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CONNECTION MAKING: WHAT IS IT AND HOW DO WE PROMOTE IT?

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IN THIS ISSUE

In this issue, we share an adapted/condensed version of a paper on connection making that we (Marc and Tina) and our guest author (Cristina De Simone) presented at the Canada Society for the Study of Education Conference that was held in May 2023 in Toronto. We share the paper here because it is all about how new teachers define connection making and envision themselves supporting students to make connections in their classrooms. Our guest author (Cristina De Simone) helps us translate the findings directly to the classroom by reflecting on her own teaching through how the research participants/teachers talked about connection making. Specifically, as Cristina is very familiar with the paper and is a current elementary teacher, she is well positioned to help identify concrete teaching moves that can support students in making connections.

We hope that readers will find their own teaching and ideas about connection making in this issue and will write to us and share more CONCRETE ideas about how we can support students to make connections in mathematics classrooms in future *Abacus* issues.

The focus of our lesson insert centres on the mathematical question 91 – 79, which emerged as a topic of frequent discussion among the research participants mentioned in the paper. Our goal is not only to provide readers with a valuable classroom lesson, but also to ignite their curiosity and encourage reflection on making connections in mathematics.

P-K THEORY THROUGH THEIR EYES: TEACHERS' CONCEPTUALIZATIONS OF CONNECTION MAKING

The Purpose

Although there seems to be agreement on the importance of connection making in math (e.g., Hiebert & Carpenter, 1992), there is no single definition or explicit advice about how teachers can support connection making. To shed light on the nuances surrounding connection making in classrooms, we developed a professional development course for newly certified teachers, using the Pirie and Kieren (P-K) Theory (1988; 1992; 1994a; 1994b) as the primary resource for creating course activities, and the main reading of the course.

We chose P-K Theory (1994) as the main reading of the course for two reasons:

1. Although the theory was written for both teachers and researchers, there has been limited uptake by teachers, and we wanted to learn how P-K Theory could support teachers to think about connection making in their future teaching.
2. PK Theory provides the language and insights for interpreting course events that endeavour to support teachers in "build[ing] further understanding… [by] construct[ing] the connection for themselves" (p. 80), meaning that P-K emphasizes that learners cannot simply be told to connect, but must have opportunities to make connections for themselves.

About the Study

This study took place during a ten-day Additional Qualification (AQ) professional development course for new teacher education program graduates at a large Canadian University. Fifteen teachers volunteered to participate in this study. At the end of the course, teachers were asked to write a definition for connection making in classrooms and describe how teachers can support connection making.

Results

Upon analyzing the data, three distinct categories surfaced that outline participants' conceptualization of connection making:

1. Personal, but involves others' ideas: Teachers viewed connection making as a personal process that involves reflecting on their own and others' ideas.
2. Non-linear linking of knowledge: Teachers described a process of connecting prior knowledge with new ideas or images in non-linear ways.
3. Algorithms and tools: Teachers identified that algorithms and tools provide a good opportunity where they themselves were able to make connections and students can make connections. They described the ways that various tools (such as manipulatives, drawings, meter sticks) and algorithms can be discussed to promote connection making.

In regards to how new teachers can support connection making in their classrooms, they explained four key elements they would focus on:

a) Careful task and material selection to facilitate the connection-making process
b) Group work to encourage interaction and shared insights
c) Encouraging students to document their individual connections to foster a deeper understanding
d) Embracing a spontaneous teaching role, allowing them to respond effectively in the moment and adapt their guidance as needed

Key Take-Aways from the Study

Definitions of connection making can lead to ways that teachers can support students to make connections. For example, the teachers from this study indicated how connection making involved ideas from others, and identified that group work can be an important element to supporting connection making. The teachers also provided specific prompts for use in group work. Teachers added that they could encourage students to actively engage in "*agreeing or disagreeing*" while comparing and refining their work. In situations where two distinct solutions arose, teachers discussed the implementation of connection-making prompts, such as "*Find similarities and differences*" or "*How do these relate?*" Additionally, teachers can prompt groups of students to "*see numbers or ideas within them [two different solutions]*," as it can help students recognize the underlying similarities in tools and strategies, even if the solutions appeared vastly different.

HOW I HELP STUDENTS MAKE CONNECTIONS IN MY MIDDLE SCHOOL CLASSROOM

By Cristina De Simone

As a middle school math teacher (Grades 6–8), I always sensed the significance of nurturing connections in the classroom. Linking ideas between concepts and guiding students to see relationships in their strategies felt essential. Through presenting the paper on connection making at the Canada Society for the Study of Education (CSSE) Conference (the condensed version is in this edition of the *Abacus*), I could explicitly identify some of the moves the teachers/research participants spoke about, and gained the confidence to be even more intentional in supporting meaningful mathematical connections.

I humbly share my insights with fellow teachers, hoping you'll find familiar practices in your own work and discover at least one valuable addition to enhance your teaching practice!

What is P-K Theory and why is it important for connection making?

1. Non-Linearity: Mathematical understanding is not a linear process. Instead, students need to "fold back" to revisit previous layers of knowledge, such as constructing an image, in order to deepen their understanding.
2. The Power of Connection: A "connected" understanding is pivotal for the growth of mathematical knowledge. Watch for disjoint understandings.
3. Student-Centred Learning: In connection making, students play an active role. They must construct their own connections, rather than being solely told how concepts are linked. This empowers them to develop a deeper and more meaningful understanding.

For the CSSE paper, we drew upon the insights of the Pirie-Kieren Theory (P-K Theory)—a powerful tool that was created for teachers and researchers to examine the dynamic evolution of mathematical understanding. There are three key take-aways from P-K Theory that will guide us in helping students make connections.

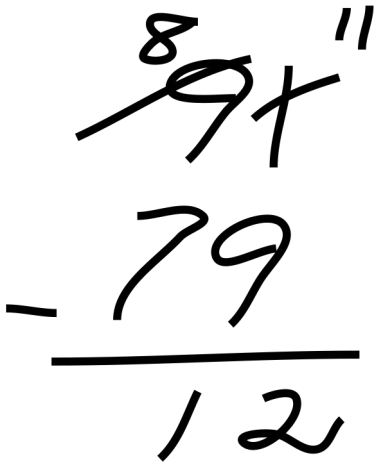
Through the lens of P-K Theory, we can think about how connection making can revolutionize our classroom practices, enriching the mathematical experience for both educators and students.

LESSON IDEA FOR CONNECTION MAKING IN SUBTRACTION: 91 – 79

Links to Ontario Curriculum Expectations:

Grade 2:	Grade 3:	Grade 4:
B2.4 use objects, diagrams, and equations to represent, describe, and solve situations involving addition and subtraction of whole numbers that add up to no more than 100	B2.4 demonstrate an understanding of algorithms for adding and subtracting whole numbers by making connections to and describing the way other tools and strategies are used to add and subtract	B2.4 represent and solve problems involving the addition and subtraction of whole numbers that add up to no more than 10 000 and of decimal tenths, using appropriate tools and strategies, including algorithms

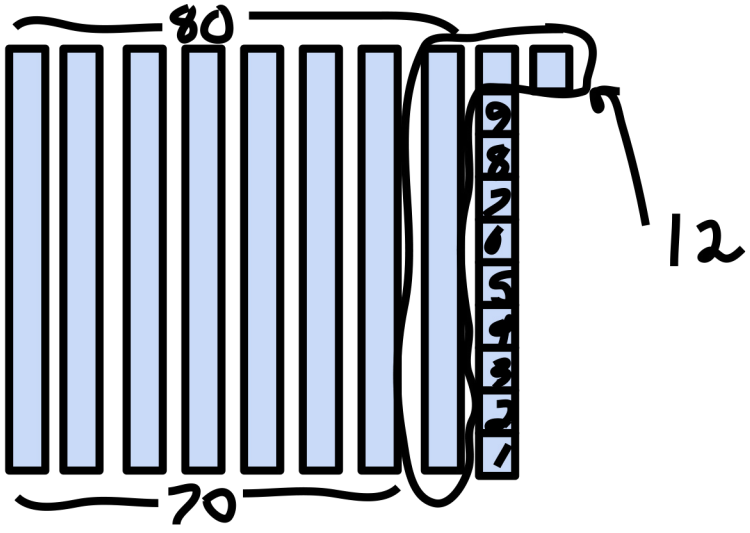
Standard Algorithm



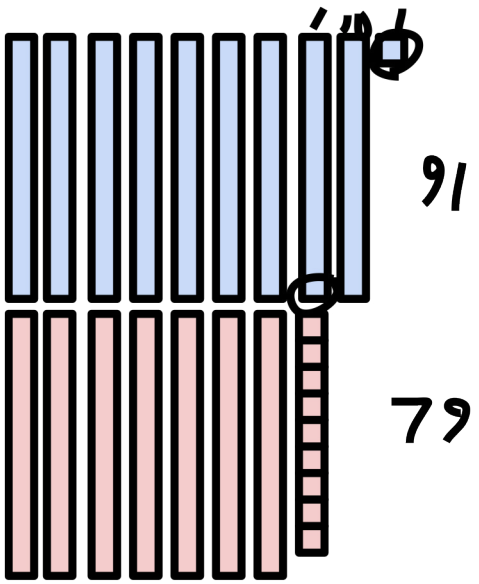
Connecting question:

Can you find the numbers (e.g., 11, 12) in the algorithm and in the blocks?

Regrouping Blocks



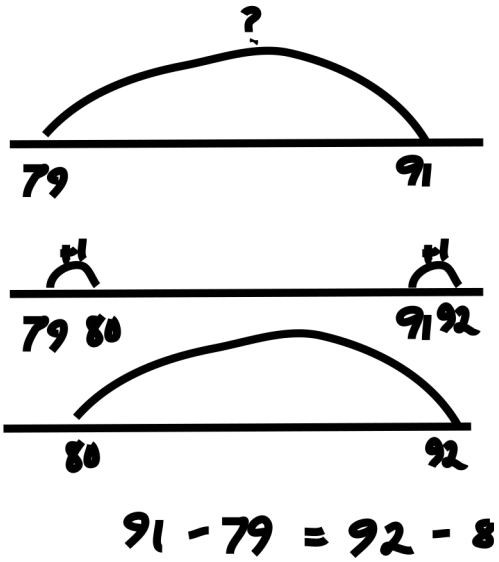
Finding the difference between 91 and 79 blocks



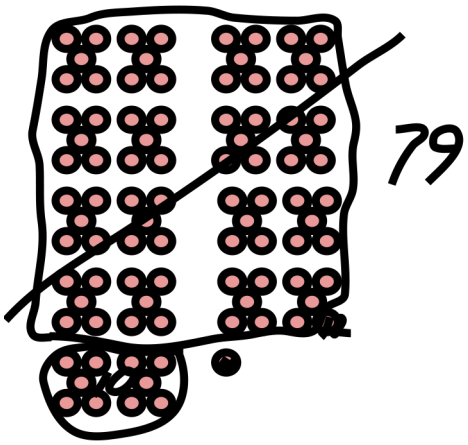
Connecting question:

What's similar and different about seeing 12 as the difference in each solution?

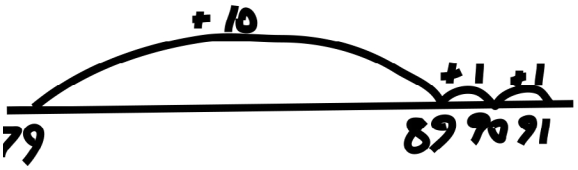
Finding the difference using easier numbers



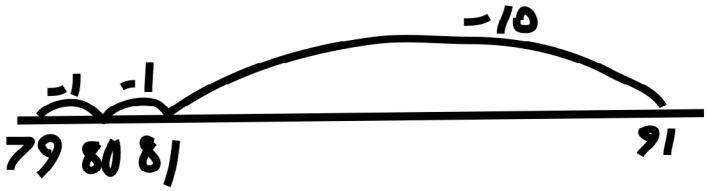
Counting 91, taking away 79, and seeing what's left over



Counting forward by 10's and 1's



Counting backward by 10's and 1's



Connecting question:

What is the same in all of these?

What does it look like in my middle school classroom?

I am going to use the recommendations outlined in the condensed paper, and offer some practical ways you can do this in your classroom today.

a) Careful task and material selection to facilitate the connection-making process

- Think about how to link new ideas to what you have already explored in class.

> *For example:* When looking at ⅓ x ½ (multiplying fractions), invite students to fold back onto the idea of the multiplication sign representing the word "of" so that the question now becomes "one-third of one-half" instead of "one-third times one-half."

- Select tasks that offer multiple representations of a solution (e.g., algorithm, visual representations such as drawings, manipulatives).
- Have materials in the room that are accessible for all students to model ideas—don't just give manipulatives to particular students.

Here's a task you can use with students:

You have 5 m of ribbon, and each bow takes ¾ m of ribbon.

How many bows can be made with 5 m of ribbon, and how much is left over?

Materials you can have out for students to use to represent their thinking:

Meter sticks, string, whiteboards and markers, chart paper, curling ribbon, etc.

(It is important to let students decide what materials they will use to represent their thinking.)

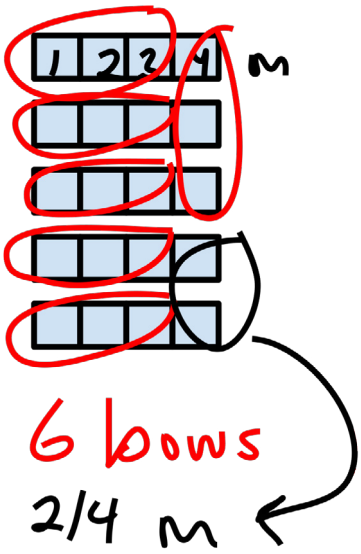
b) Group work to encourage interaction and shared insights

- Have students work together to share ideas, strategies, solutions, and insights.
- Support students in actively listening to each other's ideas.
- Working on vertical spaces allows for students' ideas to be freely shared throughout the room and encourages them to look at others' thinking.
- Use students' ideas to facilitate discussion (rather than using teacher-generated examples).

Here are some prompts you can use:

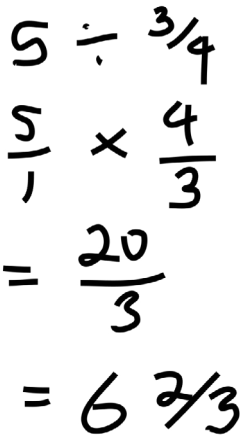
- Which strategies were most similar to yours?
- Which strategies were most different from yours?
- How are these strategies related?
- Do you agree or disagree with…?

c) Encouraging students to document their individual connections, fostering a deeper understanding



Here are some prompts you can use:

- "Can you find all the numbers (e.g. ⅔) produced from using the algorithm in the visual representation?"
- "How do you think this other group thought about the problem?"
- "Where can you see your thinking in this other group's ideas?"



Encourage students to think about their own ideas by:

- providing a few moments of quiet think time before having them move into groups to start the task
- offering time to journal a few ideas at the end of a task (e.g., "What ideas related to the way you thought about the problem?" "What did you agree/disagree with?")
- documenting students' thinking on the whiteboard during math talks and leveraging it to facilitate discussions about different solutions and strategies.

d) Embracing a spontaneous teaching role and allowing students to respond effectively in the moment and adapt their guidance as needed

This is about being in the moment with your students!

Your job is to listen actively to the ideas in the room and find opportunities to prompt students to make connections. You can bring groups of students together for small discussions or have whole-class discussions.

A final thought…

To foster meaningful connections, let's move beyond students merely "presenting" their ideas and encourage them to **engage in dynamic mathematical discussions and constructive argumentation**. As teachers, our vital role is to prompt and wholeheartedly support these enriching conversations.

MOVING FORWARD

Check out the following links to continue your professional learning about connection making!

Hatisaru, V. (2023, March 10). Making connections when teaching mathematics: The role of teacher knowledge and beliefs. *British Educational Research Association.* www.bera.ac.uk/blog/making-connections-when-teaching-mathematics-the-role-of-teacher-knowledge-and-beliefs

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NZ Maths. (n.d.). Theme 5: Making connections. www.nzmaths.co.nz/theme-5-making-connections

Primary Learning. (n.d.). Making connections in mathematics. www.primarylearning.com.au/2018/08/27/making-connections-in-mathematics/

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Pirie, S.E.B., & Kieren, T.E. (1988). A recursive theory of mathematical understanding. *For the learning of mathematics, 9*(3), 7–11.

Pirie, S.E.B., & Kieren, T.E. (1992). Creating constructivist environments and constructing creative mathematics. *Educational Studies in Mathematics, 23*(5), 505–528.

Pirie, S.E.B., & Kieren, T.E. (1994). Beyond metaphor: Formalising in mathematical understanding within constructivist environments. *For the Learning of Mathematics, 14*(1), 39–43.

Pirie, S.E.B., & Kieren, T.E. (1994). Growth in mathematical understanding: How can we characterize it and how can we represent it? *Educational Studies in Mathematics, 26*, 165–190.

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