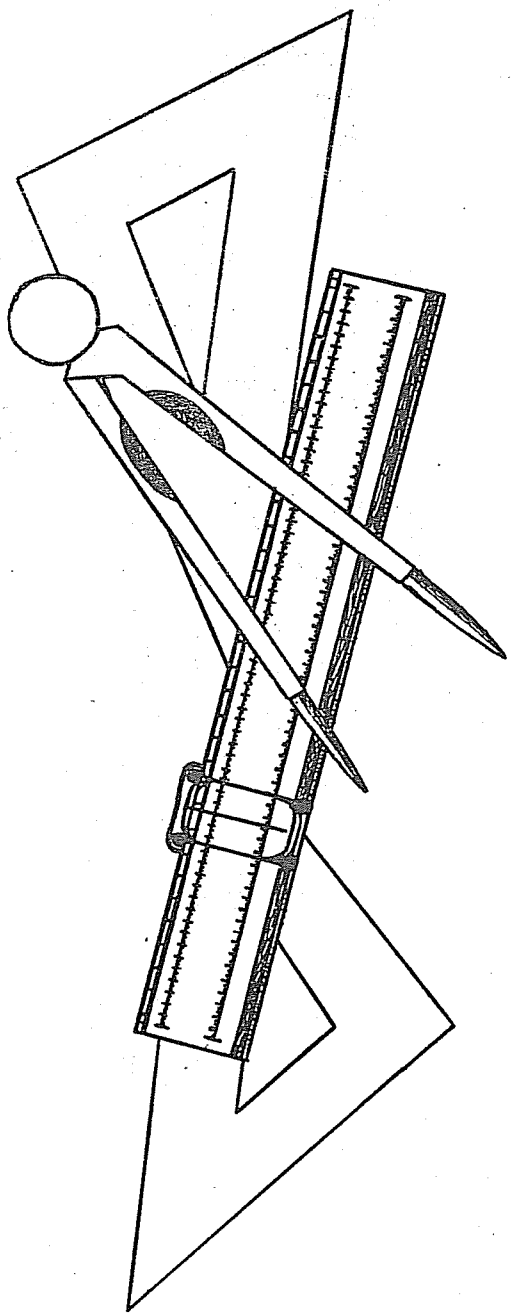
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RELATING MATHEMATICS AND SCIENCE AT THE SECONDARY LEVEL*

D. H. Crawford, Faculty of Education, Queen's University

Introduction. Perhaps I should begin by stating how inadequate I feel in approaching this topic. On the one hand, my experience in the last ten years has taken me farther and farther away from experimental science - on the other the topic is so great and I have had little time to reflect on it. What compelled me to attempt this challinging task was the critical importance of it. I am convinced that for many pupils there is little realisation of how mathematics and science are related. Probably mathematics has more to gain from being related to science than vice versa and so I venture to suggest that we need to make the first move.

What do I hope to accomplish? My intentions are as follows:

1. Discuss briefly the nature of mathematics and science, and how they are related.

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* Text of a talk given to an O.A.T.M. Seminar-workshop at St. Lawrence College, Kingston, on 26 September 1970.

2. Outline briefly some of the mathematics content which is basic to the study of the three disciplines, physics, chemistry and biology.

3. Discuss some possible ways in which we may begin to work together with science teachers to ensure that students have better understanding of how mathematics and science depend on each other and that they develop a deeper understanding of each.

4. Finally, since this is meant to be a workshop I intend to involve you in some investigations and problems which may serve as sample situations which you can use yourself in your own teaching.

Mathematics and Science: What are they and how are they related?

Mathematicians usually decline to define mathematics. They tend to say "mathematics is what mathematicians do". However, we may settle for some such definition as "the science of number, shape and relationship". This particular definition at least indicates the original existence of mathematics as comprised essentially of arithmetic and geometry - both strongly related to the real world - yet takes into account the increasing emphasis on the properties of numbers and shapes and the relationships which emerge when these are studied.

The evolution of mathematics into the subject which Bertrand Russell described as "the subject in which we never

know what we are talking about nor whether what we are talking about is true" is a development of only the past century or so. It began with the discovery that self-consistent geometrical systems other than Euclid's existed which described the real world just as well and with the realisation that the systems represented by natural and real numbers and the operations by which we combine or manipulate them were only a few of the rich possibilities among many related and more abstract mathematical systems such as groups and fields. Mathematics then became at its advanced levels the study of if-then reasoning and of axiomatics.

Scientists are not quite so coy as mathematicians and are more prepared (and let's agree better placed) to attempt a definition of their discipline. Thus a reasonable definition of science might be "the search for order in nature". Thus science is "the process of enquiry with which we attempt to look for this order and with which we question our accepted notions of the physical and biological world".

How then are the two disciplines related? Well, it is perfectly clear that they are related historically. As I have indicated, until about the middle of the 19th century, mathematics was regarded as a description of the real world. In this sense, one might view mathematics as a result of scientific enquiry: that is, mathematics provided the concise langu-

age and techniques whereby the order in nature discovered by science was codified and described. Laws of Physics and Chemistry such as Newton's Law of Gravitation $F = K \frac{M_1 M_2}{R^2}$ and the Gas Law $PV = RT$ exemplify this phase and aspect of the relationship between mathematics and science.

Further, the two disciplines are strongly related in terms of their objectives. Both are concerned with order: science specifically with order in nature, mathematics with order and pattern in any context. Viewed in this way one might conclude that science is only part of mathematics! This the scientist would strongly deny. We will see shortly why; but for the moment let us consider a third similarity between the two, namely the methods used.

Each uses the if-then approach. Some hypothesis is proposed and then its validity established or destroyed by deductive reasoning. For the scientist the hypothesis is the result of observation of some aspect of the real world, physical, chemical or biological, but this is not the case in mathematics. The second step - the deductive part - is what is common to both; this is where mathematics enters science. For it provides the logical part of the process of doing science, by furnishing in a readily comprehensible manner the consequences of accepting certain hypotheses. But science then has to go one step further. These consequences have

scientific validity only if they agree with sensory experiences of the real world, so that these logical consequences have to be tested experimentally. Thus the relationship PV = RT might be deducible logically from certain hypotheses yet when its consequences are tested experimentally, the relationship might not be validated scientifically.

So far in comparing the two disciplines their similarities and common aspects have been discussed. Let me now point out a significant difference. It concerns the nature of scientific as opposed to mathematical knowledge.

The most that can be said of a scientific statement is that its truth possesses a certain probability. Probability depends on evidence. Thus any proposition in science has an initial probability which is raised to near certainty , or lowered to negligibility by subsequent evidence. Mathematics on the other hand, is not primarily concerned with truth but rather with logical consistency and the logical consequences of given assumptions. Now we can see why the scientist denies that his discipline is included in mathematics for he sees mathematics as not concerned with the real world at all.

To sum up,

1. Mathematics provides the language and precise symbolism for expressing scientific relationships.
2. Mathematics evolved from a description of the real world

to its present state of axiomatics, and thus has strong historical links with science.

3. The two disciplines have common objectives in that both emphasize a search for order.
4. They use similar but not identical methods, for the validity of knowledge differs (logical and scientific validity).
5. Scientific validity implies a measure of probability other than 0 or 1.

Basic Mathematics Used in School Science

To ensure that the teaching of mathematics will benefit from the teaching of science and vice versa, it is clearly desirable that teachers of each discipline be aware of the major mathematical concepts and the common scientific contexts in which they are used. This can best be achieved by teachers of each sitting down together to look at school curricula in mathematics and science in a detailed manner, and after identifying concepts and contexts, working out policies and approaches which will best achieve good correlation.

I do not propose any such detail in my remarks here. Rather I shall attempt to list what I feel are some of the significant mathematical concepts, topics, and branches, and where they occur in school science.

Firstly, mathematical symbols and algebraic manipulation occur very frequently in all three sciences. It is thus obvious, though sadly neglected, that thorough familiarity and understanding of symbols, and basic ideas and topics such as variables, graphs, equations, formulae, functions, rates, ratios, proportion and limits are essential. Much more attention must be paid to using varied symbols, and scientific contexts for the work in mathematics. Moreover, the science teacher needs to know much more accurately what he can legitimately expect from students at a given point in their schooling. This means serious sustained consultation. Only by concentrated efforts can we ensure that students know exactly what is meant by such symbolic statements as $v_1 = n_1\lambda_1$ and $F\Delta t = m\Delta v$ and can solve for specific variables such as d in the equation

$$y = \frac{d}{2} + \frac{b^2}{12d} .$$

At an even more fundamental level, students need to become skilled in ordinary arithmetic particularly the use of large and small numbers using exponential and scientific notation. In physics and chemistry particularly, the discussion of nuclei, atoms and molecules and the continuing discussion of time, distance, weights, and volumes demands this.

Further, measurement is an integral part of all science. Thus, estimation, approximation, rounding, significant digits, tolerance and error all must be studied and thoroughly understood. There is a tremendous need, in my judgment, for good textual material relating mathematics and science in these areas.

In particular, the idea of experimental error, and true measurement leads one to statistics and probability and the need to develop curricular material to illuminate how intricately this branch of mathematics is interwoven with the main stream of science. In biology especially, where one is concerned with the many variations produced by environment and inheritance, continuing needs exist; firstly, to group data in histogram form, and to attempt to make inferences about its nature, and secondly, to predict from samples with given inherited characteristics, what the probabilities associated with further generations from the same population will be.

Additionally, a knowledge of common geometrical figures, properties and relationships and their extensions to basic trigonometry is indispensable in studying science. In the discussion of motion which includes concepts such as vectors, and forces, and which may be linear, circular, elliptic, sinusoidal, or vary according to some other mathematical law, a theorem such as the Pythagorean and its extension, and

knowledge of the Sine Law and the periodic trigonometric functions is essential. Again what is needed is adequate cross-linkage achieved by continued interchange of curricula and methods on a planned basis.

Some Suggestions for Integration

Having convinced you that there is in fact a very significant overlap between mathematics and science, and having indicated what I consider the main areas where overlap occurs, let me turn specifically to the problem of how to achieve integration in practice.

I confess at once, that I have no magic remedies. My views may be summarised by the phrase "Where there's a will, there's a way". The justifications I can produce for us wanting to do it are:

(i) the intrinsic interrelationships which are there and

(ii) the fact that if we are serious about educating students, it is our duty to help them obtain a properly balanced view of both subjects. Knowledge is in fact a unity, and we have been fragmenting it at both the University and High School levels. Student dissatisfaction with this fragmentation must not continue.

How then can we achieve integration? We must plan for it actively in a number of ways. Here are some.

1. Teacher-educators in mathematics and science must integrate their work. Mathematics teachers-in-training must be exposed to work in a laboratory setting; they must acquire the attitude and habit of approaching their teaching from this point of view. Essentially this is what I mean when I talk about a discovery approach. But this is not enough. There needs to be actual team work by the mathematics and science people in teacher-education institutions. The biology specialist should design some suitable biology experiments (in consultation with the mathematics department) which illustrate the basic mathematics which high school biology students need to know, and have the mathematics students carry them out. Similar work should be carried out with respect to Physics and Chemistry. Naturally, time puts a limit on the extent of such ideal situations. A variation of this might be a common Math-Science Unit for all students in mathematics or the particular science, designed and supervised by all the Faculty in these areas. Provision should also be made for the mathematics staff to give all science students an overview of the mathematics of the high school and its objectives. In this way would-be mathematics and science teachers would begin their teaching careers with much better attitudes to each other and their separate disciplines.

This in my view is probably the one most important thing to achieve. As has been said before "Prevention is better than cure."

2. Yet this is just the beginning. Attitudes need to be nourished and maintained. Thus the reality of integration can be achieved only in individual schools between individual and specific groups of teachers. Each of you here has a responsibility, and I hope you will take it seriously. Instead of muttering and moaning at the lack of cooperation of the science teachers, mount a campaign to obtain it. I know there are many difficulties - overwork, misunderstanding, personality conflicts, even disinterested and uncooperative teachers on occasion. But these are the realities we have to contend with. At the very least, you should have outlines of all the science courses, which parallel your teaching and vice versa. Surely you can talk over some of the obvious areas of common interest informally or, hopefully, on a regular basis with the science staff. Perhaps you need leadership and some external stimulation. This is where regional consultants and professional development generally come in. This particular seminar is an excellent example of the kind of cooperation necessary. County staff, Ontario Department staff, university staff, teacher-education staff

and teachers themselves must all work together to plan productive meetings and seminars. I would therefore make the specific suggestion that a steering committee in Relating Mathematics and Science in Schools in Frontenac County be set up as a result of this seminar. Perhaps the organisers of the seminar would set this in motion, remembering that science people should have equal representation.

3. Finally may I briefly illustrate by a particular example the sort of way in which mathematics and science teachers might achieve this much desired integration. Recently Geoffrey Matthews, the Director of the Nuffield Project in Mathematics has initiated a study group at the new Centre for Science education in Chelsea, associated with the University of London. Several schools are sending a mathematics and a biology teacher to the group, which is planning modules of work to cut across the subject boundaries.

He suggests that a module might consist of

(a) half-a-dozen problems for small group work

(b) hints, commentaries and solutions to be available when needed

(c) teacher notes giving the purpose, previous knowledge required, test material, etc.

(d) any relevant visual aids.

As you can see, this is in line with the current package approach to curriculum development in the U.S.A.

Clearly, such modules will require careful design. However, it should be possible for groups of teachers to design suitable projects of an appropriate length to assist students in seeing mathematics and science in perspective.

For example, estimating the size of a molecule by using an oil film might form a project associated with delving into the meaning and value of π. If a drop of oil of known volume is dipped into a tray of water, it forms a more or less circular patch. The thickness of a volume can then be investigated by focussing on how to find the area of the patch. At the same time, one might consider including some work on the physical and chemical properties of oil, and even a study of the environmental control problems associated with oil slicks in the lakes or oceans.

Conclusion. I hope that what I have said has not been too general, and too far away from the practical. I felt it necessary first to get some of my own basic thoughts in order.

With this background of what I believe is involved in relating these disciplines, I now invite you to participate in viewing a film and some film loops, and to do some problems*

---

* For these please write to Dr. Crawford.

which I feel will give you some basis for attempting more consciously to integrate mathematics and science in practice.

The following films were shown:

1. Mathematics of the Honeycomb, Moody Institute of Science.
2. Areas of Rectangular Borders, M29, National Film Board of Canada.
3. Corresponding Lines, M26, National Film Board of Canada.

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WE SHOULD INSIST THAT STUDENTS LEARN THE BASICS OF MATHEMATICS

D. E. Scovell, Head of Mathematics Department,
David and Mary Thomson C. I., Scarborough

Every teacher of mathematics has had the experience, many times over, of marking a test or examination on which one or more questions has not been attempted, or futile efforts to answer them serve only to reveal that the student has virtually no comprehension whatever of the ideas involved in the questions. Yet, when all of the marks are added together the student has obtained 51, or thereabouts, which by the generally accepted criterion permits him to pass. Thus our student moves on to new material, much of which depends on what has gone before. Only recently the K-13 Geometry Committee has stated that "nothing whatever can be built on a knowledge of 51 percent of a branch of mathematics".(1)

In this article will be presented a scheme that can do much to offset the inherent shortcomings of a promotional system based on a mark of 50 percent. First though, let us briefly examine the categories of mathematical learning as described by Avital and Shettleworth in 'Objectives for

Mathematics Learning'.[2] The first and most elementary level is 'Recognition and Recall', in which a student simply recalls or recognizes material essentially in the form in which it is taught. The second level concerns solution of problems by following routine techniques. There would, of course, be some generalization necessary because the student would have to recognize the problem as being similar to a type that he had encountered before and for which there exists an established solution procedure. This level of mathematical performance Avital and Shettleworth call 'Algorithmic Thinking and Generalization'. The third and last category, referred to as 'Open Search', deals with the kind of mathematics for which the student has not been specifically prepared and which might require a fair degree of intuition or the ability to handle abstract concepts. It should be obvious that successful performance in this last level of mathematics depends on mastery of the type of knowledge and skills involved in the first two categories. The majority of students in high school (in the opinion of the writer) lack sufficient mathematical talent to perform well at the 'Open Search' level. Nevertheless, even though all students may not have the potential to become capable problem-solvers, it is important that they acquire as many basic mathematical skills as they can. What can we do to help close those 'holes' in that

51 percent paper? What we have done in the mathematics department at David and Mary Thomson Collegiate will now be described.

Basic Skills Test Program:

The plan at present involves grades 9 to 12, 5 year (Advanced Level) students, but it could easily be applied to the General Level. In each term the subject material usually consists of (i) 'must know' items and (ii) 'can know' items. In the latter category would be material that is not essential (at this stage at any rate) to subsequent discussions. The Basic Skills Test program is concerned with the 'must know' material. The members of the department meet to discuss the course and decide what they feel the students must know, and this is what will be included in a Basic Skill Test. A detailed and precise description of what the test will consist of is prepared and the students are informed of this. Deliberately tricky questions are not permitted. It should be clear then that only mathematics of Avital and Shettleworth's first two categories will be involved. Since the test consists of 'must know' items the standard of performance expected from the student is perfection, or close to it. A small mechanical error might be overlooked in some instances, but in other cases not. For example, if the test is on the solution and verification of equations, their work

is either right or wrong. An important feature of the program is that the student is permitted three attempts to reach an acceptable standard of performance. Those students who are judged unsatisfactory after the first attempt are then interviewed, preferably individually, but because of the numbers involved this first interview may be done in small groups. They are permitted to see their papers but may not retain them. Their errors are analysed and further examples may be given for practice. After all the students who must rewrite have been seen, a second test similar, but not identical, to the first is given. There are usually a number who are still not performing acceptably. These are seen individually and coached further. Then they write for their third time.

In addition to one or two Basic Skill Tests in a term, we give conventional tests primarily on the 'can know' items, and which are marked in the normal manner. The marking for the Basic Skill Tests, however, is based on a different scheme. A fixed mark (see table below) is awarded for successful completion of a Basic Skill Test, the same mark whether accepted after the first, second or third try. If after the third attempt the work is still deemed unacceptable a reduced mark is assigned. No mark is given unless three sincere attempts have been made. The Basic Skill Test mark, or marks, plus conventional term test marks total

60. A further 40 marks comes from the general term examination. It will be observed that students could have 50 or more marks before they write the general examination. Since the 'must know' material has already been tested the general examination is made up of somewhat harder questions, more in the category of 'Open Search', but dependent on the ideas covered in the course. The mark obtained on this examination is added on to their mark out of 60 referred to above. It is made clear to the students that there is no pass or fail evaluation placed on them on the basis of the general examination, and indeed we do not expect the majority of students to get 50%, i.e. 20 marks, on the paper. For example, a student who has passed Basic Skill Tests and who has done reasonably well on the term tests might have 50 marks before writing the exam. If he, or she, gets only 12 out of 40 on the exam, the report mark would be 62.

The following table shows how report card marks are determined.

(a) Term with one Basic Skill Test

B.S.T. passed	30
B.S.T. not passed (but 3 attempts made)	20
Term Tests	30
General exam.	40
	100

(b) Term with two Basic Skill Tests

for each B.S.T. passed	18	36
for each B.S.T. not passed (but 3 attempts made)	10	
Term Tests		24
General exam.		40
		100

Recommendation for promotion without writing in June, in addition to a specified overall year average, is contingent on successful completion of all Basic Skill Tests given throughout the year. (A final opportunity to write any tests not satisfactorily completed is provided for those students who would otherwise be eligible for recommendation.)

Conclusion:

The Basic Skill Tests provide an excellent basis for remedial attention. They could be used as a major criterion for promotion. The general examination at term's end provides a challenge to students with better than average ability, but because of the marks structure a low score on these examinations does not penalize unduly any student who works hard, but who is limited in problem solving ability. The scheme should work well for a semestered system. Most of the interviewing and retesting has been done before and after school.

An arrangement permitting some of this remedial instruction to be done in regular class time, while the better

students work on assignments, would constitute a desirable improvement.

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1. From page 14 of Geometry - Kindergarten to Grade Thirteen
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2. Objectives for Mathematics Learning
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SIR? DID I PASS?

J. T. Walker, Durham College of Applied Arts and Technology

Three years ago I had set two tests for my first-year College class in Business Mathematics. The first was a fairly simple test, and the mean score was 78%. The second was much harder and contained one question out of the five which called for more than mere repetition of a thoroughly-taught algorithm. On the second test, the mean score was 41%.

These results started a train of thought which has continued until now, and which has caused me many hours of perplexing thought on the whole subject of evaluation and testing in Mathematics.

Most of my colleagues envy me at marking time, since they consider that marking mathematics is one of the easier chores in teaching. "After all, they're either right or wrong; there's no half measure, is there?" Burdened as they are with the marking of essay-type questions, I can understand that they regard the clean-cut nature of the mathematics discipline as a marker's Utopia.

But is it really as clean-cut as they are led to think?

In my submission, it is very far from clean-cut; indeed, I consider mathematics one of the hardest subjects to mark in the entire curriculum - to mark conscientiously, that is. To describe it as a 'Right-or-wrong' situation is to display a concern with the arithmetic accuracy of answers and to ignore the principles involved. A student can produce the 'right answer' to multiplying ½ by ½ if he adds the denominators to get ¼. But should he get the marks?

The mathematics teacher who marks on a 'right answer' basis is courting disaster. Because of the beautiful order displayed by our number system, there are many occasions on which a student can get the right answer to the particular question purely by the coincidence of an ordered system, and I am sure mathematics teachers can think of many examples of this type of occurrence. But what may be true by coincidence in one case may not be true in the general case, and unless the student has mastered the principles he will be unable to solve the problem if the numbers are changed a little.

The reader should not jump to the conclusion that I attach no significance to the correctness of the answer. Far from it. Since the Colleges are concerned largely, if not primarily, with job-relating training, the arithmetic accuracy of answers ranks high in importance in any evaluation system. Woe betide the graduate who tells his employer that

12% of $100 is $120: Or the graduate who charges a customer $145 for a $150 suit in a 10% discount sale! Arithmetic accuracy is important, but it is not the whole story.

We cannot allow a preoccupation with arithmetic accuracy to obscure the need for checking on principles. This means that the marking of mathematics tests must include a full reading of the student's answer; it involves also a complete check of the entire answer in the event that the student has made a minor error in the second line of an involved problem. It is totally unfair if a decimal point has been misplaced in line 2 for the student to be given a mark of zero when, apart from the slight slip, he has properly carried out a number of difficult manipulations later in the question. Teachers with experience in teaching Matrix Algebra - particularly Matrix Inversion - and Markov Chain Analysis must already be all too familiar with this situation. One of my students, recently faced with a Matrix Inversion, copied the original Matrix wrongly from the test paper. It was a 4 x 4 and the time allotted was 45 minutes. For the rest of the 45 minutes he did a masterly job of the Inversion, and eventually came up with the correct arithmetic answer to the problem he had set himself, which, incidentally, was considerably more difficult than the official problem. His answer was arithmetically 'wrong', but I gave him the marks.

It took me 28 minutes to work out 'his' problem. At that stage I would cheerfully have traded jobs with my colleague marking 57 essays.

The question each teacher must decide for himself is how much weight to give to arithmetic accuracy. I have resolved the dilemma in my own case by allowing inaccuracies in tests and exams subject to a relatively light penalty, but punishing inaccuracies in home assignments severely. The student has time at home to check his work. It he doesn't,he pays.

I am frequently engaged in discussions with my colleagues - and on occasion in heated argument with them - on the mathematics teacher's responsibilities in the area of grammar and syntax. It is claimed by the Communications and English teachers that that is exclusively their province. The fact that Johnny can't put a decent sentence together, or can't spell, or both, doesn't seem to upset them unduly - unless that fact is pointed out by a mathematics teacher. Then watch the fur fly!

In my second-year Statistics course, I place a great deal of stress on the necessity for finished work to have a look of professionalism. Most of my students can draw clean and accurate graphs and perform accurate calculations; almost all of them can draw Tables fit for photographic reproduction in print. Recently I set them the task of doing a

complete Time Series Analysis of 11 years' data by months. One of them did an immaculate analysis well up to professional standards, and presented it properly bound, documented, and indexd. But in the Footnote to one of his Tables he wrote:

"Footnote: There is no figures avalible to covering this months' sales."

This student has a four-generation English Canadian background. If his name had been Stanisczuk or Hrenkovitch, there might have been some excuse. (Incidentally, if anyone doubts the truth of this, I will be pleased to send him a Xerox copy of the original which I have kept as a souvenir.)

I do not consider that my responsibilities end with the figures. Perhaps his boss two or three years from now will not be able to understand the calculations or read the graphs and tables. But he will be able to read the footnotes. If I am to accept responsibility for the professionalism of the student's work, and that work includes the footnotes, then I am responsible for them too.

So I go on correcting the English, and my colleagues in Communications can go whistle!

Once we have corrected the mathematics - and the English - we assign the student a mark. I am bitterly opposed to percentage marks. Who am I to say that one student deserves

66% and another 67%? Can I really judge ability so accurately? Even in a quantitative subject like mathematics, how can I differentiate to the extent of one, two, or for that matter, five, percentage points? Ten, perhaps, and twenty, almost definitely. But one? Who am I kidding?

I wonder what would happen if I turned in a list of final marks to the administration for 50 students with 3 at 30%, 10 at 50%, 25 at 60%, 10 at 70%, and 2 at 80%, with no intermediate marks at all. Questions would certainly be asked. On the other hand, so long as I turn in the familiar 'bell curve', even if some of the marks are expressed as percentages to the first decimal place, all will be assumed to be normal, and I will be re-hired next year, provided the mean lies between 60% and 65%. This is one of the facts of life with which all teachers have to live. However, so long as the authorities stick to this antiquated concept, what else can we do?

As mathematicians, we have a distinct advantage over our fellows in this respect. We can 'cook' the distribution - honourably, of course, and with due mathematical ceremony. I circumvented this problem for a time by the simple expedient of adjusting marks on the pretext that I was trying to remove the element of difficulty as between tests. I used the device of setting the 'pass-mark' at 2/3 of the mean mark. On an

easy test with a mean of 80%, the pass-mark was set at 53-1/3; any student with 54 or better passed. On a difficult test with a mean of 40%, the pass-mark was 26-2/3, and any student with 27 or more passed. After all, it was the same group of students, presumably working at much the same level of intensity, and the only difference was that my questions varied in difficulty.

I was doing the students a big favour. I was letting them set their own pace in marks. If they all, or almost all, did poorly, the pass-mark was reduced accordingly. If they all did well, the pass-mark went up. I consoled myself with the thought that a student who scored 60% on a difficult test deserved as much credit as a student who scored 85% on an easy test. I was being thoroughly 'objective'.

Being nothing if not versatile, I even divided the difference between my pass-mark and 100% into four equal stages, and awarded letter grades A, B, C, D according to the stage in which the student's mark fell. I was so proud of myself! I even boasted aloud to my colleagues that I had invented a 'Dual-Grade System', and actively promoted it on the basis that the student would know, according to his numerical mark, how he was doing relative to what I expected, and according to his letter grade how he was doing relative to the performance of his classmates.

Not content with this, I invented the so-called 'Letter Profile'; if over six tests the student showed a Letter Profile of DDCCCB, I would demonstrate the he was 'creeping up through the class'; that is, that he was improving relative to the typical performance of his classmates. He therefore deserved sympathy in the form of benefits awarded subjectively. If, on the other hand, his Profile showed AABCCC, he was 'doing a slide down through the class', and his work must be carefully reviewed.

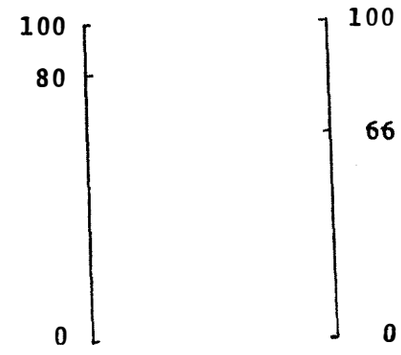
It was all so scientific that I was glowing with pride. Then the bomb fell. After a series of stiff tests with low pass-marks in which one of my students had been scoring in the 65-68 range and thereby earning a B-type Profile, he clocked his usual 67 in the Final Examination which, according to the College's official grading scheme, qualified him for a C Grade. He could not understand how, having been a consistent B student, he could wind up with a C. It took a lot of argument to make him see the College's inflexible system made no allowance for the difficulty of exams, or for differences in difficulty between the exams of two different teachers. To get a College-type B Grade he required a mark between 70 and 79 inclusive.

So the famous but short-lived Dual-Grade System went by the board.

Nothing daunted, I returned to the drawing board to yet another 'scientific' scheme. First, I identified the problem. The College marking system calls for the awarding of grades based on percentage marks: 49% or less = F; 50-59% = D; 60-69% = C; 70-79% = B; 80% or more = A. Further, the re-hiring process seems to depend on a bell-curve with a mean of about 66%. All right; I wanted to be re-hired, so that's what they were going to get.

First, I hypothesised a test in which the mean mark was 80%. The statistical textbooks told me that with a class of 30 I needn't have to worry about creating a bell-curve; the chances were that it would happen anyway. In smaller classes I would have to review the marks before I finalised them, and rearrange them to get a 1-4-6-4-1 distribution from F through A. That didn't seem to present any major problem.

Now the question of adjusting the mean arises. I drew two scales, the first showing my mean of 80% and the second showing the College mean of 66%:



I reasoned that if a student scored 90% on my test, he was halfway between the mean and 100%, so on the College scale he should lie halfway between the mean and 100% which works out to 83%. If the student ever finds out that he has been docked 7 marks to fit him into another marking scheme.....well, that's another problem I'll face as and when it comes up.

From this, of course, it is relatively easy to work out a formula which can be used to produce an 'adjusted' mark, once the student's mark and the mean mark for the class are established:

From the relationship

$$\frac{X - M}{100 - M} = \frac{X' - \bar{X}}{100 - \bar{X}}$$

we get:

$$X' = \bar{X} + \frac{(100 - \bar{X})(X - M)}{100 - M}$$

where X' is the student's 'adjusted' mark

$\bar{X}$ is the required College mean (=66%)

M is the actual mean mark of the class

and X is the student's actual mark.

By this formula, the student who scores 80% will be given a mark of 66%. So far, so good. A student who scores 90% will end up with 83%. A student who scores 65% will be marked as 41%. Not so good. One of the trying things about mathematics, as distinct from some of the more literary disciplines, is that it is fairly easy for a student to tell whether he has a written a 65% paper or a 41% paper, and no teacher on earth is going to have the courage to try to persuade him that he scored only 41% in a paper where he got more than half of the problems correctly finished.

The system works better if the exam was difficult, and the mean mark was below 66%. In that case the penalties imposed on the less able students are lighter. But in any case, if the mean of the adjusted marks comes out to be 66% it is coincidence. So the system is really self-defeating. Scientific, yes, but nevertheless self-defeating. Aligning the means and adjusting on that basis didn't work.

What about aligning the pass-marks instead? If I set my pass-marks at two-thirds of the class means, and align them, using the same system of scales, with the College pass-mark of 50%, will that work?

Again we run into the same trouble; the system works well for students above the line, but punishes those below the line severely. The only saving grace is that students

below the line can all be given a mark of 40% or 45%, and they have definitely failed. After all, is there any real difference between a mark of 30% and one of 45%? Yet, say my colleagues, there is the question of elegibility for supplementals. My answer to that is that the teacher can assess the student's chances on supplementals; those who have a chance get 45%; those who don't get 25%, or just a plain F grade without a mark.

If you want to try it, the adjusting formula is:

$$X' = 50 + \frac{50(X - P)}{100 - P}$$

where X' is the adjusted mark

X is the student's actual mark

and P is the class pass-mark (=2/3 of the class mean)

Incidentally, you have a much better chance with this system of getting a mean after adjustment in the region of 66%. Or maybe I've just been lucky.

I've been re-hired for next year. My bell-curves were really something to see. The means of my four classes were 65.2%, 65.7%, 66.1% and 66.2%. My distributions were strictly 1-4-6-4-1. In one class of 32 students I allowed -

before marking started - for 2 F's, 8 D's, 12 C's, 8 B's, and 2 A's. The mean - after marking - was 68.2%, so I subtracted 3% from each student's mark. Resulting Class mean, 65.2%. Nobody complained. In fact, I was asked to advise some of my mathematically inept colleagues on my 'marking system'. I explained the Dual-Grade System and the Matching Systems for means and pass-marks, and acquired a reputation as an expert on Evaluation and Testing.

All joking apart, so long as the Colleges stick to the idea that all exams are equally difficult - or easy - and all instructors are euqally competent to set and mark exams, what else can we do? The mathematicians and statisticians are obviously the ones who should be able to come up with the ideas. Meanwhile we can only ask ourselves what we are doing. If you stop to think seriously about it, I can tell you that you are in for some head-breaking and some soul-searching.

At the moment I am struggling with a guilt complex. Can anybody come up with a remedy?

EVALUATION IN THE MATHEMATICS CLASSROOM, GRADE 13

H. W. F. Appleton, Head of Mathematics
Carleton Place High School.

It was a sound scheme. If someone had a problem he could not work out, it was his privilege to get up and begin it on the blackboard for me to get round to, as soon as possible.

"Jim," I said, "it's your turn. Let us see you do this one". Jim got it out.

"Good work, Jim, now for a little critique. Bill, what do you think about this?" ..... Soon we had it in good form and everyone got to work again.

I marked down a "stroke" for Jim in my record book. It counted for "marks" in classwork.

After a while fewer people came up to the board to begin problems they could not work out.

"Why?" I asked a few days ago.

In grim honesty came the answer, "Because it is giving some one else a mark if you go up and begin a problem you can not do."

I like my Grade 13's even if it is difficult at times. They are, at least, honest. They must not be blamed; for

The aim of education, of course, is "marks". The search for truth, independence of mind, greatness of spirit, self-reliance. All that jazz, is nothing compared to getting 80% at some one else's expense! We teachers have no one else to blame but ourselves- Just watch the tiny ones sometime helping one another in a kindergarten. Then, reflect on what we have done to them.

MATHEMATICAL MINUTIAE

M. Cooper

Once upon a time in a far distant land there was a very popular exam question: Evaluate 5 + 2 x 3. It was beutiful in the eyes of mathematicians, such elegance and economy in the statement and no ambiguity about the answer.

But ehere grew up a generation of teachers who became exceedingly wrathful and declared, "In order to make certain our students answer well, we have to engrave BODMAS in their minds. Until we are paid six times as much we will only teach them B and we will achieve our elegance and economy in the rules of the game. We will write the question as 5 + (2 x 3)."

The high priests wept and gnashed their teeth and declared - "The bracket makers will make a fortune, and the computers will lose a micro-second, but worst of all even the common people will be able to understand the mysteries we so jealously guard." But they were helpless before the resolve of the teachers and the question disappeared from the face of the earth - or almost.

PROBLEMS AND SOLUTIONS

Edited by W. Fraser

Please send solutions and problems you would like to suggest for inclusion in this section to Professor W. Fraser, Department of Mathematics and Statistics, University of Guelph, Guelph, Ontario.

Problems

The first problem below appeared in the eleventh International Mathematical Olympiad held in Bucharest in July, 1969. There are two four-hour sessions with three problems per session. These problems appear to be of varying degrees of difficulty, but the scores published would certainly indicate the existence of some very able young high school mathematicians.

Problem 71.03

Prove that there are infinitely many natural numbers q with the following property: the number $z = n^4 + q$ is not prime for any natural number n.

The following problem was submitted by Mr. E. Evans, Guelph, Ont.

Problem 71.04

Given a square ABCD, and any point H on the side AD. Let BA and CH meet at F, and CD and BH meet at E. Let CF and AE meet at G. Show that the angle at K, where DG meets BE, is a right angle.

Solutions

Problem 70.01, March 1970

A solution to this problem has been submitted by J. D. Lammin, Fergus High School. The following solution uses much the same ideas, but is a bit briefer:

Given $E_q = |1/\sqrt{2} - p/q|$ where p, q are integers, $q > 0$, p/q is in its lowest terms and p/q approximates $1/\sqrt{2}$ better than any other fraction with denominator q.

$$E_q = \left| \frac{q - p\sqrt{2}}{q\sqrt{2}} \right| = \frac{|q^2 - 2p^2|}{q\sqrt{2}|q + p\sqrt{2}|} \geq \frac{1}{q\sqrt{2}|q + p\sqrt{2}|} .$$

Since p/q approximates $1\sqrt{2}$ more closely then (p-1)/q,

$$p/q - \tfrac{1}{2}(1/q) < 1/\sqrt{2}.$$

Manipulation of this inequality leads to $1/(q + p\sqrt{2}) > 1/(2q + 1/\sqrt{2})$. It then follows that

$$E_q > \frac{1}{q\sqrt{2}(2q + 1/\sqrt{2})} = \frac{1}{2\sqrt{2}\,q^2 + q} \quad .$$

Problem 70.08, November 1970

Solutions to problem 70.08 have been submitted by R. Singh, Arthur D.H.S., and T. J. Griffiths of North Lambton Secondary School.

Given three non-concurrent lines $a_i x + b_i y + c_i = 0$, $i = 1, 2, 3$, the solutions of the equations $a_i x + b_i y + c_i = \pm r\sqrt{a_i^2 + b_i^2}$ for x, y and r provide the coordinates of the required point, and the radius of the circle. The problem is to select the signs. With three equations and a plus or minus sign appearing in each, there are essentially four different combinations. (The solution with all plus signs is effectively the same as that with all minus signs, involving only what is to be considered the positive or negative side of a line). The four combinations correspond to the four circles which can be drawn so as to touch the sides of a triangle. Of these, the inscribed circle has the radius of smallest absolute value. This fact can be used to determine the proper selection of signs. If we write $q_i = \sqrt{a_i^2 + b_i^2}$, our equations can be written

$$a_1x + b_1y + s_1q_1r = -c_1$$

$$a_2x + b_2y + s_2q_2r = -c_2$$

$$a_3x + b_3y + s_3q_3r = -c_3,$$

where $s_i = 1$ or -1, the exact choice still to be decided.

If we solve these equations for r, we get, writing

$$a_2b_3 - a_3b_2 = M_1,\ a_3b_1 - a_1b_3 = M_2,\ a_1b_2 - a_2b_1 = M_3,$$

$$r = \frac{-(c_1M_1 + c_2M_2 + c_3M_3)}{s_1q_1M_1 + s_2q_2M_2 + s_3q_3M_3}.$$

The s_i appear only in the denominator. For the given a_i, b_i, c_i we will find the smallest solution in magnitude for r when the denominator has the largest possible magnitude. This will occur when the signs of the individual terms are all the same. Since the q_i are all positive, this requires only that we should choose s_1, s_2, s_3 so as to have all the same signs as M_1, M_2, M_3 respectively, or the signs of s_1, s_2, s_3 should be opposite to those of M_1, M_2, M_3 respectively.

BOOK REVIEWS

Mathematics 9, by J. J. Del Grande, H. A. Mulligan and H. E. Totton, W. J. Gage Limited, Toronto, 1966, 500 pp.

This being my first look at this text, I was quite favourably impressed. According to the authors, the book has been planned to give students an opportunity to participate in the discovery of new ideas. To a large extent this aim has been achieved. The use of developmental questions to learn new ideas from basic concepts previously studied is generally well done. However, there are some areas where a student might get bogged down unless he had the Teacher's Commentary.

With regard to the above I would suggest that the text could be easily used by a good student for individual study, whereas poorer students would require the direction of a teacher. I can see the book being valuable as either a text or a reference.

The material in the book seems to be well laid out, and the progression is good. The colour bands used to highlight rules and definitions are helpful. The question sections would appear to be quite adequate to give a good grasp of the material covered. The book is suitable for students in Grade 9

who are in the 5 year course. Even students at the 4 year level would benefit from sections of the book. The inclusion of the sections on puzzles and problems is good, and there could probably have been more of these. Granted, these can be gotten out of other books, but how many students ever go beyond their text to other sources. The style and type are both quite pleasing.

The following errors were observed.

Page 143 section C (c)

the sentence that results from the trial of 20 is

$4 \times 20 - 5(38 - 20) = 167$.

This should have been $4 \times 20 + 5(38 - 20) = 167$.

Page 230 section B

The teacher's guide failed to include in its answer $1:\frac{5}{3}$.

Page 233 section K

Teacher's guide should be 20%, $20:100 = \frac{20}{100}:1$.

Page 248 Exercise no. 9

If $a(x) = 5$, what is the value of $a(0)$?

The answer in the back is 3. It should be 5.

Page 251 section A (a)

Teacher's guide should read

$3x \; 4x \; (3x \times x \; 4)x$

$(3x \; 4 \times x)x$

$12x^2$.

Page 284 Question 21.

The answers in the back for parts (b) and (c) should be interchanged.

Page 367 section G

The diagrams of the congruent triangles are not accurate and hence can be very misleading. If two triangles are supposed to be congruent they should at least look it.

Page 400 Heading in Teacher's guide should read 'Some Properties of Tangents'

Page 452 section F (a), answer should read 7, 12 instead of 7, 2.

In conclusion I would agree with the authors that the student who works through this text carefully under the guidance of a stimulating teacher should gain a sure and satisfying command of the fundamental ideas of algebra and geometry.

P. Klassen,

John Diefenbaker Secondary School.

Elementary Mathematics 2, by M. O'Keefe and J. Routledge, W. J. Gage Limited, Toronto, 1967, 182 pp.

This book presents a basic course for grade two mathematics. It incorporates the aims outlined by the Department of Education in Ontario, in the Interim Revision Mathematics Curriculum P1, J1, into a series of sixteen units or topics.

Each unit, accompanied by worksheets for pupils, deals with a single topic e.g. "Basic Facts to Six". The objectives of each unit are outlined in point form at the beginning of each topic. Teacher background material and comments follow. Many good, practical ideas are used in the lesson plans which contain a step by step method for presenting the worksheets to pupils. Extra activities and suggested drill are included in the units where this work is needed. A series of evaluation questions at the end of each unit greatly assists the teacher in her evaluation of the pupils.

The pupils enjoy the varied methods of presentation given in the worksheets. The vocabulary is kept to a minimum so that pupils of age seven and up should be able to read the problems without difficulty.

Many of the units may be used out of order without losing the coherence of the programme. This Canadian book is geared to the needs of the teachers and pupils in Ontario schools.

A. Coupar,
Sherwood Heights School.

Contemporary Mathematics 5, by H. A. Elliott, K. D. Fryer, R. G. Dunkley, and N. J. Hill, Holt, Rinehart and Winston, 1970, 598 pp., $6.30.

In these days of open-ended curricula where "core" material is reduced to a minimum and free rein is given to the choice of optional topics, the development of a single, comprehensive text to cover the grade eleven year is a formidable task. It is one, however, which the authors seem to have met with enthusiasm. They state in the preface that this book develops the close integration and coordination of geometry and algebra, introduces trigonometry and stresses its essential usefulness in promoting the interrelationship of the other topics. This is done; moreover, within the scope of each of the three areas is a careful development of the basic "nuts and bolts" topics and concepts, with lots of problems and examples which lead on to work which may be included, postponed to a later year, or omitted from the high school curriculum altogether.

The geometry section starts with a review of the basic congruency and parallelism theorems, and introduces in some depth the concept of locus. The Euclidean theorems on area and volume, ratio and proportion, and the geometry of the circle. At every opportunity, the topics are enlarged to include a study of 3-space. Pythagoras and Heron are both

included, and proved by Euclidean methods. There is an excellent section on spherical geometry. Chapters on Vectors and Transformations are included, with an emphasis on their applications to geometry, which lead to subsequent applications in the trigonometry section.

A thorough discussion on statements, their converses, methods of indirect proof and the logical requirements for "if and only if" are presented, but the formal Euclidean proof has returned to the traditional format of "Given-Required Proof". The "Analysis" has been dropped as has (thankfully) the insistence on authorities for all statements, regardless of their self-evident nature.

The section on Algebra under the basic headings of Rationals and Reals is well ordered with lots of exercises and should give ample opportunity to develop algebraic skills, and ability to handle radical expressions. A whole chapter is devoted to the factor and remainder theorems, radical equations and manipulation of polynomials. Also included are several side-trips into Structures, including those involving matrices and permutations.

Cartesian geometry of straight lines is presented along traditional lines. The basic formula patterns for slope, equations of lines and distances (including point-to-line) are established. Consistency, dependence, and solution of systems are treated exhaustively, including matrix represen-

tation of the systems. Emphasis is placed on the sketching of graphs, including circles and parabolas and other relations and functions, using a transformations approach.

The trigonometric functions are first developed on a co-ordinate basis, plotting points on the circumference of a circle on a grid. Numerous exercises are given to allow the student to fully understand the behaviour of these functions in the different quadrants, their properties, and to plot their graphs. The triangular definitions are given later preceding the development of the sine and cosine laws as methods of solving triangles. There is also a section of more advanced applications of trigonometry to vector problems.

In general the tone of the book is scholarly without being pedantic, tending neither to talk up or talk down to the student. The type, print, layout and artwork in the book are excellent. A random sampling of the answers in the back revealed no errors, though a few were found in diagrams in the body of the text.

Though I have described the material in the book in terms of "sections", I do not want to give the impression that the book was written or should be used that way. Wherever possible the topics have been interrelated, both at the level of the basic material, and at what could be called the "optional" level. There could be many starting points, and an

infinite number of "orders" of course development.

After the gyrations of the last few years, I hope that the latest phase of math teaching has been reached, where full advantage can be taken of the "new" topics and concepts that have been added to High School math programs, and at the same time revive what was traditionally considered worthwhile.

I think that this text represents this phase. I recommend it and plan to use it with my university-bound grade 11's next year.

D. Clark,
Georgian Bay Secondary School.

Programmed Statistics, by R. Bellman, J. C. Hogan and E. M. Scheuer, Holt, Rinehart and Winston, 1970, 115 pp.

This book - or booklet, as it might more profitably be described - can be absorbed by the typical second-year College student in about a week. Its authors are not joking when they say in the Preface that the work contains "the minimum amount of information about statistics and measurement .....". However, they must surely be joking when they go on to say, "No mathematics beyond the simplest arithmetic or algebra is used". Any student taking that remark at its face value is going to be in trouble as early as page 3.

Chapter 2, 'Programmed Statistics', contains most of what meat there is in the book, and even so, is of limited value. If the authors' intention was to sell programmed instruction to teachers using this Chapter as a sample, they should not be surprised if their cause is already lost.

Their discussion of the Digital Computer in Chapter 3 is exactly what they say it is, 'a brief and elementary discussion'. Chapter 4 is an undisguised 'plug' for Computer-assisted Instruction. The plugging theme is worked out to its rational conclusion in Chapters 6 and 7, after a minor digression into Random Number Selection in Chapter 5. Chapter 6, New Computational Aids for Educators, and Chapter 7, The Next Decade in Education, have nothing to do with Statistics. The former is undisguised publicity material for JOSS, the RAND Corporation's personalised computer service, and the latter is a mediocre short essay on the need for educational reform; the authors' purpose in including it in this publication escapes this reviewer.

It is difficult to see any useful purpose in carrying this book in a College Library, and impossible to see any reason why a student should be asked to buy it, either as a main text or as supplementary reading. It has all the appearance of having been thrown together by academics desperate to publish, without regard either for the merit of the pub-

lication itself or the intelligence of its potential readers.

J. T. Walker,
Durham College of Applied Arts
and Technology.

Elementary Mathematics 1, by M. O'Keefe and J. Routledge, W. J. Gage Ltd., 1966, 179 pp.

Elementary Mathematics 1 has much to offer to the first year mathematics program. The teacher's commentary has been thoughtfully organized providing the classroom teacher with objectives, background, comments, teaching procedures, and evaluations.

The teaching procedures provide numerous detailed suggestions for both introducing and re-inforcing skills and concepts. The objectives of each unit are clearly defined and testable. Evaluation of each unit indicates whether the objectives have been reached or whether further instruction is required.

In the accompanying pupil's worksheets, the child is allowed to discover the concepts of number and sets with delightful pictures which are both meaningful to his experience and appealing in their design. A unique method of interpreting the concepts of addition and subtraction has been given. The illustrations actually show the action

taking place. The child is allowed to apply the concepts of union and separation of sets to incidents familiar to his environment. Equally as important is the idea of greater than and less than. Ample reinforcement has been given in this area.

Considerable material is provided for developing the concepts of 10's and 1's. The teacher's commentary offers many excellent teaching procedures. However, the accompanying worksheets appear cluttered with small diagrams which have the possibility of being confusing or distracting to slower learning children.

A new element in primary Mathematics is given in the form of geometry and measurement. The program includes recognition of common geometric shapes both in three dimensional and plane figures. The child is allowed to discover congruence by fitting disks together.

Included in the pupil's workbook is a set of disk cards. With these manipulative materials, the child is able to compare his own mathematical sentences adding variety to checking children's responses.

We have used many of the procedures given in our primary classes. The book is well organized and crammed with excellent teaching suggestions. It would certainly enhance any first year Mathematics program.

J. Joyner and A. Prokich,

St. Joseph's School.

Mathaction 1, by W. W. Bates et al, Copp Clark Pub. Co. Ltd., 1969.

Our first reaction when asked to review this book was "Just another textbook for enrichment work," but after implementing this program, in a somewhat limited fashion we are cautiously enthusiastic about it.

If need be, this book can be a source for enrichment work, but to get its full value it is recommended that the program, as detailed in its Introduction, be followed. A word of caution, though, in implementing this particular type of program, the teacher will have to guard against being carried away by it to the exclusion of all other subjects.

In view of the limited budget problem we now face, the activities listed in this book may or will have to be improvised. This the teacher should anticipate, involves a large chunk of spare time. Furthermore, keeping a checklist of the child's progress can get to be a bit harassing and, to say the least, confusing.

However, if one can successfullly overcome these niggardly, albeit pressing, problems, we recommend the book and the program outlined.

S. Pillay and A. Prokich,
St. Joseph's School.

Computer Programming: A Total Language Approach, by D. W. Washburn, Holt, Rinehart and Winston Inc., 1970, 490 pp. $9.85.

The author states in his Preface that "the purpose of the text is to provide a complete and introductory treatment on the subject of computer programming". Later he continues, "Since the objective is to provide a complete understanding..." The book certainly lives up to the promise of 'completeness', in that almost every topic dealt with in Computer Courses is mentioned somewhere, but what the author has provided in completeness he has skimped on thoroughness.

The book is 'thin', and suffers from the inclusion of topics no longer considered relevant. For example, considerable time is spent on SPS and Autocoder, both belonging to the 'horse and buggy' era of computer science. Perhaps the author is not aware that the 1401 will be phased out in the near future, since he includes a substantial amount of material on that system.

Most educational institutions, if they have access at all, have access to only one installation, and for training purposes the book would require access to several systems. The odds against the student becoming, as the author promises, 'quite sophisticated' are heavy. The author might more

profitably have concentrated on one or two languages, curtailing the breadth of coverage and increasing its depth.

The book might form a useful addition to a Library, but its usefulness as a text in a Computer Course is doubtful, for the reasons given above. Despite the complex appearance of the material to a novice, the text is well written and amply illustrated and is suitable for Library purchase as a reference book.

L. E. Edwards,
Durham College.

The ADN Binary Teacher: (Education Division, ADN Business Systems Ltd.*, 64-66 King Street, Hammersmith, London).

Typical of the plastic age, any simple pen and paper operation is being converted to unbreakable plastic. For $3.80, a plastic set of wheels and 0's and 1's will teach you how to add, subtract, multiple and divide in Binary. An accompanying instruction book (with its pages indexed in binary - complete with incorrect hand symbols on pages 1001 and 1010) details its limited uses.

This little gizmo is suitable only for the junior high

* Distributed by Bellhaven House, 1145 Bellamy Rd., Scarborough 707, Ontario.

school mathematics department that has <u>everything</u> - a very flimsy excuse as a teaching aid.

K. McLean,
McMaster University.

PROGRESS REPORT

OF THE

OMC-OATM COMMITTEE

March 1971

At the annual meeting of the Ontario Mathematics Commission, September 19, 1970, a proposal for a single mathematics education organization in Ontario was presented by the incoming president, Mr. Ross Leigh, and after some discussion the following motion was carried:

> "that the incoming executive be empowered to use this proposal in prinicple as a basis for discussion with OATM with a view to having a progress report made to the Commission in the spring."

Since that time the joint OMC - OATM negotiation committee has had three meetings, held in Toronto on December 12, 1970 and January 22, February 19, 1971. The Commission has been represented by Mr. Ross Leigh, President, Dr. Douglas Crawford, Past President, Mr. Andrew Vertesi and Mr. James Tannant, executive members. Secretary-Treasurer Gene Durrant attended the February meeting. The OATM has been represented by Mr. Don Fox, Mr. David Alexander, Mrs. Joan Routledge,

and Mr. Art Dayman.

General agreement has been reached by the committee on the following points:

1. A single organization concerned with mathematics education in Ontario is desirable.
2. The aims of a mathematics education organization should include
   (i) evaluation, research, design of mathematics curriculum and progress in the schools.
   (ii) professional development of mathematics teachers at pre-service and in-service levels.
   (iii) development of experimental teaching methods and materials.
   (iv) provision of coordination, communication, and exchange of ideas in mathematics education at all levels through publications, conferences, and personal involvement.
   (v) liaison with educational organizations in Ontario and mathematics education organizations beyond the province.
   (vi) maintenance of a representative mathematics educator's "voice" in Ontario.
3. Membership in the organization should be of three types

(i) individual membership, carrying the right to vote, act on any committee, hold any office, receive all publications.

(ii) Institutional membership, open to organizations with an interest or involvement in mathematics education, and carrying the right to representation on specified committees, and to receive all regular publications.

(iii) Associate membership, open to individuals and organizations who wish to promote mathematics education, and carrying the right to receive all regular publications.

4. The organization should have an affiliation with the Ontario Education Association for the purpose of liaison with other disciplines and aspects of education.

The joint committee recognizes the present commitments of OMC and of OATM and agrees that there need be no diminution of effort in these directions. In fact, the strength of any future organization would stem from the vitality of present programs.

It is the intention of the committee to continue to discuss tentative constitution, financial structure, possible transition stages, etc.

Members of the Commission are invited to comment on this report if they so desire.

Respectfully submitted

Ross Leigh, President,

Ontario Mathematics Commission.

O.A.T.M.

ONTARIO ASSOCIATION OF TEACHERS OF MATHEMATICS

President's Report to the General Membership

80th Annual Convention, Niagara Falls, Mar. 21-23, 1971

Introduction

The Ontario Association of Teachers of Mathematics received its new name when the Revised Constitution of the Mathematical and Physical Association of Ontario was approved at its Annual Meeting on April 21, 1965.

> "To trace in detail the threads to progress in the affairs of this Association from 1891 requires more time, deeper insight, and much better records than are presently available. Nevertheless, the following paragraphs may revive in the memories of older members, the friendly battles of the past and, for those who are younger, provide some background for comparing the old days with the new ..."
>
> -Foreward, Historical Highlights of the O.A.T.M.P. 1891-1960 (published in 1961 to mark the 100th Anniversary of the O.E.A.)

Lest the foregoing quotation be misleading, I hasten to add that Mr. H. E. Totton compiled an excellent manuscript from the records of the Association. In fact, after reading it I was so proud of the past accomplishments of our organization that I was moved to formulate the slogan which I shall recommend later to the incoming Executive Council. During this past year in office as President of the O.A.T.M., I have examined activities of the past as well as tried to forecast the future in an attempt to determine the best means of fulfilling our role for the current and future years. In this regard, I invite you to add your appreciation to my personal thanks to all members of the Executive Council for their support and sincere efforts during the past year. Our organization this year has required that they not only attend meetings in Toronto but assume a leadership role in their geographical region as well as be active members of committees, in liaison with other organizations or special projects.

A special vote of thanks is also due Mike McKenna, Bruce Jay and the members of their Convention committee which worked almost exclusively outside the regular Council activities.

Review of Council Activities in 1970-71

At its first meeting on May 23rd, the O.A.T.M. Council held a post-mortem of the Ottawa Annual Conference and then decided to (1) move towards a committee organization, (2)

plan for another Annual Conference, and (3) arrange for as many regional conferences as desirable. Two non-executive members were elected as the 1971 annual conference co-chairman, and requested to organize their committee and plans outside of regular Council meetings and activities. In this way the vice president and other councillors were freed to devote more time and effort to goals (1) and (3) in an attempt to provide more services and better communications for the mathematics teachers of Ontario.

Any failure to achieve these goals is due to the limiting factor of time, rather than lack of effort on the part of your elected representatives. They worked diligently through the spring and fall terms to arrange for a large number of Regional Conferences attended by a total of more than 1300 teachers of mathematics*, while independent plans were being made for this week's Annual Conference. In the process I am certain that much valuable information was exchanged, but we have not yet solved the problem of publicizing and reporting these conferences to all of our members - hopefully this problem will be resolved in the coming year. It is

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* See the notes on some of these meetings, submitted by Joan Routledge on p. . Editor.

worth pointing out that on October 3 the Northern Ontario Mathematics Association was formed; and that two similar motions were formulated simultaneously at different regional conferences re the future of Grade 13 Single Credits. A copy of the motion was mailed to the Ontario Department of Education, c/o G. A. Scroggie; and your executive was prepared to present it as a resolution to the O.E.A. Senate which, if passed by that body, would in turn be presented to the Minister of Education. However a satisfactory answer to teachers' concerns seems to have been effected in other ways.

During the Fall term, due to heavy regional activity, our provincial committees did not have an opportunity to meet (except as a council on September 19); but councillors were grouped according to their interest in Curriculum (content or materials), Communications, Conferences, or Teacher Education (including Summer Courses). Your executive found it valuable to issue a Newsletter on November 25 in order to keep councilors posted about the foregoing as well as the following devopments - it was our hope that each in turn would communicate with the teachers in his/her region.

Many of the Councillors were active working as liaison people with other organizations:

CAMT Lloyd Auckland was appointed as our representative, for the next two years, to the Canadian Association of Math-

ematics Teachers which met last month in Montreal when the decision was made to publish a Newsletter - copies to be mailed to all members of O.A.T.M.

NCTM The N.C.T.M. sponsored a Leadership Workshop (Toronto, October 31) attended by J. Routledge, K. Hill, M. MacGregor, A. Duffield, E. Glyn-Jones, D. Bastedo and A. Czempinski, who act as regional representatives of N.C.T.M.

NCTM 73 On November 25, the names of two people were submitted to N.C.T.M. - to act as convention and programme chairman of the N.C.T.M. Regional Conference to be hosted by O.A.T.M. in Toronto, November 7-10, 1973. The Executive Council directed a letter to N.C.T.M. in January suggesting the name of our Past President, Dave Alexander, for consideration as a member of the N.C.T.M. Board of Directors. (Current policy is always to have a Canadian member of the Board).

OEA On your behalf, as president of the O.A.T.M., I joined forces with the president of the Ontario Society for Education through Art and we circularized all sections of the Ontario Educational Association. We presented our findings of discontent to the O.E.A. Executive

(December 23) and then the Senate (January 15). From that beginning all sections were invited to participate in defining the Role for O.E.A. in the 1970's.

OMC 1) Six members of O.A.T.M. were appointed as representatives to the Ontario Mathematics Commission and have been actively working as members of various sub-committees - D. Fox, R. Leigh, M. MacGregor, W. Sokoluk, D. Bastedo and A. Dayman.

2) Mrs. Joan Routledge was elected as the O.A.T.M. representative to the editorial board of the Gazette.

3) A committee of four O.A.T.M. representatives met with representatives of the O.M.C. to continue discussions prompted (on the part of O.A.T.M.) by a motion of the council at its meeting of January 24, 1970*. It is the intention that this report, stating the principle points of agreement to date, be discussed during a Seminar - Monday P.M., March 22, 1971. Also written comments will be invited from O.A.T.M. members present at the Business Meeting (March 23). Unless instructed otherwise, the O.A.T.M. represen-

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* Editor's note - see the Progress Report on the OMC-OATM Committee submitted by Ross Leigh, and printed on p. of this issue.

tatives to the committee have been directed by Council to continue with further discussion during the school year 1971-72. These same representatives have been very pleased with the support received for the recommendation that a new organization should consider a strong regional or branch basis for most of its activity.

After reading the above details of our activities you probably realize why it was not until January that your Executive Council was able to focus attention on committee activities of its own. Even then because of the time required to exchange information and coordinate our efforts, we were able to break away from business matters for only a few short sessions to seriously discuss the state of mathematics in Ontario. However, we are very proud of the work done by the members of our Teacher Education Committee in surveying the post-secondary institutions and the teachers of Ontario to determine not only which in-service courses are being offered but what courses are "needed and desired" by teachers. A separate report of their findings will be available at this conference.

Out of a concern for mathematics education in Ontario, the Council also arranged for a small representative group to to meet (March 21/71) and informally discuss its present

state; the current mathematics programmes, K to post-secondary.

Looking to the future

In this 'instant society' and age of rapid change, it is impossible to forecast very far into the future. By reviewing our past and examining present problems it is possible to form at least a few impressions with respect to the needs of our association based on the needs of mathematics education in Ontario. Over the last few years there has been a significant lack of knowledge of our association, pride in our association, and activity in our association - this can only be remedied by organization and communication.

We have the history, ability among our members, and the organizational structure that will permit us to provide significant services to mathematics teachers of this province. Whether we work alone or join with other groups, the time is opportune for leadership and responsibility in Ontario education. I am sure that you are as alarmed and concerned as I am about the new jargon that has been crossing the border and appearing in articles lately - 'accountability', 'performance contracting', 'educational engineering' and 'management support group' (O.S.S.T.F. Bulletin, December, 1970). Implications for mathematics can be read in an article about 'performance contracting' as it is being tried

in Gary (American School Board Journal, January, 1971) - the disconcerting factor is that school boards in 16 states are trying this approach at the present time. The only consoling feature about these trends is the knowledge that "accountability in the whole process can be encouraged only if accountability grows within individuals - that is, me and, maybe, you" - comforting, because our association can be a key instrument for the development of individuals to a state of interdependence with others, without feeling any loss of personal independence.

However, the thirst for innovation in education seems to be working in opposition to the cumulative and sequential nature of our subject discipline. President Ralph Staal stated his concerns succinctly in his report to the Ontario Mathematics Commission (September 19, 1969):

> ... Concern for the individual seems, quite unnecessarily, to be resulting in a rising tide of anti-intellectualism in educational circles: polemics against the fixed linear arrangement of furniture in the classroom appear to have developed into polemics against "linear (i.e. organized) learning" and mystically tinged panegyrics of "random learning" (these are the latest "in" concepts). All structuring in education is suspect - assumed dangerous unless proven safe. It is ironic that, just as opportunities in

mathematics education are rising to new heights, the rug may be pulled out from under it all. While we are progressing toward our goals, we had better keep a wary eye on the rug underneath!

Specific Recommendations

1) For the reasons outlined above, I feel that this is no time to sever relations with other educational groups mathematics education can best be served by the realization of their 'interdependence'. Hence I recommend that we continue our negotiations with the O.E.A. and the O.M.C., working towards a restructuring of organizations if and when necessary. It should be recognized that unlimited funds are not required to maintain a 'voice' with which to express our concerns to the public as well as other teachers; nor are they necessary to produce materials at the local level that will be of direct assistance to teachers.

2) that the O.A.T.M. Executive Council use a calendar of activities:

April-June: assignment of new councillors to committees (both provincial and regional).
planning of year's activity including conferences.

Sept.-Dec.: regional conferences (supplying input and increasing feedback.

Jan.-Mar. : committee work based on feedback and the preparation of any materials to be shared.

provincially at an Annual Conference.

3) that each councillor strive to:

Know our organization.

Be proud of our organization.

Be active in our organization.

Hopefully all members of the O.A.T.M. will join us in the challenge to find more and better ways of serving the needs of the students and teachers of mathematics in Ontario.

Respectfully submitted,

Don A. Fox.

O.A.T.M.
TEACHER EDUCATION
COURSE SURVEY

Foreword

This is the age of "accountability" and "performance contracing". However, in Ontario the direction of expended energies is different than in some other countries. Our resources are directed more toward the development of people than of "teacher-proof" materials.

Without promising "results or money refunded", the 1970-71 Executive Council of the O.A.T.M. have strived very hard to be accountable by showing leadership and by providing

services to its members. You are probably already aware of our many regional conferences held during this past year and attended by a total of more than 1300 teachers of mathematics. Less publicized, than our regional activities, have been our efforts at reorganizing Council into a committee structure to study Curriculum, both content and materials; Communications; Conferences; and Teaching Methods and Summer Courses.

On behalf of present and future members of the O.A.T.M., I extend sincere thanks and congratulations to the members of this committee for the fine leadership they have provided in determining the in-service "needs and desires" of Ontario teachers. Hopefully their efforts will be rewarded by the development of a high degree of correlation between the courses that have been requested and the courses that will be offered. Under these conditions in-service education will play an important role in the development of people, who are essential to successful mathematics programmes.

D. A. Fox

President

O.A.T.M.

Purpose

The materials of this brochure were prepared by a subcommittee of O.A.T.M. which is particularly interested in

teacher education. Norman Viire was responsible for the preparation of the questionnaire, Jack LeSage for its distribution. Keith Hill looked after collecting and analyzing the data of the questionnaire and Arn Harris collected the information about current in-service courses.

The executive and Council of O.A.T.M. commissioned this project because of a growing fear that teachers are not being given sufficient opportunities to do in-service training in mathematics, and are not even fully aware of the opportunities which are available.

The purpose of the questionnaire was to analyze needs of teachers for summer and other courses, to ascertain their present intentions and their feelings about the way courses should be handled. This information will be made available to all institutions which should have a stake in in-service education.

The other part of the brochure gives information about courses to be offered during 1971. As you will note, there is very little offered in the curriculum field for those teaching new courses in the schools. The only institutions not canvassed were the County School Boards.

The sub-committee on Teacher Education hopes this brochure will help the teachers to choose and the training institutions to plan more effective in-service mathematics

education programmes in the future.

Arn Harris

Keith Hill

Jack LeSage

Norman Viire.

ONTARIO ASSOCIATION OF TEACHERS OF MATHEMATICS

RESULTS OF QUESTIONNAIRE RE SUMMER COURSES

1. Teaching experience:

0 to 2 yrs.	18.61%
3 to 5 yrs.	25.24%
6 to 10 yrs.	27.36%
More than 10 yrs.	28.77%
No answer	.02%

2. Teaching categories:

K to grade 6	42.21%
Grade 7 - 10	33.07%
Grade 11 - 13	18.27%
Post secondary	.09%
No answer	6.36%

3. Teachers definitely planning to attend a summer course this year:

Yes	50.72%
No	47.05%
No answer	2.23%

4. Purpose of course you intend to take

Upgrading	43.66%
Updating	4.44%
No answer	2.62%
Total	50.72%

5. Location of course you intend to take:

University	34.28%
Dept. of Education	8.55%
College of Education	4.40%
Teachers' College	.43%
Community College	.14%
Other	.91%
No answer	2.01%
Total	50.72%

6. Teachers unable to find a suitable course this year:

Yes	4.69%
No	38.05%
No answer	4.31%

7. Teachers planning to attend summer course in future:

	Upgrading	Updating
Yes	62.13%	43.61%
No	12.33%	15.76%
No answer	25.54%	40.63%

8. Desired length of course:

	Upgrading	Updating
2 weeks	2.12%	28.72%
4 weeks	19.87%	29.11%
6 weeks	52.17%	10.25%
8 weeks	4.54%	.82%
No answer	21.30%	31.10%

9. Desired sponsors of course:

	Upgrading	Updating
Universities	62.81%	17.45%
Dept. of Educ.	30.02%	38.87%
Colleges of Educ.	19.43%	22.77%
Local Boards	10.73%	24.95%
Teachers'College	10.15%	16.92%
O.A.T.M.	6.72%	16.29%
Federation	5.85%	12.57%
Community Colleges	8.55%	7.78%
Others	2.22%	4.11%

10. Desired starting date of courses:

	Upgrading	Updating
July 1st	70.45%	47.72%
July 15th	11.31%	12.81%
August 1st	1.45%	3.52%
August 15th	.72%	3.96%
No answer	16.07%	31.99%

11. Reimbursement by board for courses:

	Upgrading	Updating
Yes	46.66%	62.37%
No	37.37%	24.27%
No answer	15.97%	13.36%

12. Interested in summer correspondence courses:

Yes	34.91%
No	58.70%
No answer	6.39%

13. Possible topics for summer updating courses:

Activity approach to mathematics	44.6%
Programme for lower achiever	41.4%
Enrichment topics	39.7%
Integration of math with other subjects	32.7%
Recreations in mathematics	31.7%
Evaluation of students	23.8%
Evaluation of achievement	22.9%
Teaching application of mathematics	21.8%
Audio Visual Aids	21.2%
Developing Curriculum	20.9%
Core topics from the curriculum	16.2%
Evaluation of programme	14.6%
Computer - assisted mathematics	11.4%
Four year mathematics programme	11.2%

First month at school	9.9%
Probability and statistics	6.9%
Transformation geometry	6.6%
History of mathematics	6.6%

14. Possible attendance at vacation summer update course:

Yes	48.64%
No	38.58%
No answer	12.78%

15. Attendance if preferences of 13 and 14 met:

Definitely	7.35%
Probably	44.53%
Uncertain	27.85%
Unlikely	11.60%
No answer	8.67%

Total no. of replies used in sample was 2,068.

NOTES ON MEETINGS

Some Highlights from 1970 O.A.T.M. Fall Conferences, Sept. 26

Joan Routledge
St. Lawrence College of Applied Arts and Technology, Kingston

About two hundred teachers (kindergarten to University) attended this conference.

Sessions of interest to elementary teachers were on mathematical games, integration of geography and mathematics, graphing probability and statistics, teaching mathematics to low achievers, and the use of mathematics text books. Professor Whitley (Queen's) explained how the adoption of the metric system of measurement could be thought of as a return to the old decimal system of measurement used in the British Isles and northern Europe before the Roman conquest. Professor Ripley (McArthur) illustrated with materials and slides of field trips, how mathematics can be made to "come alive" to young children through practical applications in science.

At the secondary level, there were sessions on the teaching of mathematics to disinterested students, the role of McArthur College in the future of Mathematics teachers, oper-

ating a Mathematics Club, the use of calculators, and the topic of transformations. The computer staff of St. Lawrence College conducted all day seminars in computer programs in the secondary school.

Oct. 3, North Bay

Two hundred and twenty-five teachers attended a conference at which the Northern Ontario Mathematics Association (N.O.M.A) was established with D. Henderson (Sudbury) as President, A. Czempinski (Iroquois Falls) Vice-President, R. Dionne (Sudbury) Secretary, and A. Falconi (North Bay) Treasurer.

Oct. 3, Strathroy

About eighty elementary teachers attended this conference.

Oct. 24, University of Western Ontario, London

About three hundred secondary teachers attended.

Oct. 31, University of Ottawa

About two hundred and fifty teachers (Kind. to University) attended. Sessions for elementary teachers included ones on Geometry Gr. 1-3, Aids and Approaches Gr. 3-4, an activity oriented mathematics programme at the primary level, mathematics continuum (1-6), and a workshop approach to problem solving (5-8). Dr. Zoltan P. Dienes spoke of his work in the Sherbrooke Pilot Schools.

At the secondary level, there were sessions on remedial mathematics, Grade 13 single credit mathematics courses, "micro-teaching" and its applications (in English and French) a mathematics programme for occupational students, and full day workshops on computer science at Carleton University.

Mr. Arnold Harris, Althouse College of Education and representatives from Ottawa and Carleton Universities invited teachers to join in a discussion of post graduate courses and degree programmes for teachers of mathematics.

Nov. 14, St. Catherine's Teachers College in conjunction with O.E.A.

A very interesting programme of mathematical games and Geometry activities was presented. Due mainly to a lack of publicity on the part of O.E.A., the attendance was disappointingly small.

The Metro Mathematics Teachers' Association has held two meetings in the 1970-71 school year. Hosted by the Department of Mathematics, Board of Education for the Borough of North York, the October meeting at A. Y. Jackson Secondary School was a smashing success. Ove five hundred teachers attended. The second meeting was held on a snowy day in January at Northern Heights Secondary School. The main session speaker was Dr. Coxeter, University of Toronto, who chose the topic "Cyclic Sequences and Frieze Patterns". Dr. Coxeter was

presented with an illuminated scroll in recognition of his dedication to mathematics education.

Joan Routledge.

ANNOUNCEMENT

HIGH SCHOOL MATHEMATICS CONTEST 1971

Top 35 Schools in Ontario

Rank	School	City or Town
1.	*Grantham High School	St. Catharines
2.	Cedarbrae Collegiate Institute	Scarborough
3.	Ridgemont High School	Ottawa
4.	John F. Ross Collegiate & Vocational Institute	Guelph
5.	University of Toronto Schools	Toronto
6.	Burlington Central High School	Burlington
7.	Grand River Collegiate	Kitchener
8.	St. Clair Secondary School	Sarnia
9.	Newtonbrook Secondary School	Willowdale
10.	Beamsville D. S. S.	Beamsville
11.	Westmount Secondary School	Hamilton
12.	Runnymede Collegiate Institute	Toronto
13.	Bramalea Secondary School	Bramalea
14.	Upper Canada College	Toronto
15.	Merivale High School	Ottawa
16.	Forest Hill Collegiate	Toronto
17.	York Mills Collegiate Institute	Willowdale
18.	Riverdale Collegiate Institute	Toronto

* Placed second in Region VI, which comprises high schools in Ontario, Illinois, Michigan and Wisconsin.

Rank	School	City or Town
19.	Kingsville District High School	Kingsville
20.	Westminster Secondary School	London
21.	Neil McNeil High School	Toronto
22.	Lisgar Collegiate Institute	Ottawa
23.	Aldershot High School	Burlington
24.	St. Jerome's High School	Kitchener
25.	Michael Power High School	Islington
26.	Burnhamthorpe Collegiate	Islington
27.	MacKenzie High School	Deep River
28.	Sir John A. Macdonald High School	Ottawa
29.	Sarnia Central Collegiate Institute	Sarnia
30.	Parkdale Collegiate Institute	Toronto
31.	Thornlea Secondary School	Toronto
32.	West Ferris Secondary School	North Bay
33.	Forest Heights Collegiate	Kitchener
34.	North Park Collegiate Vocational School	Brantford
35.	Kingston Collegiate & Vocational Institute	Kingston

Outstanding Students for 1971

Rank	Name	Age	Grade	School
1.	*Edward A. Severn	17	13	Cedarbrae Collegiate Institute, Scarborough
2.	Michael J. Boyle	17	13	Westminster Secondary School, London
3.	William J. Breedyk	18	13	St. Clair Secondary School, Sarnia
4.	Peter G. De Buda	17	13	University of Toronto Schools, Toronto
5.	Bruce A. Desserud	18	13	Grantham High School, St. Catharines
6.	Joanne K. Vandermaarel	17	13	Grantham High School, St. Catharines
7.	Peter F. Schneider	16	12	Grand River Collegiate, Kitchener
8/9.	Diane E. Dressel	18	13	Grantham High School, St. Catharines
8/9.	Roland Koffeman	17	13	Westmount Secondary School, Hamilton
10.	Mark S. Brader	15	12	John F. Ross Coll. & Voc. Inst., Guelph
11.	Patrick R. Beirne	17	13	Neil McNeil High School, Toronto
12.	Robin Jackson	17	12	Colonel By Secondary, Ottawa

* Placed first in Region IV.

Rank	Name	Age	Grade	School
13/14.	Owen C. Leibman	16	12	Newtonbrook Secondary School, Willowdale
13/14.	James G. Morris	18	13	Kingsville District High School, Kingsville
15/16.	John R. MacPhail	17	13	Ridgemont High School, Ottawa
15/16.	Joe Y. Halpern	17	13	Forest Hill Collegiate, Toronto
17.	Jerry Zolnierlzyk	17	13	Grantham High School, St. Catharines
18.	Allan D. Shusterman	18	13	Sir Sandford Fleming Sec. School, North York
19.	Edwin Seputis	18	13	Scarlett Heights C. I., Weston
20.	Stephen C. Locke	18	13	Bramalea Secondary School, Bramalea
21.	John M. Higgins	18	13	Burlington Central High School, Burlington
22.	John R. Lawson	16	12	Burlington Central High School, Burlington
23/24.	Bill J. Ralph	17	13	West Ferris Secondary School, North Bay
23/24.	Geoffrey K. Arthur	18	13	Ridgemont High School, Ottawa
25.	Ross S. Male	17	13	Merivale High School, Ottawa
26.	Bob A. Klotz	17	12	York Mills Collegiate Inst., Don Mills
27.	Philip C. McOuiston	17	13	Victoria Park Secondary School, Don Mills

Rank	Name	Age	Grade	School
28.	Daniel Stripinis	17	12	Runnymede C. I., Toronto
29/30.	Bill C. Lalonde	16	11	Hillcrest High School, Thunder Bay
29/30.	David F. Kettle	18	13	Burnhamthorpe Collegiate, Islington
31.	Kevin F. Gibson	18	13	Loyalist Coll. & Voc. Inst., Kingston
32.	John J. Brooks	18	13	Michael Power High School, Islington
33.	Philip A. Twardawa	19	13	Beamsville D. S. S., Beamsville
34.	Gita M. Cohen	15	12	Beth Jacob School for Girls, Toronto
35.	Suzanne M. Seager	16	13	Sir John A. Macdonald H. S., Ottawa
36.	Larry D. Hammick	17	12	Georges Vanier Secondary School, Willowdale

The 1972 Senior High School Mathematics Contest will be held on Tuesday March 14, 1972. For more information regarding these contests please write to:

R. G. Boeckner,
Crown Life Insurance Co.,
Toronto.