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ROTATIONAL SYMMETRY

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In this issue, you'll find:

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LINK TO ADDITIONAL RESOURCES
oame.on.ca/main/index.php?lang=en&code=MemberFiles

- Construction of a Square, Using the Paper-Folding Method
- Constructing a Rectangle, Using the Paper-Folding Method

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ABACUS CO-EDITORS



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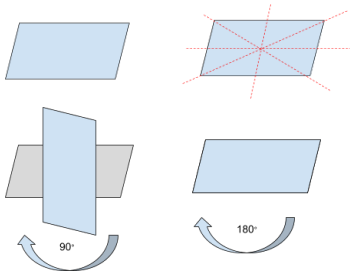
ROTATIONAL SYMMETRY

In March, we explored how paper folding and dynamic geometry software supported students' investigation of symmetry. This month, we turn to **rotational symmetry**—*when a shape is turned around a fixed point by a certain angle, it replaces its footprint.*

When students rotate a shape, and it appears unchanged, something different is happening. The question is no longer about superimposing halves. Instead, we focus on whether the entire figure maps onto itself under a specific turn.

Let's revisit the parallelogram…

In the last issue, we talked about how the parallelogram is deceptive. Students often guess it has symmetry because it looks like you could cut it into equal halves, BUT when students use paper folding to investigate their guesses, they realize there are no lines of symmetry.



Yet, the parallelogram does have a form of symmetry! Gasp!

When we rotate the parallelogram 180°, it can replace its footprint perfectly! This is what we call *rotational symmetry.*

These moments often surprise students.

As students explore these turns, they begin to connect rotational symmetry to properties of shapes. Rotational symmetry invites students to identify the properties that ensure a rotation will map a shape onto itself.

If all parallelograms have 180° rotational symmetry, what does that mean for rectangles? For squares? For rhombi?

In this issue, we continue our look at van Hiele's model of geometric thinking as a guide, while keeping classroom practice at the centre. We invite you to explore these investigations in your own classroom, and to notice how students' thinking develops as they connect rotations to properties of shapes.

COMMON MISCONCEPTIONS ABOUT ROTATIONAL SYMMETRY

MISCONCEPTION #1
"If it doesn't fold, it isn't symmetrical."

Students often equate symmetry only with folding experiences.

What to listen for:

- When talking about a parallelogram: "It doesn't have any symmetry."

Teacher move:

- Ask students to physically rotate the shape 180° and observe what happens.
- This is where you can introduce rotational symmetry!

MISCONCEPTION #2
"Turning it upside down makes it different."

Students at early van Hiele levels rely heavily on orientation.

What to listen for:

- "That's not the same shape anymore."

Teacher move:

- Rotate the shape slowly and ask: "What changed? What stayed the same?"

MISCONCEPTION #3
"All rotations are the same."

Students may not pay attention to describing the size of a turn.

What to listen for:

- "It works when you turn it."
- Students not specifying how much of a turn is needed in order for the shape to map back onto the original footprint.

Teacher move:

- Ask: "How many degrees did we rotate? Would a quarter turn work? Why or why not?"

USING VAN HIELE TO SUPPORT ROTATIONAL SYMMETRY

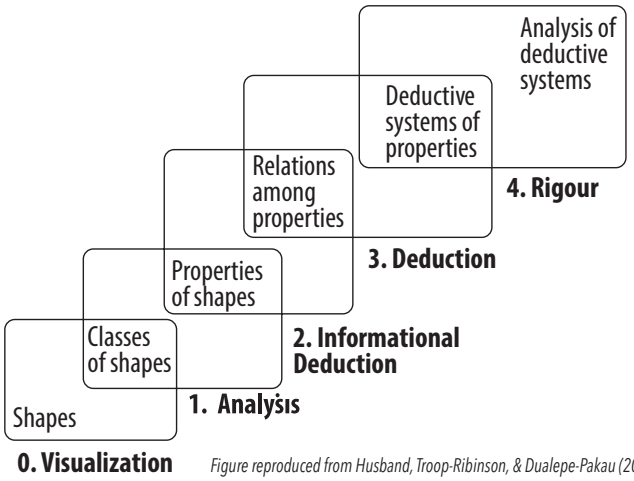


Figure reproduced from Husband, Troop-Ribinson, & Dualepe-Pakau (2025)

In the March issue, we highlighted how symmetry investigations supported movement toward relational thinking (van Hiele, Level 2).

Hierarchical maps make these relationships visible. Rotational symmetry offers an opportunity to extend this work further. At Level 2, students can identify which shapes have 180° rotational symmetry. At Level 3, students begin to reason about why certain properties guarantee rotational symmetry.

For example:
Level 2 statement: "Rectangles have 180° rotational symmetry."

Level 3 reasoning: "Because opposite sides are parallel and equal, and opposite angles are equal, a half turn maps each vertex onto a matching vertex."

This shift is subtle, but important.

Students move from identifying that something works to reasoning about why it must work. They connect differences under rotation to the defining properties of the shape.

Rotational symmetry is a good fit for inviting this kind of structured reasoning in elementary classrooms.

WHY ROTATIONAL SYMMETRY ADDS ANOTHER LAYER

Lines of symmetry asks: "Can one-half fold onto the other?"

Rotational symmetry asks: "Does the entire shape map onto itself after a specific turn?"

Rotational symmetry encourages students to visualize and investigate:

- points of rotation
- the degree to how much the shape rotated (e.g., ¼ turn, 90° turn)
- vertices and edges while rotating a shape
- the difference between "looks similar" and "maps onto itself"

CONNECTING HIERARCHICAL MAPS TO ROTATIONAL SYMMETRY

When investigating rotational symmetry, students learn how shapes are related to one another. Hierarchical maps make these relationships visible.

Students see that:

- squares are rectangles
- shapes turn into other shapes when properties are added
- some shapes can become special cases of others

Classification becomes a tool for prediction, and prediction becomes a tool for reasoning. This is where relational thinking (Level 2) begins to shift toward structured reasoning (Level 3).

CLASSROOM LESSON SEQUENCE

An Approach for Grades 4–6

Grade 4	Grade 5	Grade 6
E1.1 Identify geometric properties of rectangles, including the number of right angles, parallel and perpendicular sides, and lines of symmetry.	**E1.3** Identify and describe the congruence of shapes, including rotations and reflections.	**E1.1** Create lists of geometric properties of various types of quadrilaterals, including properties of diagonals, rotational symmetry, and line symmetry.

PART 1: INVESTIGATING SHAPES

This sequence will take place over three days, where you will repeat the same lesson with different shapes. The shapes are paired together to emphasize different properties of symmetry. In this section, we will first introduce what to highlight on each day, and then we will provide you with the two activities that take place in each lesson.

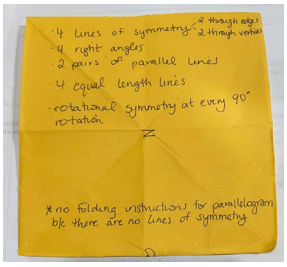
	Day 1: Isosceles Trapezoids & Kites	Day 2: Rhombi & Rectangles	Day 3: Squares & Parallelograms
What to Highlight	Both shapes have **one line of reflective symmetry**, but that symmetry appears in different locations: • In an isosceles trapezoid, the line passes through the midpoints of the parallel sides. • In a kite, it passes through opposite vertices. When rotated, neither shape maps onto itself.	Both shapes have **two lines of reflective symmetry** and **180° rotational symmetry**. However, their symmetry arises from different properties (equal sides versus right angles).	The square has multiple lines of symmetry and rotates at 90°, 180°, and 270°. The parallelogram has no reflective symmetry, yet rotates at 180°.
Teacher Focus	Help students notice that reflective symmetry does not guarantee rotational symmetry.	Invite students to compare how different structures can produce the same rotational result.	Support students in recognizing that rotational symmetry can exist without reflective symmetry. This is often the moment when reasoning shifts from identifying symmetry to explaining it.

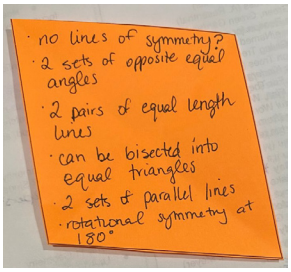
LEARNING QUESTION:

How can folding and turning help us test whether a shape has symmetry?

Materials

- Folding instructions for the respective shapes for that day (we have posted all of them at the end of this issue for you to photocopy)
- Scissors
- GeoGebra (or other dynamic geometry software)

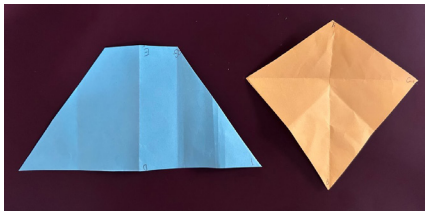


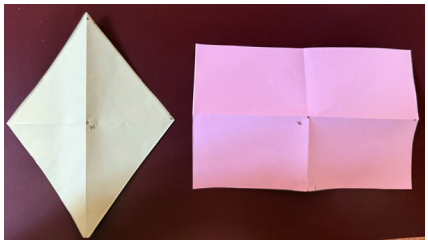


Step 1: Fold to Test Reflection

1. Ask students to predict: *"Where might this shape fold exactly onto itself?"*

2. The folding instructions are designed to emphasize symmetry.
3. Students physically fold each shape to test conjectures.
4. Have students trace the fold line and label it.





Whole-class discussion prompts:

Notice *(What do we see happening?)*

- What stays exactly on top of itself when you fold or rotate?
- Which points or edges match?

Predict *(What do we think will happen before we test it?)*

- If it works, what angle do you think will work?
- What would need to match up for it to land perfectly?

Reason *(Why does it work—or why doesn't it?)*

- What must be true about the sides or angles for this to work?
- Will this always work for this shape, even if we stretch or resize it? Why?

Step 2: Predict Before Turning

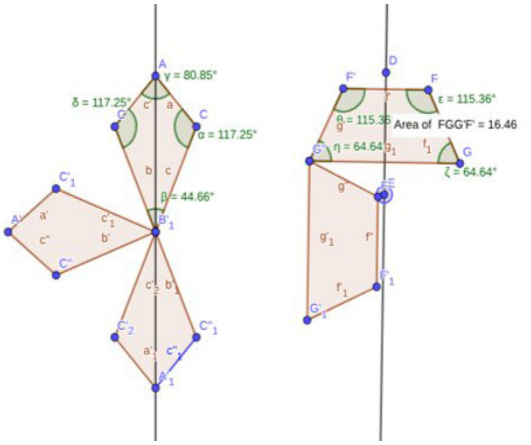
Before using dynamic geometry, ask students:

"If this shape rotates onto itself, what angle will work?"

Invite students to:

- identify the rotation point
- describe what would need to line up perfectly for the turn to succeed

Encourage students to commit to a prediction before testing.

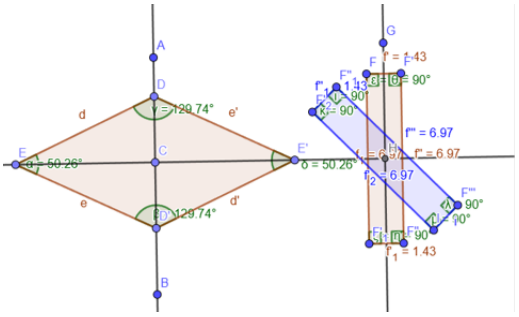


Step 3: Use Dynamic Rotation

Construct the same shape in GeoGebra (or other dynamic geometry software).

1. Use *Rotate Around Point*.
2. Rotate 180° about the centre.
3. Slow the rotation.

Teacher move: Ask students to track one vertex carefully as it turns.



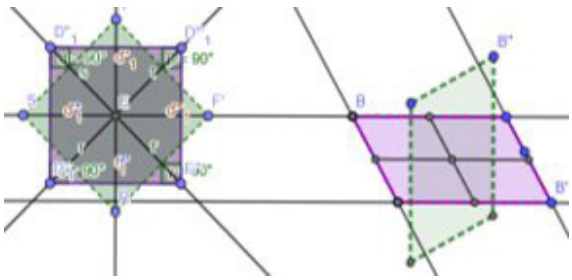
Pause and ask:

- Where did that vertex go? What must be true about this shape for that to happen?

Allow students time to describe what they notice, before formalizing language.

If appropriate, drag a vertex while maintaining the defining properties of the shape and ask:

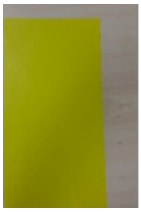
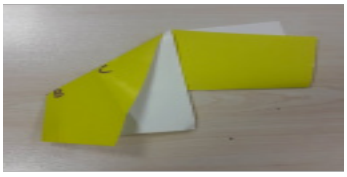
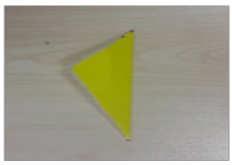
- Will this still work? Why?



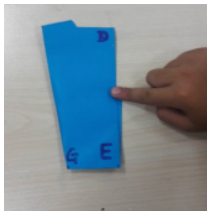
DISCUSSION PROMPTS FOR AFTER ROTATION

Day 1: Isosceles Trapezoids & Kites	Day 2: Rhombi & Rectangles	Day 3: Squares & Parallelograms
After discovering that neither rotates: • Why does having one fold not guarantee a successful rotation? • What is different about folding compared to turning? • What would have to change about this shape for a half turn to work?	After discovering that both rotate 180°: • What do these shapes have in common that might explain the half turn? • Does the rotation depend on equal sides? Right angles? Parallel sides? • If I stretch the rectangle (but keep it a rectangle), will 180° always work? Why?	After the parallelogram rotates successfully: • Why does 180° always work for parallelograms? • What do opposite sides and opposite angles have to do with that? • If I drag the vertices and keep it a parallelogram, will the half turn still work? • What must be true about any shape that has 180° rotational symmetry? The final question is your bridge to hierarchical mapping.

INSTRUCTIONS FOR BUILDING THE TRAPEZOID, USING THE PAPER-FOLDING METHOD

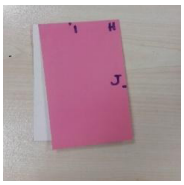
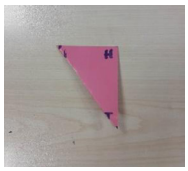
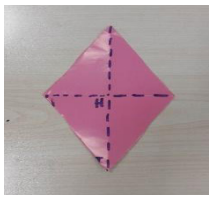
1. Take an 8 x 11 piece of paper.		5. Make the fold that forms the line segment passing through Points A and C.	
2. Fold the given paper (leaving the single-layered part less visible) in any part.		6. Similarly, make the fold that forms the line segment passing through Points B and C.	
3. Determine two points on the fold mark (Points A and B).		7. Cut out the folds obtained in steps 4 and 5, and examine the shape you obtained without opening the folds. Imagine the shape you will get when you open the paper.	
4. Determine a point outside the fold mark (Point C).		8. Write down the features of the shape you obtained, which you can find by simply folding paper, without using any auxiliary tools.	

CONSTRUCTION OF AN ISOSCELES TRAPEZOID, USING THE PAPER-FOLDING METHOD

1. Take an 8 x 11 piece of paper.		6. What would you say about the condition of the fold marks obtained in the third and fourth stages relative to each other?	
2. Fold the given paper (leaving the single layer less visible) in any part.		7. Determine a point on the fold marks obtained in the third and fourth stages (Points F and G).	
3. Determine two points on the fold (Points D and E).		8. Make the fold that forms the line segment passing through Points F and G.	
4. Create a crease that will overlap the parts of the crease on both sides of Point D. **What is the measure of the angle you obtained on the paper?** Discuss your answer without using a measuring tool.		9. Cut the folds obtained in the third, fourth, and fifth stages, and examine the shape you obtained without opening the folds. Imagine the shape you will get when you open the paper.	
5. Create a fold trace that will overlap the parts of the fold trace you obtained in the first step on both sides of Point E.		10. Write down the features of the shape you obtained, which you can find by simply folding paper, without using any auxiliary tools.	

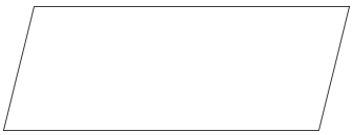
INSTRUCTIONS FOR CONSTRUCTION OF RHOMBUS, USING THE PAPER-FOLDING METHOD

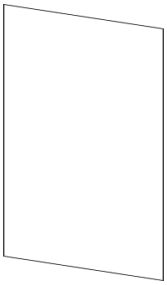
			
1. Start with a **rectangular sheet of paper**.	2. **Fold the paper once** so that part of the paper overlaps, leaving a single-layer section visible.	3. **Choose a point on the folded edge** and label it **Point H**.	4. **Create a right angle at Point H.** Fold the paper so that the two edges on either side of Point H overlap exactly. Press the fold to mark the right angle.

			
5. **Choose two points on the arms** of the right angle and label them **Points I and J**.	6. **Fold along the line connecting Points I and J.** This fold will form the defining line segment of the rhombus.	7. **Cut along the fold line created in Step 6**. Examine the shape you obtained without opening the folds. Imagine the shape you will get when you open the paper.	8. **Open the paper fully** and observe the shape you have created. **Write down the features of the shape you created.** Use folding to identify these features. Markers can help you trace them.

Parallelogram Cut-Outs

Because there is no line of reflective symmetry, it is not possible to construct the parallelogram using paper folding. Cut out these shapes to explore symmetry with parallelograms.





PART 2: BUILDING THE HIERARCHICAL MAP

Learning Question: How can we showcase our understanding of the relationships among six quadrilaterals by representing them in a single, organized map?

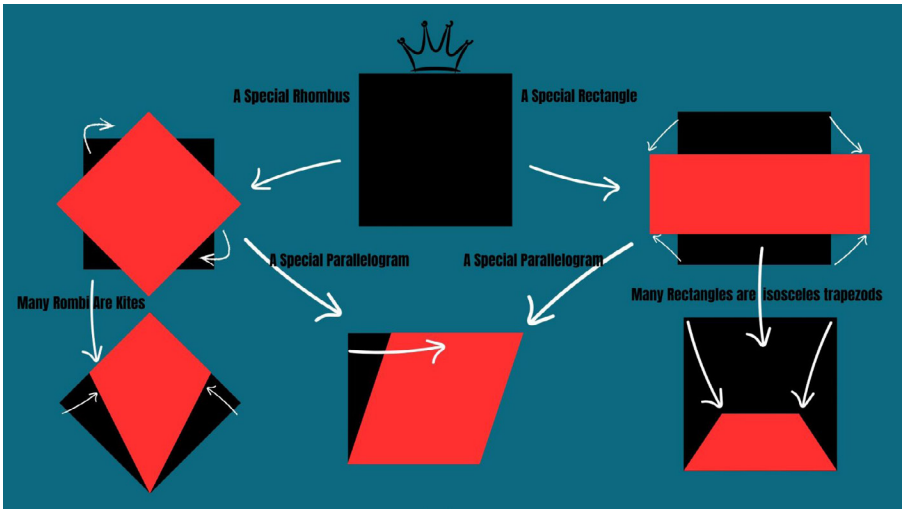
CREATING A MAP OF QUADRILATERALS

Over the past three days, we investigated the following quadrilaterals in pairs: kite, isosceles trapezoid, rectangle, rhombus, square, and parallelogram.

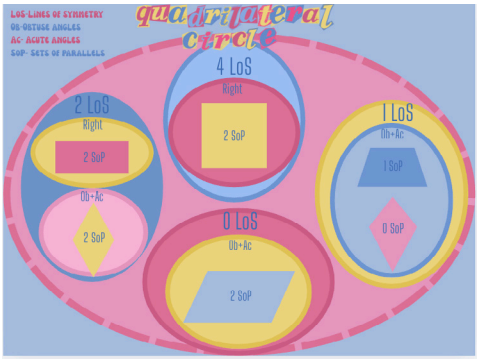
Your task is to create a map that shows relationships (e.g., hierarchical) among these six quadrilaterals, using attributes that we explored (e.g., sides, angles, lines of symmetry) through paper folding and GeoGebra.

Your map should showcase how the shapes are related (e.g., shapes are “special cases” of others). You may design the map in a format that makes sense to you, but it should try to highlight the structure of how these quadrilaterals connect.

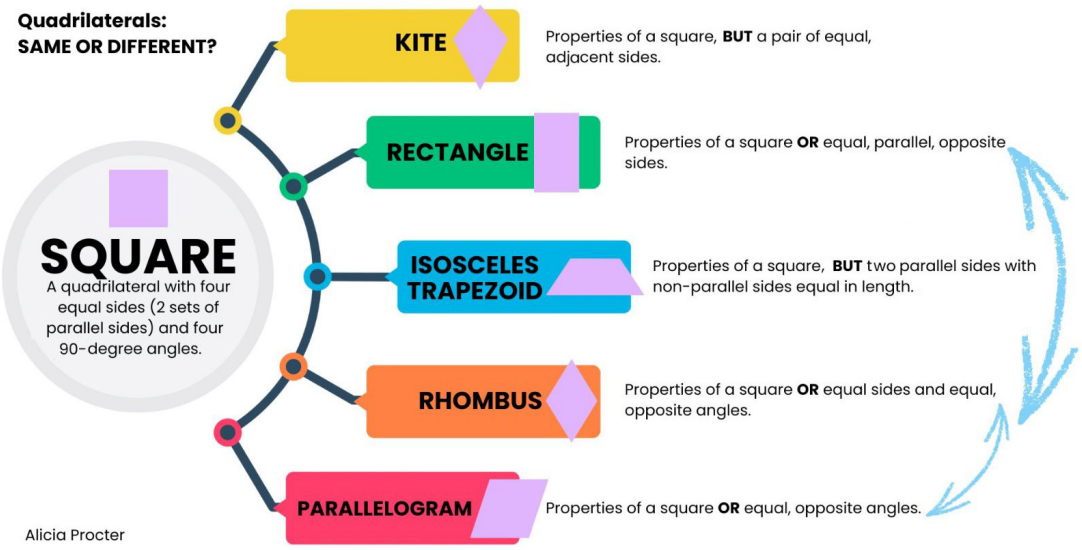
Here are some examples of hierarchical maps that have been created by teachers during a professional development course and a Grade 8 student.



Mike Sweetman



By Sekeenat Adedapo



"The blue arrows on my visual were meant to show that the square could also be a rectangle, rhombus, or parallelogram. The arrows were meant to appear disconnected from the isosceles trapezoid and the kite." – Alicia Procter

Alicia's Experience of Creating a Hierarchical Map

Originally when I created my visual connecting quadrilaterals, I really struggled with the isosceles trapezoid and the kite. I was quite certain that a square could not be considered either one of these shapes, while it could be considered a "special" representation of the others (rectangle, rhombus, and parallelogram).

I was of the mindset that a kite must have two shorter sides and two longer sides, and that a trapezoid could not have 90-degree angles. ~ Alice Procter

Invite students to create their own hierarchical maps and share them with each other.

CLOSING THOUGHTS

Paper folding helped students see symmetry through superimposing halves. Rotational investigations help students reason about invariance across entire structures.

Together, classification and rotation create opportunities for students to move toward reasoning from properties (Level 3).

We encourage you to try these investigations in your own classrooms. Send us photos of your hierarchical maps and student reasoning—we would love to feature them in a future issue.

REFERENCES

Husband, M., Throop-Robinson, E., & Duatepe-Paksu, A. (In press). *"Try folding it": Exploring symmetry through paper folding and dynamic geometry software to enhance teachers' geometric reasoning*. Proceedings of the MACAS Conference, Moncton, New Brunswick.

Husband, M., Throop-Robinson, E., & Duatepe-Paksu, A. (in press). Try folding it: Exploring symmetry through paper folding and dynamic geometry software to enhance teachers' geometric reasoning. *Mathematics Education Across Cultures and Societies (MACAS 2025),* Moncton, New Brunswick.

van Hiele, P.M. (1986). *Structure and insight: A theory of mathematics education*. Academic Press.