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PARA-IMAGING: THE BENEFITS OF MAKING STUDENTS' MOVING MENTAL IMAGERY VISIBLE

TABLE OF CONTENTS

Classroom Vignettes: Para-Imaging in Action 3

Lesson Insert: Try Para-Imaging in Your Next Number Talk 4

Tips for Teachers 6

Linking Research and Practice..... 7

Final Thoughts..... 7

Reference..... 7

ADVERTISMENTS
Neufeld Learning Systems..... 8

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ABACUS CO-EDITORS



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PARA-IMAGING: THE BENEFITS OF MAKING STUDENTS' MOVING MENTAL IMAGERY VISIBLE

In this edition of the *Abacus*, we introduc e para-imaging, a teaching move that brings students' invisible "moving mental imagery" to life during a classroom discussion. You can think of it as the secret ingredient that can energize classroom discussion, or a way of supercharging Number Talks.

What is para-imaging?

The term came from "paraphrasing" and using students' images – "para-imaging."

When teachers invite students to share what they saw in their minds while doing a mental-math calculation, para-imaging is the act of revoicing (or paraphrasing) students' strategies and using an image (imaging) to make the ideas visible for public discussion.

To make the image, the teacher can use counters or drawings, but the important part of para-imaging is to capture the movement in their ideas and images.

Para-imaging matters because:

- with para-imaging, the teacher makes mental images visible and usable for the whole class to analyze and compare different strategies for performing mental calculations.
- when teachers para-image student strategies, it sends the message that students' ideas matter, which can build confidence.
- for teachers, para-imaging is a way to honour students' voices, while also nudging deeper mathematical conversations: talking about why things work.

Para-imaging is especially powerful in Number Talks, where students' mental strategies are the focus.

CLASSROOM VIGNETTES: PARA-IMAGING IN ACTION

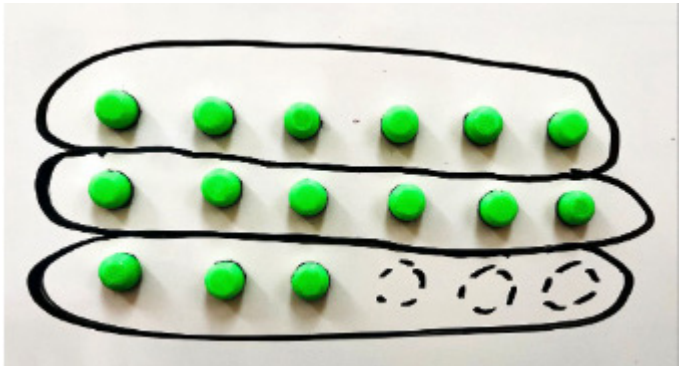
This episode comes from a Grade 5 classroom in Nova Scotia, part of our research on para-imaging in Number Talks. Students were asked to close their eyes and imagine 15 ÷ 6.

After the teacher provided students with individual "think time" and "turn and talk to an elbow partner time," two strategies were para-imaged on the whiteboard:

Strategy 1: Two Groups and a Half-Group of 6

A Grade 5 student explained: "I skip-counted by sixes—six, twelve… then I only had three left."

Para-imaging move: The teacher moved up each group of six, one group at a time, and then circled two groups of six counters. The teacher began a third group, but stopped at three. Ghost dots were sketched to show what was "missing."

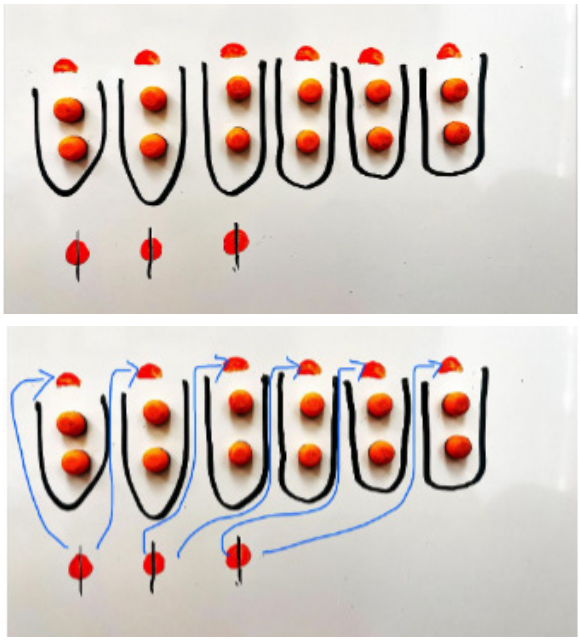


Why it matters: Instead of recording "2 remainder 3," para-imaging revealed the idea of half a group of six. This helped students shift from whole-number remainders to fractional reasoning.

STRATEGY 2: SIX GROUPS OF 2 WITH LEFTOVERS

Another student described: "I put two into each of six buckets, then I had three left."

Para-imaging move: The teacher drew six buckets with two counters each, then showed the three leftovers. Pausing to ask, "What could we do with these?," led students to suggest splitting them in half and redistributing them equally.



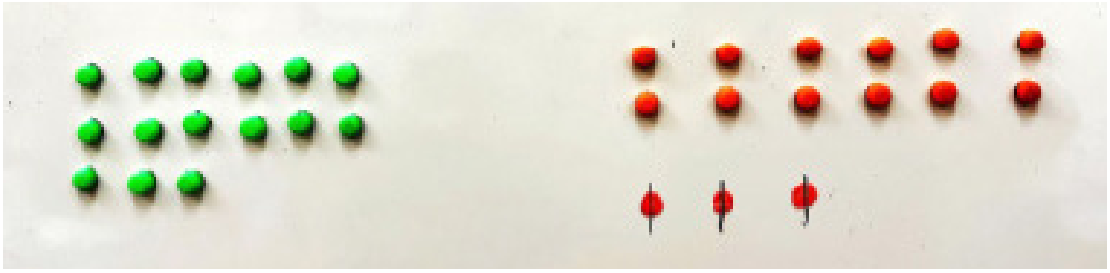
Why it matters: Para-imaging captured the movement of halving and sharing. Students could now see six groups of 2½—another fractional perspective on the same problem.

MAKING CONNECTIONS

When both images stayed visible, the teacher asked: “Can you see the first idea inside the second?”

Students realized:

- One showed 2½ groups of 6.
- The other showed 6 groups of 2½.
- Both led to the same quotient.



Research Connection: This side-by-side comparison (especially when the black marker showing the groups is removed, like in this figure) supports proportional reasoning. Research shows that para-imaging helps students see equivalence and relationships across strategies, not just isolated answers.

LESSON INSERT: TRY PARA-IMAGING IN YOUR NEXT NUMBER TALK

Ontario Curriculum Links

- Grade 4 (B2.6): Divide whole numbers, and express remainders as fractions.
- Grade 5 (B2.6): Solve multiplication and division problems, connecting to fractions.

Lesson Goal

Students explore division with remainders by sharing their moving mental imagery and seeing it represented publicly.

Set-Up

- Place 15 counters randomly on the board.
- Prompt: “Close your eyes and imagine 15 ÷ 6. What do you see?”

Think time → partner talk → whole-class discussion.

Teacher Moves (Para-Imaging in Action)

1. Listen First: Echo with “So you saw…?”
2. Animate the Motion: Move counters or draw step-by-step.
3. Check Fidelity: “Is this what you saw?”
4. Leave Traces Visible: Keep strategies side-by-side.
5. Invite Connections: “Where do you see this idea in that one?”

Extensions

- Try: 14 ÷ 4, 18 ÷ 5, 25 ÷ 6.
- Ask students to write both multiplication and division sentences that match their para-imaged strategy (e.g., 4 times what equals 14, 14 into 4 equal groups).

5 EASY WAYS TO TRY PARA-IMAGING



Start with student words.
Repeat their idea back before drawing or moving anything.



Animate the motion.
Move counters or sketch gradually, rather than jumping to the final picture.



Check for accuracy.
Ask the student, “Is this what you saw?,” to ensure the image matches their moving mental imagery.

Leave traces visible.
Keep all drawings or counters on the board so students can compare multiple strategies side by side.



Invite connections.
Encourage students to look across strategies: *“Where do you see this idea in that one?*

DO'S AND DON'TS OF PARA-IMAGING

DO ✓	DON'T ✗
Honour student words.	Pre-draw the finished diagram.
Capture movement.	Completely replace the student's idea with your own.
Keep multiple strategies visible.	Erase too quickly.
Pair with revoicing.	Skip confirming with the student.

Students say para-imaging helps them:

- see how leftovers can be shared fairly
- notice fractional ideas (like half a group)
- feel ownership: "That's exactly what I saw in my head!"

Teachers say it helps them:

- honour student thinking without imposing
- scaffold students toward fractions naturally
- keep mathematical conversation grounded in concrete images

LINKING RESEARCH AND PRACTICE

Para-imaging is rooted in the Pirie-Kieren Theory of the Growth of Mathematical Understanding (Pirie & Kieren, 1994), which highlights that learning is not a straight line, but a dynamic, back-and-forth process. As students deepen their understanding, they fold back and rework their mental images again and again. In other words, para-imaging isn't just a teaching trick—it's grounded in solid theory.

By para-imaging students' moving mental imagery, teachers make this invisible process visible. Specifically, you can use para-imagining to:

- bridge whole-number and fractional reasoning
- support proportional reasoning (e.g., 2½ groups of 6 = 6 groups of 2½)
- enhance responsive teaching—teachers build on what students already imagine and describe seeing in their minds

FINAL THOUGHTS

Para-imaging is not "one more thing" to add to your plate. It's a responsive move you can use in your next Number Talk:

1. Ask students to imagine the problem.

2. Para-image one or two strategies on the board. Don't try more than two when you first start.

3. Ask lots of "Is this what you saw?," "It moved like that?," and other questions to the student who is describing what they see in their mind.

4. Leave the images visible and ask students to compare.

In doing so, you'll honour students' voices, support fractional reasoning, and make the mathematics visible to everyone.

Share your experiences with us: abacus@oame.on.ca

REFERENCE

Pirie, S.E.B., & Kieren, T.E. (1994). Growth in mathematical understanding: How can we characterize it and how can we represent it? *Educational Studies in Mathematics, 26*, 165–190. doi.org/10.1007/BF01273662

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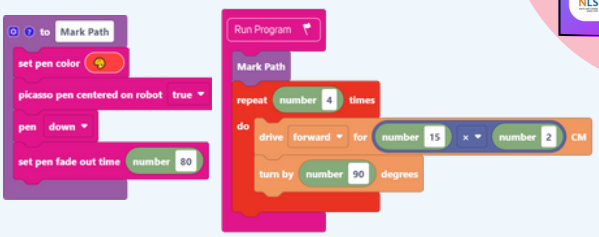
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Sample:

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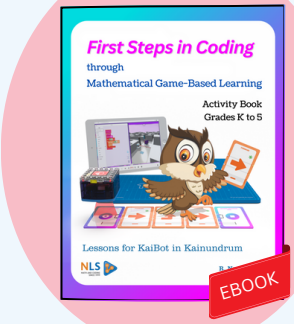


- Virtual stage with Free Block Coding



Result:

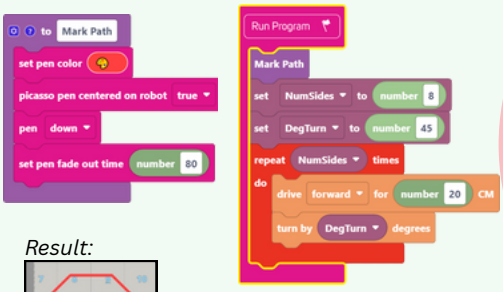




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