations, I introduce mathematical language to support their thinking (i.e., dividend, divisor, quotient, remainder, equal parts, sharing, regrouping), and encourage students to take up this terminology. In all cases, the question about what to do with the one leftover treat causes students to consider their options. Some groups decide to break the treat into three equal pieces and give each friend a "fraction" of the treat. Other groups decide to reward Rafaele for her generosity and give her the leftover treat as a whole. One group decides not to complicate the sharing and throws the treat away!

Ending Soft Starts

In the follow-up discussions with the whole class, I ask groups to share their findings and to comment on the strategies they hear from peers. It is important to ask if other groups did something differently so the students can analyze the representations and discuss the similarities and differences between groups.

Collectively, students will develop and describe their own models for dividing, based on grouping equally, subtracting repeatedly, or levelling towers. The soft-start approach allowed students to develop reasoning, rather than receiving procedures to follow. As Daniels stated in *The Curious Classroom*, "soft starts… [are] a better way to set the tone and mindset for genuine engagement and challenging inquiry" (68).

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SanGiovanni, J.J. (2019). *Daily routines to jump-start math class, elementary school: Engage students, improve number sense, and practice reasoning.* Corwin Press.

Zevenbergen, R., Dole, S., & Wright, R.J. (2005). *Teaching mathematics in primary schools.* Allen & Unwin.

MOVING FORWARD: CONTINUING PROFESSIONAL LEARNING

Ritchhart, R., & Perkins, D. (2008). Making thinking visible. *Educational Leadership, 65*(5), 57–61.

Although not specifically about math, this is a good place to start when attempting to make students' ideas visible.

Costello, D. (2021). Linking literacy and math: Using writing to support instruction and learning. *Ontario Mathematics Gazette, 60*(2), 13–14.

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