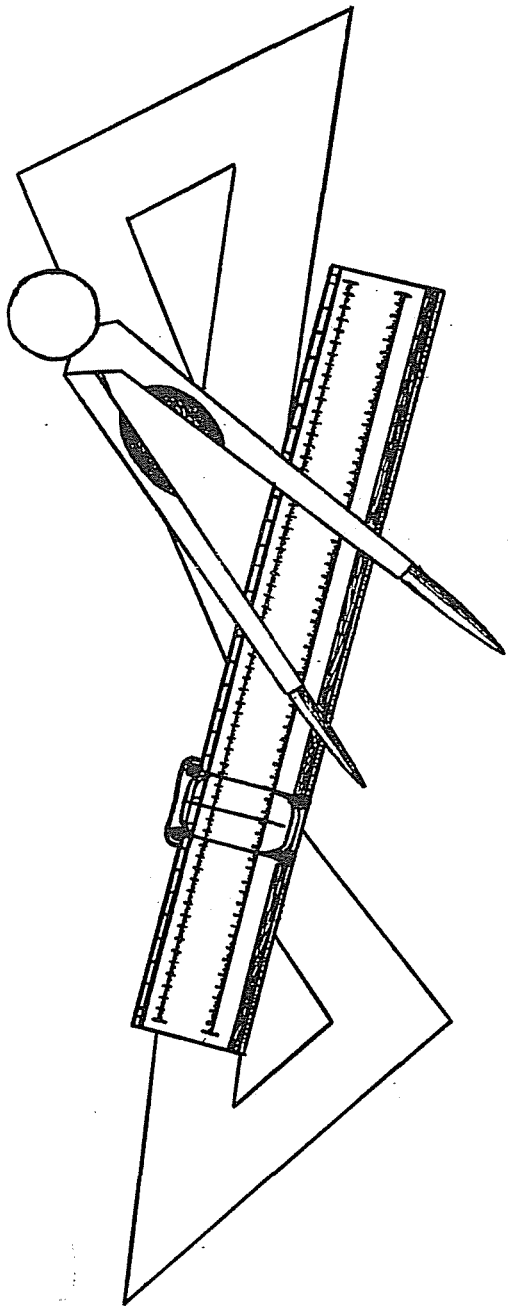
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# ONTARIO MATHEMATICS GAZETTE

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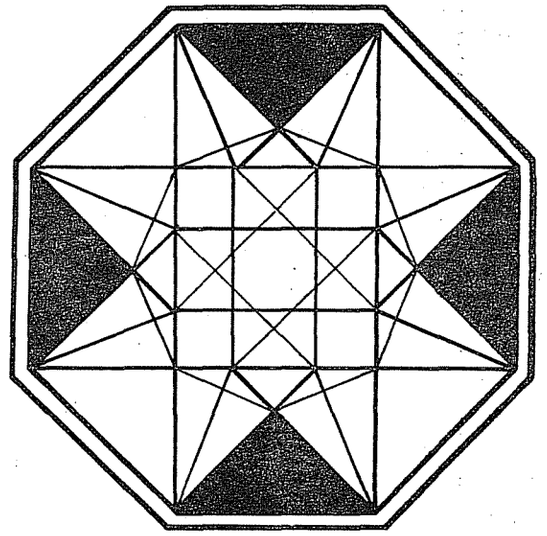
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# ONTARIO MATHEMATICS GAZETTE



- see Free Poster Offer!

Please note that the previous copy of the Gazette should be renumbered Volume 12, Number 1.

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# EDITORIAL

It is generally conceded that mathematics is rooted in reality - although 'rooted' and 'reality' may have to be interpreted rather loosely if Cartesian formalists are to be convinced of this. For mathematics is created by sentient beings - us - aware of the physical world and constrained by the physical/chemical mechanisms in our brains. We may deny reality - and in so doing, discover non-Euclidean geometry or non-standard analysis - but surely our conscious act of turning away from intuitively acceptable concepts implies our recognition of them. Just because we drive faster than the speed limit, it doesn't follow that we didn't see the signs!

Why, then, do so many students believe that mathematics is a dry, unreal, formal, intellectual exercise, a tale full of syllogism and tautology, signifying nothing. Why do they believe that mathematical concepts are not open to criticims: if the theory is logically consistent, there is no basis for arguments? Do they really hold to Russell's dictum that 'the excuse for accepting an axiom is that many propositions which are nearly indubitable can be derived from it'?

No wonder students think mathematics is just something to be endured in order to stay in school, where it's warm and dry!

Perhaps an answer will be found in the 'radical heuristic' approach of Imre Lakatos. Professor Lakatos, who is Professor of Logic at that hot-bed of student dissent, the London School of Economics, advocates 'the logic of mathematical discovery'. Not a very radical idea, really: to understand a concept, simply reconstruct the process of its discovery by example, counterexample, conjecture, and proof. Its obvious defect is that it wouldn't fit too well into a school curriculum - it might take an unconscionably long time to arrive at an examinable topic and, indeed, one might arrive at a concept not on anybody's syllabus! But it would surely reveal the processes by which mathematics is made, and give the students the opportunity to make some of their own.

The editors would like to know of any such approaches currently used in the schools, and of their success or failure. In the immortal words of the Rourke Jargon Generator, let's get meaningful indepth dialogues[1] going here.

The Editors

"Beware of mathematicians and all those who make empty prophesies. The danger already exists that the mathematicians have made a covenant with the Devil to darken the Spirit and confine Man in the bowels of Hell".

St. Augustine

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[1] 252, see p. 21.

# What is the OAME doing?

D. W. Alexander

By the time this issue of the Gazette reaches you, the first year of the OAME will be half over. What has been accomplished?

1. The basic committee structure of the Provincial Council has been established and all committees are active. The Committees are:

Chapter Relations
External Affairs
Manpower and Nominating
Meetings
Membership and Publicity
Projects
Publications.

2. The new Chapters established last year have made good starts in providing local programs for their members. To further encourage such local programs, the Provincial Council has changed Bylaw 9 of the Constitution so that a Chapter will receive the full membership fee (up to $5) for any OAME member who designates the chapter as his prime chapter. At the same time, the requirement of payment for non OAME members in a chapter has been deleted. This action should provide for strong local initiative in the OAME.

3. The organization has hosted an outstanding NCTM meeting in Toronto.

4. The Membership and Publicity Committee has stimulated interest in the OAME to such an extent that memberships now stand at 460. This is double the number attained by OATM by November last year.

5. The Publications Committee has initiated the Abacus, a newsletter for members of OAME. The first issue was available at the Toronto NCTM Meeting and has already proved itself a valuable companion for the Gazette.

6. The Meetings Committee has drafted Guidelines for Regional Meetings and for Annual Meetings. These will be available before the end of the year.

7. The Manpower Committee is organizing a leadership file. Watch the Abacus and the next issue of the Gazette for further information.

8. The Projects Committee has discussed a number of projects and established priorities.

(a) The French Language Instricution Committee of the O.M.C. has been appointed as a committee of OAME and is actively engaged in answering the needs of the teaching of mathematics in French.

(b) A committee is investigating the Qualifications and Preparation of Teachers of Mathematics K-8. Their report, expected in the spring, should provide a basis for action by the Universities, Teachers' Colleges, and Ministry.

(c) A committee is establishing the OAME's response to the Mathematics Study of the Science Council of Canada.

9. The Provincial Council, through the External Affairs Committee, is submitting a series of recommendations concerned with the study of mathematics to the Intermediate Cyclical Review Committee.

It is hoped that by the end of the year, more Chapters will be organized, more projects will be underway, and the membership will be over 1,000!

Do you have an idea for a project?

Have you obtained a new member?

Have you thought about forming a Chapter?

What are you doing to promote the objectives of OAME?

# JUST THE FACTS

Joan Routledge

What are the basic skills in mathematics? What do people mean when they tell us to forget the *new math* and get back to the basic skills? What do secondary school teachers expect from elementary school teachers when they ask that the youngsters be drilled in the *basics*.

Too often basic skills are thought to be merely the ability to add, subtract, multiply, and divide whole numbers and fractions. If a child has instant recall of numbers and facts and can compute, he is said to have the *basics*, but does competence in computation automatically guarantee excellence? Most experienced teachers can name many children who are excellent in mechanical arithmetic but who cannot use these tools to solve applications questions. Computational skills have been taught and learned too often as ends in themselves. An unfortunate consequence of this undue emphasis on computational skills is a neglect of the basic skills which must precede them. There is much more to basic skills than the regurgitation of number facts and the slick performance of algorithims. Teachers of Kindergarten and Grade 1 classes may feel pressured to *get on with teaching the number facts*, but unless a child can sort, classify, order, recognize the cardinal numbers of a set, and use the numeration system, he will be unable to understand the meaning of the operations and the symbols which represent them. In fact, too early an insistance on memorizing numbers facts and computational procedures can hinder the development of competence in the basic operations of adding, subtracting, multiplying, and dividing.

True, everyday living requires a facility for the four basic operations. Actual physical situations, however, often first require an understanding of basic geometric ideas, skill in measurement techniques, and an ability to determine which operation to use. Then the proper computational procedures can be performed.

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Most of the drill on concepts at the Kindergarten to Grade 2 level, and on number facts in later grades should take the form of games or activities. Children, in later grades, who have learned to perform addition, subtraction, division, and multiplication algorithms will need practice sessions in the form of pencil and paper exercises. It is more effective to give them a few questions every day than to give one or two pages of questions once a week. Perhaps the best way to practice computational skills is to use them in application type questions; preferably those that arise naturally and are meaningful and relevant to the children.

Prior to the extensive use of business machines in banks, offices, etc. (stores now are equipped with cash registers that not only total purchases but calculate change) it was necessary to place almost equal emphasis on speed and accuracy. Now-a-days accuracy is still important, but it would seem sensible to expect only a reasonable amount of speed in pencil and paper type calculations. On the other hand, mental agility with numbers has become even more important than it was once. For self-preservation alone, it is necessary to be able to approximate the total a machine may produce, and the amount of change one can expect to receive. Exercises in rounding, estimating, and in the use of computational tricks or shortcuts seem far more relevant and useful than do extensive sessions of speed drills.

Shoppers' manual pocket calculators are very inexpensive and will give totals to $20. More sophisticated electric and electronic calculators can be purchased for less than fifty dollars. If a teacher can obtain any of these, try playing a game of Beat the Machines: one child does the calculations on the machine while the others use pencil and paper. Allow the children to decide how many points are scored when a player beats the machine in obtaining the correct answer. Most children are surprised and delighted when they discover that they can beat the machine quite often.

It is very helpful, of course, to have the computational skills well-developed when they are needed, so that time need not be spent rediscovering them before being able to process an answer. For example, writing 49 fifteen times and adding to find the cost of fifteen 49¢ pens may be an acceptable method at the third or fourth grade level. Older children appreciate a shorter way, however. Once this algorithm has been mastered, it is very easy for them to see how they can arrive at the answer without using pencil and paper by multiplying 15 by 50 and subtracting 15.

There appears to be an optimum time when children are eager and willing to practice a skill until it becomes almost automatic. If children are to reach this stage, they must work with mathematical ideas first through concrete materials, then through pictures, graphs, and diagrams. When they are accustomed to recording with mathematical symbols the ideas that they discover through the concrete and semi-concrete or semi-abstract materials, they build a visualization of the basic concepts. This visualization will bridge the gap between work with the concrete and visual representation and work with abstract symbols.

Repetitive practice or drill is necessary to building and maintain any skill. This drill need not be always pencil-and-paper exercises. Fortunately, for both teachers and youngsters, there are hundreds of suggestions for drill games available (many commercially). These games range from the elaborate and expensive to simple card, dice, and domino games that can be made quickly and easily by the teacher, or by the youngsters themselves.

In choosing a game, the teacher should know what she wants the game to do for her students. Not all games accomplish what it is claimed they will accomplish. It makes sense, therefore, to play the games several times before wasting time, effort, and money on them. For example, a teacher is not likely to choose a game for drilling the extensions of basic addition facts, if she finds that the game allows time for the children to count on.

Practice exercises using reasonable numbers only should be given students. Seldom is it necessary to compute with unwieldy numbers of six, seven, eight, or more digits. This type of computation should be done on a machine. Well-developed computational skills with reasonable numbers will usually guarantee that a youngster will be able to apply these skills to greater numbers, if and when it becomes necessary.

It is very difficult to predict in our age of rapid change just what mathematical skills we will need in the future. It becomes increasingly apparent, however, that we are on the threshold of adopting the metric system. Even a cursory glance at Junior level mathematics textbooks from the metric countries of Continental Europe impresses one with the almost total lack of exercises on operations on fractions in common notation. Fractions in decimal notation and the operations on them are introduced at a much earlier stage than that to which we are accustomed. It would appear that we will be

freed from most of the work that we do with common fractions, especially the operations on them. Operations on decimal fractions are so much more easily understood and mastered (probably because the notation is an extension of the base ten numeration system); whereas common fraction notation presents new ideas in notation, bearing little relation to the place value system.

Children will find it quite simple to extend the multiplication and division algorithms for whole numbers to accommodate decimal fractions. If they are already skilled in the performance of these algorithms and understand the meaning of the multiplication and division of fractions they should not need very much additional practice when decimal fractions are involved.

The form a/b may lose its significance as a common fraction, but it probably will be used as a means of representing division or a two-term ratio. Work with ratio and percent still will be emphasized, and skill in multiplication and division of whole numbers and decimal fractions will be required.

In terms of skills, then, the pendulum in mathematics education is seen to swing from one extreme to the other. At the present moment we appear to be swinging away from a stress on teaching for understanding only and a deemphasis on skill building. Influenced by the peer performance of too many youngsters on standardized mathematics tests, we are in danger of returning to an over-emphasis on computational skills. Surely, the balance can be found somewhere in the middle. Committees of teachers (K-13) in consultation with mathematics education specialists and applied mathematicians are in the best position to decide what skills are basic, and when and how they should be taught to meet the needs of the students in their particular school system.

Joan Routledge is Master Teacher - Mathematics, with the York County Board of Education.

# METRICATION:
## *For the Schools and for the Nation.*

W. G. Anglin

1. Introduction

The Metric System, in one form or another, has been in existence since its original introduction in France in the late eighteenth century as the French Revolution drew to a close. At that time the French Academy of Sciences convened to develop a system of measurement which could replace the tremendous proliferation of local units which had grown up as trade within and among French communities increased. It was a part of the French Revolution that the existing order of the day must go, to be replaced by a new order. Hence the climate was ripe for the introduction of a new order into measurement.

The system introduced by the Academy was so consistent, simple and logical that its popularity has grown until today approximately 90% of the world's population lives in countries which are officially using the Metric System or are committed to a conversion to the Metric System and 75% of the world's trade is conducted using Metric specifications.

2. The Meaning of a Standard Unit of Measurement

Most primary teachers, in introducing measurement ideas to their classes, begin by establishing with the class some non-standard unit, such as the length of a chalk brush, piece of chalk or a pencil, for the measurement of length. Students then measure lengths of things such as the desk, the window, themselves, etc. in terms of the unit chosen. This standard would be perfectly adequate if the students never had to communicate the measurements made to anyone outside of their own classmates, who are acquainted with the standard chosen. Difficulties arise as soon as it is necessary to communicate with someone who is not familiar with your standard. If one class has chosen the length of a chalk brush as standard and another has chosen the length of a pencil as their standard then conversion factors are necessary to compare measurements in the two systems.

This difficulty of working with measurements based on two different standards is the problem facing the world at the moment. A large segment of the population of the world measures lengths using the metre as a standard, while a much smaller portion of the world's population uses the yard as its

standard of length. Any communication of measurements between peoples using the two different systems requires conversion factors so that each can interpret the other's measures.

World-wide communication has become so essential at this point in time that it is no longer possible to continue to functions economically with two systems of measurement based on different standards. For this reason one system must go. Unfortunately for most North Americans, it is the system with which we are most familiar that is going. However, all is not lost. The system to which we are going is simpler, more logical, more consistent and more universal than the one we are leaving. Once we become familiar with the new, we will wonder how we ever functioned effectively with the old.

3. Origins of Imperial and Metric Standards

As most students find out when they are first introduced to standard units of measure, the yard was legislated by King Henry I to be the length of his arm from the tip of his nose to the tip of his outstretched finger. This is a very arbitrary way to select a standard. The Metric System standard of a metre, however, is no less arbitrary and children should not be led to believe that there is something more sacred about a metre than there is about a yard. A metre is equally arbitrary. It was originally defined as 1/10 000 000 of the distance from the North Pole to the equator measured along a meridian of longitude near Paris, (though now its definition is in terms of the wavelength of light emitted by the Krypton 86 atom). Both systems were defined by man, neither is God- iven. The advantages of the Metric System lie in the simplicity of the relationships among the units within the System and the inter-relationships among the measures of various quantities.

4. Proliferations of Imperial Units

The Imperial System, as it has developed, has had no logical base and consistent origins. Each specialized field of endeavour has developed its own units for its own use. When we recognize that units such as light-years, furlongs, fathoms, feet and angstroms are all measures of length we see that you have to learn a specialized language of measurement if your field of endeavour is astronomy, horse racing, navigation, carpentry or physics.

We, as teachers, have met, have had to learn and/or have forced our students to learn all sorts of conversion factors between units in the Imperial System. Consider the following relationships, which may or may not be familiar to the reader:

| | | | | | |
|---|---|---|---|---|---|
| 2 pints | = 1 quart | | 7 firkins | = 1 hogshead | |
| 3 feet | = 1 yard | | 8 pints | = 1 gallon | |
| 4 rods | = 1 chain | | 9 square feet | = 1 square yard | |
| 5 American quarts | = 1 Canadian gallon | | 10 chains | = 1 furlong | |
| 6 feet | = 1 fathom | | 11 fathoms | = 1 chain | |
| | | | 12 inches | = 1 foot | |

There are many, many more relationships between Imperial units which we have learned in our lifetimes and most conversions using these relationships involve difficult arithmetic calculations.

Restricting considerations of units to those which measure length we can readily see the simplicity of conversions within the Metric System by comparing the two lists below:

| | | | | | |
|---|---|---|---|---|---|
| 12 inches | = 1 foot | | 10 millimetres | = 1 centimetre | |
| 3 feet | = 1 yard | | 10 centimetres | = 1 decimetre | |
| 5½ yards | = 1 rod | | 10 decimetres | = 1 metre | |
| 4 rods | = 1 chain | | 10 metres | = 1 decametre | |
| 10 chains | = 1 furlong | | 10 decametres | = 1 hectometre | |
| 8 furlongs | = 1 mile | | 10 hectometres | = 1 kilometre | |

At first glance, the arithmetic of the Metric System appears simpler but the language of it looks more difficult. However, on closer inspection, the language turns out to be simple, too, because it does not change significantly when you switch to the measurement of volume or mass rather than length. The six prefixes from milli- to kilo- remain unchanged. Milli- always means one one thousandth of the base unit and kilo- always means one thousand of the base unit. Thus, a millilitre is one one thousandth of a litre and a kilogram is a thousand grams.

## 5. Canada's Position

Since January, 1970, with the Federal Government's presentation of the "White Paper on Metric Conversion In Canada", Canada has been officially committed to a conversion of its measurement system to one based on metric standards. At the provincial level in Ontario the Honourable John White, Minister of Industry and Tourism in May, 1972 made a statement to the Legislature "On The Metric System of Measurement" which committed the Province of Ontario to support the Government of Canada in its move toward metric conversion. Thus, the two major levels of government for the people of Ontario have made this committment. In spite of the fact that no specific dates have been set at either level to begin or to complete

conversion industry, government and education are all working vigorously at awareness and training of their employees for the inevitable day when our Imperial units are officially phased out. One needs only to look on the shelves of any local supermarket to see the effects of the decision to convert. Most commodities on display are packaged, still in Imperial sizes, but with metric equivalents printed on the labels. Some, but not many, have actually switched their package size to a convenient metric module and have not bothered with the corresponding Imperial equivalent on the label. As time goes on more and more package sizes will change to convenient, metric sizes such as 50, 100, 200, 500 grams or millilitres or a kilogram or a litre size.

6. Effect on Education

Metric conversion in Canada will have a tremendous impact on the teaching of measurement in schools, particularly at the Grades 4-6 level, where students are introduced to ideas of length, area, volume, mass, temperature and time. Each of these six basic measurement topics will be effected by the move toward metric units. Teachers need to become aware of these effects and be prepared to introduce ideas of measurement standards clearly, unambiguously and rationally as students see the need for them.

W.G. Anglin was a mathematics teacher in Gananoque Secondary School and Queen Elizabeth C.V.I., Kingston, before joining the Department of Educations as a mathematics inspector in 1966. At the present time he is a mathematics consultant for the Ministry of Education. A previous article of his 'Making mathematics interesting for non-college bound students' appeared in the March 1973 Gazette.

# Relevance in Teaching Mathematics

Gordon Edwards
- excerpted from a letter of March 15, 1973 -

"It seems pretty clear to me that we are seeing a real change of consciousness take place in our youth - and this does not necessarily imply 'better', but just 'different'. McLuhan is right in most important respects. Young people today are not as linear-minded as they used to be. They are much more aware of what is going on in 'the global village', and because of TV, radio, movies, etc., they are used to soaking up all kinds of useful and exciting information (and misinformation) in a non-linear, spontaneous, simultaneous, non-programmed way. They are very much aware of the shortcomings of their elders (though not so aware of their own shortcomings) and are more cynical and distrustful of authority. One does not very often encounter any more the enthusiastic, trusting, wide-eyed innocence of students who are willing to believe that anything they are taught is for their own best interests and has its purpose, even though the purpose may be inscrutable. Young people today yearn for a sense of relevance, of responsibility, of involvement. They are no longer willing to take these things on faith and it is unreasonable to expect them to, given their background.

If the future is going to be an age of greater human self-realization, it must not be just a reversion to a more primitive type of mentality - using primitive in its historical sense - which is a non-linear way of thinking and apprehending the universe - but rather a fusion of the linear and non-linear ways of thinking. Our educational system, our society, our philosophy and our science have all gone overboard on 'linearity', 'objectivity', 'manipulability', 'domination', 'logicality', etc. For these things to be redeemed in the eyes of our young people they do not have to be abolished, but just 'put back in their proper place'. The new generation will recover the respect for mathematics, science, logic, and linear thinking as soon as these things are stripped of their arrogance and are presented as useful tools in the service of man for the satisfaction of man's aspirations and needs. We need whole people, who are able to think non-linearly as well as linearly, without any sense of conflict or contradiction between the two.

In this, mathematics clearly has a vitally important role to play, since it is the language of linear thinking, and yet in its creative aspects, it is very non-linear (as is the case with creativity in any field whatsoever). One of the most important things, it seems to me, is to reconsider the whole way in which mathematics is taught. I think it is taught far too linearly, far too logically, so that in fact the student comes away with a very inaccurate picture of what mathematics is all about. (Of course I'm generalizing unfairly!) It's like the way in which many mathematics paper are written - logically impeccable, but completely obscuring the creative process by which the results were arrived at. One of the worst aspects of this emphasis on linear presentation is the feeling of despair that it engenders in the student who is lost or who wants to know just 'some' advanced mathematics. He is told that to learn what he does not know, he must go through a long-drawn-out process of linearly accumulating little bundles of mathematical information which are prepackaged in a certain highly specific order and which will inexorably lead the student to the information he desires. I personally don't think this is a good or a necessary way to go about it and in my own teaching have experimented with alternatives.

For one thing, I have tutored many dozens of high school students who were failing mathematics. Most of the people I tutored ended up with A's after relatively little tutoring. Instead of saying 'Let's start at the beginning' (which is a deeply frustrating thing to most of today's young people) I start by saying, 'What do you find the most difficult thing in the entire course?' and when he/she tells me what it is, I say 'Let's start there'. This policy gets right to the heart of the course very quickly, and if the student can be led to understand this 'super-difficult' material, he has a terrific psychological boost. Any other difficulties MUST seem trivial by comparison. Once the student masters a significant portion of the language, (mathematics), the rest falls into place relatively easily.

In my approach, too, I always try to emphasize 'relevance'. This over-used word is greatly misunderstood. To my mind, relevance is primarily a psychological concept: a piece of information is regarded as 'relevant' to the extent that it impinges upon and illuminates other bits of knowledge which the person has already digested and accepted as 'sound' knowledge. Thus relevance is very much a function of the personality of the background of the person. So

if a student is having trouble with factoring, I say 'Why do you want to learn how to factor?' and if he says 'I don't' then I say 'Well why do you suppose anyone wants to factor, ever? Why would anyone in his right mind ever want to factor anything?' And the usual answer is 'I don't know'. So that's where I start. I find that once they see a reason for factoring, they learn the mechanics of factoring much much faster. Similarly with trigonometry, I talk not only about triangles and navigation, but also about vibrations and electromagnetic waves, even though it isn't on the course and is therefore a 'waste of time'. If done in just the right amounts (neither too thoroughly nor too superficially) I find that it is in fact a great time saver. My general principle is: Shoot for the most relevant aspect of the subject first, use that as a starting point, and then build backwards and forwards from that point - to fill in the background ('missing steps') and outline the consequences. Incidentally, one important aspect of this non-linear approach to a linear subject is that it gives the student confidence because he feels that it is possible to 'jump in in the middle' and actually learn something, which is a tremendous relief to someone who has always been led to believe that you have to remember literally everything, all the way down the line, and all the steps between them too, or else you're a goner.

I'm afraid I'm being too long-winded again - what I'm getting at of course is that I think there should be a complete reassessment of how mathematics is taught as well as what mathematics is taught. In this context too, I think there should be a great deal more experimentation in new methods of instruction - especially now that kids at lower levels are used to having more freedom and more responsibility in determining the thrust of their education. Let me once again lapse into 'stream-of-consciousness'. One of the most frustrating aspects of a mathematics class is that the students cannot really participate in the same way that they can in a philosophy class or an English class, because they don't know enough. The prof is usually being guided by a curriculum and knows what he wants to say and in what order. (Such is not always the case in humanities.) As a result the students are completely passive, and their response is limited to things such as 'I don't understand' or 'could you explain again' - which are basically admissions of inferiority and not exactly conducive to a sense of confidence and well-being. On the other hand, if a student asks a genuine question as to possible applications, connections, or new directions, more often than not it is treated

as a very interesting question, but unfortunately off the course, and therefore not to be pursued in any great detail.

Again, it is my opinion that this should be changed. I have found that if you forget the curriculum and adopt the attitude that anything is relevant as long as it is connected and interesting, then you really can pursue some of the students' questions quite thoroughly, leading to a fairly lengthy digression in which, however, the students often are exposed to some very significant mathematical ideas and in which THEY HAVE A FEELING OF HAVING SOME CONTROL. The latter is very important; instead of merely 'interrupting' the class with their questions, they feel that they are actually taking part in molding the direction in which the class is going. It was in Chicago that I first discovered the value of 'relevant' digressions - whenever the class started looking bored and sleepy, I figured it was time for a digression - and so I would promptly get off the topic into something quite different but connected. The interest of the class would immediately go way up; and it would stay up even when I returned to the curriculum at the point where I have left it. Again, I try to describe in general terms and quite frequently through the course, how the particular mathematical discipline being studied related to various other mathematical disciplines - so for example, people coming out of any linear algebra course taught by me know that linear transformations have something to do with providing linear approximations to functions of several variables, that linear manifolds are a special very simple case of more general manifolds, that 'null space' and 'range' fit into a much more general context, that the 'algebra of matrices' fits into a more general study of 'algebras', etc. Instead of confusing the students, I find that if it is well done, it tends to clarify and help the student to see things in a less distorted perspective.

I would like to see a breakdown of the 'teacher-student' relationship as an 'active-passive' relationship. In fact, I think that unless the universities really act in this direction they will find themselves more and more irrelevant and less and less respected. The students today must have a much bigger say in what they are learning. This does not mean that they should decide in a vacuum - it means that they should be engaged in a dialogue with their teachers, gradually getting a more and more complete and accurate picture of what it is they don't know, and steadily giving feedback as to which aspects they find most appealling and relevant. It is inadmissible in this day and age for a teacher

to impose his own interests and his own point of view totally upon the students, though he should of course have no hesitation in making these things clear to the class. In the last analysis, people are not 'taught' - they teach themselves. The only way to really learn mathematics is by doing it. A good teacher is one who provides a good environment for learning (i.e. self-learning) by pointing out obstacles, putting things in context, arousing interest and enthusiasm, but not by dictatorially manipulating his students and casting them in a pre-set mold. Since teaching is a human interchange, the curriculum must never be allowed to over-ride fundamental human or psychological problems - the teacher has to learn to be a good listener as well as a good talker. I have found that most of the stupid questions that my students ask, if pursued patiently and open-mindedly, often turn out to be quite intelligent questions after all, based upon a certain misunderstanding which may never have occurred to me even as a possibility, but which is just as 'reasonable' a pitfall as any other. In this respect, I have been particularly interested in noting how much of students' difficulties are psychological, metaphysical, semantic, or even linguistic rather than mathematical."

Gordon Edwards is the assistant director of mathematics study with the Science Council of Ottawa. He graduated from the University of Toronto in Applied Mathematics in 1961, obtained his Master's degree in Pure Mathematics from the University of Chicago in 1962, and then switched into English Literature while teaching undergraduate mathematics, earning a Master's degree from the University of Chicago in 1964. He taught Mathematics and some English at the University of Western Ontario from 1964 to 1968, and then studied for a Ph.D. in Mathematics at Queen's University, obtaining his degree in 1972.

WHY NOT FORM A

# Mathematics Study Group

TO PREPARE A BRIEF FOR THE SCIENCE COUNCIL?

There is a great deal of concern over the future of the Mathematical Sciences in Canada - not only by mathematicians, but also by users of mathematics in science, government, and industry. Unless mathematicians take the lead in assessing the current situation and proposing policies for the future, it is likely that all the important decisions affecting the future of mathematics in this country will be made by others on grounds of expediency. The MATHEMATICS STUDY which is currently underway offers a unique opportunity for mathematicians to formulate and articulate their views on this important topic.

## STUDY GROUPS

As a first step towards this goal, mathematicians throughout the country are being asked to set up small study groups which will meet four or five times during the fall. Each study group will

- identify some significant problem area which has a bearing on the future of the Mathematical Sciences in Canada;
- analyze the problem: how did it arise? what is being done about it? what existing policies (explicit or implicit) tend to perpetuate it? what changes in attitude will be necessary to overcome it?
- submit a brief to the Mathematics Study Director summarizing the deliberations of the study group and proposing a few practical mechanisms which might improve the situation.

Each study group is asked to appoint a secretary who will be responsible for submitting the final brief to the Study Director. The deadline for submission of briefs is January 15, 1974.

## DIFFERENT PERSPECTIVES

If a problem is viewed from several different perspectives, a more realistic and balanced overview will likely result. For this reason, it is hoped that each study group will go out of its way to solicit the views of others who have a professional interest in mathematics. For example, any study on undergraduate mathematical education should involve some contact with users of mathematics in other university departments as well as employers of mathematicians outside the university.

## CONSENSUS NOT NECESSARY

It is always satisfying to arrive at a consensus. In most cases, however, a consensus cannot be expected to emerge from the kind of study group envisaged here. It will be far more valuable at this stage if the various conflicting views are given full and complete exposure, so that the range of possible alternatives is clearly appreciated by everyone. A great deal of time can be wasted in trying to force a consensus which simply isn't there.

While virtually any aspect of mathematics in Canada could be the subject for a study group, it might be most appropriate for teachers to investigate the actual teaching of mathematics in the schools. Provocative questions to guide discussion might be:

1. Are students learning how to use mathematics effectively in their everyday life? (e.g. managing finances wisely, understanding graphs, recognizing extortionate interest rates, spotting fallacious numerical arguments, following simple mathematical explanations, etc.)

2. Are students acquiring an understanding of the role that mathematics plays in contemporary society? (e.g. uses and abuses of statistics, computers, mathematical modelling, simulation techniques, etc.)

3. To what extent is mathematics presented as an enriching aspect of human culture? (e.g. as a historically evolving discipline, as one of the great languages of man, as it relates to philosophy, art, and science, etc.)

4. Are students becoming "mathematically literate" in the sense that they recognize the utility of abstraction and are

not afraid of symbols? (e.g. are they able and/or willing to pick up a well-written math book and learn the contents themselves?)

5. What have been the effects (positive and negative) of the "New Math"? (e.g. are students more self-confident, are they less competent in basic skills, do they regard math as "abstraction for the sake of abstraction", etc.)

6. Is teacher training (including in-service training) adequate to the needs of mathematics teachers? (some comments on the relevance or irrelevance of various aspects of university math education would be welcome here).

7. Is there sufficient communication and cooperation between primary school, secondary school, and university teachers? (specific suggestions for improving the situation?)

8. What are the chief obstacles (administrative, pedagogical, financial, etc.) to improving the quality of mathematics education at the pre-university level?

9. Should mathematics always be taught as a skill to be mastered, or should certain topics be presented in a more descriptive way in order to acquaint students with important mathematical aspects of modern thought and modern society? (c.f. questions 2 and 3 above).

Why not consider some of these questions with your colleagues, and submit a brief to the council?

For further information contact:

The Mathematics Study
c/o Science Council of Canada

# ROURKE'S JARGON GENERATOR

At one time the only classroom aids a teacher needed were a piece of chalk, a brush, and a long stick, preferably pointed. However, in this modern and advanced age, when teachers have become educationalists and chalk is a visual aid, the challenges have become greater and more numerous, and consequently must be met quickly and with a minimum of thought. Otherwise we will never get around to coping with the next challenge.

In particular, the evaluation of educational programs and experiments is time-consuming, and since hardly anybody pays any attention to what we say anyway, an opinion generating device should be in every teacher's briefcase for quick consultation. *Robert Rourke*, in his talk on *'Some properties of Well-rounded Squares'* given at the N.C.T.M. meeting in Toronto, suggested the following jargon generator, which we reprint here with his permission:

Rule: *Think of any 3-digit number. Then select the corresponding words from the 3 columns. Example: 254 gives meaningful in depth expertise.*

| Column 1 | Column 2 | Column 3 |
|---|---|---|
| 0. *functional* | 0. *reciprocal* | 0. *commitments* |
| 1. *viable* | 1. *basic* | 1. *experiences* |
| 2. *meaningful* | 2. *monitored* | 2. *dialogues* |
| 3. *insightful* | 3. *participatory* | 3. *feed-back* |
| 4. *relevant* | 4. *grass roots* | 4. *expertise* |
| 5. *integrated* | 5. *indepth* | 5. *alternatives* |
| 6. *compatible* | 6. *computerized* | 6. *accountability* |
| 7. *meaningful* | 7. *socio-economic* | 7. *involvement* |
| 8. *optimal* | 8. *digital* | 8. *individualization* |
| 9. *balanced* | 9. *relevant* | 9. *input* |

Examples:

What is the outstanding feature of X's educational system?

*Answer:* 254 = meaningful indepth expertise

What is the greatest need in X's educational system?

*Answer:* 438 = relevant participartory individualization

(If comments cause argument or discussion, choose another number.)

In an actual consumer test, this editor - in just two weeks - was saved from nearly 30 minutes of thought by using the Rourke generator, and has had one report, prepared almost entirely by the generator, commended by his superiors. There is one danger however - don't use the generator too much, or you might believe what you are saying!

# THANK YOU, JEAN PIERCE!

The wives attending the NCTM convention in November at Toronto, could not help but be thrilled by the fashions shown after the luncheon on Saturday at the Granite Club. It was the kind of show that would inspire any woman who cared what she looked like (and what woman doesn't) to overhaul her wardrobe and update her image. We saw coats, suits, sportswear, daytime and evening dresses, and gorgeous furs. Have you ever seen a plaid mink coat?

Many of the fashions were original, by young Canadian designers, and some were exclusive to the Jean Pierce Shop, having been created by the resident designer, Everitt Staples. Some of the dresses and fabrics were imported from London, Paris, Italy and Switzerland. All of these clothes, though excitingly styled were extremely wearable, and many of them within the budget of even the most frugal shopper.

For all of you Toronto N.C.T.M. wives who didn't have the foresight to come to this lovely show, and for those from out of town who may be visiting our fair city some day, a reminder that these stunning fashions may be found at:

Jean Pierce
259 Eglinton Avenue, West
Toronto, Ontario

# Book Reviews

*For a list of books available for review, write the editors.*

*The Psychology of Learning Mathematics*, by R. R. Skemp, Penguin Books Ltd., 1971, ($2.15) pp. 319.

The first section of this book should be read and re-read by every teacher of mathematics from Kindergarten to post graduate. The fact that the author studied psychology after obtaining a degree in mathematics and teaching mathematics in a secondary school, is clearly indicated in the way he interrelates concepts of learning, mathematics and teaching. The book is an illustration of one of the author's own statements "concepts of a higher order than those which a person already has cannot be communicated to him by a definition, but, only by collecting together, for him to experience, suitable examples."

Some quotations:

"to know mathematics is one thing, and to be able to teach it - to communicate it to those at a lower conceptual level - is quite another";

"[The teacher] has to know which stages require only straightforward assimilation, and when accommodation is needed -- and he has to plan on a long-term basis the schemas which will be most adaptable to future as well as present needs."

"the teacher of mathematics must fit the mathematical material to the state of development of the learners' mathematical schemas; -- fit his -- presentation to the modes of thinking -- of which his pupils are capable; -- he gradually increasing their analytic abilities to the stage at which they no longer depend on him".

This book can help the teacher work toward these goals.

If there is any criticism to be levelled, it is that the first part of the book is too brief and that an extension of the ideas contained there would have been preferable to the second part which is more of a mathematics text. However, this second part does have the feature that it illustrates

the thesis developed in the first.

D. W. Alexander
F.E.U.T.

*Mathsense*, by Douglas S. Ailles, Ginn and Company ($3.45) pp. 141.

Looking for that interesting problem to give your intermediate advanced class? Need an exercise for addition of fractions for that remedial class? Want to have some fun yourself? Have a look at this collection of interesting ideas packaged in a workbook with stimulating art work. It is impossible to quote from the book because the interest is created by the visual impact of the pages not the words. But: Can you "Find the Key" on page 74? How's your "Recall" page 77? Try "Patchwork" and "Mosaics" pages 46 and 47 for remedial work in fractions.

Throughout the book you feel that the ideas are ones coming from a teacher who has successfully applied them in the classroom.

I needed the Teacher's Guide to understand "Park-a-Lot" on page 88, which could prove frustrating for a student "browsing" in the workbook. Usually the instructions are clear. The Teacher's Guide gives ideas for introducing the work on the pages as well as answers or solutions and still remains just a few pages long.

The only annoying feature, to me, was the page by page reminder I was not to reproduce. As necessary as this admonition may be with such tempting material, it seems rather hypocritical when Ginn is a Xerox subsidiary.

D. W. Alexander
F.E.U.T.

*The Man Made World*, by the Engineering Concepts Curriculum Project, Polytechnic Institute of Brooklyn, McGraw-Hill, 1970, ($13.15) 628 pps.

This book is full of ideas for the mathematics and physics teacher. The contents are set out to be presented as a complete course, but many of the concepts may be presented individually to supplement or provide the basis for

existing or new courses.

Basically the book presents a course on the theories and techniques that contribute to our technological civilization. Topics include:

1. Decision making, which has an excellent section on algorithms. Some classical and some mathematical algorithms are presented and these, along with the associated problems, are most enlightening and interesting.
2. Optimization. Techniques are studied through transportation problems, linear programming and Queueing Theory.
3. Modelling is presented as something which organizes and orders given information so that conclusions and decisions are more easily made.
4. Analog and Digital Computers. The techniques for using these machines, and associated problems, are included. To follow the text with any degree of success through this section would require access to each of these machines, which, incidentally, are not particularly expensive.
5. The remaining material is of an engineering and computer-oriented nature, including sections on stability and Dynamic Systems.

I have presented the course almost in its entirety as an Industrial Physics course, but the material could be used equally effectively in an applied mathematics course.

The most appropriate level of instruction would be grade 12, although students of above average mathematical ability could handle the work in grade 11.

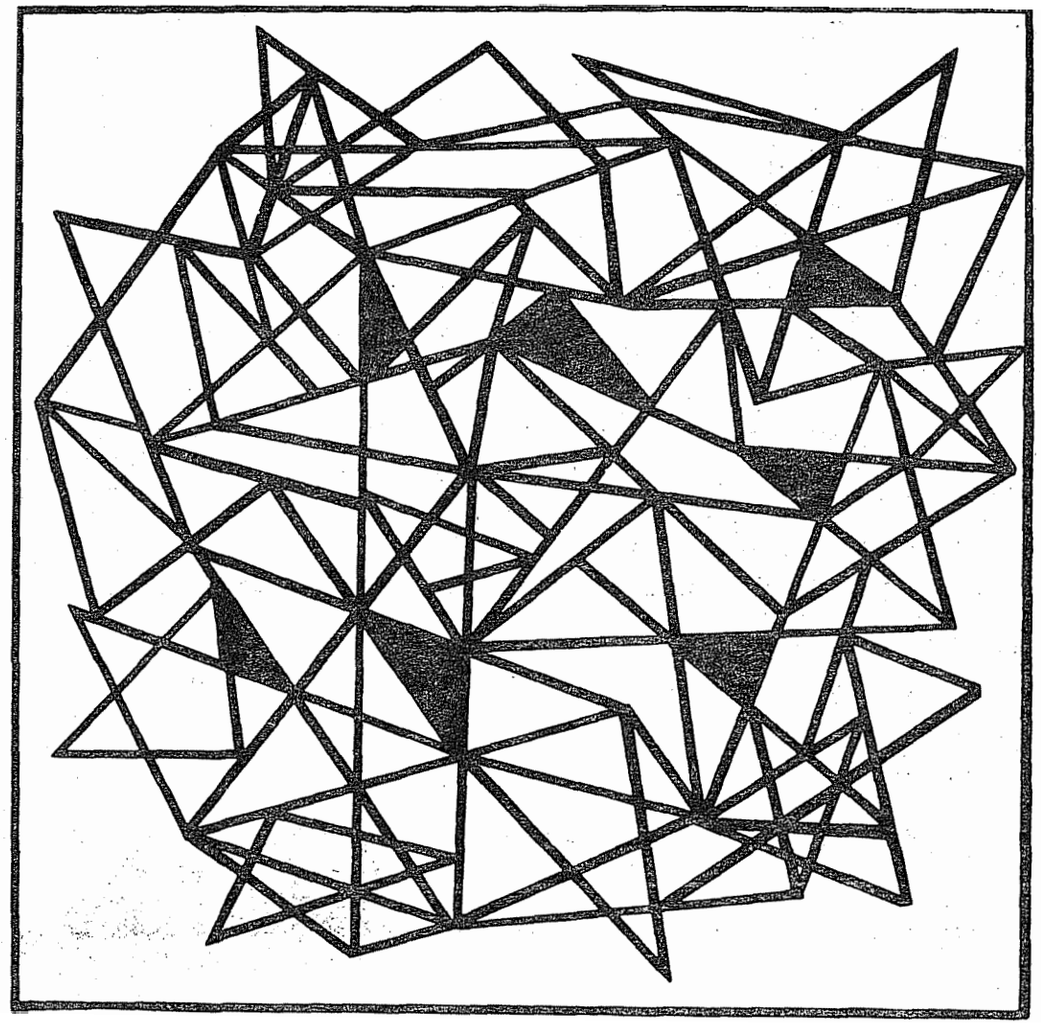
No particular area of mathematics is required, but a generally mature, logical, inquiring mind is essential for students to get anything of benefit from the course.

Without a doubt this is one of the finest texts I have ever used. The material is presented in a very readable style with numerous examples of a practical nature being cited. Some of the theory is quite complex and is too difficult for the average student, but I have found that some students really enjoy trying to understand and master these sections. Mastery of these difficult sections of theory is not essential to understanding the underlying principles and techniques.

After each chapter in the text there are two sets of questions -- one for discussion and one to be answered as problems. There is a lab manual available. A teachers' guide, including list of film transparencies and further questions, may be obtained from the authors.

This book is worthy of a place in any library and makes an excellent text for use by students in a course of an engineering or applied mathematics nature.

David J. Laurenson
Grand River C.I., Kitchener



- see free poster offer!

## SUBSCRIPTION RATES

It was resolved at the meeting of November 24 that the price of the Gazette be:

*$5.00 for one year's subscription*
*$2.50 for a single copy*

Of course, O.A.M.E. members receive the Gazette as an adjunct to their membership.

Invoices will be sent 30 days prior to expiry of a subscription, and 60 days is allowed for payment.

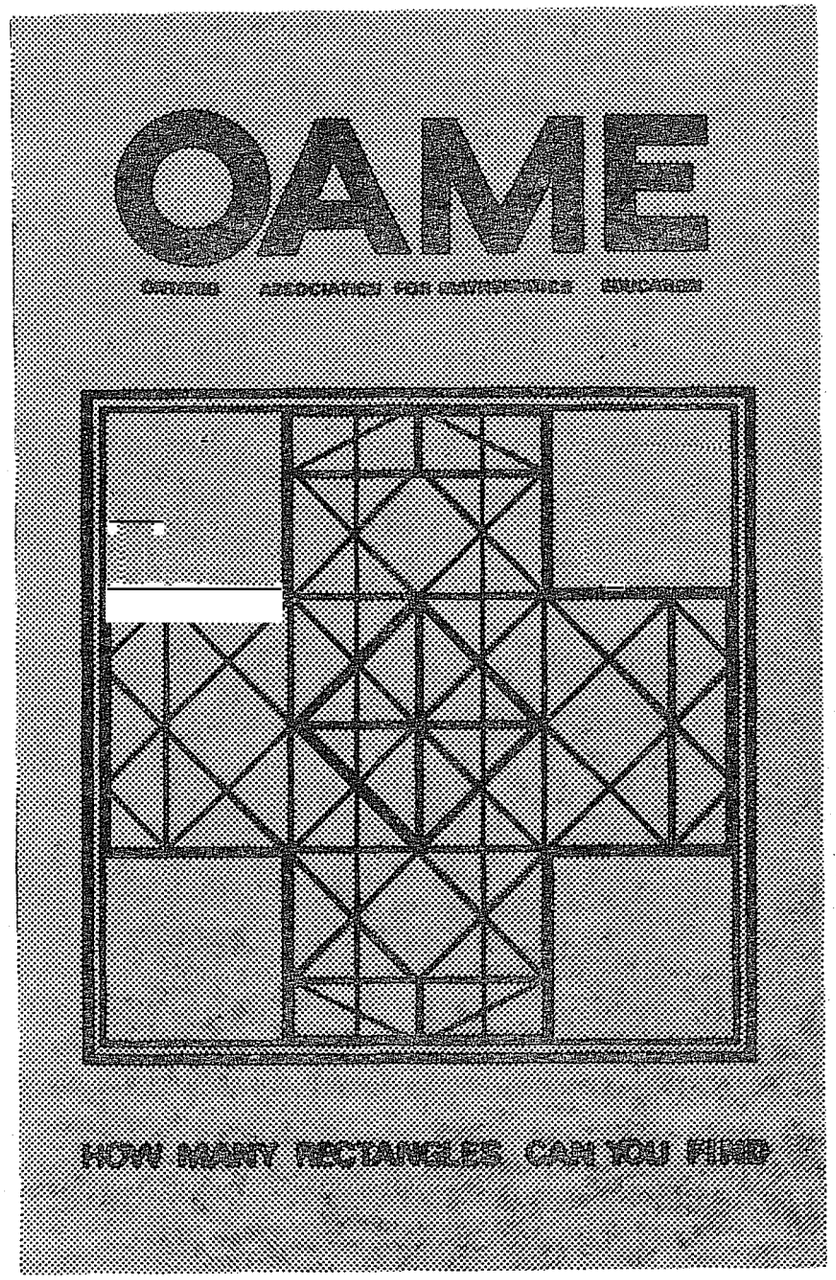
Orders for subscription and single copies should be sent to the Secretary-Treasurer

M. B. MacGregor

Winner of the Pocket Calculator at the N.C.T.M. Toronto meeting was

Mr. Lou Puccini
Brebeauf College
North York

The first annual conference of the Ontario Association for Mathematics Education (O.A.M.E.) is to be held at Queen's University, Kingston, May 10-12.


OAME
HOW MANY RECTANGLES CAN YOU FIND

# FREE POSTERS!

13" 20" Your choice of any one of three brilliantly coloured posters, suitable for the classroom.

- how many rectangles can you find? (opposite)
- how many triangles can you find? (p. i)
- how many five-pointed stars can you find? (p. 26)

Every member of O.A.M.E. can get a poster - you can get additional posters by cosigning new members - a poster for every new member you cosign.

Send your name, and the names of any new members you have cosigned, and indicate the poster you want, to:

Miss Shan Pinkerton

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