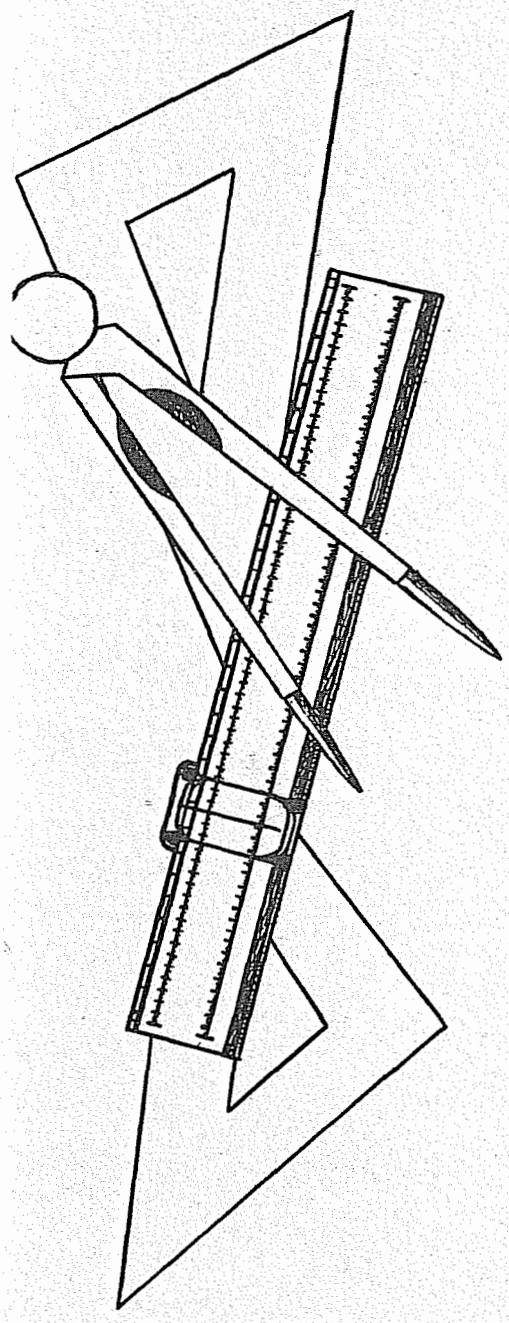
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NOTICE OF MOTION - AMENDMENT TO CONSTITUTION

It is proposed by D. Attridge and J. Routledge that

ARTICLE VIII - ELECTIONS

be amended to read

1. Elections shall be held at each annual general meeting for the positions of Elected Council Members.

2. (a) Elections of Vice-President (the President-elect)
Executive Members
Secretary-Treasurer (if not a paid employee of the organization)
shall be by mailed ballot.

(b) Ballots shall be mailed by the Nominating Committee to all members five weeks prior to each annual general meeting. Ballots shall be counted by scrutineers appointed by the Nominating Committee two weeks prior to the annual general meeting.

3. If the Secretary-Treasurer is a paid employee of the organization, the appointment is to be ratified at the annual general meeting.

Present sections 2 to 4 to become sections 4 to 6, present section 1 to be deleted.

This motion will be voted on at the annual general meeting in Kingston, May 12th, 9:00 a.m.

EDITORIAL

The reborn Gazette is suffering from a lack of suitable articles. It is from the membership that we expect ideas for articles and suggestions about possible authors. If you want to ask any questions, please do so: if you disagree with anything in these pages, please tell us.

Volume 12 of the Gazette started with Volume 11, number 4, which should be retitled Volume 12 number 1, and concludes with this issue. Interspersed in the pages of this issue you will find anonymous answers to the question put by the OAME "as you are involved in mathematics education, what does mathematics mean to you?".

The Editors

Anyone who cannot cope with mathematics is not fully human. At best he is a tolerable subhuman who has learned to wear shoes, bathe, and not make messes in the house.

From the sayings of Lazarus Long (a character created by Heinlein).

Pythagorean Proofs

Shaaron Freiman

It has been stated by Elisha S. Loomis in his book *The Pythagorean Proposition* that as the possible number of algebraic proofs of the Pythagorean Theorem has been shown to be limitless, so too are the possible number of geometric proofs through dissection and comparison of congruent or equivalent areas. The problem, however, is that many of these proofs the average junior high school is unable to understand.

The textbooks I have used at this grade level offer an illustrative case as a proof (Fig. 1). The sides of the triangle are to each other as 3, 4, 5 units. Then square ABKC contains 25 square units, AGFH 16 square units and HEDB 9 square units. Now it is evident that the number of unit squares in the square ABKC equals the sum of the unit squares in the squares AGFH and HEDB. Therefore, square ABKC equals square AGFH plus square HEDB.

I have found that students at the junior high school level have difficulty comprehending the idea of area. Explaining the Pythagorean Theorem to them in terms of the sum of areas of squares usually leaves students in a fog. Last year a group of junior high students and I toured the Boston Museum of Science. We came upon a most fascinating model demonstrating this theorem. Attached to a rotating backboard were plastic models of squares A, B, and C, (Fig. 2). One side of each square formed a right triangle with one side of each other square. Each square was actually a very thin prism whose height allowed for a layer of liquid between the bases. In Fig. 2, squares A and B contained liquid and square C remained empty. When the backboard was rotated clockwise, the liquid in both squares A and B gradually poured into square C (Fig. 3). At the completion of a 180 rotation, all of the liquid from squares A and B had poured into square C and had filled it completely. The model showed the students what area represented and how the area of the two smaller squares combined to form the area of the square attached to the hypothenuse.

So often in geometry students do not understand relationships until they are aided by concrete models. I heard

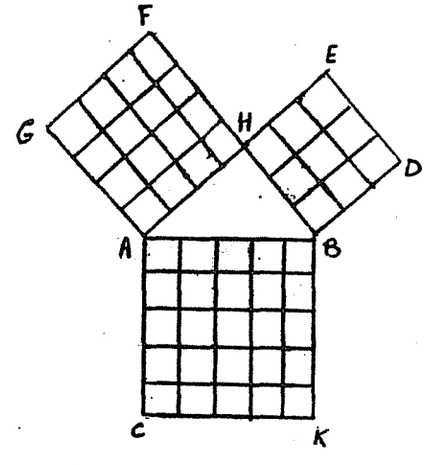


FIG. 1

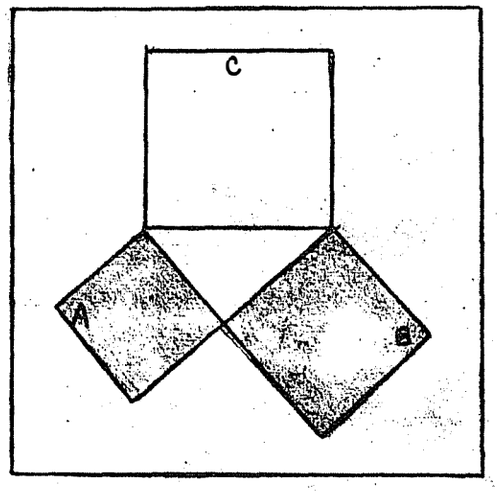


FIG. 2

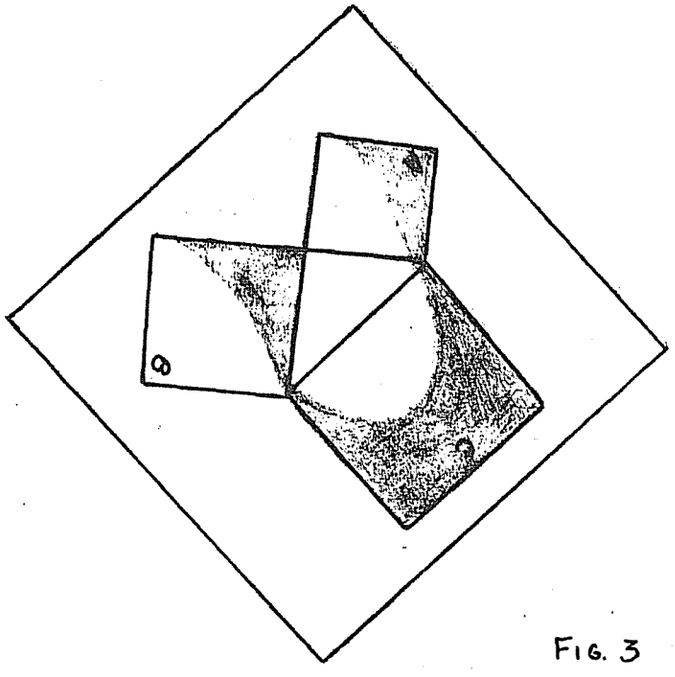


FIG. 3

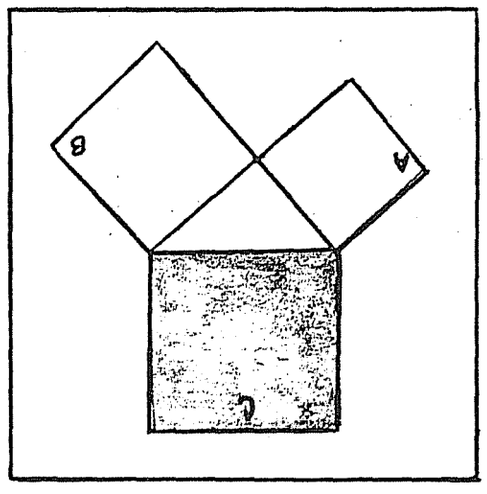
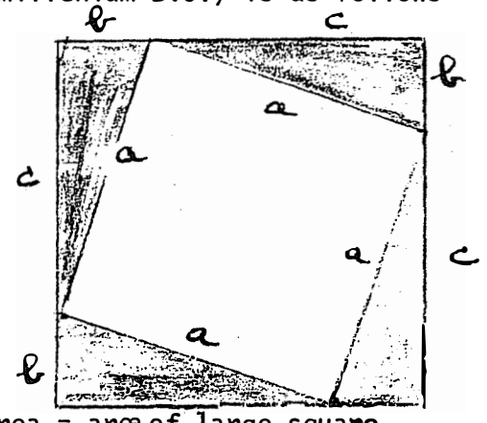


FIG. 4

from a member of my touring group, "Hey, so that's what $a^2 + b^2 = c^2$ means!"

COMMENT: A proof that appeals to an editor (Diagram from second millenium B.C.) is as follows



Total area = area of large square
= area of small square
+ area triangles

$(b+c) = a^2 + 4 \times \frac{1}{2} bc$

$b^2 + c^2 + 2bc = a^2 + 2bc$

$a^2 = b^2 + c^2$

Incidentally $a = p^2 + q^2$, $b = p^2 - q^2$, $c = 2pq$ and

$$a = \frac{1}{2}(m^2+1), \quad b = \frac{1}{2}(m^2-1), \quad c = m$$

give pythagorean triples.

Motivating Number Fumblers

Stanley J. Bezuszka

The low achievers, underachievers and reluctant learners need motivation more than anything else in mathematics. Frequently these students are mentally capable of performing the required tasks but are not interested and thus neither concentrate nor apply themselves.

For the past three years we have been gathering materials which we hope will help the number fumblers to concentrate, to acquire some of the manipulative skills, and be sufficiently motivated to take part in the class activity. These materials are to be given to the students after the regular classroom presentation of the corresponding topic and the accompanying minimal drill. In place of the extended and dull pages of unmotivated problems, may we suggest the following problems and others similar to these.

A. Problems that require concentration.

MAXI-COLUMNS

Place the numbers 1,2,3,4, ..., and so on in column 1 or in column 2. There is only one rule: no number may be put in a column if it is the sum of any 2 other numbers already in that column.

Example

Column 1	Column 2
1	2
3	4
5	

We placed 1 in column 1, 2 in column 2, 3 in column 1. Notice that we had to put 4 in column 2, since we could not put it in column 1 (1 and 3 are already in column 1 and 1 + 3 = 4). We placed 5 in column 1. Now we cannot place 6 in either column 1 (since 1 + 5 = 6) or in column 2 (since 2 + 4 = 6). With the above arrangement of numbers, we have gone from 1 to 5. Can you go higher by arranging the numbers differently? (Answer: yes, you can go to 8.)

Problem 1 How high can you go with 3 columns? (Answer 23)
Problem 2 How high can you go with 4 columns? (Answer 65)
In solving these problems, the student will see a pattern

developing as he goes from two columns to three and four columns.

B. Problems that require concentration and provide opportunity for drill.

HAPPY NUMBERS

Take a number, say 13.
Square each digit and add. $1^2 + 3^2 = 10$
Repeat the above with 10. $1^2 + 0^2 = 1$

A number for which this pattern yields finally a 1, is a happy number.

The number 13 is a happy number.

Take the number 2.
Square it and add.
Repeat the above with 4.
Square each digit in 16 and add.
Repeat.

$2^2 = 4$
$4^2 = 16$
$1^2 + 6^2 = 1 + 36 = 37$
$3^2 + 7^2 = 9 + 49 = 58$
$5^2 + 8^2 = 25 + 64 = 89$
$8^2 + 9^2 = 64 + 81 = 145$
$1^2 + 4^2 + 5^2 = 1 + 16 + 25 = 42$
$4^2 + 2^2 = 16 + 4 = 20$
$2^2 + 0^2 = 4$

Notice that we are back to the number we started with. This pattern will cycle forever.

The number 4 is not a happy number.

Problem 1 Find all the happy numbers less than 100.
Problem 2 Let A = 1, B = 2, C = 3, D = 4, ... and so on through the alphabet. Take a name: ALLEN. Add the number corresponding to each letter. Thus

ALLEN $1 + 12 + 12 + 5 + 14 = 44$

Test whether or not 44 is a happy number. If 44 is a happy number, then ALLEN is a happy name.
Problem 3 Problem 2 can be adapted to the days of the week, to the months of the year and to the years themselves. Thus, take 1973. $1^2 + 9^2 + 7^2 + 3^2 = 1 + 81 + 49 + 9 = 140.$
Continue with 140. Is 1973 a happy number year?

C. Problems that require concentration, provide drill and motivate solution strategies.

PATTERNS FROM THE ADDITION AND MULTIPLICATION TABLES

Addition Table

+	0	1	2	3	4 ...
0	0	1	2	3	4 ...
1	1	2	3	4	5 ...
2	2	3	4	5	6 ...
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.

1. Take 3 consecutive numbers in a row (column), for example | 2 | 3 | 4 |. What is the sum? Is it 3 times the middle number? (yes. $3 \times 3 = 9$)

2. Take 5 consecutive numbers in a row (or column), | 2 | 3 | 4 | 5 | 6 | What is the sum? Is it 5 times the middle number? (yes. $5 \times 4 = 20$)

3. Take 7, then 9 consecutive numbers in a row (or column). What is the sum? Did you use the same pattern as in (1) and (2)?

4. Take 4 consecutive numbers in a row (or column), for example | 2 | 3 | 4 | 5 | What is the sum? Is it twice the sum of the first and last number? (yes. $2(2+5) = 14$)

5. Take 6 consecutive numbers in a row (or column). | 2 | 3 | 4 | 5 | 6 | 7 | What is the sum? Is it 3 times the sum of the first and last number? (yes. $3(2+7) = 27$)

6. Try the above for 8 and 10 consecutive numbers in a row (or column). What is the sum? Did you use the same pattern as in (4) and (5)?

7. Discover the rule for finding the sum of the numbers in the addition tables that appear in the following patterns.

(a)

2	3
3	4

Square 2×2
Rule: Number of numbers in the square (4) times the number on the dotted diagonal.

(b)

1	2	3
2	3	4
3	4	5

Square 3 × 3
Does the rule in 7(a) hold here?

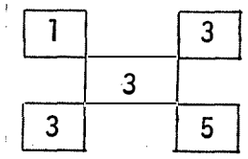
(c)

+	0	1	2	3	...	10
0						
1						
2						
⋮						
10						

Challenge: What is the sum of all the numbers in this addition table (do not consider the numbers here or here)?

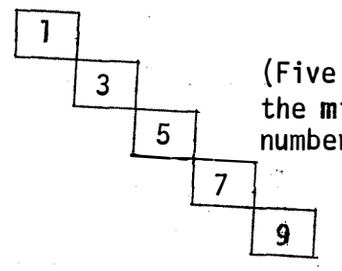
8. Find a rule for the sum of the numbers in the addition table that appear in the following pattersn.

(a)



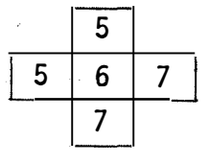
(Five times the number in the centre. 5 × 3 = 15)

(b)



(Five times the middle number)

(c)



(Five times the middle number)

9. Challenge: Find other patterns in the
   (a) addition table and
   (b) in the multiplication table.

D. Problems that inter-relate topics.

HOW DO YOU FIND THE GREATEST COMMON DIVISOR OF TWO NUMBERS?

(a) Intersection of sets of divisors.

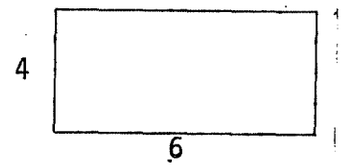
We illustrate this method by an example.

Find the GCD of 6 and 4.
The set of divisors of 6: $D(6) = \{1,2,3,6\}$
and the set of divisors of 4: $D(4) = \{1,2,4\}$

Now GCD $(6,4) = D(6) \cap D(4) = \{2\}$

(b) Geometric method

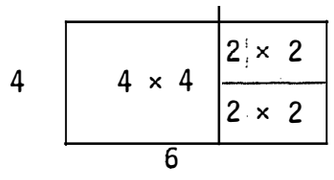
Take the numbers 6 and 4
Construct a rectangle with sides 6 and 4 units.



Partition the rectangle according to the following procedure:

1. Find the largest square in the rectangle (There may be more than one).
2. Repeat step 1 for what is left in the rectangle after each partition.

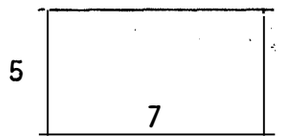
Thus,



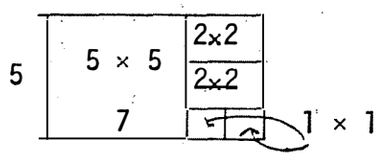
The dimension of the smallest square(s) is the greatest common divisor of 6 and 4. In the above, GCD (6,4) is 2 since the dimension of the smallest square is 2.

Example: Find the GCD (7,5)

Step 1



Partition into squares

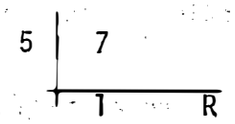


GCD (7,5) is 1 since the dimension of the smallest square is 1.

(c) Division method:

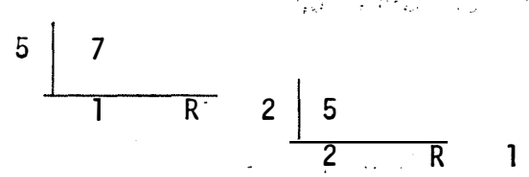
Example: Find the GCD (7,5)

1. Divide the smaller number into the larger number according to the following procedure

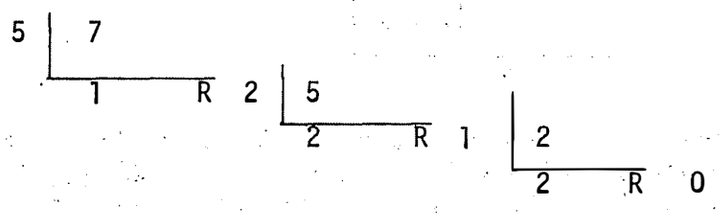


5 goes into 7 with a remainder of 2

2. Now divide the remainder (2) into the previous divisor (5):



3. Repeat the step (2)



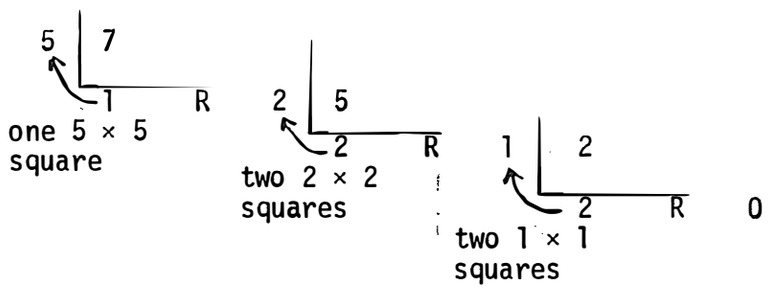
The divisor (1) which leaves a remainder zero (0) is the GCD of the two numbers 5 and 7.

Problem 1 Use the above procedure to find the GCD of

a) 8 and 13
b) 21 and 34
c) 35 and 55

Note: The above numbers are consecutive numbers in the Fibonacci sequence. These numbers give the largest chain of steps in the division algorithm above.

(d) Compare the division algorithm in (c) with the rectangle partition procedure in (b).



The division algorithm gives the partition into squares of the rectangle consisting of sides 5 and 7.

E. Problems for modular units and projects.

A student can enjoy working on a project if it is challenging and interesting. The following project is best done by using inch squares rather than trying to show the squares on graph paper.

PROJECT

1. Make a 3 χ 3 square of 1 χ 1 unit squares

   The perimeter is 12 units.

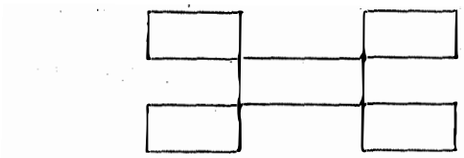
S

1	2	3
4	5	6
7	8	9

Now remove squares to make the perimeter larger. The rule is:

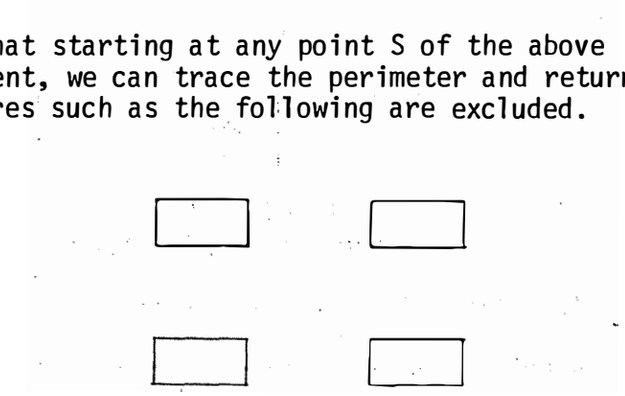
> you must remove squares in such a way that starting at any point S you can trace a path back to S.

The maximum perimeter for a 3 χ 3 square is made by removing squares 2, 4, 6 and 8 to get

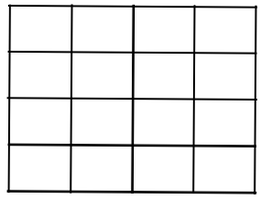


The perimeter here is 20 units.

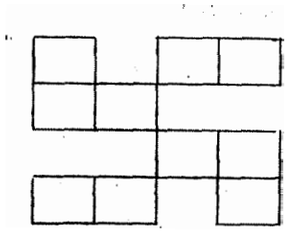
Notice that starting at any point S of the above arrangement, we can trace the perimeter and return to S. Figures such as the following are excluded.



2.



4 4 square
Perimeter 16 units



Maximum perimeter
28 units

Problem 1 Take 5 χ 5, 6 χ 6, 7 χ 7, 8 χ 8 squares. Find maximum perimeter.

Problem 2 Can you find formula(s) that will give you the answer for the maximum perimeter for an n χ n square where n is an element of 1, 2, 2, 4,....

Challenge: Apply the above to unit cubes and find the arrangement of n χ n χ n unit cubes with the maximum surface area.

Professor Bezuska is Director of the Boston College Mathematics Institute with interests in vector analysis and ultrasonics. He is the author of a large number of texts and articles. This report formed the basis of his talk at the NCTM Toronto meeting in November 1973.

A note.

What can a music teacher do when his school decides to "go metric" for a month? This was the question confronting Charles Rinehart at the Campus School in Oswago, New York. His response was to use rhyme and rhythm to teach the new metric vocabulary. His song, "Halve Your Meter," has become an instant hit due to its catchy melody and painless introduction to the metric system. From the standpoint of a music teacher looking at the metric system, Charles says, "How pleasant it is to have a spoonful of sugar to help the metric go down".

The words and music of the tune appear in the May, 1974 issue of SCHOOL SCIENCE AND MATHEMATICS. If you would like a reprint of the article, which includes the words and music to the tune, simply send a stamped, self-addressed envelope to:

School Science and Mathematics Association
P.O. Box 1614
Indiana University of Pennsylvania
Indiana, Pennsylvania 15701

Distance of point from a line

M. R. Nanjappa

Problem: To find the (perpendicular) distance from a point to a line.

Being a student of mathematics for a long time and seeing different solutions for the above problem, I have come up with the following elegant solution.

Solution:

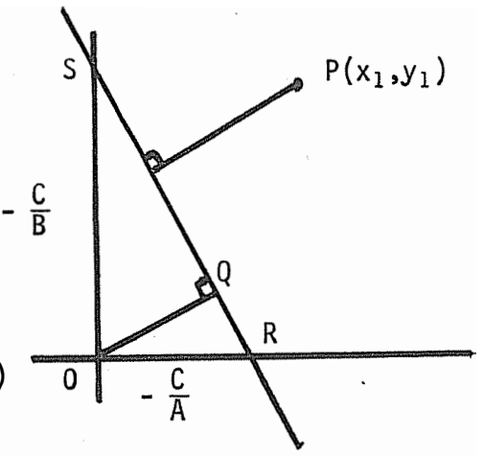
Let $P(x_1, y_1)$ be the point.

Let the equation of the line be

$Ax + By + C = 0$ - - - (1)

Let p be the distance from the point to the line.

The line (1) cuts the x-axis at $R(-\frac{C}{A}, 0)$ and y - axis at $S(0, -\frac{C}{B})$



$\Delta OSR = \frac{1}{2}$ base $\times$ ht.

$= \frac{1}{2} OR \times OS$

$= \frac{1}{2} |-\frac{C}{A}||-\frac{C}{B}|$

$= \frac{1}{2} \frac{C^2}{|AB|}$ --- (2)

$SR^2 = OR^2 + OS^2$

$= \frac{C^2}{A^2} + \frac{C^2}{B^2}$

$\therefore SR = \frac{|C|}{|AB|} \sqrt{A^2 + B^2}$

Let $OQ = p_1$ (1)

$\therefore \Delta OSR = \frac{1}{2} SR \cdot p_1$

$= \frac{1}{2} \frac{|C|}{|AB|} \sqrt{A^2 + B^2} \cdot p_1$ --- (3)

From (3) and (2), we have

$$\frac{1}{2}\frac{|C|}{|AB|}\,p_1\,\sqrt{A^2+B^2} = \frac{1}{2}\,\frac{C^2}{AB}$$

$$\therefore\quad p_1 = \frac{|C|}{\sqrt{A^2+B^2}} \qquad (4),$$

which is the perpendicular distance from the origin to the line (1).

The equation of the line through $P(x_1, y_1)$ and parallel to $Ax + By + C = 0$ is

$$Ax + By + C_1 = 0 \text{ ---(5)},$$

where $C_1 = -(Ax_1 + By_1)$ ---(6) $(\because (x_1,y_1)\varepsilon\ (5))$

$\therefore$ the distance from the origin to the line (5) is

$$p_2 = \frac{|C_1|}{\sqrt{A^2+B^2}} \text{ --- (7)} \left(\text{follows from (4)}\right)$$

$$\therefore\ p = |p_2 - p_1|$$

$$= \frac{|C_1 - C|}{\sqrt{A^2+B^2}} \qquad \left(\text{from (4), (7)}\right)$$

$$p = \frac{|Ax_1 + By_1 + C|}{\sqrt{A^2+B^2}} \qquad \left(\text{from (6)}\right)$$

Comment. Other solutions that the editors found include:

i) the square of the distance of the point (x_1,y_1) from the point (x_1,y_1) is $d^2 = (x-x_1)^2 + (y-y_1)^2$

If $Ax + By + C = 0$ then

$$d^2 = \frac{1}{A^2}(By + C - Ax_1)^2 + (y-y_1)^2 \text{ ----------(1)}$$

and this gives equal roots for y when d is a minimum i.e. when

$$\left(\frac{B}{A^2}(C + Ax_1) - y_1\right)^2 = \frac{B^2}{A^2} + 1\left(\frac{(C+Ax_1)^2}{A^2} + y_1^2 - d^2\right)$$

giving $d^2 = (C + Ax_1 + By_1)^2 /(B^2+A^2)$

ii) differentiate (1) with respect to y gives for the minimum

$$y_1 = y + \frac{B\,Ax_1 + By_1 + C}{B^2 + A^2} \qquad \text{----------(2)}$$

and hence the usual formula for d.

iii) line through (x_1, y_1) perpendicular to $Ax + By + C = 0$
is $B(x-x_1) = A(y-y_1)$ which meets the given line when
$Ax + By + C = 0$
$Bx - Ay + Ay_1 - Bx_1 = 0$
leading to equation (2) and hence to d

(iv) Vectorially if the point is $\underline{c}$ and the line passes through $\underline{A}$ and $\underline{b}$ then the equation of the line is $\underline{a} + \mu(\underline{b} - \underline{a})$ with unit vector perpendicular to the line being $\hat{k} \times \frac{(\underline{b} - \underline{g})}{|\underline{b} - \underline{a}|}$

and hence $d = (\underline{c} - \underline{a}) \cdot (\hat{k} \times (\underline{b} - \underline{a})$
$\overline{|\underline{b} - \underline{a}|}$

with $\underline{c} = (x_1, y_1, 0)$
$\hat{k} = (0,0,1)$
$\underline{a} = (-\frac{C}{A}, 0.0)$
$\underline{b} = (0, \frac{C}{B_1}\ 0)$

we obtain the usual equation for d.

Problème des trois sphères

ANDRÉ LEMAIRE

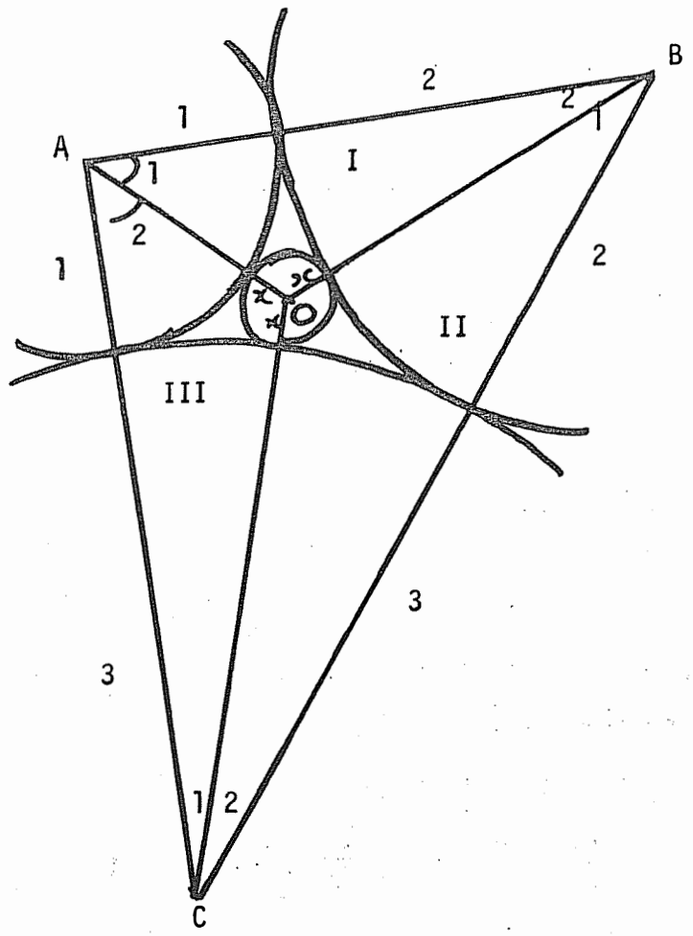
Introduction:

Le problème des trois sphères mutuellement tangentes n'est pas nouveau; nous voulons exposer ici une de ses méthodes de résolution pour le cas particulier où les rayons sont respectivement: 1" , 2" et 3".

La méthode utilisée ici offre un intérêt très particulier à l'étudiant senior de l'école secondaire. En effet, nous utilisons un triangle dans lequel se trouve l' "inconnue". En comparant deux formules différentes, donnant la surface du même triangle et en substituant le sinus d'un angle par sa valeur donnée à partir d'une formule mettant en relation les côtés d'un triangle quelconque, nous arrivons finalement a une équation du second degré.

Heureusement, le théorème de Pythagore s'applique ici à ce cas particular, c'est lui qui nous sort d'un mauvais pas, au bon moment. L'élève senior retrouve donc ici une bonne partie de l'arsenal mathématique qu'il a déjà appris et peut apprécier l'emploi à bon escient et au bon moment, des lois et théorèmes de géométries, de trigonométrie et d'algèbre.

Trois sphères de Rayons 1". 2", 3" sont mutuellement tangentes. Quel est le Rayon de la plus grosse sphère pouvant passer dans l'interstice formé par les 3 sphères?



1. $\triangle ABC$ est rectangle (côtés: 3,4,5) — Pythagore

   $\therefore \angle A = \angle A_1 + \angle A_2 = 90°$

2. Surface du triangle ABO → A(aire) — Héron

   $A = \sqrt{s(s - AB)\ (s - BO)\ (s - OA)}$

   $s = \frac{1}{2}$ périmètre

   $s = \frac{1 + 2 + 2 + x + x + 1}{2} = 3 + x$

   $A = \sqrt{(3 + x)\ (x)\ (1)\ (2)} = \sqrt{6x + 2x}$

3. La surface d'un triangle est égale au $\frac{1}{2}$ produit de 2 quelconques de ses côtés par le sinus de l'angle qu'ils comprennent.

Surface du triangle ABO — Trigonométrie

$A = \frac{1}{2} \overline{AB} \cdot \overline{AO} \cdot \sin A_1$

$\therefore A = \frac{1}{2} (1 + 2) (1 + x) \sin A_1$

4. Dans tout triangle, le carré d'un côté est égal à la somme des carrés des 2 autres côtés moins le double du produit de ces 2 autres côtés par le cosinus de l'angle qu'ils comprennent.

Considérant ΔACO

$\overline{CO}^2 = \overline{AC}^2 + \overline{AO}^2 - 2 \cdot \overline{AC} \cdot \overline{AO} \cdot \cos A_2$ — Trigonométrie

$\therefore (3 + x)^2 = (3 + 1)^2 + (1 + x)^2 - 2(3 + 1) (1 + x) \cos A_2$

5. Parce que $\angle A_1 + \angle A_2 = 90°$ (1.), $\therefore \cos A_2 = \sin A_1$.
D'après (2.) et (3.)

$\therefore \frac{1}{2} (1 + 2) (1 + x) \sin A_1 = \sqrt{6x - 2x^2}$

$$\sin A_1 = \cos A_2 = \frac{\sqrt{6x + 2x^2}}{\frac{1}{2} (1 + 2) (1 + x)}$$

D'après (4.)

$$(3 + x)^2 = (3 + 1)^2 + (1 + x)^2 - \frac{2(3 + 1) (1 + x) \sqrt{6x + 2x^2}}{\frac{1}{2} (1 + 2) (1 + x)}$$

Effectuant, $9 + 6x + x^2 = 16 + 1 + 2x + x^2 - \frac{16}{3} \sqrt{6x + 2x^2}$

$$-8 + 4x = \frac{16}{3} \sqrt{6x + 2x^2}$$

élevant au carre´ $(64 - 64x + 16x^2) = \frac{256}{9} (6x + 2x^2)$

$$\therefore \frac{9}{256}(64 - 64x + 16x^2) = 6x + 2x^2$$

$$\frac{9}{16}(4 - 4x + x^2) = 6x + 2x^2$$

$$\frac{36}{16} - \frac{36x}{16} + \frac{9x^2}{16} - 6x - 2x^2 = 0$$

$$\frac{36}{16} - \frac{132x}{16} + \frac{23x^2}{16} = 0$$

$$\therefore 23x^2 + 132x - 36 = 0$$

6. $x = \frac{(-132 \pm \sqrt{132^2 - (-4 \cdot 36 \cdot 23)}}{2.23}$ Equation quadratique
Algèbre

$$x = \frac{(-132 \pm \sqrt{20736})}{46}$$

$$x = \frac{-132 \pm 144}{46} \qquad \therefore x = \frac{12}{46} ; x = \frac{-276}{46}$$

Eliminant la réponse négative,

$x = (6/23)" = 0.\dot{2}5086956521739130434\dot{7}8$ pouce

Problème soumis par M. André Lemaire, étudiant à la Formation des Enseignants, Faculté d'Education, Université d'Ottawa 1973-74.

"

In general the mathematics education at secondary level should be directed towards the two main objectives:

(i) to provide each student with the mathematical literacy useful to him/her as an individual and a member of society.

(ii) to generate in some students an insatiable curiosity about the mathematics discipline.

Mathematics is a way of thinking; analytical, organized, logical, planned, critical.

A mathematician (and here I refer to those trained in mathematics as opposed to the mathematical elite) approaches most of life's problems in the same way as he has been trained to think, be the problem social, environmental or his work. "

Geoboards and Equilateral Triangles

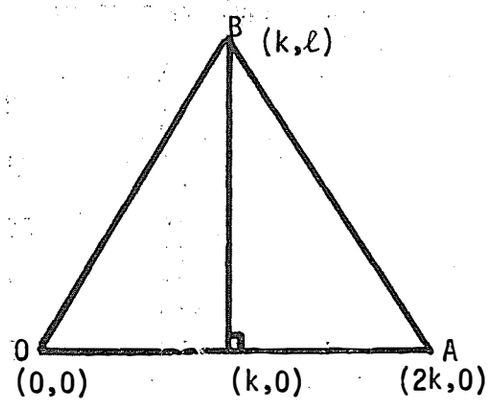
R. Koskela

If you have a geoboard whose pegs are arranged so that they form little squares, it is not possible to use elastics to make an equilateral triangle on it -- despite the claim of Sigma Scientific Inc. in their 1973 Elementary Math Materials Catalogue.

However, assuming you can obtain a geoboard with as many pegs on it as you wish and an elastic as long as you wish, you can make a triangle which is as close to an equilateral triangle as you want.

Choose as an origin the peg on the lower left corner of of the geoboard. Let the x-axis run along the bottom row of pegs and the y-axis along the left most column of pegs.

Choose A(2k,0)
Make an isosceles OAB
by erecting a perpendicular
on OA from (k,0) to B(k,l)



ΔOAB is equilateral if
base = one of the equal
sides

$2k = \sqrt{k^2 + l^2}$

$4k^2 = k^2 + l^2$

$3k^2 = l^2$

Two integers k and l can't be found such that $3k^2 = l^2$. But if we can find an increasing sequence of pairs of integers k and l such that $3k^2 = l^2 +$ constant, then the problem is solved because as k and l get bigger and bigger the significance of the constant decreases, $3k^2$ gets relatively closer and closer to l^2, the length of the base, OA, gets closer and

closer to the lengths OB and AB, and the Δ OAB gets closer and closer to being equilateral.

The next step of finding a sequence for k and l is pure pattern finding. My own progress went as follows.

First by looking at a table of squares of integers I noticed that

$$3(4)^2 = (7)^2 - 1$$

Here k = 4, l = 7 and the constant is -1. Now I started looking for k and l such that $3k^2 = l^2 - 1$ and wrote down the following table:

k	l
1	2
4	7
11	19
15	26

There is a mistake in that k = 11, l = 19 doesn't fit the pattern (because $3(11)^2 = (19)^2 - 2$) but the error is serendipitous.

I noticed that 4 + 11 = 15 and 4 + 7 = 11 and after much juggling came up with the following:

k	l
1	2
3	
4	7
11	
15	26
41	
56	97
153	
209	362

The numbers 3, 11, 41, 153, ... are not k values but they are used to compute k and l.

These numbers are part of a Fibonacci sequence.

F = 1,2,3,4,7,11,15,26,41,56,97,153,209,362,...

where $f_1 = 1$, $f_2 = 1$ and $f_n = f_{n-1} + f_{n-2}$ except for

fn |n ≥ 3, n ≡ 2 mod 3 (that is, the 5th term, 8th term, 11th term, 14th term, etc.) when fn = fn-1 + fn - 3

The k and 1 values are the pairs indicated by arcs.

F△ $f_1 = 1$, $f_2 = 1$ and for fn, n ≥ 3

fn = fn-1 + fn-2 when n ≡ 0 mod 3 and n ≡ 1 mod 3
fn = fn-1 + fn-3 when n = 2 mod 3

F also appears in the solution to the problem of making a square on a geoboard whose pegs are arrnged so that they form little equilateral triangles.

It's interesting to speculate what geometric problems are solved by considering other pairs of F or by considering other Fibonacci type sequences.

Richard Koskela is a student at the Faculty of Education, University of Toronto.

"Mathematics means a study of the practical side of the use of numbers in the abstract form. The abstractness of such has been overemphasized far too quickly in the primary grades so that children are terribly confused. We need to educate children, not teach mathematics for math's sake.

As far as education is concerned, Mathematics is the last frontier for establishing self-discipline. Math is an excellent vehicle for teaching organization. Logical thinking, and an approach to problem-solving. In this last regard (and perhaps in the last analysis) MATH IS FUN!"

Spiral approach to Math.

H. Kornelson

The spiral approach used in the teaching of "new mathematics" has produced considerable comment and discussion.

In the year three mathematics classes in Ontario, topics in Euclidean geometry, co-ordinate geometry, trigonometry, vectors and algebra are taught.

Is it possible to create exercise material using all of these topics?

Consider please the following comments:

(1) We all require that the student find exact values for sin 0°, cos 30°, tan 45° etc. using the appropriate diagram.

(2) We require that the student be able to solve oblique triangles using the sine law and cosine law for triangles.

(3) Students know how to add vectors and can calculate the magnitude of a vector from its co-ordinates.

The following problems are suggested to develop several ideas.

1. (a) Given $\vec{a} = (1,1)$ calculate $|\vec{a}|$.

(b) Use $\vec{a}$ and $|\vec{a}|$ and the appropriate diagram to calculate sin 45° as an exact value.

Solution: $|\vec{a}| = \sqrt{1^2 + 1^2}$

$\therefore |\vec{a}| = \sqrt{2}$

also $\sin 45° = \frac{1}{\sqrt{2}}$

2. Given $|\overrightarrow{AB}| = 4$, $|\overrightarrow{BC}| = 2\sqrt{7}$ and $|\overrightarrow{AC}| = 6$
Calculate $\angle A$ of ΔABC.

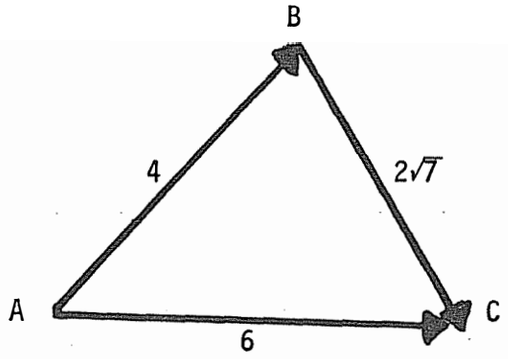
Solution:

$\cos \angle A = \frac{b^2+c^2-a^2}{2bc}$

$\cos \angle A = \frac{4^2+6^2-(2\sqrt{7})^2}{2X(4)X(6)}$

$\cos \angle A = \frac{1}{2}$

$\therefore \angle A = 60°$



3. Given $\overrightarrow{OP} = (1,\sqrt{3})$ and $\overrightarrow{OQ} = (1,1)$

   (a) calculate $\angle POQ$,

   (b) calculate cos 15° as an exact value

   Solution:

   Let R = (1,0)

   (a) $\therefore \angle QOR = 45°$
   and $\angle POR = 60°$

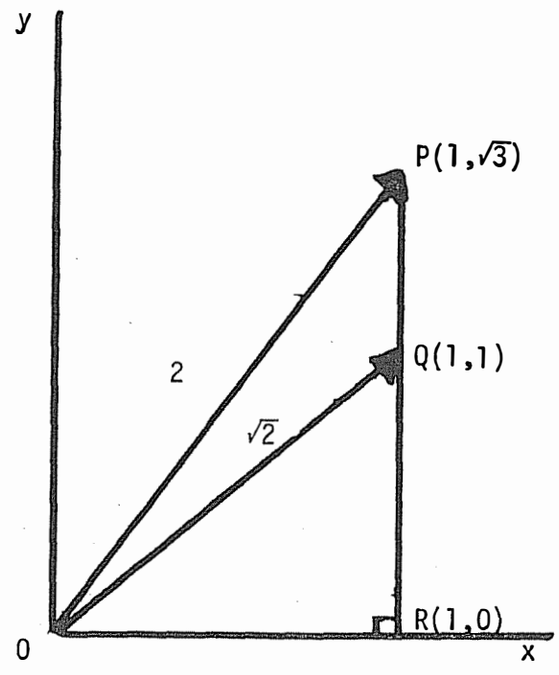
   $\therefore \angle POQ = 15°$

   (b) $\because |\overrightarrow{OP}| = \sqrt{2}$, $|\overrightarrow{OQ}| = \sqrt{2}$
   and $|\overrightarrow{OP}| = \sqrt{3} - 1$

   $\therefore$ cos POQ =

   $$\frac{2^2+\sqrt{2}^2-(\sqrt{3}-1)^2}{2X(2)X(\sqrt{2})}$$

   $\therefore \cos 15° = \frac{1+\sqrt{3}}{2\sqrt{2}}$



Note that $\overrightarrow{OP} \cdot \overrightarrow{OQ} = (1,1) \cdot (1,\sqrt{3})$

$= 1 + \sqrt{3}$

and further that cos 15°

$= \frac{\overrightarrow{OP} \cdot \overrightarrow{OQ}}{|\overrightarrow{OP}| \times |\overrightarrow{OQ}|}$.

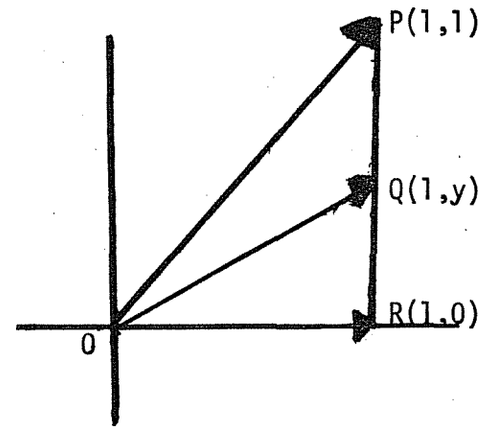
4. Can you create a similar diagram to calculate cos 105°?

Solution: Calculate cos 105° from the points $(1,\sqrt{3})$ and $(1,-1)$.

Note that the proper identities now make it possible to calculate cos 75° or sin 15° etc. using the results of problems 3 and 4.

5. How would you calculate tan 22 1/2° as an exact value from the appropriate diagram?

Solution:



Let OQ be the bisector of $\angle POR$ so that P = (1,1), Q = (1,y) and R = (1,0)

Then RQ = y, $OQ = \sqrt{1 + y^2}$ and PQ = 1 - y

In ΔQOR

$$\cos \angle QOR = \frac{1^2 + (\sqrt{1 + y^2}) - y^2}{2\times(1)\times\sqrt{1 + y^2}}$$

$$= \frac{1}{\sqrt{1 + y^2}}$$

In ΔPOQ

$$\cos \angle POQ = \frac{\sqrt{2^2} + (\sqrt{1 + y^2}) - (1-y)^2}{2\times(\sqrt{2})\times\sqrt{1 + y^2}}$$

$$= \frac{1 + y}{\sqrt{2}\sqrt{1 + y^2}}$$

$\because \cos \angle QOR = \cos \angle POQ$ then

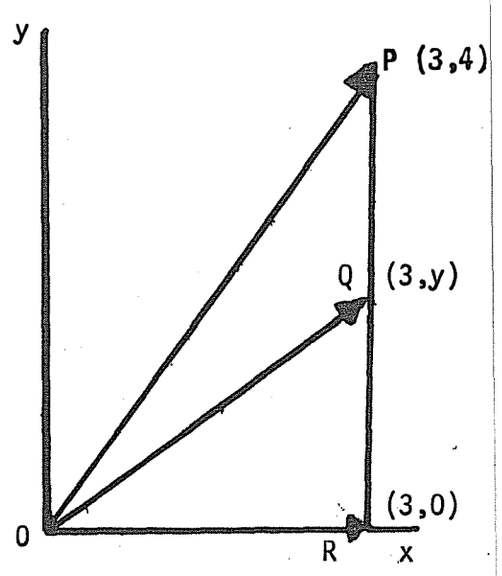
$$\frac{1}{1 + y^2} = \frac{1 + y}{\sqrt{2}\sqrt{1 + y^2}}$$

and $y = \sqrt{2} - 1$

$\therefore \tan 22\ 1/2° = \frac{QR}{OR} = \sqrt{2} - 1$

6. Given that $\sin\theta = 4/5$ calculate $\sin 1/2$ as an exact value from the appropriate diagram.

Solution:



Let $\angle POR = \theta$ and $\angle QOR = 1/2\ \theta$ so that P = (3,4), Q = (3,y) and R = (3,0).

$\therefore$ RQ = y, QP = 4 - y
and $OQ = \sqrt{9 + y^2}$

In ΔQOR: $\cos \angle QOR$

$$= \frac{3^2 + (\sqrt{9 + y^2})^2 - y^2}{2\times(3)\times\sqrt{9 + y^2}}$$

$\cos 1/2\theta = \frac{3}{\sqrt{9+y^2}}$ i.e. $\frac{\overrightarrow{OR} \cdot \overrightarrow{OQ}}{|\overrightarrow{OR}|\times|\overrightarrow{OQ}|}$

In ΔPOQ: $\cos \angle POQ =$

$$\frac{5^2 + (\sqrt{9 + y^2})^2 - (4 - y)^2}{2\times(5)\times\sqrt{9 + y^2}}$$

$$\cos 1/2\ \theta = \frac{9 + 4y}{5\sqrt{9 + y^2}}$$

i.e. $\frac{\overrightarrow{OP} \cdot \overrightarrow{OQ}}{|\overrightarrow{OP}|\times|\overrightarrow{OQ}|}$

$$\therefore \frac{3}{\sqrt{9 + y^2}} = \frac{9 + 4y}{5\sqrt{9 + y^2}}$$

and y = 1 1/2

Notice that RQ $\cong$ 1/2 RP

$$\therefore \sin 1/2\ \theta = \frac{RQ}{OQ} = \frac{1.5}{\sqrt{9+1.5^2}}$$

$$\therefore \sin 1/2\ \theta = \frac{1}{\sqrt{5}}$$

7. Can we create a diagram to calculate cos 2 θ given that $\sin\theta = 4/5$?

Solution: Calculate $\cos 2\theta$ from a diagram using points (3,4) and (3,-4).

Have I made my point? What are the other "spin offs" from the ideas in these problems and solutions? I do hope that

you will feel that these ideas are for all of your students and not just for the superior students in your advanced classes in years 3, 4 and 5.

H. Kornelson is a teacher at Lakeport Secondary School in St. Catherines.

"Mathematics is a tool for the solution of real problems. Graduates of Math porgrams who do not realize this, who feel mathematics is a mental exercise, who are unaware of the diversity of applications, who have not been exposed to the process: real problem → mathematical model → methods for tackling the math problem → comparison with behaviour in real problem → revised model, etc. ...these graduates have been mis-educated.

The 19th century (18th century?) "academic" problems (the motion of a vibrating string) have little weight in educating a student to meet the mathematical demands of modern technology. "Real problems" must come government and industry.

The "role of mathematics" and the "direction in which mathematical education should go" and the "Type A research which the government should support"... these questions become answered (without any effort) once the lines of communication have been opened between where the real problems lie and where the methods A solution are taught.

There is left only one problem: HOW TO MAXIMIZE THE COMMUNICATION BETWEEN GOVERNMENT, INDUSTRY AND EDUCATIONAL INSTITUTIONS?

After establishing a free flow of ideas and problems between these (at present, disjoint) regimes, no one will even ask "what is the rule of mathematics?"

Nominations

Once again we begin a new year with great expectations, and I trust that you will join me in wishing our Association every success in 1974.

In order that O.A.M.E. continues with good leadership I am asking you for some assistance to the Nominating Committee to suggest names for Executive and Council positions for the coming year 1974-1975. The President-elect is John Egsgard from Twin Lakes S.S. in Orillia. Our constitution states that there shall be an election for the position of Vice-President, three positions on the Executive, and for sufficient numbers to complete the Council.

Would you assist us by sending in names for:

(a) Vice-President (one to be elected)
(b) Executive (three to be elected)
(c) Councillor (approximately 10 to be elected, depending on the number of Chapter representatives selected.)

The nominating Committee will assess the nominees and investigate whether or not they will allow their names to stand for election.

Please complete the form below and send it to me at the address below. Thank you for your assistance.

Donald C. Attridge (acting Past-President)

Chairman of the Nominating Committee

.....

I,_____, nominate the following for Positions on the Council of the Ontario Association for Mathematics Education:

For Vice-President:_____

_____

For Executive: _____

_____

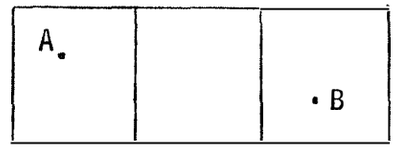
For Council: _____

_____

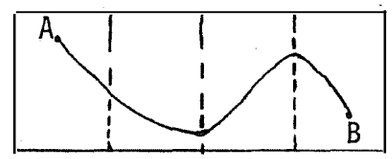
_____

Correction

On page 280 of the September 1973 issue, a figure is incorrectly shown as follows:



The correct figure is given below



SUBSCRIPTION RATES

It was resolved at the meeting of November 24 that the price of the Gazette be:

$5.00 for one year's subscription
$2.50 for a single copy

Of course, O.A.M.E. members receive the Gazette as an adjunct to their membership.

Invoices will be sent 30 days prior to expiry of a subscription, and 60 days is allowed for payment.

Orders for subscription and single copies should be sent to the Secretary-Treasurer

M. B. MacGregor

CYCLIC REVIEW

The recommendations given below were thoroughly discussed and voted on individually by the Council of O.A.M.E. They received unanimous or almost unanimous support and have been distributed to all directors of education in Ontario and to the program consultants of Mathematics.

1. Mathematics should be taught as a distinct discipline. Its applications and inter-relationships with other studies should be stressed but mathematical concepts and skills should be studied as such.

2. The importance of competence in performing basic arithmetic operations should be recognized by providing appropriate time allowance in line with the student's needs.

3. The more common algebraic manipulations should be stressed, practiced and mastered in order to establish the student's competence in handling a variety of expressions involving symbols.

4. Students should be competent in the recognition and use of basic geometric and measurement concepts. Opportunity should be given for the application of arithmetic and algebra to these concepts.

5. The guidelines should clearly indicate the sequential development of these mathematical skills and should recognize the variations in student ability and maturity.

6. Students should be given the opportunity to develop mastery in certain fields of mathematics rather than only a brief exposure to a wide range of topics. A proliferation of topics which could prevent skill development should be avoided.

7. Guidelines should be established which outline an essential core of topics and suitable additional topics for expansion and enrichment. This core content should include some topics which illustrate both structure of mathematics and mathematical reasoning.

8. The study of mathematics, appropriate to the student's program, should be required at least to the level of compulsory school attendance.

N O T I C E

With the financial support of the National Science Foundation, the National Council of Teachers of Mathematics and the Mathematical Association of America through its Committee on the Undergraduate Program in Mathematics, is engaged in producing resource materials in all the various applications of mathematics suitable for use by both teacher and student in mathematics instruction for grades 7 - 12, i.e., the last six years of secondary school. Applications of arithmetic, elementary and advanced algebra, geometry, computing, and other more advanced topics are being worked on. In addition to the uses of mathematics in other disciplines, applications of mathematics in daily life and to skilled trades will be especially emphasized. The readership of this journal is hereby requested to send suggestions regarding this project, sample problems, reference, or any other suitable materials ranging from simple exercises to extended model building and mathematical development to

CUPM
P. O. Box 1024
BERKELEY, California 94701
U. S. A.

The readership is reminded that through hobbies or previous employment it may know of special applications that might otherwise escape notice. The editors of the Gazette would appreciate copies of material sent for possible publication in the Gazette.

CHAPTERS OF O.A.M.E.

From the constitution of O.A.M.E.

> A group of members who organize and act in harmony with the objectives and programs of the provincial organization may be recognized annually as a chapter by a simple majority vote of the members present at the annual general meeting of the organization.

In order to implement this procedure as smoothly as possible the executive requests that (1) executives of present chapters (2) groups intending to seek chapter status provide

M.B. MacGregor

Secretary-Treasurer

by May 1, 1974 with an application containing

(1) Name of group
(2) Geographic area to be included
(3) Sponsoring members of O.A.M.E. (minimum of 4)
(4) Names and addresses of protem executive (at least one)
(5) Description of activities undertaken over the past year
(6) Description of planned future activities
(7) Membership fee

NOTE: By Law 9

<u>Affiliation of Chapters</u>

(1) Each chapter may extend its membership beyond individual members of O.A.M.E.
(2) Each individual member of O.A.M.E. may designate one chapter as that member's prime chapter.
(3) Each chapter is to receive from the Secretary-Treasurer of O.A.M.E. a payment equivalent to the membership fee of that chapter for each individual member of O.A.M.E. who designates that chapter as his prime chapter up to a maximum of $5.00.
(4) Each individual member of O.A.M.E. shall have the right to attend the in-service activities of any chapter on the same basis as the individual members who have designated that chapter as their prime chapter.

ONTARIO ASSOCIATION FOR MATHEMATICS EDUCATION

Past Presidents: Andrew Vertesi

Donald Attridge

President: David Alexander

Vice-President:

John C. Egsgard

Executives:

Thomas J. Griffiths

Ross W. Leigh

Joan R. Routledge

Secretary-Treasurer: Morley B. MacGregor