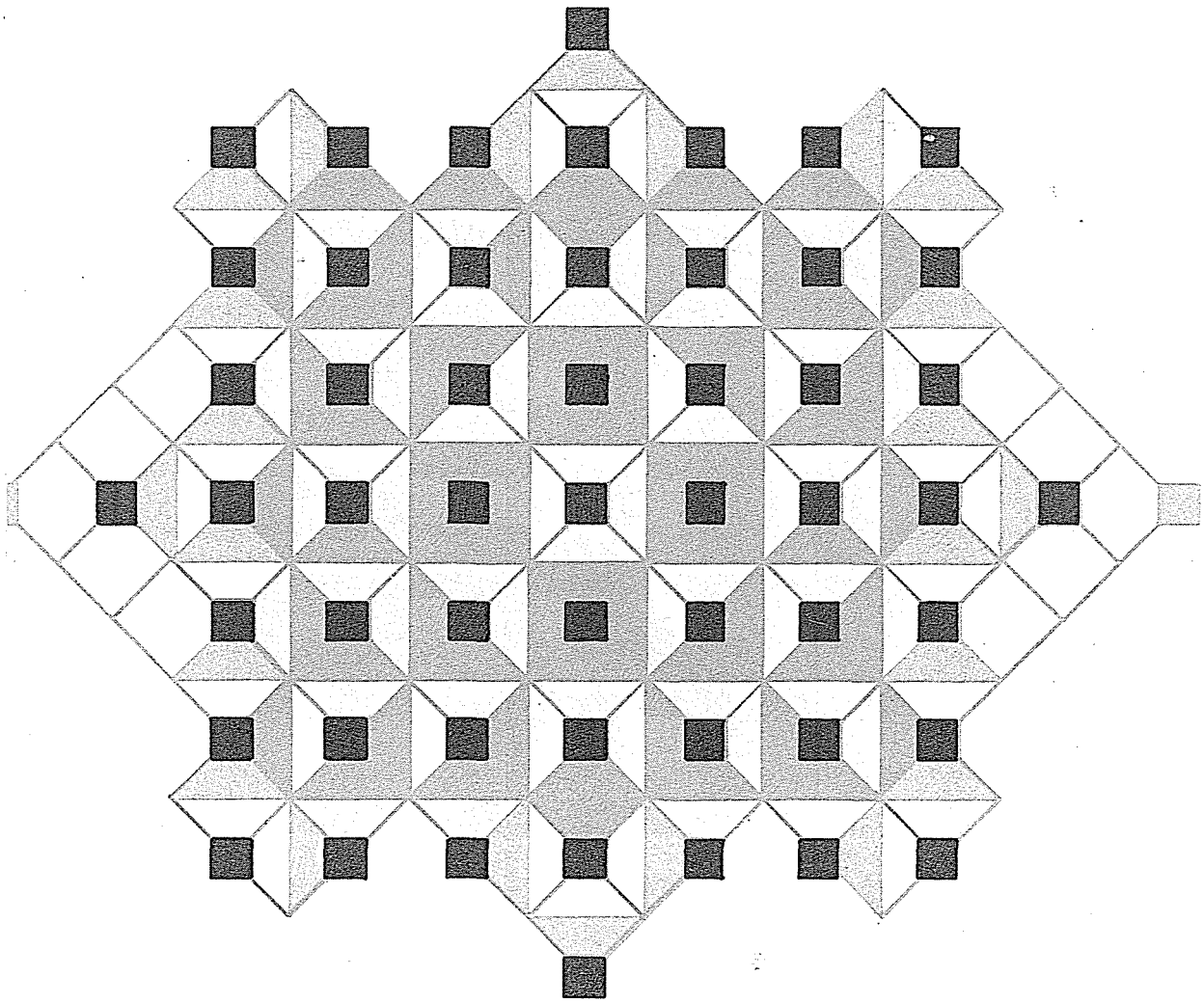
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THE ONTARIO ASSOCIATION FOR MATHEMATICS EDUCATION

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ONTARIO MATHEMATICS GAZETTE

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Editorial

Goodbye to us - it is with a great deal of regret, liberally mixed with astonishment, that we must inform you that this is the last issue of the Gazette we will edit. It was announced at the last meeting of the O.A.M.E. concil that our tenure had ended and that Mr. Arn Harris, of the Faculty of Education at Western, will be the new editor. We wish him luck - and hope that he will achieve a somewhat more effective liaison than we evidently did with the publications committee of the O.A.M.E., so that at the very least, he will be privately informed of the termination of his editorship before the public announcement.

Looking back over the three years we were editors, it is difficult to find anything was really accomplished, other than perhaps improving the superficial aspects of the Gazette. Our goal has been clearly stated - to involve the readers in an on-going discussion on matters of mutual concern. Here we have failed. No mathematical educator in this province can be unaware of the general dissatisfaction with the product of his labours - yet little of this has penetrated to the Gazette's pages. One need only look at the amount of remedial mathematics being done at the post-secondary level today to realize that curricula and teaching methods need to be severely scrutinized and - dare we say it - incompetent teachers asked to find some other profession where they can do less harm. But, reading through the reports/newsletters/journals that inundate our desk, the overall impression is one of complacency, half-understood half-truths, preoccupation with mathematical trivia, and an abysmal lack of interest in what industry, universities, and the outside world in general require.

Come on - there are good, interested teachers out there, we have met a lot of them, and a pretty dissatisfied lot they are; use the Gazette - anonymously, if you have to - before the jargon experts have completely taken over.

SHOULD WE TEACH 'MODERN' MATHEMATICS?

Jean A. Dieudonné

An affirmation from a founder of Bourbaki of the principles of the new curricula in mathematics

René Thom recently[1] presented his views on so-called "modern" mathematics in an article published in the December 1974 issue of this journal. In my opinion, his paper contains some keen insights into the nature of mathematical thought - such as would be expected from this first-rate mathematician - and some sound common sense, together with some curious misconceptions and pronouncements which, for lack of a better word, I can only label "parochial." An analysis of the article is not made easy by the fact that Thom constantly mixes three different questions which the uninformed reader may have difficulty in dissociating from one another: (1) the meaning of "rigor" in (pure) mathematics; (2) the teaching of mathematics at the university level; and (3) the teaching of mathematics at the secondary level. In examining Thom's ideas, I think it will be clearer to tackle these three questions separately.

Rigor and Axiomatization

Underlying Thom's opinions on this subject is the belief - which I fully share myself - that the important mathematical ideas come from a handful of mathematicians. I am convinced that, since 1700, 90 percent of the new methods and concepts introduced in mathematics were imagined by four or five men in the eighteenth century, about thirty in the nineteenth, and certainly not more than a hundred since the beginning of our century. These creative scientists are distinguished by a vivid imagination coupled with a deep understanding of the material they study. This combination deserves to be called "intuition," although it has nothing in common with what is usually meant by that word in ordinary language, since it applies to "objects" which, in general, have no image in the world of our senses.

One might therefore suggest the view that it is only what these people are doing that really matters - that the

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[1] Reprinted in the last issue of the Gazette. Both articles have been reprinted here with the kind permission of the American Scientist, the authors and, in the case of Professor Thom's article, Dunod Editeurs.

creation and transmission of mathematics could entirely be done within a small company of geniuses, much in the way, we are told, that Egyptian priests or the Pythagoreans transmitted their traditions by oral communication to a chosen few. In this ideal situation, the concept of "rigor" could very well be the one that Thom advocates, with few if any harmful consequences to the development of mathematics.

Unfortunately, such a utopian dream does not seem compatible either with our present social system or with any foreseeable evolution of that system. Communication between mathematicians by means of a common language must be maintained, as Thom himself acknowledges, and the transmission of knowledge cannot be left exclusively to the geniuses. In most cases it will be entrusted to professors, who, in Thom's words, are "adequately educated and prepared to understand [the proofs]." As most of them will not be gifted with the exceptional "intuition" of the creators, the only way they can arrive at a reasonably good understanding of mathematics and pass it on to their students will be through a careful presentation of their material, in which definitions, hypotheses, and arguments are precise enough to avoid any misunderstanding, and possible fallacies and pitfalls are pointed out whenever the need arises.

It is this kind of expository writing that has been, I think, the goal of those mathematicians whom Thom calls "formalists," from Dedekind and Hilbert to Bourbaki and his successors. If this may seem overly modest, it is certainly closer to the truth to interpret their efforts along these lines than to attribute gratuitously to them such extravagant ambitions as Thom does. I doubt if he is serious when he says that the Bourbaki group ever believed that the Elements would generate new discoveries, or hoped that the fundamental structures of mathematics would "arise naturally from a hierarchy of sets." If Bourbaki's collaborators do not necessarily share Thom's opinion that mathematical structures are "imposed by the exterior world," they think, as Hilbert did, that structures arise out of problems in an unpredictable way and that it should be clear that the "hierarchy of sets" is just a convenient empty frame in which structures are inserted as they are gradually discovered.

The work of systematization of mathematics is certainly not a very exciting one, but it sometimes has its rewards. In trying to make new ideas and methods clearer and more understandable, one is almost invariably led to think of other possible approaches, which sometimes open ways of thought

For instance, when one wanted to prove that from a point P on a line D, one can draw in the plane a perpendicular to D, the "proof" consisted of "rotating" a half-line Δ with origin at P, starting with one of the half-lines, D', D and ending with the other one, D"; as the angle (D, Δ) increased from 0 to the "flat" angle, 180°, and the angle (Δ, D") decreased from 180° to 0, there should be a position in which they are equal. Certainly this is a good intuitive idea, but it is far removed from the kind of strict logical argument the professor used at the same time to prove facts which, to an unprepared child, would seem just as "obvious" as the preceeding one. My own recollection of those school years is that I never understood this discrepancy, and the idea of proof remained a mystery to me for a long time.

My opinion is that before age fifteen no axiomatic system at all should be introduced. This does not rule out attempts at logical deduction; on the contrary, every opportunity to impress children with the extraordinary power of this mental process should be eagerly seized upon. But if one wants to prove that A implies B, where A is a fairly obvious geometrical fact (for example, the existence of a bisector of an angle) and B is not obvious at all (say, that the three bisectors of a triangle have a common point), it is certainly not necessary to know that A is the 426th link in a sequence of propositions starting from an axiom

available (in two or three dimensions) to high school students instead of the incredible, apparently irrelevant dissections of triangles, where every step is made to appear to be a conjurer's trick? Can Thom really believe that linear algebra in a two-dimensional real vector space is an "abstract" subject beyond the grasp of a boy or girl of fifteen, when every one of the basic concepts can be made visible on the blackboard and every axiom has an immediate intuitive meaning?

His diatribe against algebra would be justified if "algebra" were either the "blind application of arithmetical rules," from which he and I suffered during our school years, or the purely abstract theory of groups, rings, and fields. But now that the basic principle of modern mathematics is to achieve a complete fusion between "geometric" and "algebraic" ideas, opposing "geometry" to"algebra" as Thom does is simply meaningless, provided of course that "geometric algebra" (to use the happy expression coined by E. Artin) is taught as a whole, with every algebraic notion or theorem accompanied by a geometric "translation." It is also chiefly in this context that I would like to see notions such as those of group or ring introduced to the students, when one has geometric examples to illustrate them, such as the group of plane rotations (the "angles"!) or the ring of endomorphisms of the plane (as vector space). Of course, the "abstract" theory of groups or rings should have no place at all in secondary schools.

Once the basic theorems of Euclidean geometry have been established by linear algebra (without coordinates, of course!), there is nothing to prevent the bright student from tackling the classical problems on triangles or conics if he is so minded. I do not quite share Thom's belief that this kind of exercise has a great educational value. I suggest that if one wants to introduce "non-useful" material, there are plenty of other choices which certainly have more relevance to modern ideas in mathematics and physics that the traditional problems. To prove, for instance, that the rotation group in three dimensions is a simple group does not need more "geometric algebra" than to prove the existence of the nine-point circle. Yet this exercise, if not immediately "useful," has, to my mind, far more significance.

When I reflect on the curious reluctance of Thom and many of his colleagues at the university level to abandon

the traditional way of teaching mathematics in the secondary schools--even if they well realize that it lacks both relevance and rigor--I think it comes from the instinctive feeling that "modern" mathematics, if badly taught, might yield much worse results. Unfortunately, there are plenty of examples to show that such fears are not unfounded. If many university teachers already have a deplorable tendency to emphasize abstraction at the expense of intuition, one should not be extremely surprised at the fact that high school teachers, who have no possibility of grasping the deep motivation behind the introduction of new mathematical concepts, should adhere to the letter of the new curriculum instead of understanding the spirit which made it necessary. Just like their colleagues at the university, they also have discovered that it is far easier to teach the manipulation of abstract concepts, even to very young children, than to give them an intuitive grasp of the realities behind these abstractions. Hence their relish for logic in particular--and the inordinate amount of it they are prone to include in their teaching.

Of course one may contend that training in logical thought has some usefulness; while only wide-eyed idealists truly believe that logic can and will one day govern human behavior, it can readily be admitted that a little more of it might do no harm in the general populace. But the ludicrous examples that Thom stigmatizes at length should convince educators to limit Boolean algebra strictly to where it belongs, namely, mathematics and science in general. Even within those limits, educators should beware of mistaking it for a panacea instead of a tool. Logic is no more mathematics than atom smashers are nuclear physics; if you are clever enough to imagine a proof, it will help you carry it through, just as the nuclear physicist needs complex and costly apparatus to show that his insight into the nature of atomic forces is correct. In both cases, imagination always is the irreplaceable initial spark.

One could almost believe that in secondary school teaching, there is a built-in urge to run everything into scholastics, in the worst possible sense of the word. Most working mathematicians and scientists are genuinely distressed when they see the old scholastics, which they had accepted as a fact of life and learned to live with, replaced by a far more aggressive and silly brand under the banner of "modernism."

Nevertheless, I still hope that the present turmoil will eventually settle down to some moderately reasonable compromise. With the pressure of science and technology on human affairs steadily increasing, we cannot afford to let future managers and technicians spend most of their precious years of school life absorbing useless knowledge taught by obsolete methods, even if we recognize the necessity for some element of "play" in the curriculum. The anguish of parents who cannot understand their children's vocabulary will die down with the arrival of the next generation. And finally, if the people responsible for drawing up school curricula may be persuaded to consult professional mathematicians in order to understand the relevance of their decisions to science as it is practiced in the university and beyond, we may yet witness one day some sensible teaching of mathematics from kindergarten to graduate school.

Professor Dieudonné is one of the founders of Bourbaki, and one of the world's leading mathematicians. At present he is emeritus Professor of Mathematics at the University of Nice. He has spent many years as visiting professor at universities all over the world and is familiar with the teaching of mathematics both in the United States and Europe.

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THERE'S A FOREIGNER AMONG US

M A T H E M A T I Z A T I O N

David Wheeler

It is more useful to know how to mathematize than to know a lot of mathematics. Teachers, in particular, would benefit by looking at their task in terms of teaching their students how to mathematize rather than teaching them mathematics.

Let me try to put some colour into these plain statements. I'll begin with an analogy which I find suggestive. When I was taught "Art" at school, I was shown a very limited number of techniques - mainly pencil-sketching and water-colouring - and practised them in situations which were mainly imitative. I cannot remember that I ever made an association between what I was doing in art lessons and the actual examples I met in the environment or pictured in books and magazines. Many of the art lessons I see in schools now are quite different. The teachers offer a very wide range of materials together with techniques that exploit them, encourage the students to show initiative and originality, to use their own particular talents and sensitivities, to cooperate in joint projects, etc. In short, to function as artists even though everyone knows that most of them will not become artists. The "works of art" produced by the students are usually displayed as if, indeed, they were objects of value in their own right.

Why do the majority of mathematics teachers not encourage their students to "function as mathematicians" rather than try to lead them through a relatively restricted number of skills, mainly practised in a routine imitative fashion, and which bear little obvious relation to mathematics as it exists beyond school? One reason is that they may not know how to do it, and I wouldn't want to minimize the difficulties of teaching to this kind of goal. But I'm sure that a prior reason is that they don't believe it is possible, just as my art teacher didn't believe we could function as artists.

Reprinted from the Canadian Mathematical Congress Notes (Vol. 7, No. 2), with their permission, and with the permission of the author. The article is an expansion of Mr. Wheeler's lecture at the Summer meeting of the Congress.

The analogy between teaching art and mathematics cannot possibly be exact, of course. Art has an immediately available and tangible product in a way that mathematics usually does not; art is essentially expressive whereas mathematics is (in the proper sense) essentially intellectual; naivety can be a positive value in art but not, I think, in mathematics. Whether anything of the analogy remains after these elements have been removed depends on the possibility of establishing that children can function as mathematicians.

One pointer to an affirmative demonstration may come from the well-known phenomenon of mathematical precocity which disproves any necessary correspondence between mathematical achievement and chronological maturity. Although the cases are exceptional, they decisively take mathematics out of the class of activities which no one could expect children to be able to do except by imitation. I don't see children, however exceptional, being able to function as historians, or as lawyers, or as psychologists, for instance, since these are extremely complex functionings that involve subtle relationships between several frames of reference. But I would hypothesize that mathematics belongs with art, music, writing, and possibly science, as one of a class of activities that require only a particular kind of response to be made by an individual to his immediate, direct experience. To put it the other way round, these latter activities do not characteristically depend on experience-by-proxy. We remember, in our own field, the story of the child Pascal re-creating for himself the forbidden fruits of Euclid.

Now, it would be my contention that school teaching is mostly geared to learning-by-proxy - one learns at school mainly what others have achieved in each particular field. Although most kindergarten and primary grade classes relate the beginnings of mathematics to children's direct experiences of sorting, counting, measuring, etc., it does not seem long before it shifts into "passing on" mathematical skills and concepts, and by the time the students gets into a high school algebra, geometry or trigonometry course, the element of direct experience has almost disappeared.

I don't want to oversimplify this. Some learning-by-proxy is inevitable for economical reasons; and desirable because everyone needs the enrichment of his own experience via the experience of others. Also, I don't intend "direct experience" to mean only the experience of physical, tangible

mathematics is really open, wide open, and its future inexhaustible because there is no limit in principle to the situations and the problems that can be mathematized.

I want, then, teachers to think of mathematics in terms of the act of mathematization rather than the accumulated body of mathematical knowledge because (a) it will force them to think positively about the mental capacities of their students rather than negatively about the tremendous amount of mathematics there is to learn, and (b) it will, if it succeeds, be a more realistic way of educating students to face an unknown mathematical future. The shift of emphasis wouldn't mean that students would not learn a good deal of the mathematics that it is conventionally expected they will learn at school, but that this would be a by-product of the exercise of their own mathematical powers.

The special mental functioning that I call mathematization may be operating even when there is no recognizable mathematical product coming out of the activity. I am not sure if Boole was the first to propose that the natural operations of the mind are algebraic in character; I am sure that this is what he meant and that it is unfortunate that his discovery has been debased, for many people, into the theorem that two-valued sentence logic is isomorphic to an algebra of sets. Much more recently, the Swiss epistemologist Piaget has asserted that the structures of mental operations are isomorphic to the "mother structures" used as foundations by Bourbaki. The evidence for an algebra of the mind is even easier to find than in these two sources. It has been repeatedly stressed by observers that within a very short time of beginning to speak, young children utter grammatically correct phrases and sentences that are not copies of any that they have heard others speak. It is clear that they could not operate autonomously within the grammar of the language without the capacity to handle classes (nouns, verbs), inclusion and intersection (adjectives, adverbs), relationships (prepositions), transformations (tenses) and substitutions (pronouns). I mention only the simplest and most obvious features: in fact the transformational requirements of a confident use of grammar are very complex and still defy complete analysis. But the chief points at issue are that (a) the child's mastery of grammar can only be adequately described in terms of mathematical operations, and (b) this mastery is not derived by imitation. The algebraic character of mental functionings seems proved by this one example even though the evidence can be multiplied by considering other

functionings - visual perception, for instance.

The bonus accruing from the example of speech is that it shows the mental functionings of children are already algebraic long before they attend school and start "learning mathematics", and that teachers can assume from the beginning that the mathematical powers of their students are in operation. Indeed, the teachers' job becomes more of providing children with the raw material that will yield familiar mathematics when it is mathematized. A number of articles and books discuss in some detail how this may be done, particularly with children in elementary schools*, so I will end with a sketch of one example suitable for secondary school level which may serve to illustrate how this approach can parallel the coverage of the traditional method but by-pass the usual build-up.

A short silent animated film shows a fixed circle with a fixed point in its interior and a variable circle which touches the given circle and passes through the given point. The trace of the centre of the variable circle is shown. (This is the first part of the scenario of a film made by the Swiss educator, J. L. Nicolet.)

The teacher gives technical vocabulary - ellipse, major and minor axes, foci - if and when necessary. (These words are not related to the mathematizing process.) In oral discussion or short periods of group or individual investigation, the teacher asks the students to consider certain questions - e.g. what are the symmetries of the trace? what would be the effect of changing the position of the given point? how would one construct an accurate drawing corresponding to a frame of the film? study the behaviour of particular radii of the two circles during the motion.

Investigation of these questions will throw up new questions - e.g. shifting the given point generates a family of curves yet the length of the major axis of each member of the family is the same; why? the sum of the distances of the centre of the moving circle from the fixed point and the fixed centre appears to be constant; why?

* For example, C. Gattegno, The common sense of teaching mathematics, New York: Educational Solutions, 1974, and M. Goutard, Mathématiques sur mesure, Paris: Classiques Hachette, 1970.

The last question can become the chosen basis for a definition. The lengths of the axes and the positions of the foci are determined by the situation and can be formalized.

Further exploration of the situation can yield, for example, the "reflection property". Opening out the situation by asking what would happen if (a) one point of the circumference of the "given" circle remained fixed but the length of its radius were increased indefinitely, or (b) the given point were placed exterior to the given circle, gives an entry into the related generations of the parabola and hyperbola.

All this ground can be covered with a minimum of technical equipment of the part of the students, essentially through their use of their perceptions and the control of their own geometrical imagery. At each stage the teacher only asks questions and uses the students' answers to determine how much more he can ask of them.

The O.A.M.E. Publications Committee have made arrangements for the circulation of some materials from the Grand Valley Math Association. These are:

(i) Grade 9 Preparedness Test
(ii) Grade 13 Sample Examinations.

The publications are available at no cost to O.A.M.E. members and for a fee of $2.00 to non members of O.A.M.E. To obtain your copy, simply write

The Grand Valley Mathematics Assoc.,
161 William Street W.,
Waterloo, Ontario.

AN INEXPENSIVE METHOD OF RAPIDLY SCORING MULTIPLE CHOICE TESTS

J. B. McGreevy

After a student has written a test, he is entitled to know in the shortest possible time interval how he fared. The marking of papers is a burdensome load to all teachers and often classes will have to be carried on and the marking done in spare time. In some cases, tests are given daily or weekly in which case rapid marking is a must. In large centers, tests may be marked on data processing equipment or by a desk top calculator which has a mark-sense attachment such as the one used by Wang calculators or Hewlett-Packard equipment or the 3M Company, but due to the cost, many teachers do not have quick, close access to such equipment. The following gives an inexpensive method of rapidly marking tests of up to 34 multiple choice test questions.

The equipment required is:

1. Data processing answer cards.
2. A deep throated one-hole paper punch.
3. A felt tipped pen with red ink for marking.
4. The students should have a soft-lead pencil or a pen (preferably not red ink).
5. Multiple choice test papers.

Procedure for Marking

1. Issue multiple choice test papers to the students.
2. When the student has completed the test have him bring the test paper and his card to the invigilator.
3. The person scoring the card superimposes a punched master answer card over the student's card. Each answer that does not correspond with a punched hole is incorrect. Thus, a no choice or incomplete card is scored as incorrect. The number of wrong answers is subtracted from the number of questions to obtain the test score.

General Discussion

1. A data processing quiz answer card is shown in Example 1; while the master answer card which is of a different colour is shown in Example 2.

short tests are involved, the tests can be given consecutive numbers.

3. For a small group of students, only one answer card is required. However, as many cards as desired can be made, thus, a group of markers could score a large number of students in a short time.

4. The answer card may be used in several ways:
   (a) It may be carried around by the teacher for quick checking in the classroom. This is handy for cases in which the students do a quiz and then are allowed to try again the questions that were answered incorrectly.
   (b) If the instructor wishes to indicate to the student the correct answer (for the latter's follow-up), a red marker pen can be used to highlight the correct choice.
   (c) In order to steady the card, a jig may be used. A suitable jig is an "IBM Portapunch".
   (d) If a test containing more questions is desirable, additional answer cards may be used.

In scoring data processed answer cards by a computer, it is usually the practice to weight each question equally. If some questions have a value of one each, and other questions have a value of two or three, etc. the computer must be specifically programmed. However, with a manually scored card, the value for each question can be marked on the answer card and added up by the marker.

Quizzes of the true-false type may be answered on these cards by using choice (a) for true and choice (b) for false. The cards can be readily adapted to tests in which students are required to do alternate, or every third question or even random questions.

The system is very flexible. For instance, a teacher may have a quiz made up for two classes of different calibre of student. The advanced class would be asked to answer all questions; the other class might be advised not to attempt certain questions. In such a situation, two master answer cards would be required - one with all the correct answers punched out, the other with the deleted question numbers not punched.

If an incorrect punch has been made on the master answer card, a correction may be made as follows:
1. Punch out the correct answer choice.
2. Place the round bit of card from (1) in the spot in which the error was made and hold in position with a small piece of scotch tape.

A few blank cards may be obtained for evaluation by contacting:

Mr. J. B. McGreevy

Although a special deep throated one hole punch is normally used, for a trial evaluation any one hole punch could be adapted. If you have a particular test you wish evaluated by this card method, the writer would be pleased to punch out the master card and enclose it with your sample of student cards.

Sources of Components:

1. Cards No. CDC 5919: Contact the writer at address given.
2. IBM Portapunch: I.B.M.
3. Card Punch No. 464 - ¼ inch round hole ticket punch:
   Torch Sales Corporation
   64 Stratford Road
   Montreal, Quebec
   Phone: 514-486-6866

Some Aspects of Multiple Choice Test Papers

A great deal of thought must be given to the design of good multiple choice test questions. For instance, in mathematics the choice of answers should be based on possible incorrect computations so that the teacher may know at which stage an error crept into the student's calculations. The ingenious teacher can devise many steps in the process of quizzing so that the test may be objective.

The use of "none of the preceding" choices must be carefully considered in order to reduce the odds in the guessing game. By normal probability, a student could get 50% or better on a true-false quiz and know very little of the subject matter. The value of the true-false method of enquiry is to detect points on which the student has not grasped the concept or terminology correctly. Further study is hence indicated.

Many teachers consider that the minimum requirement on multiple choice questions should be 80%.

Others have various standards to suit the setting. For instance, the minimum requirements are sometimes set up as:

1. Five options, 70% correct
2. Four options, 75% correct
3. True-false, 80% correct

On the other hand, some teachers insist that students do their math calculations on the test paper or answer booklet to show that they actually do have mastery of the subject. The students are then requested to transfer the answer choice to a test scoring card. The advantage of this process is that it allows quick scoring but also allows the teacher to discuss the student's work with him plus showing the student's true capability.

Obviously, there are several alternatives for marking multiple choice questions, but the method presented herewith is effective for all of them.

Preferably, the student should be made aware of the minimum requirement for each test - either as a percent or as a ratio.

Suggested Readings

1. The Writing of Multiple Choice Mathematics Tests Items by Dr. Dorothy M. Horn. The Ontario Institute for Studies in Education, 252 Bloor St. West, Toronto, Ontario.

2. Mathematics Test Item Pool. This is a set of four booklets issued by the Ontario Institute for Studies in Education.

3. Preparation of Computer Programmer Aptitude Tests. J. J. Jensen, Cowles Book Co., New York.

4. Multiple choice questions to accompany textbooks and text workbooks are available from most publishing companies.

THE SCARBOROUGH JUNIOR MATH LEAGUE

Paul Zolis

Many students taking mathematics in our schools today seem dissatisfied with the mere classroom learning which they are presently receiving. There is, however, an alternative for many of the students where they can display their individual differences and mathematical ability in a different way. It is unlike a mathematics club or a project oriented venture where the student may once again be "asked" or "told" to produce something. On the contrary, it is a learning experience whereby each student is encouraged to think for himself and widen his horizons above and beyond the classroom experience; it is a competition where students can display their mathematical ability beyond their immediate grade level.

This is not a "new" or revolutionary method to make mathematics more interesting but it is an excellent way of showing students that mathematics can be challenging and rewarding.

It is with this viewpoint that we in the Scarborough Board of Education have attempted to create the Junior Math League. Our aim is not only to make mathematics more interesting and rewarding but also to broaden the student's knowledge beyond the "text book" questions or assignments encountered in everyday mathematics classes.

Our Junior Math League's format and rules are as follows:

LEAGUE FORMAT:

1. The League consists of 14 collegiates in three divisions.

2. Each school will play one game with each of the other 4 teams in its division.

3. The winners of the EAST, WEST and CENTRAL divisions will meet in the playoff game to decide the championship.

4. A team must consist of five players from year 1 (grade 9) or year 2 (grade 10) and may not have more than 3 players from grade 10.

5. Each game is worth 2 league points for a win, 1 for a tie and 0 for a loss.

6. A winner from each division will be determined by league points. Accumulated game points will be used to break a tie.

7. The names of the four top players in each grade will be published (based on accumulated game points).

8. Identical question papers will be supplied for each game in sufficient numbers.

9. Papers will be sent to the home school coach, by courier, on the day of the game.

GAME FORMAT

1. A typical game consists of two written papers consisting of 5 questions each and 10 minutes is allowed for each paper. The third paper is usually a team question where students work as a team to arrive at an answer to a "logic" problem. Finally, the last part of the game is the oral section consisting of 15 questions where the students are required to answer as quickly as possible.

2. Each written test contains 5 questions worth 2 marks or zero. Tests should be marked by the coach of the opposing team and team totals listed on the board.

3. Oral questions score 2 for a correct answer and -1 for a wrong answer. Each team is allowed one chance to answer and points go to the team with the first correct response. A responsible time limit should be set for each question (eg. 1 min.). One coach should keep a running score on the boards and call the time limit. Students should be allowed pencil and paper but must work individually.

Each game lasts for about an hour, and coffee and donuts are supplied by the home school. At the end of the season (games are scheduled from January 15 to February 28) the three students with the highest standings from the three divisions are awarded Mathematics pins at a special presentation. Furthermore, the teams winning in their respective divisions are invited to compete for the Scarborough Championship in a final sudden death game. The winner of this match is then invited to compete with the winning school from the Etobicoke Board of Education for a trophy and pins. These final two competitions are also televised on ETV a few weeks later and the students have an opportunity to survey their achievements or errors.

In the final analysis, all of the participating students find the competitions very rewarding and are forever asking questions and arguing about the various test papers and their solutions. They also find these competitions very valuable when they come to write the Junior and Senior Mathematics Contests scheduled for March or April of each year.

A typical game might consist of the following test items:

Part A: 10 Min.
(2 marks each)

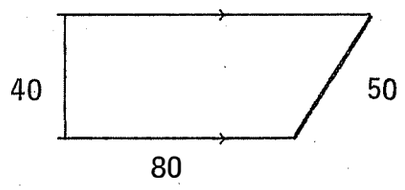
1. A 3-inch block is painted and then cut into twenty-seven 1-inch blocks. How many of the 1-inch blocks have:
   (a) 3 sides painted? (1 mark) _____
   (b) 1 side painted? (1 mark) _____

2. A house worth 9000 dollars is sold by Mr. Smith to Mr. Jones at 10% loss. Mr. Jones sells the house back to Mr. Smith at a 10% gain. Is the result of the two transactions a loss or a gain and by how much?

   _____

3. The square of $5 - \sqrt{y^2-25}$ is _____

4. What is the perimeter of the trapezoid?



   _____

5. In the equation $y = mx-2$, when $x = -8$, $y = 24$; what is the value of y if $x = 10$?

   _____

PART B: 10 Min.
(2 marks each)

1. $\frac{1}{3}$ per cent of a number is 24. What is the number?

_____

2. The cost of importing five dozen china dinner sets, billed at $32 per set and paying a duty of 40% is _____

3. If $9x + 5 = 23$, the numerical value of $18x + 5$ is

_____

4. A motorist travels 120 miles to his destination at the average speed of 60 miles per hour and returns to the starting point at the average speed of 40 miles per hour. His average speed for the entire trip is _____

5. A snapshot measures $2\frac{1}{2}$ inches by $1\frac{7}{8}$ inches. It is to be enlarged so that the longer dimension will be 4 inches. What is the length of the enlarged shorter dimension?

_____

PART C: 10 Min.
SCORE: Correct Answer 5
No answer 0
No penalty for wrong answer

This questions is to be solved by your team working as a group. (Do not allow the opposition to overhear your reasoning.)

During lunch hour, a group of five bosy from Mr. Zolis's ninth grade homeroom had visited a nearby grocery store. One of the five had taken an apple.

Jim s d, "It was Hank or Tom."
Hank said, "Niether Eddie nor I did it."
Tom said, "Both of you ar lying."
Don said, "No, one of them is lying, the other is speaking the truth."
Eddie said, "No, Don, that is not true."

When Mr. Zolis was consulted, he said, "Three of these boys are always truthful, but two will lie every time."

CAN YOU DETERMINE WHO TOOK THE APPLE?

_____

Orals

1. If $\frac{x}{y} = \frac{3}{5}$, find $\frac{6x-5y}{4x+2y}$ _____

2. If x is a real number, what is the smallest value of
$$\frac{77(x-2)^2 + 177}{77}?$$ _____

3. If $\frac{3ab}{a+b} = 2$, find $\frac{1}{a} + \frac{1}{b}$ _____

4. Given $x - \frac{1}{x} = 3$, what is the value of
$$x + \frac{1}{x^2}?$$ _____

5. Evaluate $\frac{321^2 - 123^2}{222}$ in simplest form _____

6. How many more edges than vertices does a cube have? _____

7. The angles of a triangle are in the ratio 2:3:5. What is the size of the smallest angle? _____

8. Express as a mixed radical in simplest form
$$\sqrt{72} - \sqrt{98} - \sqrt{50}$$ _____

9. What is the value of $ab^3 - a^2b$ if $a = 2$ and $b = -3$? _____

10. How many different sets of 3 cards could you choose from a set of six different cards? _____

11. If $\frac{3}{4} - \frac{2}{3} - \frac{2}{3} - \frac{1}{n}$, what is the value of n? _____

12. If x is negative, which of the following is the greatest?
$$\frac{26}{48}x, \frac{22}{48}x, \frac{22}{42}x, \frac{22}{40}x, \frac{2}{4}x$$ _____

13. What is the length of the hypotenuse of a right angled triangle if the other two sides are $3\sqrt{3}$ and $3\sqrt{6}$? _____

14. Evaluate $\sum_{n=3}^{5} (-2)^n =$ _____

15. If $xy = 12$ and $x^2 + y^2 = 64$, find the value of

$$\frac{x}{y} + \frac{y}{x}$$

_____

If you are interested in obtaining more information and other tests, please write to:

Mr. Paul Zolis
Department of Mathematics

or

Mr. J. W. Fencott
Coordinator of Mathematics

C H A P T E R S O F T H E O. A. M. E.

John Johnston, Russell Prescott, et al
St. Lawrence College of Applied Arts and Technology

Ray Miske
Grand Valley Mathematics Association

Frederick Maskell

Robert Tuck

M A G I C S Q U A R E S

John Ebsary

Any person who has taken mathematics in High School has probably, at one time or another, encountered magic squares. In this essay I will analyze magic squares, with the emphasis on 9 squares, but I will also analyze magic squares with 25 and 49 squares and show some of the formulas I used to get the solutions.

We will begin by examining the magic square with 9 squares and the numbers 1 to 9 inclusive.

Example 1:

8	1	6
3	5	7
4	9	2

The middle number is five and in every direction the numbers add up to fifteen.

Is there a way to calculate what the middle number will be and what each row will total?

Yes, if you take the sum of the numbers 1 to 9

$$(1 + 2 + 3 + \ldots + 8 + 9) = 45$$

and divide by the number of squares in any row (i.e. 3 squares) you will have the total of each and every row, (RULE #1). Also if you divide the sum of the numbers 1 to 9 inclusive (i.e. 45) by the number of squares (i.e. 9) you will get the number in the middle square (RULE #2). I have tested both Rule #1 and Rule #2 and they appear to hold true in all cases, so we will assume that they are correct unless we find exceptions to these rules further on in this investigation.

Now we will examine three magic squares to find some other rules.

Example 1:

8	1	6
3	5	7
4	9	2

(Numbers are written down in increasing order)

(Middle Number)

1 2 3 4 5 6 7 8 9

Example 2:

85	15	65
35	55	75
45	95	25

15 25 35 45 55 65 75 85 95

Example 3:

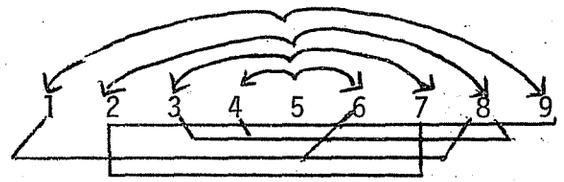
16	1	10
3	9	15
8	17	2

1 2 3 8 9 10 15 16 17

The sum of the numbers symmetric about the middle number in each case equals twice the middle number. (RULE #3a)

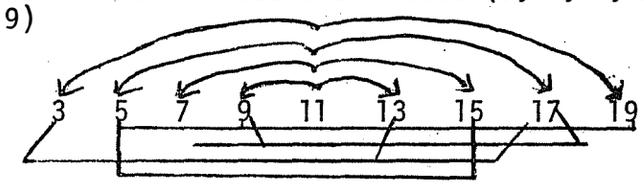
We can also use a diagram to show which three numbers will equal the total (in this case 15). (RULE #3b)

Rule #3a and 3b.



We could use this as a method for deciding whether the numbers will work in a magic square.

Let us try it on an example that works and on one which does not work. We will use the numbers (3, 5, 7, 9, 11, 13, 15, 17, 19)

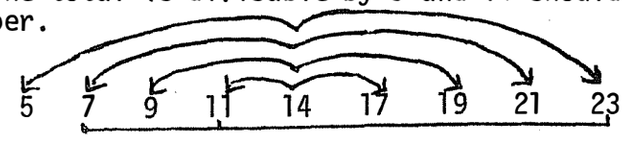


These numbers should work in a magic square as they will follow the pattern of Rule #3.

Example 4:

17	3	13
7	11	15
9	19	5

Now we will try the following numbers which can follow rules 1 and 2. The total is divisable by 9 and 14 should be the middle number.



The above pattern obeys Rule #3a but does not follow Rule #3b.
These numbers should not work in a magic square and I found no possible solution for these numbers in a magic square.
We now have a method for deciding whether or not the numbers will actually work at all. (RULE #3a and 3b)

Let us look at example #1 again. If you put a border square along the top and right side and fill in the corner square, the square will look like this:

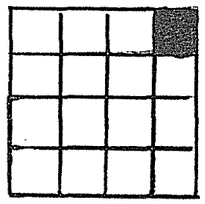
Example #1:

8	1	6
3	5	7
4	9	2

"Put the smallest number in the middle top cell of the original square. Then proceed diagonally upward toward the right with successive integers. When the operation takes us out of the original square we get back into the square by shifting clear across the square either from top to bottom or right to left as the case may be, then continue going upward diagonally toward the right. When this operation leads us into a square which is already occupied put the next number below the other one and continue."[1] (For example, when 3 meets 1 put 4 below 3 then continue going diagonally upward to the right with 5, etc.). (The shaded square is to be regarded as being occupied.)

RULE #4.

	9 ↓	2 ↓	
8	1	6	←8
3	5	7	←3
4	9	2	



If we try this with the other magic squares we have used so far, we will find Rule #4 to hold true.

But I have found this magic square that works but it does not follow the above rule #4 or the Rule #3 where we write out all the numbers in a straight line in increasing order.

Example 5:

15	2	13
8	10	12
7	18	5

2 5 7 8 10 12 13 15 18

We will say that Rule #4 functions only on numbers that increase by a constant.

Now let us try Rule #4 on a magic square with 25 squares.

Example 6:

	18	25	2	9	
17	24	1	8	15	←17
23	5	7	14	16	←23
4	6	13	20	22	←4
10	12	19	21	3	←10
11	18	25	2	9	

Sum of (1-25) = 325
each row should be = 65
middle number should = 13

Let us go back to 9 magic square for a moment.

Example 1:

8	1	6
3	5	7
4	9	2

In Example 1 we see that (1) going from the bottom left hand corner diagonally to the top right hand corner that the numbers differ by 1, (2) going horizontally through the middle the numbers differ by two, (3) going diagonally from the bottom right hand corner the numbers differ by 3 and (4) going vertically through the middle, the numbers differ by 4.

If we check example 6 we see that:

(1) Going from bottom left corner to the top right hand corner the numbers differ by one.

(2) Going vertically through the middle the numbers differ by six. But the other two directions are inconsistent. So I thought that maybe I could find another solution with each direction increasing by a consistant amount. This is what I found:

Example 7:

3	22	5	20	15
24	8	9	14	10
25	19	13	7	1
2	12	17	18	16
11	4	21	6	23

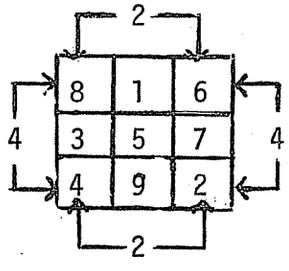
← difference of 6

difference of 1 | difference of 4 | difference of 5

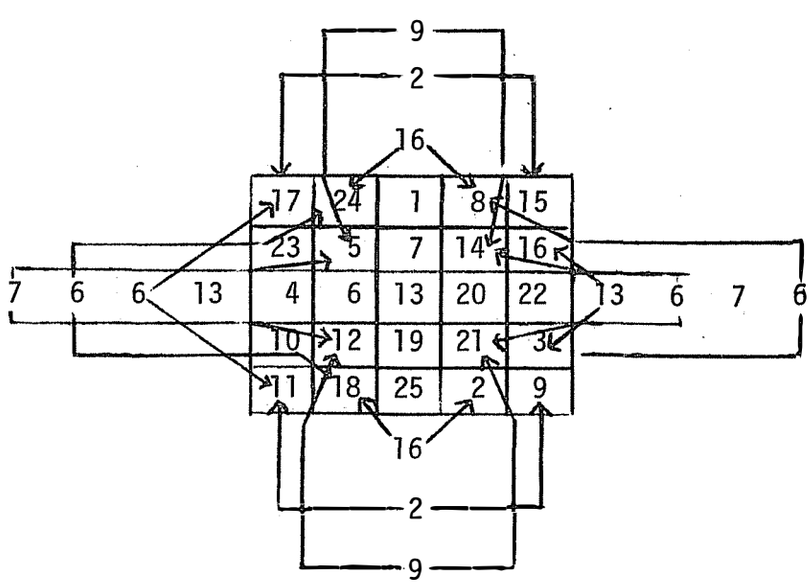
Unfortunately this solution does not seem to follow any pattern like Rule #4. But in any magic square with 25 numbers, if they will work for Rule #4 they will also work and follow a pattern like example 7.

Before going on to 49 magic squares, let us see if there is anything else we can use to help solve it. If we check all of the examples so far we can see that the numbers in the magic squares have a difference compared to other numbers which is equal to the difference between the opposite pair or numbers.

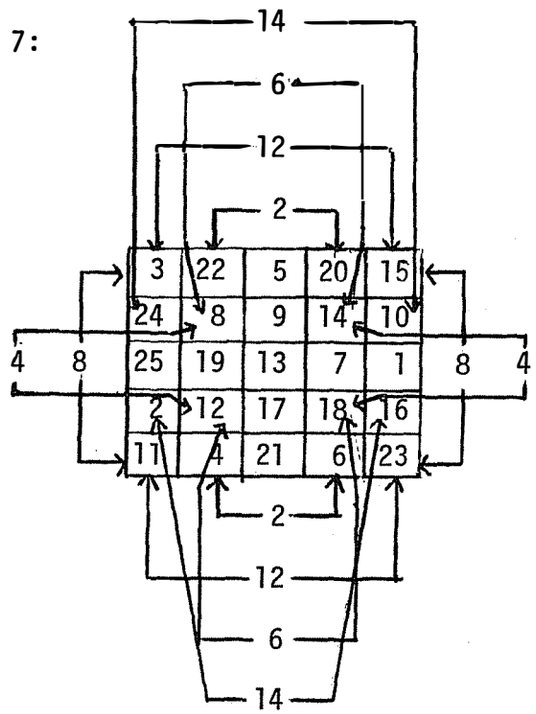
Example 1:



Example 6:



Example 7:



Now let us see of we can get two or more possible solutions for the 49 magic square. Let us try using rule #4 first:

Example 8:

	31	40	49	2	11	20	
30	39	48	1	10	19	28	←30
38	47	7	9	18	27	29	←38
46	6	8	17	26	35	37	←46
5	14	16	25	34	36	45	← 5
13	15	24	33	42	44	4	←13
21	23	32	41	43	3	12	←21
22	31	40	49	2	11	20	

rule #4 works.

Now let us try to get a solution following example #7 where the numbers differ by a constant in the four directions passing through the middle square:

4			7			28
	11		13		27	
		18	19	26		
48	41	33	25	17	9	1
		24	31	32		
	23		37		39	
22			43			46

This is the skeleton of the solution following example #7. But there are 24 spaces left to fill and the chances of getting the correct number in the correct square are very slim (24:1). Let us try comparing the 9 numbers of the magic square (eg. #1) to the positions of the numbers in the inner 9 squares of the 25 magic square (eg. #9).

Example 1:

8	1	6
3	5	7
4	9	2

Example 7:

3	22	5	20	15
24	8	9	14	10
25	19	13	7	1
2	12	17	18	16
11	4	21	6	23

1 2 3 4 5 6	7 8 9	10 11	12 13 14	15 16	17 18 19	20 21 22 23 24 25
6 numbers	7 8 1		4 5 6		9 2 3	6 numbers
	3 nos	2 nos	3 nos	2 nos	3 nos	

Now let us compare the 25 magic square (example 7) to the skeleton of the 49 magic square that we have:

3	22	5	20	15
24	8	9	14	10
25	19	13	7	1
11	4	21	6	23

4			7			28
	11		13		27	
		18	19	26		
48	41	33	25	17	9	1
		24	31	32		
	23		37		39	
22			43			46

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	
8 nos.								1		3		5	2 nos			7	8	9		2 nos.		
										5 nos.								5 nos.				

23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41				
11	12	13	14	15	2 nos		17	18	19			2 nos		21		23		25				
5 nos.							5 nos.							5 nos.								

42	43	44	45	46	47	48	49
8 nos.							

the numbers 10, 12, 16, 20, 30, 34, 38 and 40 should go in the inner 8 spaces and the rest of the numbers should fill the outer part of the square:

Example 9:

4	48	36	7	8	44	28
29	11	12	13	38	27	45
15	30	18	19	26	20	47
49	41	33	25	17	9	1
35	16	24	31	32	34	3
21	23	10	37	40	39	5
22	6	42	43	14	2	46

We have found two possible solutions to the magic square with 49 squares and there may possibly be one or two more solutions but this is as close as I could come to another solution:

Example 10:

28	8	47	1	45	42	4
14	27	34	9	38	11	36
15	30	26	17	18	40	29
7	13	19	25	31	37	43
21	10	32	33	24	20	38
44	39	12	41	16	23	6
46	48	5	49	3	2	22

= 169

= 181

Every direction equals 175 except for the two indicated and no matter how I juggled the numbers around I could not find a possible solution.

"Mathematicians have never pretended that magic squares were anything more than amusement, however much time they spent on them, although the continual study devoted to this puzzle form may incidentally have cast some light on relations between numbers. Their chief appeal is still mystical and recreational."[2]

REFERENCES

(1) Howard Eves. An Introduction to the History of Mathematics. p. 46.

(2) James R. Newman. Machines, Music and Puzzles. p. 2436.

THE SOCIAL SCIENTISTS' VIEW OF HIGH SCHOOL MATHEMATICS

Ian B. MacNeill

INTRODUCTION

Within recent times a substantial interest has developed in the study of the social sciences and a large percentage of college-bound high school students will register for courses in one or more of these disciplines. Hence it it appropriate to consider more seriously than has been the case in the past the mathematical needs of the social-science-bound student. For this reason, the Secondary School Statistics Project at The University of Western Ontario has conducted a poll of the social scientists of Ontario Universities for the purpose of discovering their opinions on mathematics curricula. It seems that the opinions and sensibilities of social scientists have never yet been directly considered by those who have been responsible for designing mathematics curricula for the Ontario secondary school system.

DESCRIPTION OF SURVEY

Each of the Ontario universities that was public as of 1972 was canvassed. Within each university all departments of Anthropology, Economics, Geography, Political Science, Psychology and Sociology were polled. The results for Anthropology and Sociology were combined since most universities have a single department of Anthropology and Sociology. The first mailing of the questionnaire brought a response of approximately 60%. A second mailing resulted in a further response, increasing the total percentage to just under 80%.

The types of questions asked were of several kinds. First of all, opinions were solicited as to the present and future importance to a high school mathematics program of various aspects of mathematics. Secondly, a number of questions were asked about specific aspects of statistics and how valuable they would be as additions to the mathematics program of the secondary schools. Then comments were requested on how the social scientists felt about the present mathematics background of their students.

This research was supported by the Ministry of Education, Ontario through a grant-in-aid of research.

IMPORTANCE OF VARIOUS MATHEMATICAL TOPICS FOR SOCIAL SCIENCE DISCIPLINES

The first set of questions asked the social scientists to indicate how valuable to their undergraduate programs were each of 11 major mathematics topics. The individual social scientist's response consisted of a number of a scale from 1 to 5 with 1 indicating "of no use" and 5 indicating "indispensible". The average response, by discipline, is tabulated in Table 1* and the overall average response is graphed in Figure 1 where the scale is also indicated. The graph presents the subject matter areas arranged from largest to smallest. The tabulation indicates a substantial range in the average responses with Statistics, Algebra and Probability at the top of the range and Analytic Geometry, Euclidean Geometry, Trigonometry and Actuarial Mathematics at the bottom. If one regards the respondents as being equivalent to a random sample from the population of all social scientists in Ontario universities then sampling fluctuations need to be taken into account. The question that then needs to be asked is "Which, if any, of the differences in the averages is large enough to be considered statistically discernable?" To obtain an indication as to the significance of the differences among the five disciplines and the differences in perceived importance among the eleven difference branches of mathematics a statistical analysis was performed using multivariate analysis of variance. At the 5% level of significance it was found that there are significant differences among the disciplines. The data for the disciplines were combined and the various responses for mathematical topics were tested for equality. Equality was rejected at the 5% level. Having decided that there are differences in the perceived usefulness to the social sciences of the various mathematical topics one next asks "Which differences are large enough to be considered statistically significant?" We let $\bar{X}_i$ be the average response for the i^{th} topic of Table 2 and then indicate in Table 2 which of these average responses differ significantly from others. For example, the fourth row indicates that enough data is available to conclude that statistics is perceived as being more useful than computer programming which in turn is perceived as being more useful than trigonometry and actuarial science. Other differences involving computer programming are not large enough to be considered statistically significant (at the 5% level) although some of the differences for particular disciplines may be significant. The inference to be drawn from this set of data is that the social scientists perceive the basics of algebra, statistics and probability to

* The tables are to be found at the end of this article.

be more valuable to the secondary school mathematics program than such subjects as actuarial mathematics, trigonometry, Euclidean geometry, analytic geometry and even the calculus. Statistics, probability and basic algebra are percevied as being "very useful".

PREDICTIONS OF FUTURE IMPORTANCE OF VARIOUS MATHEMATICS TOPICS FOR SOCIAL SCIENCE DISCIPLINES

The second set of questions posed to the social scientists dealt with the anticipated future mathematical needs of the social sciences. The social scientists were asked to predict whether or not they anticipate an increasing usefulness for the same 11 mathematics topics discussed in the first set of questions. A prediction of increasing importance was scored as +1, a prediction of decreasing importance was scored as -1 and a prediction of not much change was scored as 0. The average responses to predicted future importance of mathematics topics for the various social science disciplines is to be found in Table 3. A statistical analysis was performed and the disciplines were not found to differ significantly. All responses for the various mathematical components were averaged and appear in Figure 2. The hypothesis of equality of future importance of these mathematics topics to the social sciences is rejected at the 5% level and multiple comparisions are reported in Table 4. The import of this analysis is that the more modern aspects of the mathematics curriculum, such as computer science, probability, statistics and matrix algebra, will be of increasing importance to those students planning to study the social sciences. Analytic geometry, trigonometry, actuarial mathematics and Euclidean geometry, more traditional components of high school mathematics programs, are perceived as being low, no or negative growth areas for the social sciences. This analysis, combined with that for the perceived importance of these same topics, would be of interest to anyone concerned with secondary school mathematics curricula.

TOPICS IN STATISTICS AND PROBABILITY

The next set of questions deals specifically with statistics and probability. A lengthy list of topics in these areas was presented to the social scientists and they were asked to rate each topic on a scale of from 1 to 5 as to its usefulness as a topic of study for social-science-bound students. A score of 1 indicates "of no use", and a score of 5 indicates that the topic is "indispensable". (See

figure 1 for the complete scale.) The preamble to this section of the quesionnaire asked the respondents to ignore the difficulties of teaching the topics to high school students and concentrate only on usefulness. It is highly unlikely that all of the topics in the list could be taught to secondary school students but in order to make wise choices as to which to include and which to leave out one should know which are considered useful. A survey carried out on a broader scale asking about the usefulness of various topics of the traditional mathematics subject matter might be revealing.

The statistics and probablity topics are arranged in six parts. The first set of topics dealing with descriptive statistics is as follows:

A. Data analysis and Presentation

a. Tabulation of data, graphical representation of data, histograms, cumulative frequency polygons.

b. Measures of central tendency for raw and grouped data. Arithmetic mean, median, mode.

c. Measures of variability for raw and grouped data. Variance and standard deviation, coefficient of variation, range, interquartile range.

d. Measures of skewness and kurtosis.

e. Quantiles, percentiles, deciles, quartiles.

Table 5 contains the averages of the responses to these topics for each discipline. An analysis of the data does not detect significant differences among the social science disciplines but, overall, topics (a), (b), and (c) are found to differ significantly from topics (d) and (e). There is substantial agreement among the disciplines as to the importance of each topic and the first three topics are regarded as being in the "very useful" to "indispensible" range, whereas the other two topics are in the "useful" to "very useful" range. It seems plain that social scientists feel that plotting of data and measures of central tendency and of dispersion are topics that should be part of any course in high school statistics.

The second set of topics that the social scientists were asked to assess deals with probability.

B. Elements of Probability Theory

a. Sample space, events and basic manipulations using union, intersection and complements.

b. Probability measures.

c. Equally likely events and the combinatorics associated with counting such events.

d. Conditional probabilities and Bayes' rule.

e. Independent events.

f. Random variables - discrete and continuous. Probability distribution, mass and density functions.

g. Jointly distributed random variables. Joint, conditional and marginal probabilities.

h. Specific discrete random variables: binomial, Poisson, geometric hypergeometric, multinomial.

i. Specific continuous random variables: uniform, exponential, gamma, normal.

j. Mathematical expectation: mean, variance, higher order moments, covariance, correlation coefficient, moment generating function.

k. Central limit theorem, law of large numbers.

Table 6 contains the averages of the responses to these topics for each disciplines. Additionally, the bottom margin contains combined averages. No disagreement can be detected among the disciplines as to the importance of each topic. The scores, by and large, fall in the "useful" category. The only topics which can be regarded as being substantially more useful than any others are (b) and (j) over (h). Some of these topics are presently taught in the relevant Grade XIII mathematics course.

The third group of statistics topics deals with the elements of statistical inference. The topics are:

C. Elements of Statistical Inference.

a. Notion of a population and a sample from a population. Characterization of a population variable by a

probability distribution which in turn may be characterized by one or more parameters.

b. Estimation of population parameter(s) by sample statistic(s). This section might include some of the following topics. Desirable properties of estimators such as unbiasedness, consistency, and efficiency. Methods of obtaining estimators possessing desirable properties i.e., method of maximum likelihood, method of moments and method of least squares.

c. Interval estimates. This section might include some of the following topics. Distribution of the sample mean from a normal distribution, and the standar ized normal variable. Distribution of the sample variance from a normal population and the χ^2 distribution. The t-distribution. Use of standard tables. Confidence intervals for the mean and variance of a normal distribution. Use of nomograms in obtaining confidence intervals for the binomial parameter. Use of the central limit theorem in obtaining large sample confidence intervals.

d. Hypothesis testing. This section might include some of the following topics. Null versus alternative hypotheses. Test statistics. Critical regions and the level of significance of tests. Type I and Type II errors. Power of a test as a criterion for deciding which test to use. Likelihood ratio tests: their optimality and their large sample distribution. Tests for mean and variance of normal observations including one and two sample t-tests.

As can be seen from Table 7 these topics generally fall in the "very useful" to "indispensible" range. There are no significant differences among the disciplines. Among the topics, (a) differs significantly from (b) and (c).

If it can be demonstrated that these topics can be taught to secondary school students, this data would indicate that, using the criterion of usefulness to the social sciences, these topics should be given high priority by those planning statistics curricula.

D. Correlation and Regression Aanlysis

a. Correlation. This section might include some of the following topics. Population and sample correlation

mathematical skills <u>well</u> than to try to even teach statistics which would inevitably degenerate into plugging numbers into formulas.

(iv) I would prefer a greater understanding of traditional mathematics at secondary school level, leaving statistics until university. The relevance of statistics to the Social Sciences is clearer at university rather than secondary level.

(v) Yes, I do think statistics should be taught in high schools, but I would recommend a social-problem or social science applications approach to teaching it as against the usual "marbles-in-the-air" approach of mathematics departments.

(vi) It is every bit as effective as, for example, geometry, as mental exercise and has far more practical value than all high school mathematics except perhaps algebra. Should at least be available as an option for those interested in social rather than natural science.

(vii) Yes, but it would be better to concentrate on a few topics and cover them well (eg. descriptive statistics) than to attempt to duplicate a college course by spreading out over a large number of topics. Many of these topics make more sense to the student in the context of a data-gathering situation, such as a laboratory associated with a discipline.

(viii) In my opinion a mastery of statistics is an essential prerequisite to becoming a social scientist. I think that if students are introduced to statistics and realize it was an integral part of most social science programs before they choose to study the social sciences, then students choosing to study the social sciences would be doing so on a more realistic basis than they often seem to be doing now.

(ix) Our department was more concerned with secondary schools just teaching <u>all</u> students the most basic algebra and arithmetic <u>first</u>; then worry about statistics courses.

(x) Statistics not taught in the context of a specific area of application is of limited value. To the extent that statistics could be integrated into a specific course, say geography, then it might prove successful.

(xi) It would save us the work.

(xii) Not if a trade-off must be made basic mathematics (algebra,

geometry, set theory and calculus).

(xiii) Statistics is an indispensable part of Social Science today. A knowledge of statistics will be necessary for anyone in the Social Sciences in the future.

How do the social scientists feel about the present secondary school mathematics program? To a certain extent their answer is contained in the ratings given to the various mathematics topics in previous questions. To add colour to these figures, the socail scientists were asked to indicate what they perceive to be the strongest and the weakest points of the present mathematics program. There were some positive comments suggesting that the introduction into the mathematics curriculum of the calculus, linear algebra, set theory and probability was a step in the right direction. Some of the negative comments are as follows.

(i) Students cannot do basic algebra.

(ii) One of the weakest points is that mathematics courses in Grade 12 and 13 are not compulsory anymore. This is why some students have so much difficulty.

(iii) In my experience students do not possess even the most basic ideas about probability or the ways data can be used or arranged to test simple hypotheses.

(iv) One of the weakest points is the application of mathematical techniques to social problems and to data.

(v) Primarily that students who contemplate entering psychology apparently are often advised by secondary school mathematics teachers that mathematics is largely unnecessary training for such students. This advice dooms many students to years of only the vaguest understanding of empirical methods of experimental psychology, and encourages many students to avoid, as well, courses in computer programming and other mathematical topics.

(vi) For psychology and perhaps some other areas in Arts the continuance of the image of mathematics as useful to science and engineering but not relevant to the behavioural sciences. Many of our students unwittingly buy themselves trouble at university by avoiding mathematics in secondary school.

(vii) I would like to see a compulsory minimum high school mathematics curriculum. Many people attempt to get into

Psychology with almost no high school mathematics and the result is usually a disaster.

(viii) Social science and environmental science applications are not stressed in high school mathematics. Students come inadequately prepared to take our statistics course - many are allowed to come into programs without sufficient mathematics. Students come with inadequate training in logarithms.

ACKNOWLEDGEMENTS

The author would like to acknowledge the generous assistance in the preparation of the survey of his colleagues in the Secondary School Statistics Project at The University of Western Ontario. These colleagues are L. K. Chan, A. W. Harris, F. B. Rachar and S. Rinco. The author would also like to acknowledge the outstanding contributions of A. Ciavarella to the preparation of the questionnaire and to the statistical analysis of the survey data.

Professor MacNeill is a statistician in the Department of Applied Mathematics at the University of Western Ontario. He is a coordinator of the Secondary School Statistics Project. He is also chairman of the committee for high school statistics of the Statistical Sciences Association of Canada. Professor MacNeill, who was born in Saskatchewan, where he spent 8 years as a high school teacher of mathematics, received his B.Ed. and B.A. from the University of Saskatchewan. He graduated from Queen's University with an M.A. in mathematics and obtained his Ph.D. in Statistics from Stanford University.

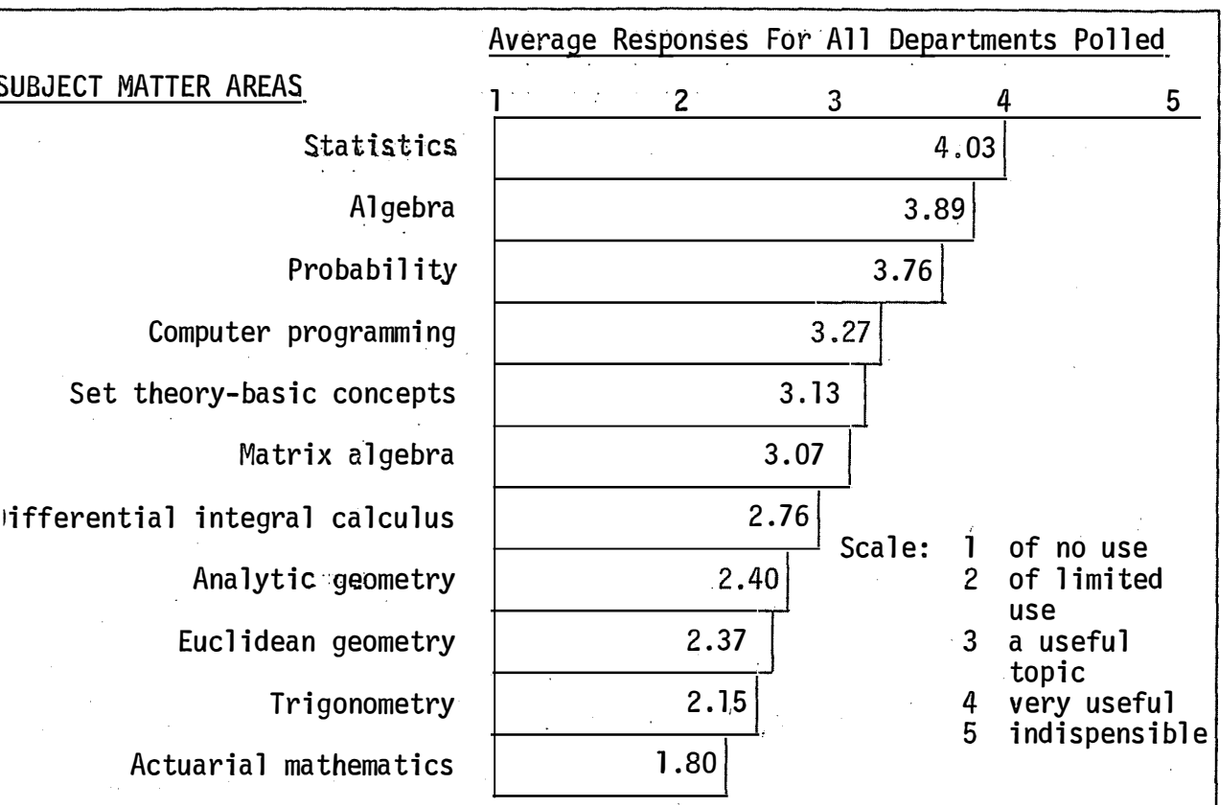


Figure 1

Ordered plots of average ratings given by social scientists to the usefulness of various topics in mathematics.

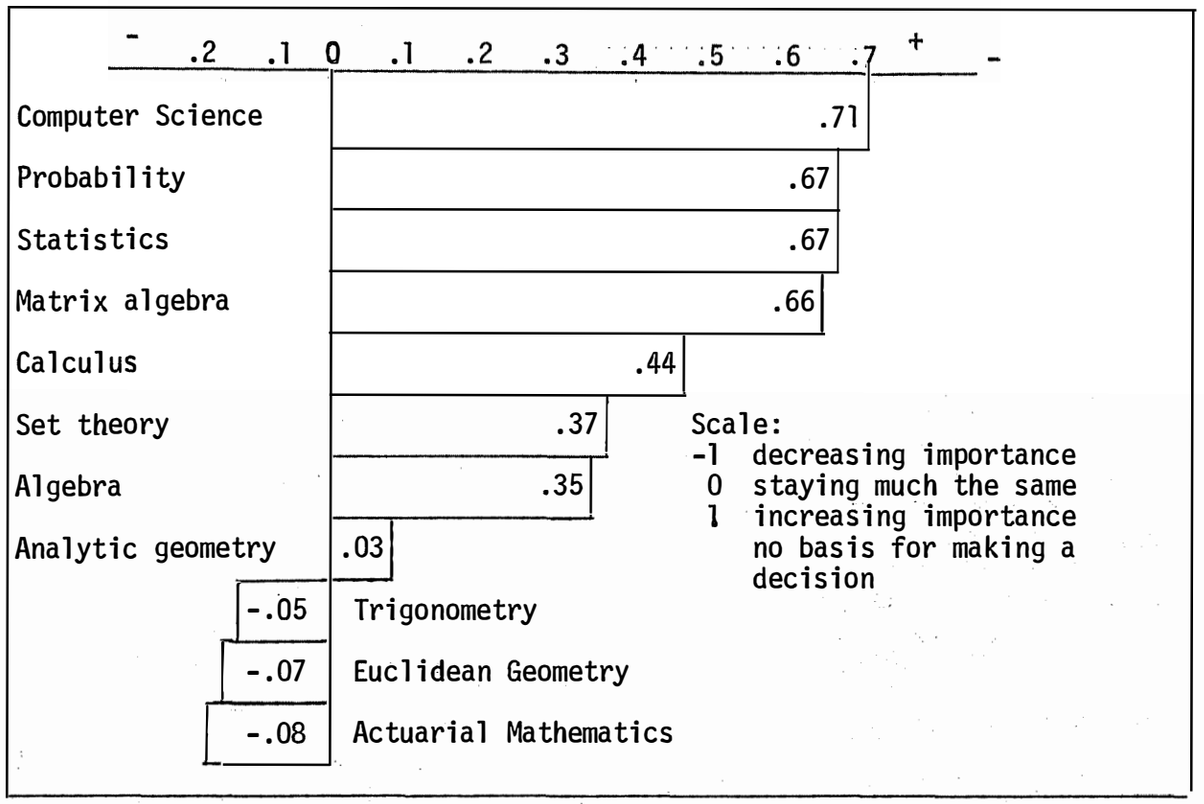


Figure 2

Ordered plot of average predictions given by social scientists to the future usefulness of various topics in mathematics.

Scale: 1 of no use
2 of limited use
3 a useful topic
4 very useful
5 indispensible

TOPIC NO.	MATHEMATICS TOPICS	Economics	Geography	Political Science	Psychology	Sociology & Anthropology	All departments combined
1	Algebra. Basic concepts.	4.27	4.38	2.73	4.00	3.92	3.89
2	Set theory. Basic concepts.	3.45	3.46	2.55	3.13	3.00	3.13
3	Euclidean geometry	3.09	3.50	1.36	2.19	1.92	2.37
4	Analytic geometry.	2.91	2.92	1.45	2.25	2.42	2.40
5	Trigonometry.	2.20	3.45	1.18	1.94	1.92	2.15
6	Differential & integral calculus.	3.55	3.00	2.18	2.56	2.58	2.76
7	Matrix algebra.	3.64	3.54	2.55	2.60	3.08	3.07
8	Statistics.	3.82	4.00	4.09	4.13	4.08	4.03
9	Probability.	3.55	3.85	3.73	3.88	3.75	3.76
10	Computer programming.	3.00	3.54	3.00	3.25	3.50	3.27
11	Actuarial mathematics.	2.09	1.64	1.50	1.81	1.92	1.80

Table 1
Average response to importance of mathematics for social science disciplines.

8 Statistics				$\overline{X}_8$	>	$\overline{X}_{10}$	$\overline{X}_2$	$\overline{X}_7$	$\overline{X}_6$	$\overline{X}_4$	$\overline{X}_3$	$\overline{X}_5$	$\overline{X}_{11}$
1 Algebra				$\overline{X}_1$	>		$\overline{X}_2$	$\overline{X}_7$	$\overline{X}_6$	$\overline{X}_4$	$\overline{X}_3$	$\overline{X}_5$	$\overline{X}_{11}$
9 Probability				$\overline{X}_9$	>			$\overline{X}_7$	$\overline{X}_6$	$\overline{X}_4$	$\overline{X}_3$	$\overline{X}_5$	$\overline{X}_{11}$
10 Computer programming	$\overline{X}_8$	>	$\overline{X}_{10}$	>								$\overline{X}_5$	$\overline{X}_{11}$
2 Set theory - basic concepts	$\overline{X}_8$	$\overline{X}_1$	>	$\overline{X}_2$	>							$\overline{X}_5$	$\overline{X}_{11}$
7 Matrix algebra	$\overline{X}_8$	$\overline{X}_1$	$\overline{X}_9$	>	$\overline{X}_7$	>						$\overline{X}_5$	$\overline{X}_{11}$
6 Differential integral calculus	$\overline{X}_8$	$\overline{X}_1$	$\overline{X}_9$	>	$\overline{X}_6$	>							$\overline{X}_{11}$
4 Analytic geometry	$\overline{X}_8$	$\overline{X}_1$	$\overline{X}_9$	>	$\overline{X}_4$	>							
3 Euclidean geometry	$\overline{X}_8$	$\overline{X}_1$	$\overline{X}_9$	>	$\overline{X}_3$	>							
5 Trigonometry	$\overline{X}_8$	$\overline{X}_1$	$\overline{X}_9$	$\overline{X}_{10}$	$\overline{X}_2$	$\overline{X}_7$	>	$\overline{X}_5$	>				
11 Actuarial mathematics	$\overline{X}_8$	$\overline{X}_1$	$\overline{X}_9$	$\overline{X}_{10}$	$\overline{X}_2$	$\overline{X}_7$	$\overline{X}_6$	>	$\overline{X}_{11}$	>			

Table 2

Significant differences among the average responses (averaged over all responses) for the subject areas.

TOPIC NO.	MATHEMATICS TOPIC	Economics	Geography	Political Science	Psychology	Anthropology & Sociology	All Departments Combined
1	Algebra. Basic concepts.	0.455	0.500	0.300	0.133	0.333	0.350
2	Set theory. Basic concepts.	0.364	0.417	0.273	0.429	0.364	0.373
3	Euclidean geometry.	-0.182	0.167	0.000	-0.214	-0.091	-0.070
4	Analytic geometry.	-0.100	0.000	0.111	0.000	0.000	0.035
5	Trigonometry.	-0.091	0.091	-0.100	-0.143	0.000	-0.052
6	Differential & integral calculus.	0.636		0.300	0.200	0.667	0.441
7	Matrix algebra.	0.636		0.500	0.667	0.750	0.661
8	Statistics.	0.455	0.833	0.800	0.600	0.667	0.667
9	Probability.	0.364	0.909	0.600	0.688	0.750	0.667
10	Computer programming.	0.364	0.750	0.700	0.813	0.833	0.705
11	Actuarial mathematics.	-0.300	0.143	0.000	0.000	0.000	-0.083

Table 3

Average responses to predicted future importance of mathematics topics for social science disciplines.

10	Computer programming		$\overline{X}_{10} > \overline{X}_4, \overline{X}_5, \overline{X}_3, \overline{X}_{11}$
9	Probability		$\overline{X}_9 > \overline{X}_4, \overline{X}_5, \overline{X}_3, \overline{X}_{11}$
8	Statistics		$\overline{X}_8 > \overline{X}_4, \overline{X}_5, \overline{X}_3, \overline{X}_{11}$
7	Matrix algebra		$\overline{X}_7 > \overline{X}_4, \overline{X}_5, \overline{X}_3, \overline{X}_{11}$
6	Differential integral calculus		$\overline{X}_6 > \overline{X}_5, \overline{X}_3, \overline{X}_{11}$
2	Set theory - basic concepts		$\overline{X}_2 > \overline{X}_5, \overline{X}_4, \overline{X}_{11}$
1	Algebra		$\overline{X}_1 > \overline{X}_5, \overline{X}_4, \overline{X}_{11}$
4	Analytic geometry	$\overline{X}_{10}, \overline{X}_9, \overline{X}_8, \overline{X}_7$	$> \overline{X}_4$
5	Trigonometry	$\overline{X}_{10}, \overline{X}_9, \overline{X}_8, \overline{X}_7, \overline{X}_6, \overline{X}_2$	$> \overline{X}_5$
3	Euclidean geometry	$\overline{X}_{10}, \overline{X}_9, \overline{X}_8, \overline{X}_7, \overline{X}_6, \overline{X}_2, \overline{X}_1$	$> \overline{X}_3$
11	Actuarial mathematics	$\overline{X}_{10}, \overline{X}_9, \overline{X}_8, \overline{X}_7, \overline{X}_6, \overline{X}_2, \overline{X}_1$	$> X_{11} >$

Table 4

Significant differences among the average responses to the anticipated future importance of mathematical topics.

Topic / Dept.	(a)	(b)	(c)	(d)	(e)
ECONOMICS	4.33	4.44	4.44	3.00	3.33
GEOGRAPHY	4.83	4.67	4.58	3.75	3.92
POLITICAL SCIENCE	4.36	4.27	4.00	2.82	3.09
PSYCHOLOGY BEH. SCIENCES	4.69	4.75	4.75	3.13	3.94
SOCIOLOGY ANTHROPOLOGY	4.50	4.67	4.67	3.25	3.67
TOTAL	4.57	4.58	4.52	3.20	3.63

Table 5

Average response to topics in descriptive statistics.

Topic / Dept.	(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)	(i)	(j)	(k)
ECONOMICS	4.11	3.89	3.50	3.88	4.25	4.11	4.25	3.25	3.88	4.38	4.33
GEOGRAPHY	3.46	4.09	3.18	3.18	3.55	3.83	2.91	3.67	3.33	3.91	3.46
POLITICAL SCIENCE	3.36	3.82	3.46	3.64	3.73	3.18	3.18	3.18	3.46	4.00	3.55
PSYCHOLOGY BEH. SCIENCES	3.38	3.76	3.56	3.50	3.81	3.56	3.31	3.13	3.19	3.76	3.56
SOCIAL ANTHROPOLOGY	3.25	3.92	3.25	3.42	3.58	3.42	3.17	2.75	2.58	3.75	3.00
TOTAL	3.48	3.88	3.40	3.50	3.76	3.60	3.31	3.19	3.24	3.91	3.54

Table 6

Average responses to topics in probability theory.

Topic / Dept.	(a)	(b)	(c)	(d)
ECONOMICS	4.33	4.00	4.00	4.33
GEOGRAPHY	4.75	4.27	4.09	4.27
POLITICAL SCIENCE	4.09	4.09	4.00	4.18
PSYCHOLOGY BEH. SCIENCES	4.44	4.06	4.06	4.31
SOCIOLOGY ANTHROPOLOGY	4.58	3.83	4.08	4.25
TOTAL	4.45	4.09	4.09	4.27

Table 7

Average responses to topics of Statistical Inference.

Topic / Dept.	(a)	(b)	(c)
ECONOMICS	4.00	4.22	4.13
GEOGRAPHY	4.55	4.46	3.64
POLITICAL SCIENCE	4.09	4.00	3.90
PSYCHOLOGY BEH. SCIENCES	4.25	4.06	3.5
SOCIOLOGY ANTHROPOLOGY	3.92	3.92	3.36
TOTAL	4.17	4.12	3.66

Table 8

Average responses to topics in correlation and regression analysis.

Dept. \ Topic	(a)	(b)	(c)	(d)	(e)	(f)	(g)
ECONOMICS	2.50	2.50	2.00	2.75	2.50	2.75	2.38
GEOGRAPHY	3.55	2.82	2.27	3.09	2.91	2.82	2.82
POLITICAL SCIENCE	3.20	2.36	2.36	2.91	3.18	3.18	3.27
PSYCHOLOGY BEH. SCIENCES	4.25	3.75	3.31	3.50	4.00	3.25	3.69
SOCIOLOGY ANTHROPOLOGY	3.00	2.58	2.25	2.83	3.00	2.92	3.00
TOTAL	3.42	2.83	2.53	3.07	3.21	3.02	3.12

Table 9

Average responses to topics in experimental design.

Dept. \ Topic	(a)	(b)	(c)	(d)
ECONOMICS	3.11	2.75	2.63	2.75
GEOGRAPHY	4.46	4.00	3.46	4.36
POLITICAL SCIENCE	3.91	3.27	3.27	4.18
PSYCHOLOGY BEH. SCIENCES	4.38	3.19	3.56	3.94
SOCIOLOGY ANTHROPOLOGY	4.25	3.00	3.33	3.75
TOTAL	4.09	3.26	3.31	3.86

Table 10

Average responses to topics in non-parametric statistics.

Book Reviews

For a list of books available for review, write the editor.

Learning and Teaching Metric Measurement, by Louis C. Fillinger, The Interstate Printers and Publishers, Inc., x + 45 pp., $2.50 (American).

In the review data the publishers point out that Dr. Fillinger assisted in writing metric materials for the schools of Uganda. This fact is reflected in his practical approach to the strategy of presentation. He is one of the educators who feel that measurement is an activity; e.g. "make a metre" on page 4, "make a weight" on page 5, and "make a litre" page 26.

In setting the stage Dr. Fillinger notes the presence of a conflict between the social pressure to use customary measures and the market place demands of a technology that would like to replace traditional British units with metric ones. He includes brief accounts from history to support the need for a changeover.

He suggests that the nomenclature for one table of measures should be presented after students understand place value and the decimal system. The prefixes are a way of expressing decimal fractions and decade multiples. As a good educator he frowns on changing metric quantities to British ones; that is, children should not be forced to play the adult game of seeing what this new quantity is in the old terms. He extends the idea of not converting to using one unit per job. He feels 9 cm and 4 mm is not as concise, clear and decimal-oriented as 9.4 cm.

He continues the practical approach by suggesting places where it would be practical to use the various measures. An example of this is kilograms in a play store. Piling and counting activities extend the category to things to do.

He presents area by tiling and counting, by the use of a transparent gridded overlay and tracing on graph paper. He

uses a leaf and the outline of a hand for teasers. He mentions the use of non-standard units.

In introducing volume he checks on vocabulary and terms. He also draws attention to a semantic problem that pupils have. His resolution of terminology is to define capacity in litres as what an object holds, while volume is the space that it occupies. In marking bottles at the litre level he uses a bottle cutter.

He finally brings in exponents as a matter for higher intermediate grades. An outline by grades is included.

The booklet could have been longer. It is probably constraints of size and publishing costs that raise some questions: why did he omit perimeter; has he a step by step plan in mind for the presentation of a topic involving vocabulary, comparison, ordering, use of non-standard units and the need for standard units; did he introduce prisms or just drop the term on an unwary reader.

It probably is fair to criticize the publisher for using customary American spellings liter, ton, etc. in place of the agreed S I litre and tonne. This restricts the circulation of the booklet.

G. Andress
Co-ordinator Pure Science &
Mathematics
The Valley Way Centre
Niagara Falls

Sourcebook of Fundamental Mathematics Series: Arithmetic, by W. Zlot, et al, Burns and MacEachern Ltd., $6.95.

This book covers the basic operations of addition, subtraction, multiplication and division for whole numbers and positive fractions. It also discusses decimal notation and percent. Some chapters do not have many worked examples and problems to assign while others have more than enough. The book could not be used as a text.

The book is of no use to a secondary school mathematics teacher but an elementary or occupations teacher might find a new or different approach for one or two topics in it.

D. Evans
Smith Falls District Collegiate
Smith Falls

B I R T H O F S. W. O. A. M. E.

The founding meeting of the Southwestern Ontario Association for Mathematics Education was held at Tilbury District High School, on Saturday, January 25, 1975, from 9:00 a.m. to 12:30 p.m., with more than 60 persons present.

The following officers were chosen to act until the first annual conference:

President:	*Professor Arn Harris* *Faculty of Education* *University of Western Ontario*
Vice-President:	*Mr. Dave Skoyles* *Coordinator of Pure & Applied Sciences* *Windsor Board of Education*
Secretary-Treasurer:	*Mr. Bill Scott* *Head of Mathematics Department* *Sarnia C.I. and T.S.*

It was also agreed that a Council be appointed, representing the elementary and secondary schools, the Universities of Western Ontario and Windsor and Fanshawe, Lambton and St. Clair Colleges of Applied Arts and Technology.

S.W.O.A.M.E. intends to apply to O.A.M.E. for admission as a chapter at the London conference in May, 1975. It is hoped that the first annual conference of S.W.O.A.M.E. will be held in Sarnia, November 1, 1975.

After the business meeting a very interesting panel discussion took place on the topic "Instructional Objectives for Mathematics Education". Dr. Carole Crealock, Assistant Professor, Faculty of Education, U.W.O., gave a brief history of the behavioral objectives movement and the rationale of this approach. Mr. George Robbins, Area Superintendent, Middlesex County Board of Education, discussed the writing of instructional objectives for mathematics in the elementary schools of Middlesex County. Mr. Dave Skoyles, Coordinator of Mathematics for Windsor, discussed some of the implications for his system. Mr. Bill Scott, Head of Mathematics for Sarnia C.I. and T.S. illustrated some of the challenges, difficulties and limitations of stating objectives in advance, for Grade 10 Four Year and Grade 13 Algebra. A lively discussion followed. The meeting adjourned at 12:45 p.m.

It is hoped that S.W.O.A.M.E. will serve the cities of London and Windsor, the counties of Essex, Kent, Elgin, Middlesex, Lambton and Huron, U.W.O. and U. of Windsor, Fanshawe, St. Clair and Lambton C.A.A.T.'s and any others who care to join. The intention is to continue and if possible improve the excellent conferences held in London annually since 1965, and as well, to initiate other professional development and curriculum activities.

O. A. M. E. C O N F E R E N C E I N 1976

The Mathematics teachers in the Lincoln County area are pleased to announce that the O.A.M.E. Conference in 1976 will be held in St. Catharines at Brock University on May 14, 15 and 16.

Co-Convenors	- Mr. Don Roberts, Governor Simcoe S.S. - Dr. Eric Muller, Brock University, Mathematics Department
Program Chairman	- Mr. Mike McKenna, Ministry of Education, Region 6
Publicity	- George Vinc
Publications	- John Russell, Niagara College
Exhibits	- Bob Robinson, Hamilton Board of Education

O.A.M.E. CONFERENCE

TO BE HELD

LONDON, ONTARIO

MAY 23RD, 24TH, 25TH, 1975

The second annual conference of the Ontario Association for Mathematics Education will be held in London, Ontario on May 23rd, 24th, and 25th. The location of the conference will be the University of Western Ontario.

The theme of the conference will be . . . . *"The Old With The New"* . . . . in mathematics education. Over 100 hours of valuable workshops, demonstrations, prominent speakers, and displays are planned.

More than 1000 persons are expected to attend and professionals from the elementary and secondary schools, universities, community colleges, and faculties of education will be present to exchange ideas.

Anyone involved in mathematics education at any level should plan now to attend since enrolment will be limited for certain presentations.

PLAN AHEAD . . . . BE THERE!

CONFERENCE CO-CHAIRMEN ARE:

MR. T. GRIFFITHS &
NORTH LAMBTON DISTRICT S.S.,
FOREST, ONTARIO, CANADA.
NON 1JO

MR. B. RACHAR
CITY MATH CENTRE,
OAK PARK SCHOOL,

FIFTY–THIRD ANNUAL MEETING OF THE N.C.T.M.
IN DENVER
APRIL 23RD – 26TH, 1975

APRIL 23, 1975
Opening address:

8:00 p.m.
Eric McPherson, University of Manitoba, Winnipeg, Canada.
"The Neo-Orthodox Position."

Delegate Assembly during the day.

APRIL 24, 1975
Presidential Address:

7:30 p.m.
E. Glenadine Gibb, University of Texas At Austin, Austin, Texas and NCTM President
"Aspirations – Actualities – Anticipations."

APRIL 25, 1975
Banquet:

7:30 p.m.
The speaker is Lt. Col. Ben M. Pollard, United States Air Force Academy, Colorado Springs, Colorado.
"Aerodynamics, Hanoi Hilton Style."

Over 200 section meetings, classified as elementary, junior high, secondary, two-year college, general interest, teacher education, and research in mathematics education. Section meetings will be held during the day on April 24th and 25th and on the morning of April 26th.

Approximately 115 workshops, including computer workshops. Workshops will be held during the day from noon of April 23rd through noon of April 26th. Workshop spaces are available through pre-registration.

Programs will be mailed to all NCTM members about the middle of February. Non-members of NCTM who wish copies of the meeting program may obtain one by writing to the National Council of Teachers of Mathematics, 1906 Association Drive, Reston, Virginia. 22091.

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$5.00 for one year's subscription
$2.50 for a single copy

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Invoices will be sent 30 days prior to expiry of a subscription, and 60 days is allowed for payment.

Orders for subscription and single copies should be sent to the Secretary-Treasurer

M. B. MacGregor

BRIEF TO MINISTER OF EDUCATION

Copies of a brief on the qualifications and preparation of Teachers of Mathematics (K-8) are available from M. MacGregor, 247 Bright Street, Sarnia, N7T 4E9. This brief was prepared from comments from OAME members and Faculties of Education who were sent a preliminary draft. The brief has now been approved by the OAME Council and sent to the Minister.