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HELPING STUDENTS MAKE SENSE OF SYMMETRY: WHAT PAPER FOLDING AND DYNAMIC GEOMETRY CAN TEACH US

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- VISUAL SUPPORTS AND INFOGRAPHICS DESIGNED FOR QUICK REFERENCE

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WELCOME TO THE MARCH ISSUE OF THE *ABACUS*!

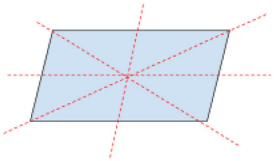
This month, we turn our attention to *symmetry*.

Symmetry appears early in the mathematics curriculum and seems straightforward at first glance. Many students think they can quickly identify symmetry in familiar shapes just by looking at them. Yet researchers suggest that investigating lines of symmetry requires hands-on experiences that allow students to *see* symmetry by mapping one-half of a shape onto another (superimpose). Actions like paper folding really help students to *see* which quadrilaterals have lines of symmetry and which ones don't!

For example, this is how it shows up in the parallelogram.



We might look at it and say, "Yes, it looks like it has a line of symmetry."



But when you begin to paper-fold, you'll see that there are no lines of symmetry.

A helpful word for your symmetry lessons!

Children love to play with big words! The word *superimpose* is a helpful way to test out symmetry.

su·per·im·pose

/ˌso͞op(ə)rəmˈpōz/

verb

Definition: **Place one thing directly on top of another so that they line up or overlap.**

Why is the word *superimpose* helpful?
When we paper-fold, we are able to superimpose one-half of the image onto the other. If the pieces match when they are superimposed – we have symmetry! If they don't, no symmetry is present!

COMMON MISCONCEPTIONS ABOUT SYMMETRY

MISCONCEPTION #1
"If the pieces are equal, there must be symmetry."

Students associate symmetry with fairness and equal parts, especially through early folding experiences.

What to listen for:

- Saying "both sides are the same size," rather than describing how the shape matches itself

Teacher move:

- Invite students to test whether the whole shape maps exactly when folded, not just whether the pieces are equal.

MISCONCEPTION #2
"That shape isn't a square – it's tilted."

Students rely on familiar visual prototypes (upright shapes), rather than properties.

What to listen for:

- Hesitation, disagreement, or relabelling when a square is rotated

Teacher move:

- Ask students to name the properties they see, and consider whether rotation changes any of them.

MISCONCEPTION #3
"Squares are their own shape, not rectangles."

Shapes are often taught as separate categories, so overlap isn't expected.

What to listen for:

- Language like "it's a square, not a rectangle" or "it can't be both"

Teacher move:

- Encourage comparison: Which rectangle properties does this square have?

VAN HIELE: A MODEL FOR UNDERSTANDING HOW GEOMETRY LEARNING DEVELOPS

The van Hiele model describes how learners' understanding of geometry develops in stages, particularly how they come to understand shapes, properties, and relationships. It's not about age; it's about designing the right tasks. These ways of thinking can overlap and appear within the same lesson.

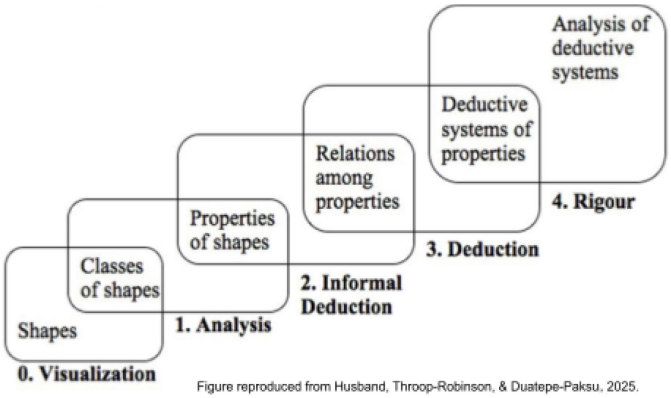


Figure reproduced from Husband, Throop-Robinson, & Duatepe-Paksu, 2025.

THE VAN HIELE LEVELS OF GEOMETRIC THINKING

Level 0 – Visualization

Students recognize shapes by how they look, not by their properties.

- "That's a square because it looks like a box."
- Shapes are identified holistically.
- A square and a diamond may be seen as *different* because they "look different."

Teaching activities that focus on sorting, naming, and informal talk about shapes.

Level 1 – Analysis

Students begin to notice and describe properties of shapes.

- "A square has four equal sides and four right angles."
- Properties are listed, but not yet connected.
- Students may not see relationships between classes of shapes.

Teaching activities that focus on identifying attributes, measuring, and describing.

Level 2 – Informal Deduction (Relational)

Students understand relationships among properties and among classes of shapes.

- "All squares are rectangles, but not all rectangles are squares."
- Definitions matter.
- Shapes are seen as nested categories.

Teaching activities that focus on comparing, classifying, mapping relationships, and justifying.

Level 3 – Deduction

Students can work within a formal system of definitions, axioms, and proofs.

- Can follow and construct logical arguments.
- Understand why definitions are structured as they are.

Teaching activities that focus on formal reasoning, proof, and logical structure.

Level 4 – Rigour

Students can compare different **axiomatic systems** (e.g., Euclidean versus non-Euclidean geometry).

- This level is typically reached in advanced mathematics.

Teaching activities that focus on abstraction and systems of logic.

WHY THIS MATTERS FOR TEACHING ELEMENTARY STUDENTS

The van Hiele model offers teachers a way to observe **student thinking** in the tasks they select. It helps to explain why students can recognize shapes and identify their properties, but may have difficulty when shapes are rotated or used in new ways. *These ways of thinking can overlap and appear within the same lesson.*

1. It explains why "clear explanations" don't always work.

Here's an example of a teacher explaining an idea at Level 2, while the student has not had enough experiences to recognize these properties.

Teacher: *"A square is a rectangle because it has four right angles."*	*Level 2 – Teacher understands generalizations about rectangles.*
Student: *"But it doesn't look like one."*	*Level 0 – Has not yet experienced enough rectangles with different side lengths to recognize a square as a special rectangle*

2. It shifts teaching from telling to designing experiences.

Students move levels through:

- sorting and comparing shapes
- manipulating, folding, rotating, and transforming
- discussing and justifying similarities and differences
- seeing multiple representations of the *same* idea

This means that **rich tasks, mapping activities, documentation, and spatial play** are *necessary*.

3. It reframes student errors as part of development, not deficits.

When a student says:

- "That's not a rectangle—it's a square," or
- "A diamond isn't a square,"

they are reasoning **appropriately for their current level**.

For teaching, this means we:

- **listen** to *how* students are thinking
- **respond** with prompts, not corrections
- **design** tasks that create productive tension

This aligns closely with your work on *listening as a teaching disposition*.

4. It explains why definitions and hierarchies are challenging.

Formal definitions live at **Level 2 and above**. If students are still at Level 0 or 1:

- definitions feel arbitrary
- hierarchies feel like rules to memorize
- proofs feel meaningless

The van Hiele model tells us: *Don't start with definitions—end with them*.

5. It guides task selection and questioning.

Knowing the levels helps teachers ask better questions:

Level 0 → *What do you notice? What does it look like?*

Level 1 → *What properties does it have? How do you know?*

Level 2 → *How are these shapes related? What must be true?*

This makes instruction **intentional, rather than reactive**.

6. It supports equity and access.

When teaching jumps too quickly to higher levels:

- only some students can follow
- others disengage or memorize without understanding

Designing experiences that honour the van Hiele progression creates **multiple entry points**, allowing *more students* to participate in geometric reasoning.

Connecting van Hiele to a Lesson

The symmetry lessons below are designed to move students beyond identifying symmetry lines, create situations where "equal pieces" are not enough, and support reasoning about how properties connect across shapes.

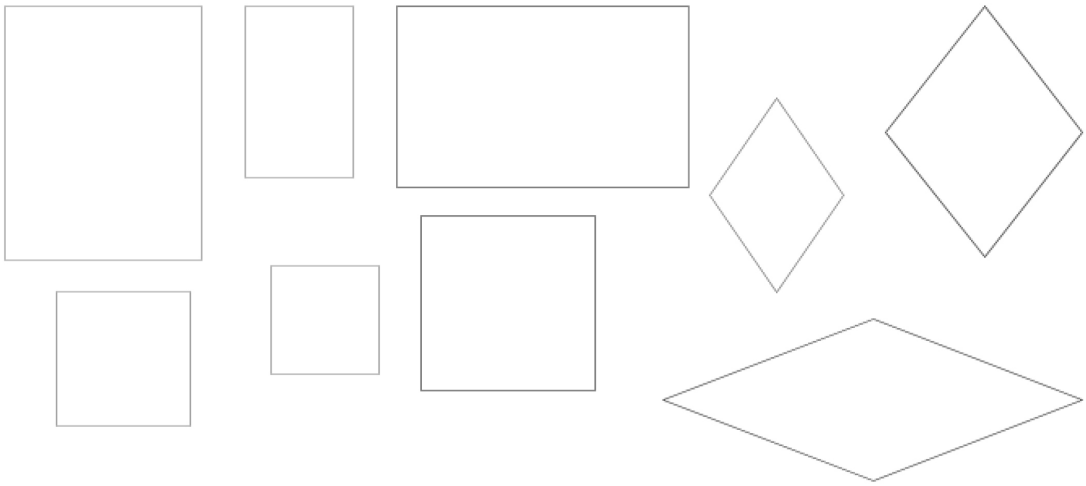
CLASSROOM LESSON SEQUENCE

A three-day approach for Grades 4–6

Curriculum Expectations

Grade 4	Grade 5	Grade 6
E1.1 Identify geometric properties of rectangles, including the number of right angles, parallel and perpendicular sides, and lines of symmetry	E1.2 Identify and construct congruent triangles, rectangles, and parallelograms	E1.1 Create lists of the geometric properties of various types of quadrilaterals, including the properties of the diagonals, rotational symmetry, and line symmetry

EXPLORING SYMMETRY THROUGH PAPER FOLDING



You will need: cutouts of different-sized rectangles, rhombi, and squares. Feel free to use the ones above!

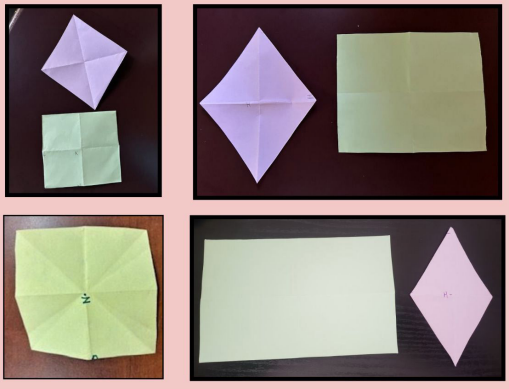
Once the class has cut out various-sized rectangles, investigate and record properties, including lines of symmetry, through superimposing one-half of the paper onto the other. Repeat this process with **rhombi**, and then with **squares**.

Encourage comparison across shapes, focusing on similarities and differences in the placement of symmetry lines.

Symmetry Depends on Shape, Not Size

Here are examples of squares, rectangles, and rhombi that students have cut out and folded.

These shapes are intentionally different in both shape and size, to encourage attention to structure, rather than scale.



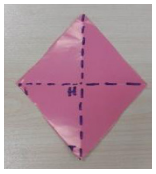
FOLD A RHOMBUS!

Ask students to notice:

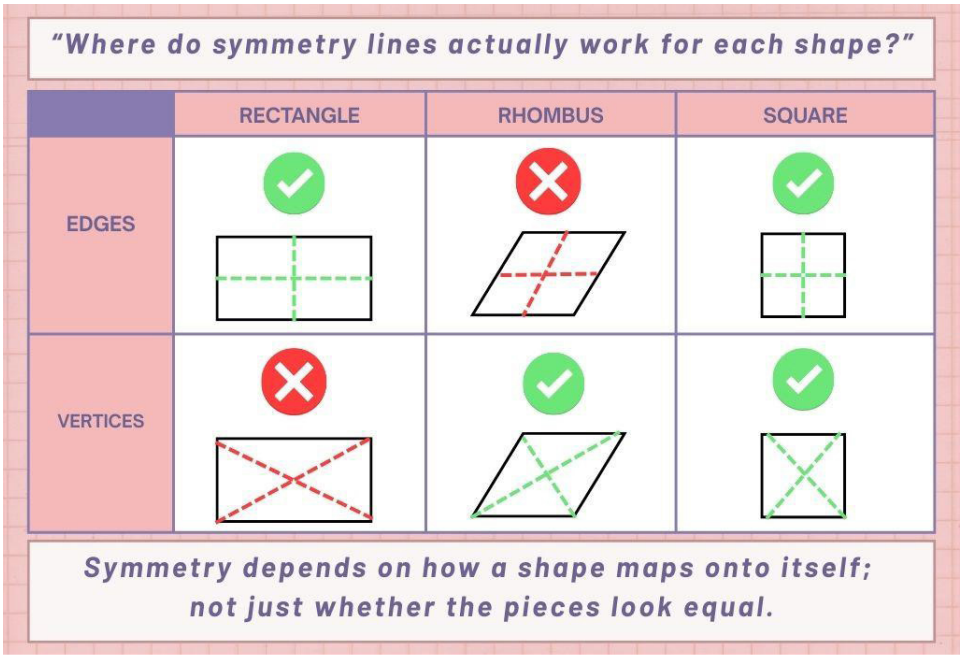
- Where are lines of symmetry possible?
- Where are they not possible?
- How were we able to make a rhombus when we started with an asymmetrical fold?

Instructions for Construction of Rhombus Using Paper-Folding Method

During this process, parts of the paper will overlap. These overlaps are intentional and are used to construct right angles.

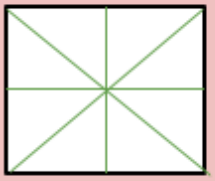
			
1. Start with a **rectangular sheet of paper.**	2. **Fold the paper once** so that part of the paper overlaps, leaving a single-layer section visible.	3. **Choose a point on the folded edge** and label it **Point H**.	4. **Create a right angle at Point H.** Fold the paper so that the two edges on either side of Point H overlap exactly. Press the fold to mark the right angle.
			
5. **Choose two points on the arms** of the right angle and label them **Points I and J.**	6. **Fold along the line connecting Points I and J.** This fold will form the defining line segment of the rhombus.	7. **Cut along the fold line created in Step 6.** Examine the shape you obtained without opening the folds. Imagine the shape you will get when you open the paper.	8. **Open the paper fully** and observe the shape you have created. **Write down the features of the shape you created.** Use folding to identify these features. Markers can help you trace them.

As a class, record observations in a comparison chart:

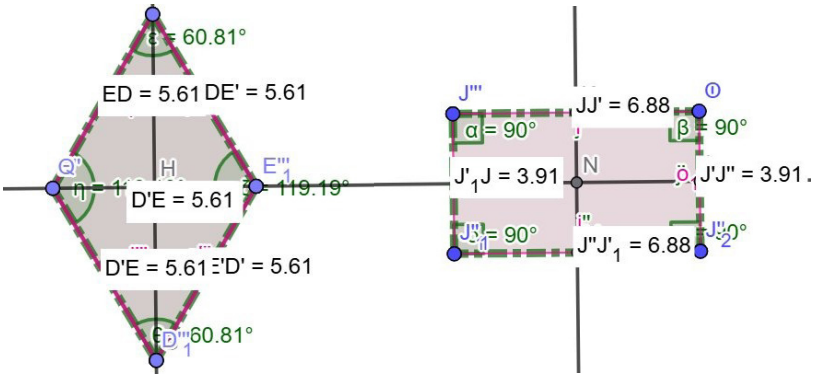


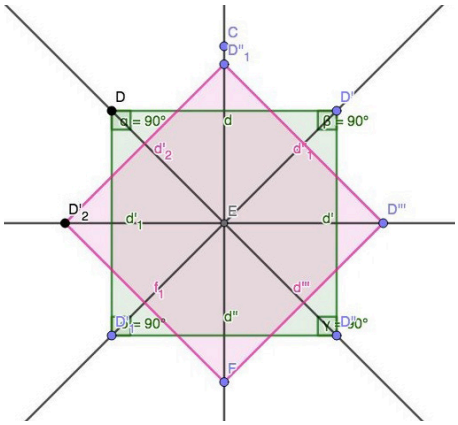
USING DYNAMIC GEOMETRY

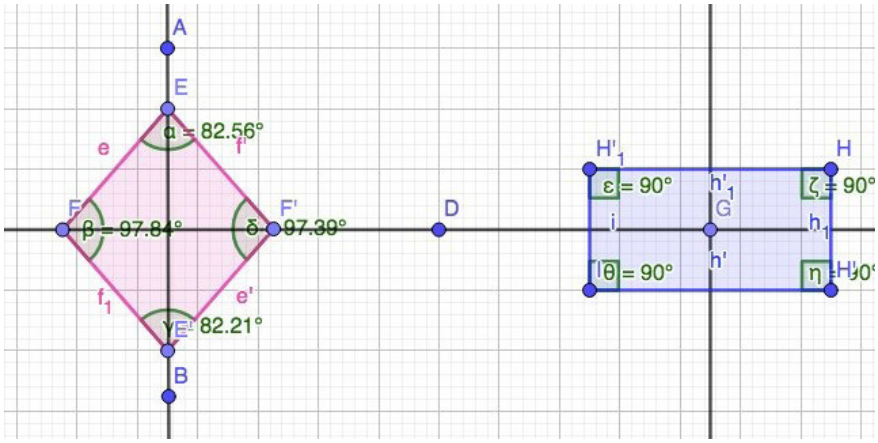
Notice: The square is the **only shape** that has lines of symmetry on both the edges and the vertices!



Students use dynamic geometry software (e.g., GeoGebra) to **construct** rectangles, rhombi, and squares; then investigate properties including lines of symmetry. The real benefit of using dynamic geometry software like GeoGebra is the drag tool feature. Encourage students to use the drag tool to increase and decrease the size of the shape. Ask them to notice what properties change and which ones stay the same.







Investigating rectangles, rhombi, and squares through paper folding and then through dynamic geometry software such as GeoGebra supports students' movement through the Van Hiele levels of geometric thinking.

Paper folding provides a tactile, visual way for learners to notice and describe properties such as equal sides, right angles, and lines of symmetry, supporting work at the visualization and analysis levels. Focusing on where lines of symmetry pass through a square (e.g., through midpoints of sides as well as vertices) gives students relational evidence that a square satisfies the defining properties of a rectangle.

Using GeoGebra's drag tool then allows students to test whether these symmetry relationships and properties remain invariant, supporting a shift toward relational (van Hiele, Level 2) thinking about class inclusion, rather than isolated shapes.

We encourage our readers to try this out in their own classes. Send us photos of your students' work! We would love to share it in a future issue of the *Abacus*.

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