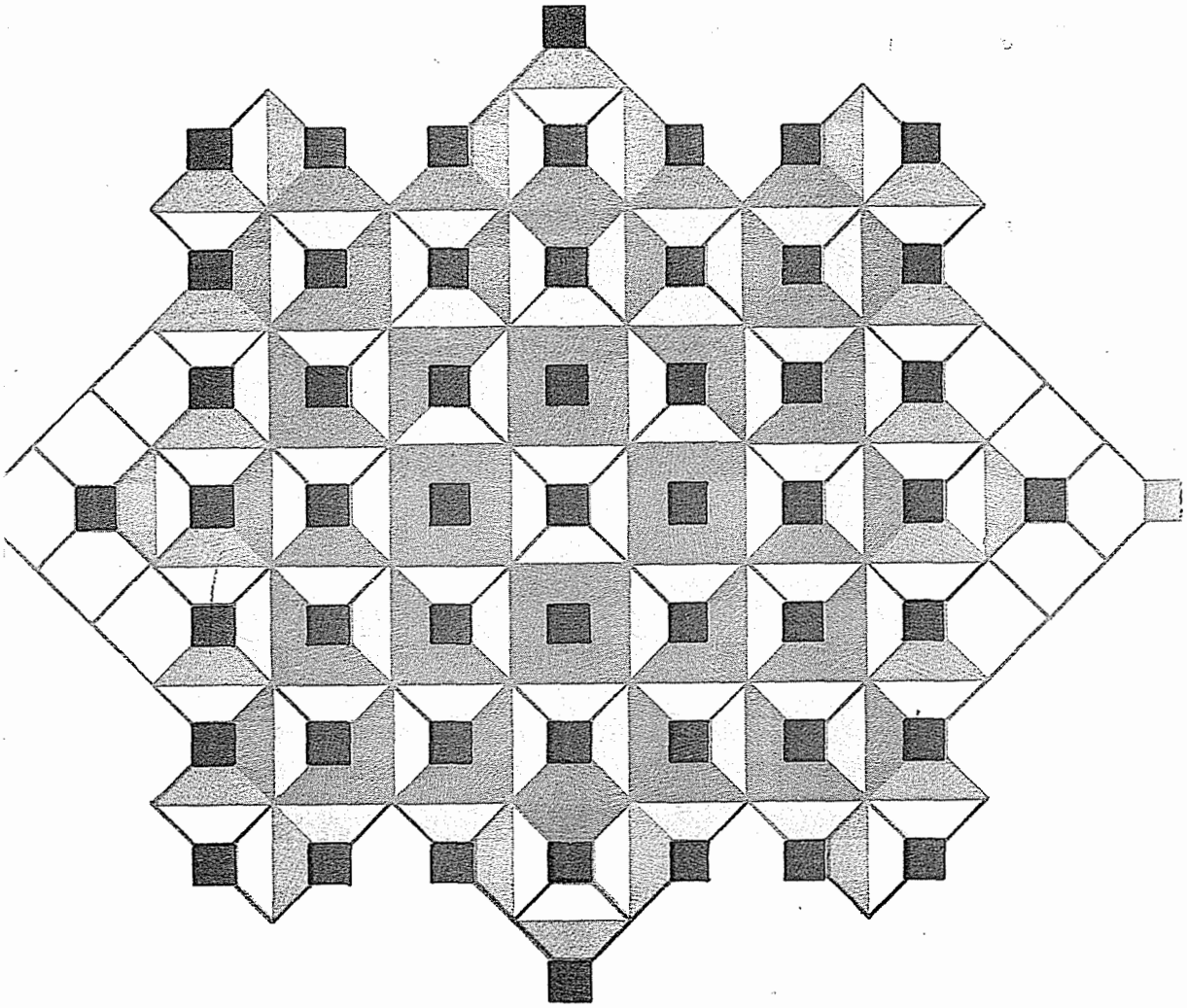
ontario mathematics gazette

VOLUME 14 NUMBER 1 SEPTEMBER, 1975



THE ONTARIO ASSOCIATION FOR MATHEMATICS EDUCATION

Editorial correspondence and articles should be submitted to one of the editors listed below.

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EDITORIAL

As you know, publication of the Gazette has been moved to Southwestern Ontario. Editorial work will be centred at U.W.O. in the Faculty of Education. Printing will be done in Forest [where, by our efforts it is hoped that more than the trees will be exposed to view!]. The Forest Free Press Printing is owned by Jamie Pole, who has been of great assistance in the production of our first issue.

Tribute should be paid to Dr. Bill Eames of Lakehead U., who has retired from the editorship. He and his colleagues have made important improvements in the format and contents of the magazine during its Thunder Bay period. Bill is still very much with us. He has already submitted an article which will be printed in the December issue.

It is the hope of the new editorial staff that we can continue the good work done in the past and gradually improve the contributions by and for the mathematics teachers of the elementary school panel. The whole staff recognizes this need and pledges to work toward this end.

By the time this issue reaches you the provincial election will have come and gone. How much did the policies of the various parties concerning mathematics education, or education in general, affect your vote? For example, the O.A.M.E. Committee on the Senior Division, which reports in this issue, favours a return to externally set examinations. What were the policies of the Conservatives, Liberals and N.D.P. in this matter? Did you hear any candidate speak about this issue knowledgeably? Probably not.

The controversy between candidates I have heard is more like ships passing in the night than a duel between protagonists. Perhaps the face-to-face T.V. debates of the leaders will provide us with some clearer insights into opposing policies.

One of the parties seeks a return to core subjects for the secondary school. Does this mean that mathematics should be a core subject for prospective elementary school teachers'? If so, how much?

It was, at any rate, healthy that the Opposition parties attacked the Conservatives on educational policy, since education is the most important responsibility of the provincial government. Whether you agreed with all of the criticisms or not, education is what the parties should have been discussing, not T.V. violence, hanging, federal riding boundaries and other irrelevancies.

My London colleagues and I hope you attended the May Conference and enjoyed yourself. What are your reactions to the proceedings booklet? A few papers are published in this issue as well. Now its on to the next. Does anyone wish to join me in a walk along the Bruce Trail, timed to make the "early-bird" meetings in St. Catharines? In keeping with the desire to bring Statistics into the school curriculum, we could gather data along the way!

Ann

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HELP!

For a considerable period of time now, we have believed that the Elementary Teachers of this province have been somewhat short-changed when it came to information relating to the teaching of Mathematics at their particular level. Few publications have been available, and the materials presently in print other than text books or subject curricula have been geared primarily towards the single subject teachers of the secondary school or they have been the results of the findings produced by the subject specialists at the post secondary school level. Why has this been the case?

Some believe that the reason for this is that Elementary Teachers are not subject specialists, but have to be versed in many subject areas in order to function. Therefore the result is that some elementary teachers may feel that they do not have the expertise in a single subject area such as Mathematics, to share some of their ideas with other professional teachers, fearing criticism from those who have access to their materials.

We believe that there are many exciting things happening in our elementary schools that are Mathematically oriented and that these could be shared. We fully realize however why some of these are not being shared to the degree that they should be. Thus the reason for having an elementary section in forthcoming Gazettes.

If you have any ideas at all about a topic, a theme, a game, or if you know of any people who have Math approaches that should be shared with others please submit these to us for use in future publications. A special request is made to those teachers who are working in a supervisory role in Mathematics and may see many Math teachers in their travels. - If each of you submitted at least one article our response would be overwhelming.

Help make Elementary Mathematics even better. - Only with your help can we achieve our goal. Regardless of how **you** may feel about your special topic, others may find it to be the catalyst to even greater ideas. Remember, we tend to be over critical of those things that we do ourselves.

Submit your articles, ideas, and names of possible authors to:
David and Sharon McPhail

A PLEA FOR SOME PRACTICAL CLASSROOM RESEARCH

It is our intention through this department to provide practical information of interest to the classroom teacher in the nature of research articles.

While thoroughing research studies are welcomed, we do not wish to exclude those very practical observations accumulated over a period of time by the observant educator.

Anything which will add to the knowledge of the practicing teacher of mathematics which can be backed up with factual information and which may lead to a more formal research study might be considered for publication.

In addition we would like to include some simple surveys which might lend support to ideas we now use in the classroom or at least suggest worthwhile changes in direction. For example: How many teach subtraction to children by regrouping, yet "borrow and pay back" themselves?

Consider the following:
The ratio of the area of a triangle to the area of the smallest rectangle containing it is 1:2. The ratio of the area of a circle to the area of the smallest rectangle containing it is π:4. Does the ratio of the area of a leaf from any one tree to the area of the smallest rectangle containing it approximate any such constant value? Non-perfect research by a grade eight Math/Science class indicated this was possible true. What does your research say?

As the by-line of **Abacus** indicated: **Mathematics Educators Share Their Ideas.**This part of the **Gazette** can only be as good as the amount you are willing to share with others. Please write to the Research Editor about research you have carried out, either formally or informally. Your suggestions about research you feel should be conducted might also spark something of value to all of us.

Brock

A MEMORIAL TRIBUTE

FATHER DONALD T. FAUGHT

Mathematics in Ontario has undergone a great change in the last fifteen years. The initiating of this change was due to a rather small group of people. One of these people was the late Father Donald T. Faught of the University of Windsor, who was the first President of the Ontario Mathematics Commission.

Father Faught, was one of a rare breed - a University Professor who had taught in high school, as he did for fourteen years at St. Michael.s College School in Toronto from 1940 to 1954. His high school background played an important role in his work with teacher groups, as he taught courses in the "New Math", led them in the marking of the old grade thirteen examinations that he set, or acted as Honorary President of the Ontario Association of Teachers of Mathematics. It was during this latter time that OMC and OATM were discussing their amalgamation to form the Ontario Association for Mathematics Education.

In the classroom, Father Faught, was a teacher who was determined that everyone should understand the Mathematics that he was teaching. Through his dynamism and ability to explain the most abstruse topic, they usually did.

To honour his memory, the OAME has established the Father Faught Memorial Award, to be given annually to the Ontario School that obtains the highest standing on the Junior Mathematics Contest of the University of Waterloo.

LOGARITHMS ARE IMPORTANT
SOME APPLICATIONS

Eric R. Muller,

Because electronic calculators are now so readily available it is more difficult to motivate students in the use of logarithms. Some even question the value of teaching the concept. Most elementary but tedious numerical computations, which involve multiplication and division, can be performed with more significant figures and often more rapidly on the cheapest calculator. (There still remains some interesting challenges, for example the calculation of C(126, 51) without the use of log factorials will keep the student and his calculator humming for a while!). Of course if the computation involves non-integral powers it is necessary to invoke the use of logarithms. It therefore seems logical to look in this area for stimulating applications.

Logarithms are particularly useful in cases when functional relationships between pairs of measurements relating to a single subject show a non-linear trend which is either monotonically increasing or decreasing in a given range. Consider the following set of measurements: -

Eg 1. Table I presents, for the solar system the relative mean distance D of the planets from the sun (where earth = 1.495×10^8 km), and the relative time of revolution T (where earth = 1 year). When plotted in Figure I (T versus D) we see evidence of a nonlinear monotonically increasing trend.

TABLE I

The relative mean distance D (where earth = 1.495×10^8 km), and the relative period of revolution T (where earth = 1 year) are given below for each planet in our solar system.

	Mercury	Venus	Earth	Mars	Jupiter	Saturn	Uranus	Neptune	Pluto
D	0.387	0.723	1.00	1.52	5.20	9.54	19.2	30.1	39.5
T	0.241	0.615	1.00	1.88	11.9	29.5	86.0	165	248

Eg 2. Table II presents the population (in millions) for Canada and the U.S. as reported at the beginning of each decade from 1850 to 1970. The graphs in Figure II for Canada and Figure III for the U.S. again clearly indicate nonlinear monotonically increasing trends.

TABLE II

The populations (in millions) of Canada and the United States for the period 1850-1970 as recorded at the beginning of each decade are given below.

Year	1850	1860	1870	1880	1890	1900	1910	1920	1930	1940	1950	1960	1970
Canada	2.44	3.23	3.69	4.33	4.83	5.37	7.21	8.79	10.4	11.5	14.0	18.2	21.6
U.S.	23.2	31.4	39.8	50.2	62.9	76.0	92.0	106	123	132	151	179	203

If our interest lies in a better understanding of the functional relationships between the variables what kind of functions should we try? For a linear model between two variables x and y, $y = mx + c$, we think of two parameters m and c to be chosen so as to give the "best fit" to the set of measurements. For the examples at hand if we wish to restrict the nonlinear model to two parameters we can try one of the two general forms either

$$y = ax^b \text{ (power type)} \quad (1)$$

or $$y = cd^x \text{ (exponential type)} \quad (2)$$

We note that the exponential type is often written as

$$y = ce^{kx} \quad (3)$$

where

$$e = 2.718281828...$$

is the base of the natural logarithm (i.e. we have put $d = e^k$). Taking logarithms of equations (1) and (3) we underline the basic difference between the power and exponential functions*, for from (1) we find

$$\log_{10} y = \log_{10} a + b \log_{10} x,$$

or $$y = A + bX \quad (4)$$

and from (3) $\log_{10} y = \log_{10} c + (k \log_{10} e)x$,

or $$y = C + K'x, \quad (5)$$

where a capital letter indicates the logarithm of the small letter. Although we have used logarithms to the base 10 any other suitable base could be used. We have now reduced equations (1) and (3) to linear relations but whereas (4) is linear in the variables

$$\log_{10} x \text{ versus } \log_{10} y$$

(5) is linear in the variables x versus $\log_{10} y$.

Thus if there is no indication from the system under study as to which model (1) or (3) is to be preferred we would proceed by plotting both

$$x \text{ versus } \log_{10} y \text{ (x versus Y)} \quad (6)$$

and $$\log_{10} x \text{ versus } \log_{10} y \text{ (x versus Y)} \quad (7)$$

The plot which then is closest to a straight line is used to determine the two parameters of the nonlinear model.

For the examples presented here I have only reproduced the preferred graph between (6) and (7). In the case of the planets we see a near perfect straight line when we plot $\log_{10} D$ versus $\log_{10} T$ (Figure IV). The slope of this graph is estimated at 2/3 with intercept (0,0). Therefore we can

* To get a feel for the difference between these two function types let the students calculate and plot $y = 2^x$ and $y = x^2$ for a range of integral values of x.
$y = 3^x$ and $y = x^3$ etc.,

represent the data by equation ④ , namely

$$\log_{10} D = \frac{2}{3} \log_{10} T$$

or $$D = T^{2/3} \qquad (T = D^{3/2})$$

which is Kepler's law.

In the case of the population data there is no clear cut linear relation for either ⑥ or ⑦. Nevertheless, it is usual to prefer exponential functions to reflect population changes because the simplest model of population growth is: -

the rate of change of population size p(t) at any instant of time t is proportional to its size at that instant

i.e. $\frac{dp(t)}{dt} \propto p(t)$

or $\frac{dp(t)}{dt} = kp(t)$

where k is constant.

This first order differential equation has the solution

$p(t) = de^{kt}$. (d = constant)

Plotting t versus $\log_{10} p$ for the Canada data Figure V we can fit a line

$$\log_{10} p = 0.007641\ t - 13.74$$

or $$p = (1.819 \times 10^{-14})\ e^{0.01759\ t}$$

The dotted line on Figure V indicates a possible alternative trend based on post 1930 data.

Plotting t versus $\log_{10} p$ for the U.S. data Figure VI we can see two definite trends emerging with a break somewhere between 1900 and 1920. We can fit the data prior to 1900 with the line

$$\log_{10} p = 0.01024\ t - 17.55$$

or $$p = (2.815 \times 10^{-18})\ e^{0.02356\ t}$$

and for the post 1900 data the line

$$\log_{10} p = 0.005626\ t - 8.781$$

or $$p = (1.656 \times 10^{-9})\ e^{0.01295\ t}$$

The above examples demonstrate the importance of logarithms in determining trends.

The following exercises can also be used to motivate student interest.

1. In a study of repair costs of transit buses a city transit authority calculated the following repair cost c(m) cents per mile for vehicles having traveled m miles.

m	120,000	170,000	220,000	260,000	270,000
c(m)	0.148	0.263	0.401	0.527	0.561

(a) What functional relationship best represents the repair cost per mile as a function of miles traveled?

(b) The authority believes that when buses exceed 0.6 cents it no longer pays to repair them. If this is

FIGURE I

T → D

RELATIVE MEAN DISTANCE VERSUS RELATIVE PERIOD FOR PLANETS

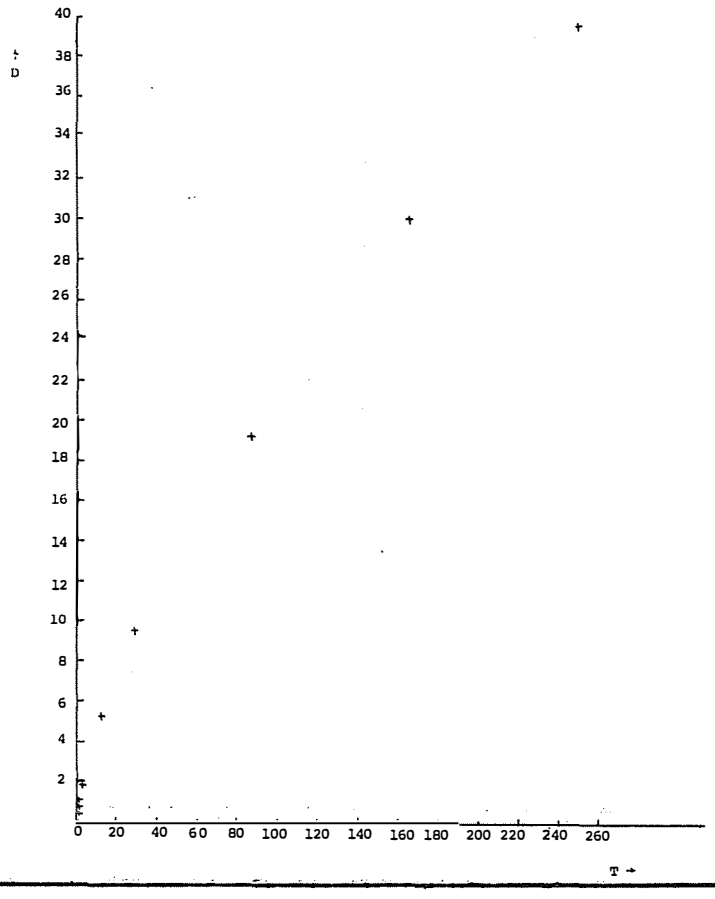
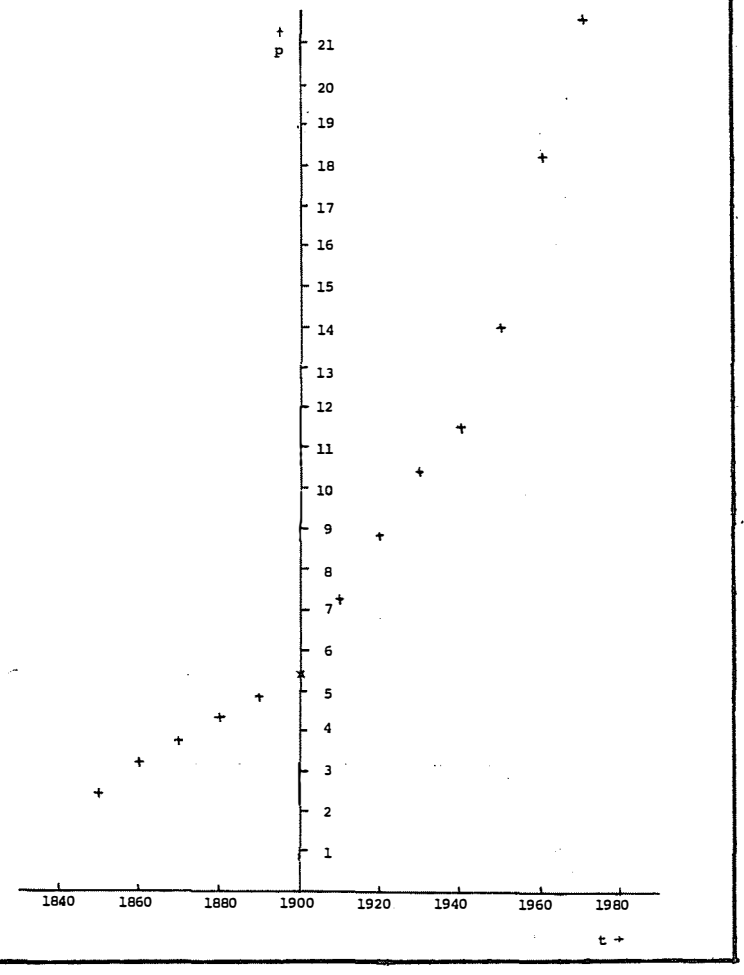


FIGURE II

t → p

CANADA POPULATION (MILLIONS)



the case at what mileage should they be discarded?

(c) What would be the estimated total repair bill for a bus which has traveled 300,000 miles?

Answers

(a) $c(m) = (6.94 \times 10^{-10})m^{1.64}$

(b) 280,000 (approximately)

(c) area underneath the graph between m = 0 and m = 300,000, total repair cost = $76,700 (approximately)

2. In a study on group behaviour a psychologist hypothesizes that the time t(x) it takes a group to perform a task the xth time decreases as the number of times, x, the task is accomplished increases. He makes the following observations on a group of students

x	2	5	8	15	30
t(x)	100	37.2	30.8	26.0	25.3

What functional relationship best fits his results?

Answer

This example presents a generalization to our discussion. We have a monotonically decreasing function. Furthermore we expect t(x) to tend to a positive value as x becomes very large, for however much practice the group gets it will still take the group a finite time to perform the task. How can we best estimate this time? A plot of x versus t(x) appears to indicate a value close to 25. We will have to select this as a guess, for from the above theory we can (from a straight line) estimate only two parameters and yet our model is expected to have three

either $t(x) = ax^b + c$

or $t(x) = cd^x + f$

Best fit $t(x) = 300x^{-2} + 25$

The following examples give two important applications of the exponential function and can be used to motivate the use of logarithms.

3. The rate at which radioactive material changes to lead is proportional to the amount of material present (the same model as the simple population model)

$$f(t) = Ce^{kt}$$

where f(t) is the amount of material present at time t, c and k constants. Living plant and animal tissues contain radioactive carbon C-14 which has a half life of 5,568 years i.e. if f(0) is the amount of material at time t = 0, $\frac{f(0)}{2}$ is the amount of material at time t = 5,568 years.

An analysis of ashes found in an unearthed Egyptian settlement reveals that one-fifth of the C-14 present in

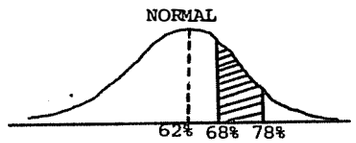
the original ashes has decomposed. Date the settlement.

Answer

Approximately 1730 years old.

4. Some important areas of statistics theory are based on the notion of probability density functions. These non-negative functions have the properties that the total area underneath the curve is equal to 1 and the area underneath the curve between two fixed values gives the probability of the variable falling within that range. Example, for a normal probability distribution of class marks centred (mean) at 62% (and given standard deviation) the area between 68% and 78% represents the probability of finding a student within that class who has attained a mark within that range. Another probability density function

$p(x) = ae^{-ax}$, $a > 0$ constant



finds applications in quality control (control of the production of defective articles), studies of accidents reported on a given stretch of highway, demand for an item in an inventory system etc. Example: The number of accidents reported on a certain stretch of highway per year has the probability density function defined by

$$p(x) = 0.02\, e^{-0.02x}$$

Estimate the probability that 10 or less accidents will be reported in the following year.

Answer

Plot x versus p(x) for x = 0, 1, 2, ..., 10 and calculate the area underneath this curve from 0 to 10 by some approximation. Note that in order to reflect the fact that we assume x to be continuous although the number of accidents is discrete a more accurate estimate can be obtained by taking area between 0 and 10.5. Answer (by integration) probability (10 or less accidents in next year) = 0.19.

The author's interests are in the areas of Applied Mathematics and statistics. He recently spent a year with the Federal Ministry of Transport developing analyses and mathematical models for a number of transportation systems.

REPORT OF THE

OAME COMMITTEE

FOR THE REVIEW OF

SENIOR DIVISION MATHEMATICS CURRICULUM

June 1975

Committee Members: Tom Griffiths, Bill Higginson, Ross Leigh, Ron Scoins, Norm Viirre.

This report which follows will be discussed at the November 8, 1975, Council meeting. The Committee and the Executive of O.A.M.E. urgently request your reaction to the report and the results of the survey, prior to November 8. Your submissions should be mailed to:

Neil Williamson

Following its appointment in the spring of 1974 the committee decided to obtain opinions concerning Senior Division mathematics curriculum from O.A.M.E. members and others.

For this purpose a questionnaire was designed. A sample of the questionnaire is attached to this report. Distribution was made to all O.A.M.E. members. The survey also extended to many University representatives, and to mathematics teachers not presently members of O.A.M.E.

Approximately 150 questionnaires were returned representing the opinions of about 200 persons. A summary of the results is given in this report. Many comments and suggestions were received in addition to the particular questions asked and these are available and included with the report.

On November 27, 28, 29, 1974, the Grand Valley Mathematics Association conducted a seminar on the topic of this report. It was attended by 31 secondary and 7 university representatives. The recorded results of that seminar are included with this report.

Other group discussions have taken place throughout the province on professional development days and in meetings of mathematics departments or other groups.

Articles which relate to the topic of this report have appeared in at least two recent issues of The Ontario Mathematics Gazette:

W.W. Sawyer, December, 1974
I.B. McNeill, March, 1975

The committee has considered all of the material mentioned above and makes the recommendations which follow.

1. There is a multiplicity of topics and a 'spiral' approach involved in the present curriculum, therefore

It is recommended that the number of core topics in mathematics courses be reduced in order to devote the time necessary to achieve a greater depth of understanding of the remaining topics and to develop the skills required to apply them when required.

2. The variation in standards from classroom to classroom and school to school results in the lack of uniformity in students' background as they undertake university courses which assume some mastery of Grade 13 core topics. Therefore

It is recommended that an externally prepared, standardized examination in mathematics be given to Grade 13 students.

3. Most students appear unable to write complete and acceptable solutions to problems or proofs of theorems. Present high school text-books contain mainly exercises requiring one-step solutions based only on the concept just taught. Therefore

It is recommended that support materials be prepared which would give the teacher alternative methods of presentation of topics and a selection of useful exercises. Text-books should be revised to inlcude greater variety of problems, especially those which require 2-step or 3-step solutions, developing the student's ability to organize and write complete solutions. These materials and texts are particularly needed with respect to general level courses.

4. Teacher training is a continuing process throughout a teacher's entire career. Therefore

It is recommened that Faculties of Education be requested to introduce programmes to assist the continuance of training for teachers at all levels of qualification and experience.

5. Students proceeding to study in the field of social sciences need some understanding of and facility in certain mathematical topics. Therefore

It is recommended that a single course be composed of topics selected from the present guidelines to suit the needs of students who will study the soical sciences after secondary school.

6. *It is recommended that O.A.M.E. appoint or coordinate specific task groups to implement the preceding recommendations of this report.*

ONTARIO ASSOCIATION FOR MATHEMATICS EDUCATION

(Questionnaire on Senior Division Mathematics Courses)

NAME (optional) ----------Level Taught ----------

(You may find it useful to separate your comments about general courses from those about academic courses. Please feel free to use the back of the page or to attach pages for longer comments.)

a) Which topics are **most useful?**

b) Which topics are **least useful?**

c) Which topics are **well mastered** by the students?

d) Which topics are **most poorly mastered** by the students?

e) What **new topics** would you like to see included in the senior division mathematics curriculum?

f) What present topics would you suggest be **dropped** from the senior division mathematics curriculum?

PART II

NAME(optional) ---------- Level Taught ----------

1. Is there a need for a single Grade XIII course for students intending to study social sciences at university? YES NO

Comment:

2. Is there a need for a Grade XIII course for students not intending to go on to university? YES NO

Comment:

3. Should the topics in the senior division be reorganized into full-credit or half-credit courses, e.g. Trig., Analytic Geom., Euclid ...? YES NO

Comment:

5. Is there a need for new courses geared to the student in Grade 11 or Grade 12. Who is not going on to higher education? YES NO

Comment:

6. Would you like to see more information made available regarding the teaching of topics in the present curriculum? YES NO

Comment:

General Comments:

SUMMARY OF SURVEY REPLIES - PART 1

Figures indicate percentage of replies in each category.

Topic	Most Useful	Least Useful	Well Mastered	Poorly Mastered	Add	Drop
Deductive Geometry	17	25	2	30	4	10
Algebraic Operations	52	7	23	53	8	4
Relations & Functions	15	7	44	6	0	10
Analytic Geometry	50	7	24	13	4	1
Trig Functions	60	4	17	35	2	5
Quadratic Functions	17	1	15	1	0	1
Expon. & Log Functions	15	8	7	23	2	8
Series & Sequences	10	1	2	6	0	1
Statistics & Probality	18	1	1	6	23	1
Transformations	7	10	3	7	0	6
Matrices & Linear Tr.	5	5	1	0	2	4
Groups	0	14	0	1	0	11
Complex Nos. & Polar Co.	1	1	1	1	0	3
Logical Reasoning	2	9	0	2	2	4
Math of Investment	13	4	2	1	13	0
Calculus	31	4	22	11	1	6
Sets	1	6	2	1	0	4
Subsets, Permutations	6	1	2	5	2	0
Binomial Theorem	6	1	1	3	0	0
Vectors	8	8	6	5	1	8
Equations of Lines & Planes	0	0	0	1	0	0
Systems of Equations	6	0	1	0	0	0
Mathematical Induction	1	0	0	4	0	0

Summary of Survey Replies - Part II

Numbers in brackets indicate frequency of response.

QUESTION No. 1 - **Is there a need for a "single" Grade 13 course for students intending to study social sciences at University?**
YES 81% NO 14% UNCOMMITTED 5%

Comments

Yes (5) Such a course may be designed by rearranging present courses.
(3) An average sized school has difficulty offering a large number of courses.
(29) Should contain probability and statistics.

QUESTION No. 2 - **Is there a need for a Grade 13 course for students not intending to go on to University?**

NO 54% YES 46%

Comments

No (5) Grade 13 is a university preparation year therefore the question is improper.
No (5) Grade 12 courses are already sufficient.
(6) Present Math of Investment is good enough for these students.
Yes (6) Should contain statistics.
(20) Could be same course as in question 1.

QUESTION No. 3 - **Should topics in the senior division be reorganized into full-credit or half credit courses? e.g. Trigonometry, Analytic Geometry, Euclid .....etc.?**
NO 53% YES 32% UNCOMMITTED 15%

Comments

No (5) Present organization is adequate or preferable.
(3) Topics should not be isolated as many are inter-related.
Yes (5) to get away from the "spiral" approach.

QUESTION No. 4 **(a) - Should there be external exams for all students?**
NO 55% YES 34% UNCOMMITTED 11%

Comments
No (3) County-wide is enough.
(2) Just have good teachers.
(5) Let's not play that game again.

Yes (3) But term work should also count about 30 - 40%.
(3) If curriculum more common.
(6) It would provide some standards.
(3) But not compulsory.

Uncommitted
Not necessarily "external".
Maybe Grade 13 will disappear.

QUESTION No. 4 **(b) - Should there be external exams for university bound students?**
YES 72% NO 22% UNCOMMITTED 6%
Comments
Yes (6) It should govern admission. (16) The purpose should be to improve curriculum and teaching standards and standardize evaluation.
(7) Set, administered and marked by the universities. Set externally, marked locally.

No (3) Controls curriculum.

QUESTION No. 5 - **Is there a need for new courses geared to the student in Grade 11 or 12 who is not going on to higher education?**
NO 48% YES 41% UNCOMMITTED 11%

Comments
No (29) Present 4 year courses are adequate and flexible enough. The question is ambiguous.

(5) Present 4 year courses are not adequate.
(2) It would be possible under present guidelines.

Uncommited (4) Present courses are O.K. just the textbooks are bad.

QUESTION No. 6 - **Would you like to see more information made available regarding the teaching of topics in the present curriculum?**

YES 66% NO 12% UNCOMMITTED 22%

Comments

Yes Canadian data for statistics.
Circulate data for statistics.
(2) Innovative methods.
Visual aids.
Interesting applications.
New Math films.
(2) Two week refresher courses in summer.
(2) Saturday seminars co-sponsored by OAME & OSSTF P.D.
Review sheets, tests, arranged by topics.
Programmed materials.
Exams from other schools.
Ministry of Education should provide leadership in this area.
(2) More collaboration between (tertiary & secondary) & (secondary & elementary)
e.g. "Alg. & Trig. - Functions and Applications" by Paul Foerster, Alama Hts. High School, San Antonio, Texas.
The more the merrier!
Packaged topics with several methods of presentation, texts, games, activities.

NO I know what to do.
(2) It's the teacher's individual responsibility.
There's plenty available if you look.
I don't have time to read.

UNCOMMITTED

No comments

CANADIAN EDUCATION SHOWPLACE

Resources and Communications for Education and Industry
March 2, 3, 4, 1976
Automotive Building, Exhibition Place, Toronto
For more information contact Lloyd Haines, Manager, Canadian Education Showplace **or** Robert McBeth and Associates, 406 Jarvis Street, Toronto (416-924-1585)

A BELATED APOLOGY

Elaine Harvey of the staff of Sydenham High School has written to point out that we inadvertently omitted an important acknowledgement.

John Ebsary, whose article **Magic Squares**, appeared in the March, 1975 issue was a Grade 13 student of Sydenham H.S., at the time he wrote the article. He is now enrolled at Queens University, in second year Commerce.

Congratulations John, on a good article. The Gazette is anxious to encourage mathematical writing by students as well as teachers. Sorry we ommitted the credit line.

Ed

OLD MATH, NEW MATH, GOOD MATH, BAD MATH

Irwin Pressman, Department of Mathematics, Carleton University. Text of an address given at the O.A.M.E. Conference in May, 1975.

A hue and cry against the New Math has arisen in recent times. Parents, teachers, chemists, and even world famous mathematicians have spoken out against the changes that have been made in mathematics taught in our schools. Many of their complaints are justified. I shall give some examples of these. There are those who would have us return to the Old Math, that is - the former curriculum. This would be a great mistake, and we shall try to recall the tedium of the old program. Examples of how the newest textbooks seem finally to be succeeding where the earlier ones so badly failed will be given. Finally, as a University Professor, I shall indicate why I believe that the best students of today are among the best that Ontario has ever produced.

I shall use textbooks as my examples, so that my comments can be based on some concrete evidence. I shall also draw on my personal experiences in the Winnipeg schools. One such recollection is of a detention for the entire class, supervised by the principal, in which we were required to do a horrendous long division exercise. I believe that this question of mathematics and punishment still survives to this day in the minds of many principals.

The Old Math was never as successful as anyone thought it was. Only the very cream of the crop could go to university in the good old days, and of course they had a far higher mathematical aptitude than the average student of today who represents an enormously greater cross-section of the population. When aging university professors write letters to editors decrying the falling High School standards, they are forgetting this significant demographic expansion. Whenever I speak to doctors, lawyers, Members of Parliament, Professors of Humanities, or the man on the street, I come back with the conviction that more than 98% consider themselves mathematically ignorant. These people were all taught by the old system. In World War II there was a desperate shortage of artillery officers who could do the elementary calculations for computing the range and direction for their guns; and crash programs had to be instituted for them, navigators, and other Armed Forces personnel to learn what supposedly had been taught in the schools.

The endless, mindless drill was a crashing bore. I spent most of grade 5 writing out the 12 x 12 table. My grade 4 text; Lennes, Rogers, and Traver had over a thousand division problems alone - all the same - and we probably did them all. This book taught us such goodies as:

Multiplicand x Multiplier = Product
Minuend - Subtrahend = Remainder
How to point off

Nevertheless, the entire book contained nothing of any real interest.

We also spent much time learning about various different units of measure that metric has saved us from. For example, do you recall that

20 grains = 1 scruple
24 scruples = 1 ounce
12 ounces = 1 pound
} in Apothecaries measure

With the arrival of the metric system our apothecaries need not have any scruples any more.

One forgets today the large agricultural input that there once was in the schools. Our society has changed so that one need no longer know that up until about 1300 an acre was the amount of ground that a man could plow in a day with a yoke of oxen (2). It then become 160 square rods. This was the only interesting comment in Gage's 1933 Everyday Arithmetic, used for grade 5 in Saskatchewan. Almost the entire book was devoted to manipulation of fractions, without ever mentioning decimals or the number line.

Mathematics was chosen for its appropriateness for producing people ready to march into employment as clerks who have all been replaced today by adding machines. Do you recall the

thousands of bills such as:

compute the following butcher's bill:

3¾ lb. steak 48c
5½ lb. roast 42c
3½ lb. pork 46c
2 lb. ham 75c

This example is from Arithmetic for Public Schools, Authorized by the Ministry of Education, Ontario, 1936. Price 10c. This book swarms with such gems - repeated ad naseum.

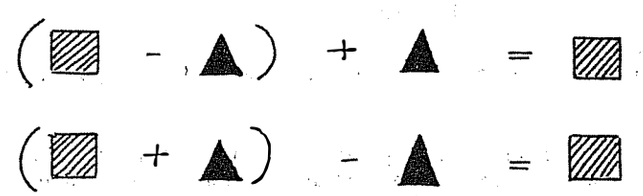
Many fundamental concepts were never taught or taught wrong. How many of us were told that the square root of -1 does not exist? If it didn't most electrical engineers today would be out of work because there would be no way to describe impedance of a circuit. Nor is it imaginary. It is a "real" a number as 93. It is simply a mathematical quantity with certain well defined algebraic properties. We all learned how to compute the GCD(or HCF) and LCM of a pair of numbers, but nowhere was it ever mentioned that the product of this pair was equal to the GCD times the LCM. Fractions were almost never explained to be numbers on the number line. Their connections with long division and the division algorithm were never mentioned. They were simply a secret process of taking parts of a "whole" and adding them in a manner that created more frustration than enlightenment. Only slight justification was ever given why $1/2 = 2/4 = 3/6 = 79/158$.

It was also found that students on this continent were lagging a year or two behind their equals in Europe. Moreover it was felt that our students would be left too far behind the rapidly progressing sciences, especially after Sputnik, to continue adequate scientific progress. Finally, the mathematical sophistication of our society was increasing and the importance of learning some new mathematical tools was felt at many levels.

Unfortunately the first leaps forward landed in the mud. Many textbooks launched in the name of the New Mathematics Curriculum should have been torpedoed. Distinctions between cardinal numbers and ordinal numbers; the language of set theory; and the mathematical operations and their commutative, distributive, and associative properties are indeed important mathematical concepts. There is a good argument that all teachers should learn and understand these concepts thoroughly; but there is no mathematical justification for force-feeding these to children at an early age. Something should be left for the universities to do. Definitions simply harden the mind and provide obstructions to learning the mathematics. Every teacher and every writer of a mathematics text should have a sign in 8 foot letters on his wall stating "SIMPLIFY".

Distinctions between numerals and numbers are not really needed. Signed numbers are a contrivance that must be unlearned. If it is not absolutely necessary to define something then it is best left undefined. And yet many recent texts are choked full of definitions. Examples of this are common. Other texts introduce and define concepts far beyond the level needed for the course. In a Grade 13 book one finds universal quantifiers - which could easily be omitted until 4th year university. In a 1967 Grade 4 text, one finds all the defintions of the properties of associativity, commutativity, etc., as well as questions of the following type:

"Are these number sentences true?



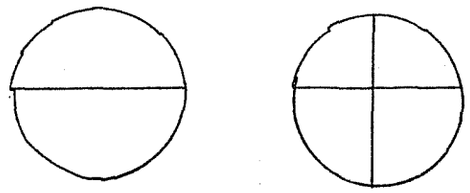
The author's interest in abstraction is commendable but surely this is not Grade 4 material. Fractions are introduced as follows:

"The fraction 2/3 tells that you have taken 2 of 3 equal parts.

Question: How many boats have white sails?



Ans. 2/3

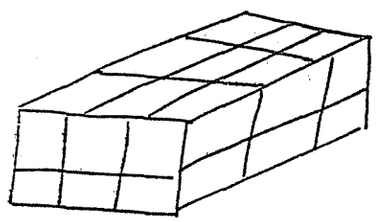


"Two quarters is the same as one-half and we write 2/4 = 1/2 or 1/2 = 2/4".
I find this all somewhat confusing. I believe the students who used this text may have shared my opinion.
In a Grade 6, 1967 book, one reads: "A boy weighs 80 lbs. and his father 160. Compare their weights. The father weighs 2 x 80 lbs. Think of 80 lbs. as 1 unit. The boy weighs 1 unit and the father 2, so the boy's weight is 1/2 of the father's weight."

This is how ratio is introduced. It is not the clearest or best choice. Let us compare it with the Grade 5 text by Dilley, Rucker and Jackson (Heath 1973); p. 260 "There are 7 dogs in the picture. Five are pups. We say that the ratio of pups to grown dogs is 5 to 2. You can write this ratio in several ways: 5 to 2; 5:2; 5/2 which is read as five halves or 5 to 2.

There are no extra units as in the previous example to distract or confuse the reader. I would simply call the latter a good definition. In their series of texts for grades 1 - 6 there are some superb questions posed by these authors. For example, in the Grade 6 text, alongside a picture of the Apollo II Saturn V rocket there are questions such as: "Apollo II weighed 2950000 kgs. A medium-sized car weighs about 1800 kgs. Did Apollo II weigh more or less than 1600 medium-size cars?
Apollo II was 109 meters tall. If a 32 story building has each story 3.3 meters high, which is taller?"

This is good pedagogy and should genuinely excite the students. Questions such as:



"How many cubes cannot be seen?"

are a delightful way to stimulate spacial perception and geometric intuition, and to practice multiplication.

In the Grade 5 text they present a picture of 3 apples a sign saying apples 6c each, and give an answer of 42c and ask the students to make up a question to go with the answer. This is a far cry from the 1930's.

A sequence of ratios is given 1/2, 2/3, 3/4, 4/5, ... and the students are asked to verify, as a project, that the ratios get larger and larger; that no term of the sequence is equal to 1; and to find to which number the fractional numbers of this sequence is equal to 1; and to find to which number the fractional numbers of this sequence get nearer and nearer. Here the deep concept of limit is cleverly introduced to those students willing to attempt the project, in a mathematically non-trivial fashion.

In the Grade 4 text they firmly place addition on the number line - where it belongs. A cartoon

multiplying machine is introduced in such a way that it is clear that multiplication is an operation. Yet machines are not used excessively or unnecessarily.

This is not the only text with good new ideas that I found. The one by Eicholz, O' Daffee and Fleener published by Addison-Wesley 1973, asks the students to cut out slips of paper with numbers 2, 3, 5, 8, +, -, and = written on them, and to see how many different equations they can write using these slips. This is the proper way to teach the laws of arithmetic - not by stating long incomprehensible defintions. They ask also how many rectangles of different shape one can find with area 36. Questions like this, together with a good teacher, will produce the kind of thinking that any mathematician would appreciate. Mathematics is appreciated more by a flexible, intuitive mind than by one rigidified by definitions, intimidated by tests, terrorized by fear of failing, or humiliated by a loss of self-esteem for getting 17X in the last quiz.

I feel that this analysis of textbooks, though dull, is of great importance because of the great dependence of most teachers on them. Few teachers have the time or training to establish their own independent program in mathematics. The amount of Faculty of Education mathematics training for most elementary school teachers is quite inadequate in my view, and leaves the new teacher confused as to the reasons for this new curriculum and the objectives of their course. This confusion is soon transmitted down to their class - and few come out unmarked. The right to teach mathemetics should not be given to every teacher regardless of his or her qualifications or interests, even in the elementary grades. In Ottawa, today, not everyone is allowed to teach French, so there is adequate precedent.

In order not to simply remain a critic but to offer some positive suggestions let us venture some ideas about the teaching of the new curriculum. Arithmetic must by firmly attached to the number line from the earliest grades. The property of "less than" should be introduced there in the sense of $2 < 3$ means that 2 is to the left of 3 on the line, as soon as the concept of left-right is understood rather than the more than-less than concept alone. Fractions should begin with division of integers, i.e.:

$2 \times 1 = 2 \quad 2 = 2 \div 1 = 2/1$
$2 \times 2 = 4 \quad 2 = 4 \div 2 = 4/2$
$2 \times 3 = 6 \quad 2 = 6 \div 3 = 6/3$
$2 \times 4 = 8 \quad 2 = 8 \quad 4 = 8/4$
$2 = 2/1 = 4/2 = 6/3 = 8/4$

This makes the concept of equivalence class quite clear. Only then should one proceed to

$1/2 = 2/4 = 3/6 = 4/8 \ldots$

which one can introduce by turning the above expression upside down. Division by 2 can be seen from the above.

$1 = 2/2,\ 2 = 4/2,\ 3 = 6/2,\ 4 = 8/2 \ldots$

to be simply halving the intervals (0,2), (0,4), etc. Thus 1/2 must be half of the interval (0,1). Similarly 3/2 is half of the interval (0,3), so it must represent some number on the number line between 1 and 2. Thus fractions are seen to be **numbers** - which is often a well-kept secret.

The hand calculator should be used from the first grade upwards as a device for verifying computation. It is a marvelous tool for teaching counting, by ones, fives, or tens; for learning the laws of arithmetic; etc. It should not replace all hand computation. Drill and mental arithmetic are very important and cannot be neglected.

Set theory is really a very minor piece of mathematics until the latter years of university. It should be left until then. There is no justification for the amount of mathematics time wasted in learning about sets and Venn diagrams.

Problems must be chosen to stimulate and encourage the students - not devastate them. Drill is important - but there must be a substantial pay-off in terms of exciting, fun, and challenging problems to positively reinforce the students for their efforts. Open-ended tasks should be found so that students of different ability can each progress as far as they can. Testing should be more to establish that the student has achieved a satisfactory level of competence than to simply punitively assign grades.

The good students in my present First Year Mathematics class at Carleton University are among the best First Year students I have ever seen. They are capable of absorbing concepts, such as those of sophisticated linear algebra, at a level that twenty years ago was given in Graduate School or the last year of an undergraduate majors program. For these students I would say that the new curriculum has been of considerable benefit. Unfortunately the benefits have not yet percolated down to all of the students but I am hopeful for the future. I believe that much of the criticism that has been voiced has been premature or has not fully considered other factors such as sociological change. Ontario can still be proud of its good teachers and good students.

WHITHER CANADIAN MATHEMATICS?

This is an address given at the O.A.M.E. meeting, May 24, 1975.

A.J. Coleman - Head,
Department of Mathematics, Queen's University

Our response to the question posed by the title of this talk should not be a crystal-ball prediction. It should be a resolution on the part of parents, teachers and officials of the Ministry of Education, to greatly improve the teaching of mathematics at all levels of the educational system. The study of the Role of Mathematics in Canada, which was financed by the Science Council of Canada and of which Professor Coleman was the Director, discovered much evidence

- of an extraordinary increase in the application of mathematics in business, industry and government in recent decades;
- that the diversity of mathematical techniques which have been utilized appears to be limited chiefly by the failure of persons with wide-ranging mathematical competence being present at the right moment to assist in the formulation and analysis of the problem at hand - the conditions for the proper symbiosis did not obtain;
- of widespread dissatisfaction with the mathematics programmes in the schools. They are probably as good or better than they ever were, but they were never adequate and certainly are now not sufficient given the ever increasing importance of mathematics in our life;
- that managers are distrustful of mathematically trained persons, do not sufficiently encourage their employment in analyzing their problems and are often unwilling to make any real effort to communicate with them;
- that there is widespread anxiety about mathematics among teachers which tends to enhance psychological blocks to the efficient study and utilization of mathematics.

On the other hand, there are many positive signs:

- a widespread acknowledgement of the importance and value of mathematics;
- an improvement of the formal qualifications of mathematics teachers in the Province of Ontario;
- the success of the O.A.M.E., and in particular the participation of a large contingent of elementary school teachers in the London meeting is most encouraging.

We need to work steadily at (i) improving the curriculum (ii) in-service training and mathematical discussion programmes for teachers (iii) encouraging the Ministry of Education and School Districts to appoint mathematical consultants with power to initiate innovative experiments (iv) inviting university professors of mathematics to interact creatively with teachers in elementary and secondary schools.

Above all, we must feel that **to mathematize is to joy**. Insist that the mathematical way of thinking is a valuable and satisfying power of the truly human person.

APPLICATIONS OF MATHEMATICS: A NATIONWIDE SURVEY

Alan C. Madgett, Mathematics Department, Laurentian University

The aim of this project is to produce an extensive collection of resource material on "applications" for teachers of Mathematics at the secondary school and introductory unversity levels throughout Canada. The finished product will consist of real-life mathematical problems (with background information and solutions) from industry, commerce, government agencies, etc. which illustrate the uses of some of the mathematical techniques currently being taught in classrooms at these levels. In this regard two aspects of the project must be noted.

i. The collection of illustrative problems is **not** tied to a particular instructional approach. It should be useful to most teachers -- as a source of supplementary problems for those using the traditional teaching approaches, as a source of small unstructured problems for those preferring a project oriented or modelling approach, or in any other manner the teacher desires.

ii. This survey is only a start in, hopefully, an ongoing effort to assemble a collection of examples; it is not intended to be "all inclusive".

The material emanating from this survey will provide information on which parts of the mathematics currently being taught at these levels are actually being used in the ***non-academic*** world and, equally important, how they are being used. In this way, we will obtain some indication of adequacy of the current Mathematics curricula in the secondary schools and insight into our Mathematics needs for the future.

The first phase of this project involved 10 months of travel across Canada unearthing the various applications. Mr. Peter Weygang (Levack District High School, Levack) and the author started the visitations in British Columbia in August 1974 and by the end of February 1975 had covered the four western provinces, northern Ontario and southwestern Ontario. The project director, Dr. J.F. Scott-Thomas (Laurentian University), and Prof. Madgett spent the Spring months visiting establishments in Quebec, the Atlantic provinces, and Ottawa.

We have attempted to concentrate on the distinctive economic features of each geographical region -- the forest products and fishing industries in B.C.; the petroleum, natural gas, and agricultural activities in Alberta; dairy, grain, and potash mining in Saskatchewan; resource management, agriculture, and government services in Manitoba; mining and resource processing in Northern Ontario; the manufacturing and chemical industries of Southern Ontario; the transportation industry and its related activities in Quebec; the fishing industry, coal mining, forest management, and the generating of hydro-electric power in the Atlantic provinces.

In June 1975 the team began compiling the raw material and re-working it into a form that will be of maximum value to the classroom teacher. Sections on Fishing, Food Production, Food Processing, Petroleum & Natural Gas, Transportation & Communications are currently being developed. It is hoped that material on these topics can be made available to teachers in Spring 1976. Resource material on Biology and the Health Sciences, the Environment, Wildlife, Forestry, Construction, the Chemical Industry, Heavy Industry, General Manufacturing, Finance, etc. should be ready for Summer or early Fall 1976.

The reaction of participants from industry and government has been most enthusiastic and cooperative. There are two reasons for this. Many have children in the present education system and are aware of the difficulties their children are encountering in their Mathematics courses. Secondly, they have welcomed the opportunity to contribute material, not just their opinion, to the education system. These participants are also assisting in the final preparation of the resource material. That is, once the original material has been collated and re-worked, the final draft is being returned to the contributors for their perusala and comments before being released to teachers. Many contributors have expressed confidence in our work by making available material that might be considered proprietory. In this regard they have requested, and quite justifiably so, that it be carefully disguised and returned to them for final clearance.

We feel that this project is unique in the field of mathematics education. It is the first organized attempt by teachers of Mathematics to carry out a visitation program to put together resource material on applications of mathematics that involves real-life problems (as opposed to purely theoretical, academic or contrived problems). The project is national in scope and covers a wide range of academic levels (secondary school and introductory university). Furthermore, most of the material will be authenticated by people working in the field of application.

This nationwide survey evolved from a project begun in 1970 by Peter Weygang at Levack District High School. With the assistance of members of the Mathematics Department at Laurentian University (including Dr. Scott-Thomas and Prof. Madgett), Mr. Weygang designed, and subsequently offered, a senior level mathematics course entitled **Broad Spectrum Mathematics.** In the BSM course the mathematics is developed from a consideration of mathematical problems and situations encountered in the non-academic world. The scarcity of real-life applications readily available for classroom use motivated the project participants to undertake putting together a compendium of such applications which could be used in the overall high school and introductory university curricula.

SOURCES OF FUNDING

Phase I (1974 - 75)

1. Ontario Ministry of Education - Grants in Aid of Educational Research Program $ 20,000.
2. The President's Research Fund, Laurentian University (A.C. Madgett) $ 3,000.
3. Ontario Secondary School Teachers' Federation Travelling Scholarship (P. Weygang) $3,000.

Phase II (1975 - 76)

1. Ontario Ministry of Education - Grants in Aid of Educational Research Program $ 7,500.

2. The President's Research Fund, Laurentian University (A.C. Madgett) $ 2,000.

Editor's Note: The author has also submitted a list of the institutions visited during the survey. Additional information may be obtained from the editor or by writing to Dr. Madgett.

THE ADVANTAGES OF INCREASING THE ROLE OF PROBABILITY AND STATISTICS IN THE SECONDARY CURRICULUM

Barbara Elaine Bernstein

Dr. Bernstein teaches mathematics at Bowie State College, Bowie, Maryland.

A lot of mathematics that students learn beyond the fifth grade is not related to immediate experience. But even basic material in probability and statistics has obvious relevance to practical matters. We speak of a "10% chance of rain" or a "50% chance of having an even license plate." We should capitalize on the worldliness of this field by giving it more emphasis than we now do in secondary school.

One problem for example which is suitable for high school students is that of determining the chance of getting x heads and y tails on (x+y) flips of a coin. This can be approached several ways. If (x+y) is sufficiently small, students can solve by simply listing all the possible outcomes. Or students can use the formula:

$$\frac{\frac{(x+y)!}{x!\,y!}}{2^{(x+y)}}$$

Finally, Pascal's triangle can be generated to find the solution.

Furthermore, this topic lends itself to encouraging students to discover patterns for themselves. For example, students, can inductively determine that the number of total possible outcomes in n flips of a coin is 2^n. They can find the relationship between successive lines in Pascal's triangle. Also students can verify for themselves the truth of certain probabilities. For example,, students' intuition often tells them that the chance of getting one head and one tail (in any order) on two flips is 1/3 since there are three possibilities (2 heads, 2 tails, and one of each). Students can correct this notion by flipping two coins many times to discover that this probability is really 1/2.

Statistics, too, could play a greater role in high school curriculums. The design of a statistical test is precisely the same as that of an indirect proof. Typically indirect proofs are covered in tenth grade. Teaching some simple statistical test (for example, λ^2) would reinforce the concept of indirect proof. More important, it would demonstrate that this method of proof is fruitfully applied to human questions.

These are just suggestions of the many exercises suitable for high school students in these fields. Compared to algebra and geometry, probability and statistics is more "relevant," probably more interesting and just as useful to a student's mathematics education. Especially with the current emphasis on applications of mathematics, we owe it to our students to increase the role of probability and statistics in the secondary curriculum.

ARE OUR SECONDARY SCHOOL GRADUATES LACKING IN BASIC MATHEMATICAL SKILLS?

Lawrence May, Department of Mathematics, Carleton University

For the last three years I have acted as coordinator for the basic first-year calculus and algebra course for science and engineering students (at Carleton University). It has been an accepted fact -- probably for a decade -- that the failure rate for this course is about 40%. This figure is roughly the same in most Ontario universities. The pass level for this course is set realistically in the sense those that fail would have little, if any, chance of achieving a pass in a second-year course. However, this failure is just about the highest for any first-year university course and it is reasonable to ask if the students who entered the course are lacking in preparation. I will give some time to this question and what should be done to improve their preparation.

About two years ago a basic twenty-question mathematics quiz was given to our science and engineering students in first year. This quiz was not intended to be difficult but we found that the average mark was seven out of twenty. (This quiz appears as an appendix.) My own feeling is that the vast majority of our entering students have the intellectual capability to perform well on such a test (say, better than 75%), and, in fact, after only four weeks of teaching the average performance moves up to about 60%, which supports this contention. However, it is definitely desirable that the basic material with which the quiz is concerned becomes almost second nature to students. Until this has been achieved I will continue to feel that students lack in basic skills and, in addition, that they are not well-prepared for first-year uniersity courses. What students need, especially if university bound, is a deep understanding of basic skills. This is, of course, only obtained if students (and necessarily teachers too) work hard. Any basic material which is to be worked on must be studied in depth. As an example I would like to consider just how much can be said about the quadratic equation.

Let us write $y = Q(x) = Ax^2 + Bx + C$, where $A \neq 0$.

1. The student can practice function notation (or be introduced to it, if he has not seen it before). Initially, the teacher gives examples with specific numerical values for A, B, and C (and then, later, Q can be evaluated where the coefficients are left in letter form).
2. Now that the student understands function notation he can graph a few specific quadratics -- x^2, x^2-4; x^2+1; x^2+2x-3; x^2+2x+5; x^2+2x+1; $4x^2+4x-3$; $9x^2+1$; $2x^2+2x-5$; x^2+x-1 -- and the negatives of all of these.

There are twenty examples listed:

(a) Eight of them are symmetric about the y-axis; twelve are not;

(b) Ten of them have a positive leading coefficient; this must be interpreted graphically.

(c) Six of them have "nice" distinct roots; four of them have repeated roots; six of them have no real roots; four of them have real "messy" roots.

(d) The student will almost certainly notice that if $Q(x) = x^2+2x-3$, then $Q(1) = 0$. Can he prove that $(x-1)$ is a factor of Q, preferably by himself?

(e) Since we can easily solve $x^2-4 = 0$, it is reasonable to ask if the quadratics x^2+x-1 and $2x^2+x-5$ can be written as a difference of squares. The commonly used "trick" of "com-

pleting the square" is thus motivated.

3. We may now solve an arbitrary quadratic by completing the square.

4. We can now see that as $B^2-4AC=\Delta \gtrless 0$, the quadratic has two, one or no real roots. More examples should now be given to illustrate use of the discriminant and quick qualitative sketches made using the sign of Δ and the sign of A. (Use of the sign of Δ is made when proving the Cauchy-Schwarz inequality; few students grasp this point immediately.)

5. If p, q are roots of $Q(x) = Ax^2+Bx+C = 0$, then $Q(x) = A(x-p)(x-q)$. It follows that $pq = \frac{C}{A}$ and that $p+q = -\frac{B}{A}$. If p, $q = \frac{-B \pm \sqrt{B^2-4AC}}{2A}$, then the student should check that $A(x-p)(x-q)$ really does give back Q. The student should do this, <u>not the teacher.</u>

I do not apologize for going into this topic so thoroughly. It is essential that you understand how much can be said about one seemingly simple topic. Many books, (even books on remedial mathematics) give such shallow coverage to quadratic equations or any other topic that it's not surprising that little stays with the student. If a person understands and can reproduce the material in 1-5 then he must have a grasp of basic algebraic rules.

I have chosen quadratic equations as an example. My contention is that any deep study of a non-trivial but elementary topic will help reinforce the facility with which students are able to use the basic tools. One cannot believe that $\frac{1}{x} + \frac{1}{y} = \frac{1}{x+y}$ and, at the same time be able to derive the quadratic formula. A good proportion of students in their early teens are capable of using basic algebra. It is by applying basic algebra to a problem, either of a pure or an applied type, that one learns how to use it correctly. As alternatives to the quadratic example here considered, one can take a detailed look at logarithms, basic trigonometry, "word" problems, and a lot can be done, I suspect, in teaching quick approximate methods in arithmetic. These are certainly not the only topics that can be covered. But in order to learn basic algebra the student must apply it; to study algebraic manipulation by itself is just boring.

APPENDIX

Please mark the answers to each question on the test paper and hand in both the question paper and the exam booklet.

<u>NOTE</u>: <u>I WOULD MUCH PREFER TO SEE A FEW WELL DONE QUESTIONS THAN A LOT OF BADLY DONE QUESTIONS.</u>

1. Solve for y: $1 - \frac{2}{y} = x$.

2. Evaluate: $\log_a a^3 + \log_a (\frac{1}{a})$

3. $a(a+b)^2 = ?$

4. Given that
   $\sin(A+B) = \sin A \cos B + \sin B \cos A$
   $\cos(A+B) = \cos A \cos B - \sin A \sin B$,
   find an expansion for $\tan(a+B)$. Simplify.

5. Factor: $x^2 - 4xy + 3y^2$

6. Show that: $\frac{4}{x^2 - 4} - \frac{1}{x - 2} + \frac{1}{x + 2} = 0$

7. $a\sqrt{a^{2-a}} = ?$

8. Simplify the function: $\dfrac{\frac{1}{x} + \frac{1}{y}}{\frac{1}{xy} + \frac{2}{x}}$

9. Prove that:

$$\frac{\frac{x}{2\sqrt{x+1}} + \sqrt{x+1}}{x+1} = \frac{3x+2}{2(x+1)^{3/2}}$$

10. What is the equation of the circle which has the points (1,2) and (7,2) as ends of a diameter?

11. Evaluate the following:

    (i) $\sin 90°$ (ii) $\log_3 9$

    (iii) $\cos 45°$ (iv) $\log_2 (1/2)$

12. $\frac{45\,xy^2 - 9\,x^3\,y + 18\,xy^2}{9xy} = ?$

13. Solve for I: $R = \frac{P}{I^2}$

14. Eliminate the root signs: $\sqrt{x+1} - \sqrt{x} = \frac{1}{2}$

15. A straight line has a slope of (-1) and passes through (2,3). Find its equation.

16. Rationalize the denominator of: $\frac{49}{\sqrt{11} - 2}$

17. Solve for z: $\log_2 3 = \log_8 z$.

18. What is a if: $x^2 + 3x + a = 0$ has only one root?

19. $(3x + 2)(3 - x) = ?$

20. Factor: $x^4y - 3\,x^3y + 2x^2y$.

THE PRACTICAL JUSTIFICATIONS FOR PURE MATHEMATICS

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Pure mathematics is put on the offensive often enough and, indeed, the schoolroom is surely the most likely place in which the (Unfortunate!) teacher is confronted with the question: "What good is mathematics, anyway?" It goes without saying, of course, that when such a question is asked, the interlocutor is not questioning the authority of applied mathematics (my, no!) for he will usually appreciate the relevance of physics and engineering! What the teacher must do then, (if he is not to follow the example of my Eighth Grade Teacher, who made me write lines for asking silly questions) is justify **pure** mathematics.

Depending upon the level of sophistication of the students, this is more or less a difficult issue to take on, but the justification is there and only needs ferreting out and then simplifying to the appropriate level of juvenility. It is recognized, of course, that school pupils are generally not patient with answers given in the context of history, but the nature of the problem allows us very little option. Possibly one could apologize to the student first. At any rate, some or all of what follows is better than not answering him at all.

A famous British mathematician, G.H. Hardy, is said to have proposed the toast - "Here's to Pure Mathematics - and may it never be of any use to anyone!" This description of Pure Mathematics, if true, would, in this utilitarian age, raise the question - "Why should anyone study Pure Mathematics?" But in fact, even if "usefulness" is taken in a very narrow "practical" sense, Pure Mathematics has been, and is, of great practical use; and present indications are that this will continue, and even increase. I remember a schoolmaster who used to introduce his class to Complex Numbers by asserting "The Jacques Cartier Bridge could not have been built without $\sqrt{-1}$ This remark may by an exaggeration, but the essential idea is true, that very practical, technological, results often depend on the "useless" abstractions of Pure Mathematics. The point of Hardy's maxim is, therefore, not in what it says about mathematics, but rather in what it implies about mathematicians, and the origins and motives of mathematical discovery.

A classical example of a pure mathematical topic finding an application is the theory of conic sections. If a cone, standing on a circular base, and extending indefinately in both directions through its vertex, is cut by a plane, then the curve in which the plane intersects the surface of the cone is the "conic section". These include, in particular, ellipses, hyperbolas, and parabolas. The study of these curves was begun by a Greek mathematician Appolonius (?260 -?200 B.C.), out of pure intellectual interest, unrelated to applications. This study was a labour of love, especially since, in view of the definition, the geometrical proofs had to be three-dimensional, and consequently were mostly long and difficult. Conic sections were found to have many beautiful properties, perhaps the most notable being the focus-directix property, which enables the curves to be defined without going outside the plane in which the curve lies; a conic section is the locus of points P in the plane, whose distance from a fixed point, the focus, is in a constant ratio to the distance of P from a fixed line, the directrix.

And there the subject rested for about 1500 years, until Kepler. Although Copernicus had put forward the theory (known long before to some of the ancient Greeks) that the planets, including the earth, moved in orbits around the sun, he had assumed circular orbits, which could only be a crude approximation to observations. Kepler, after monumental labours at both observation and computing (there were no computing aids, not even logarithms, in Kepler's time) discovered that, although various oval curves fitted the planetary orbits approximately, an exact fit (within the precision of his data) was given by ellipses; "the planets move around the sun in ellipses, with the sun in one focus". Kepler's discovery could not have been made, had not the theory of conic sections already existed.

The specification of planetary orbits in precise geometric terms enabled Newton to establish his inverse square law of gravity, and to confirm Galileo's concepts of dynamics, from observations of planetary orbits. The resulting ability to **predict** planetary positions was, and remains, of great importance for navigation. The understanding of mechanics which stems from Newton's work is of much greater importance, since it underlies a large part of modern science, with very many applications to engineering, and other matters. Whether or not Jacques Cartier Bridge could have been built without $\sqrt{-1}$, it certainly could not have been done withou Newtonian mechanics. Of course, such uses of mathematics are "applied", rather than "pure" mathematics. But without the preceeding "pure" mathematics, in this case going back to Apollonius's conic sections, it seems unlikely that the "applications" would ever have happened.

A more recent example of a pure mathematical topic finding a, quite unexpected, practical application is the theory of complex numbers. Several eighteenth-century mathematicians interested themselves in solving cubic equations, and found a procedure which will solve any cubic, whatever the coefficients. There was only one thing wrong with this procedure; sometimes it required one to operate with the square root of a negative number. This concept was so meaningless to the mathematicians of the time that they named such things "imaginary numbers" - an unfortunate name which is still with us. (The name reminds one of the gibe that a pure mathematician "does not know what it is that he is talking about, nor whether what he says is true"). But if these "imaginaries" were fed into the equations, and treated **as if** they were ordinary (real) numbers, they led to a correct answer, which could be verified.

So far, all this was abstraction, meaningless to the "practical man". The theory was later put on a firm logical ground, but was still highly abstract. But it found an application, late in the nineteenth century, in the then new field of electrical engineering. Alternating current generators, transformers, transmission lines, and motors had been invented, but the question of how to design a complicated electrical system, how to calculate the alternating currents which would flow in its several parts, remained difficult. This problem was solved by an American engineer, Stenmetz, by use of "imaginary" numbers, to represent alternating currents and voltages in both amplitude and phase. The mathematical problems involved can, indeed, be solved by other means (solutions of differential equations), but these are considerably more complicated, and give less insight into what is going on.

These two historic examples suggest that an important discovery has often been made by investigating some other, apparently quite irrelevant field out of pure intellectual curiosity, unconcerned with any immediate application or profit. This has, in fact, happened a number of times - too often to be without significance. A famous discovery in Physics, that of x-rays by Roentgen, will illustrate the point. If Roentgen had deliberately been seeking an instrument to render bones visible, without breaking the skin, as an aid to surgery, it is highly unlikely, in 1897, that he would have been experimenting with discharge tubes and similar electrical apparatus. Yet his discovery has been of the greatest medical value. Of course, to deliberately look in what is, ostensibly, the wrong direction does not guarantee a world-shaking discovery, or we would all be famous. Of course, if you want, for example, a better motor car it is appropriate, and often fruitful to confine your attention to cars; but you won't invent the hovercraft, or the helicopter, if your imagination is so restricted.

What is the stimulus, or motivation, for scientific discovery? Of course, this depends on the individual, but it is certainly true that many discoveries have come out of curiosity, a love of knowledge for its own sake. W.H. Whyte, in his book "Organization Man", has some relevant comments on this aspect of scientific research. "In pure research, half the trouble is in finding out **that there is a problem** - that there is something to explain ... By its very nature, discovery has an accidental quality. Methodical as anyone can be in following up a question, the all-important question itself is likely to be a sort of chance distraction of the work at hand. At this moment you neither know what practical use the question could lead to nor should you worry the point".

Whyte's remarks were written as general comments on pure scientific research, but they apply as well to Pure Mathematics as to Physics or Chemistry. While it is true that the kind of problem which a researcher is likely to think of is influenced, and to a serious extent bounded, by his circumstances, by the climate of opinion among which he lives, and by current political and economic factors, these factors are only a part of the story. A society which fails to encourage some of its best minds to"free curiosity" will, in time, cease to have major advances, whether in knowledge, technique, or social organization. While, in some other times, and places, the

extension of knowledge, for knowledge's sake, was reckoned a respectable pursuit in its own right, it is somewhat ironic that today, when more people have the opportunity to undertake scientific research than ever before, the motivation for doing so should so often be thought to need justification on grounds of utility. But, as already discussed, such justification can readily be given.

After this digression on Scientific Research in general, we may ask, what is distinctive about mathematics, compared with other intellectual disciplines? Since Euclid (in fact since Pythagoras), the postulation approach has been an important aspect of mathematics; one starts with a set of postulates, and deduces their logical consequences by a process of rigorous proof. This seemingly arid method is analogous to the physicist's approach to a complicated situation far too complicated to describe in full detail, by abstracting from the real situation certain features reckoned essential, for the immediate purpose, and then considering their implications. For example, in analysing the movements of the balls in a game of billiards, it is not necessary to consider their atomic structure. This process of abstraction, or simplification, is essential in the experimental sciences; a mathematician likes to think that the experimenters learned it, in the first place from Euclid.

Another aspect of mathematics, which arises out of the postulational approach, is the pursuit of generality. Mathematicians talk ad nauseam about generalising theorems, to hold under less restrictive conditions; why? If a set of conclusions are based soley on a given set of postulates, then they will be true for every system to which the postulates apply. If a theorem is true for triangles in general, then it is true, in particular, for triangle drawn with chalk on the blackboard, triangles measured, though not drawn, by a land surveyor, and triangles optically sighted by an astronomer, in order to determine the distance to a star. A method for solving simultaneous equations, if valid, applies whatever the equations may represent - whether the internal structure of an atom, or the production of fertiliser. This generality permits economy of thought, precisely because it focuses attention on what is, for the purpose, essential to the exclusion of irrelevant details.

A third significant feature in mathematics is th idea of "structure" or "pattern". What are the essential features which several systems have in common? Can we formulate postulates for these common features? The logical consequences of these postulates will then apply to all the systems. Inherent in the idea is that every mathematical system or subject, has a given structure, by which it may be recognized, and distinguished from other systems which differ "essentially" from it. For example, the system of integers ... -3, -2, -1, 0, 1, 2, 3... and the system of rationals (vulgar fractions, with either sign) e.g. 3/8 - 1/2, ..., differ in the structural feature that the quotient of two rationals is a rational (except that division by zero is not allowed), whereas the quotient of two integers may not be an integer (e.g. ¾). It is possible to formulate these structural differences, by setting down a system of postulates for each system, concerning the operations of addition, subtraction, multiplication, and division. If this is done, then it is found that there are other objects than integers, and rationals, respectively, to which the postulates apply. For example, the system of polynomials in a variable x, e.g. x^2 , 2 + x, etc., behave similarly to the integers, in that the sum, difference, and product of two polynomials are polynomials, but the quotient generally is not.

The mathematician's concept of "structure" is closely related to the experimental scientist's endeavours to find "order" or "pattern" in the observations which he makes of the universe. For this reason, the "structural" language and ideas of mathematics have so often been found the more suitable form in which to express, and summarize, the conclusions of the experimental scientist. And also, the labours of the mathematicians, to explore the logical consequences of their postulational systems, make it possible to deduce logical consequences of the models of physical systems proposed by experimentalists.

One final mathematical aspect deserving mention is that feature of a mathematical argument, which the mathematician calls "elegance". A proof which is neat, economical of argument, makes no more assumptions than necessary, and leads to the conclusion in a manner which leaves the reader enlightened as to **why** the conclusion is valid, would be called "elegant". This concept is somewhat vague, since it is partly an aesthetic one. In this age, when so much that is labelled art conveys no meaning at all, it may be encouraging that a concept, related to beauty of a more classical kind, is still alive, and significant, in mathematics. And the pragmatist may note that a mathematical proof which is attractive in the aesthetic sense is usually also clear, concise, and relevant.

THE FACTOR TRIANGLE

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Since long before the time of Pascal, mathematicians have been studying the triangular array of numbers which bears his name, Pascal's Triangle, and other 'similar' triangles. Its history dates back to about 1100 A.D., when it was studied both by Omar Khayyam and by several Chinese scholars (Hogben 1940, p.237). Pascal, of course, studied this array of numbers in depth, it is also called the Arithmetic Triangle, which is more suggestive. Relationships which have been discovered in this triangle abound in the literature. (See, for instance, the bibliography following the article by Toschi in the MATHEMATICS TEACHER 1974, p.210.)

Arithmetic Triangle

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1

Another very old, but much less famous triangle of this type is called the Harmonic Triangle. The patterns in the Harmonic Triangle fascinated Leibniz. The two outer lines are formed by the terms of the Harmonic Sequence: 1, 1/2, 1/3, 1/4, 1/5, ... A few rows are shown below.

Harmonic Triangle

1/1
1/2 1/2
1/3 1/6 1/3
1/4 1/12 1/12 1/4
1/5 1/20 1/30 1/20 1/5
1/6 1/30 1/60 1/60 1/30 1/6

Call the horizontal rows of numbers 'rows', and for each of these triangles, begin numbering with 0. Call the diagonal lines of numbers descending to the left 'columns'; the 0th column is of course the left-hand edge. In the Harmonic Triangle, each term is the difference of two terms, the one above and to the left, and the one to the left. In the first column, the entries are one-half the reciprocals of the triangular numbers: 1/2, 1/6, 1/12, 1/20, ... In the second column the entries are one-third the reciprocals of the pyramidal numbers, and so on. The series associated with each of the columns except the 0th is convergent. The 0th column, the Harmonic Sequence, has an associated series which is well-known to be divergent. An interesting relationship between the two triangles is that the entries in the nth row of the Harmonic Triangle are the reciprocals of the corresponding entries in the Arithmetic Triangle, divided by (n 1) (Boyer 1968, p.439).

Recently, students have been increasingly encouraged to search for patterns and to discover relationships for themselves (e.g. Polya 1957 and Toschi 1974). In studying these triangles, probably the most frequently posed problem is to find the pattern and to produce more rows of the triangle, and then, if possible to prove the validity of the work already done. With these ideas in mind, another triangle of a similar type provides a new set of patterns for students to discover, and is therefore presented below.

In the Arithmetic Triangle, the entry in the nth row and kth column is $\binom{n}{k}$, which adds a good deal of practicality to the intrinsic beauty of the triangle. Several patterns have been found in a triangle in which the entry in the nth row and kth column is $\left[\begin{smallmatrix} n \\ k \end{smallmatrix}\right]$ (Toschi 1974).

Consider a triangle in which the entry in the nth row and kth column is (n,k,), the highest common factor of n and k. In order to 'finish off' the triangle, put an '*' in the 0th row where (0,0) ought to go. Part of this Factor Triangle is shown below.

Factor Triangle

```
                *
              1 1
            2 1 2
          3 1 1 3
        4 1 2 1 4
      5 1 1 1 1 5
    6 1 2 3 2 1 6
  7 1 1 1 1 1 1 7
 8 1 2 1 4 1 2 1 8
 9 1 1 3 1 1 3 1 1 9
10 1 2 1 2 5 2 1 2 1 10
```

A lot of patterns are readily apparent:
- the two sequences of natural numbers down the outsides.
- the sequences of 1's just inside them.
- the sequences of alternating 1's and 2's just inside them.
- the triangle is symmetric about a line drawn down the middle.

Of course there are more patterns still.

Let us prove first of all that in the middle of the 2nd, 4th, 6th, ... rows appear the natural numbers 1, 2, 3, and so on. First, observe that the even-numbered rows have an odd number of entries (for instance, in the 0th row there is just one entry, an '*'). Let k by any natural number. In the 2kth row there are 2k + 1 entries and the kth one (the one in the kth column) is in the middle, since the leftmost entry is the 0th one. Therefore the middle entry in the 2kth row is (2k,k) which is k. So every natural number k appears in the middle of the 2kth row.

Let us prove next that the triangle is symmetric about a line drawn through these natural numbers. We shall need the following simple result:

Theorem: $(a, b) = (a-b, b)$ (1)

and $(a, b) = (a+b, b)$ (2)

To prove (1), let d be a factor of a and b. Then for some m and n, $dm = a$ and $dn = b$. Then $d(m-n) = a-b$ so d is also a factor of a-b. Similarly, any factor of a-b and b is also a factor of a. So (1) is proved. (2) is proved similarly.

The symmetry follows from the following:

$(n,k) = (n-k,k)$ by (1)

$(n-k,k) = (k,n-k)$

$(k,n-k) = (n,n-k)$ by (2)

so $(n,k) = (n,n-k)$, and these last two are to be entered in positions in the nth row symmetric with respect to the line in question.

The natural numbers appear in the 0th column because $(n,0) = n$ for all natural numbers n. The first column contains alternate 1's and 2's because alternate whole numbers are even. These rows get a 2 and the others get a 1. An application of (2) in the Theorem gives $(n,k) = (n + k, k)$. This leads to an alternate method of reading (n,k) from the triangle. (n,k) can be found at the bottom corner of a parrallelogram whose vertices contain the entries (0,0), (n,0), (n + k, k) and (k,k) where, as usual the first number represents the row number and the second, the column number. Together with the above remarks, the identity $(a,b) = (b,a)$ yields an alternate proof that the triangle is symmetric.

The most common and enticing problem, though, remains yet to be solved: to formulate a rule for producing an arbitrarily large part of the triangle. The solution to this problem can be derived from the observation that the previously mentioned sequences of 1's, of alternate 1's and 2's and of 3,1,1,3,1,1,3,... and so on, appear not only in lines descending to the left and right, but that each of

them has exactly one cycle appearing horizontally, joining the tops of its other two appearances. The triangle can be generated from the following rules:

- First, enter the natural numbers in two diagonal lines, as necessary.
- Then, for n = 1,2,3,..., the 1st through nth entries of row n are copied, beginning in the n + 1st row and 1st column, in a line going diagonally down and to the right. They are repeated as often as necessary.

The reason behind this is as follows:

$(n+1, 1) = (n,1)$
$(n+2, 2) (n,2)$
In general $(n+k, k) = (n,k)$, which is true by (2).
Also $(2n+k, n+k) = (n, n+k)$ by (1)
$(n, n+k) = (n+k, n)$
$(n+k, n) = (k,n)$ by (1)
$(k,n) = (n,k)$

Similarly, (mn n k, mn k) = (n,k), where the above are true for k= 1,2,3,...n, and for m= 0,1,2,... This last statement justifies the repetition of the sequences which appear horizontally, as in the second rule above. Clearly, the entire triangle can be generated from the two rules given.

Because of the three directions in which these number sequences appear, an interesting pattern can be obtained by placing four copies of the triangle around a central asterisk.

```
4 1 2 1 4
1 3 1 1 3 1
2 1 2 1 2 1 2
1 1 1 1 1 1 1 1
4 3 2 1*1 2 3 4
1 1 1 1 1 1 1 1
2 1 2 1 2 1 2
1 3 1 1 3 1
```

An arbitrarily large rhombus (or hexagon) can be produced in this way. In it, each of the number sequences proceeds forever in its several directions.

Another way of considering these sequences in the original triangle is to regard each sequence as a ray coming up from the lower left, reflecting off the right hand edge of the triangle horizontaliy to the left, and reflecting off the other edge down to the right.

Some more observations are obvious: a number p is prime if the pth row is of the form: p,1,1,...p; perfect numbers are those which equal the sum of all the distinct entries in their rows.

In conclusion, I should like to suggest that the Factor Triangle, used either alone, or in conjuction with the other three triangles mentioned, provides a new situation in which students can search for and discover patterns. The Factor Triangle has the advantage that the mathematics used in constructing it is perhaps the most elementary.

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THE USE OF THE HISTORY OF MATHEMATICS AS A TEACHING TOOL

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Text of an address given at the O.A.M.E. conference in May, 1975

The major pedagogical purposes served by the history of mathematics can be gathered under four headings: (1) the "humanizing" of mathematics and mathematicians, (2) the enriching of the teaching of mathematics, (3) the increasing of pupils' understanding of the nature and role of mathematics, and (4) the increasing of pupils' understanding of mathematics, per se.

The need for "humanizing" was repeatedly voiced at the Second International Congress on Mathematical Instruction held at Exeter, England, in 1972 although the term had not been suggested in the preliminary planning and publicity. Marshall Bye of Calgary told of the concern of the schools there for a general loss of interest and a decline in enrollment in later courses. A city-wide questionnaire survey showed that students viewed mathematics as mechanical, dull, unrelated to real people or their activities. A city-wide remedy in the form of required project activity in mathematics at the grade 7-9 level was tried with substantial success. Many of the projects were historical. One of the most successful and extensive involved many students in a study of mathematics and art with especial emphasis on perspective drawing and its history.

Stanley Cotter of Foothill College, a two-year College in California, had stressed historical and historical-philosophical ideas in a liberal arts course for non-science students.

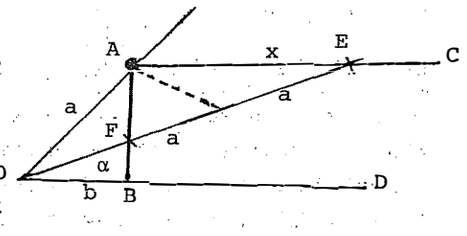
Dr. R. Stowasser of the Federal Republic of Germany told of the use of historical material in teaching mathematical induction. He felt that this topic, as usually taught, is an extreme example of unmotivated problems picked out of thin air and formulated by an author or teacher to fit an ill-understood algorithmic proof process. He found in a variety of original sources many old problems, often formulated in story form and recreational in nature. His students found fun and a more real problem solving experience in formulating, as well as proving, the theorems.

The writer cherishes a copy of a radio script on Galois's life which was the product of a former, non mathematically inclined student. Several years after the student's passage through college algebra she sent the script with a note of thanks for the historical discussions that had accompanied the unit in theory of equations and had excited her interest in such personalities as Cardan, Lagrange, Gauss, and Galois. She had transferred to a school which trained people to produce materials for radio programs, but expressed her thanks for her experience in a mathematics class.

Of course "humanizing", the conveying to students of the fact that real human beings find mathematics an exciting and rewarding creative activity, can be achieved in many ways. However, historical problems, projects, and personalities are often useful for this purpose.

There are also many ways to "enrich" the teaching of mathematics. Something is added to instruction by the mere recounting of historical developments of famous problems, interesting persons, the search for relationships and solutions. However, some recapitulation of the process of development can enrich the pedagogical process itself by using **"discovery"** (or rediscovery) teaching methods, by bringing out the roles of **generalization**, of **analogy**, of **counterexample** in discovery and proof. The story of the trisection problem illustrates this. The origin of the following "verging" solution is uncertain, but antedates Archimedes who developed a similar one.

Select any point A on one side of an angle AOB and draw AC and AB respectively parallel and perpendicular to OD, the other side. Let the length OA = a and mark



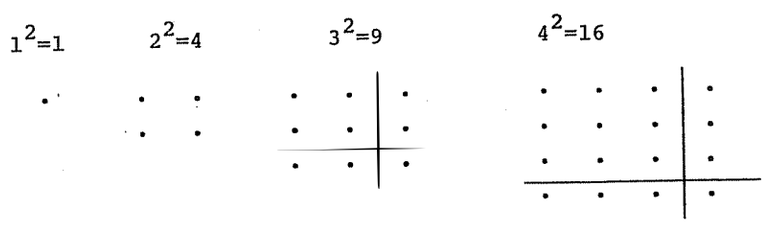
off the length FE = 2a on a straight edge. Slide this straight edge over the angle with point E on $\overrightarrow{AC}$ and point F on $\overline{AB}$ until the straight edge also passes through O. This determines $\angle\alpha$ which is $\frac{1}{3}\angle AOB$ as it is fairly easy to lead students to prove. Some student will likely ask, "why then do people say trisection is impossible?" (If no one does, then sketch the story of the three famous Greek problems.) Here is an opportunity to stress (1) the importance of clearly formulating a problem and (2) the "if-then" nature of mathematics. The problem may be formulated as: If we restrict ourselves to specified operations with an unmarked straight edge and compass, then it is possible to trisect all angles. Now we have an opportunity to stress two additional important concepts: (3) that real mathematics begins with conjectures which may or may not be true, and (4) that although a million examples will not prove a theorem, one "counter-example" will disprove it. Anyone who can construct a given angle, say 45° or 60° and can bisect an angle, can thereby trisect an infinite number of angles (i.e., 45° is one third of 135° and one half of 60° is one third of 90°, etc.) However, if we can show that one angle, say 60° can not be trisected, the conjecture is disproved.

This immediately produces an opportunity to teach the interrelatedness of mathematics and to review (or teach) some interesting theory of equations. It is fairly easy to show using similar triangles that if we let AE = x, for convenience take a = 1, and OB = b, then $x^3-3x-2b = 0$. If we can construct the length x we can locate E and draw OE and conversely. We now have reduced an ancient geometry problem to a fairly modern algebra problem. If we take m $\angle AOB = 60°$, $b = \cos \angle AOB = \frac{1}{2}$ and our equation becomes $x^3-3x-1=0$. We next introduce the theorem not to be proven here (or in an elementary class) due to Pierre Wantzel in 1837, If a cubic equation with rational coefficients has no rational roots, then it has no "constructible" roots. "Constructible" refers to our earlier restriction on the use of geometric tools. We now pull out of our hat the theorem which we first motivate and then prove, If the a_i are integers and $\frac{p}{q}$ is a rational root of $a_nx^n + a_{n-1}x^{n-1} + \ldots a_2x^2 + a_1x + a_0 = 0$, then p divides a_0 and q divides a_n. According to this theorem the only possible rational roots of $x^3-3x-1=0$ are ± 1. Since neither of these is a root, the equation has no rational root and therefore it has no constructible roots, and finally, a 60° angle can not be trisected. The Wantzel

theorem can also be used to show that $x^3-2=0$ has no constructible roots and that therefore duplication of the cube is also impossible.

The story of Pythagorean triples can be used (1) to get students involved in discoveries, (2) to dramatize the role of intellectual (mathematical) curiousity, (3) to suggest generalization and specialization as devices for extending mathematics. In my experience someone in every secondary school will volunteer "3, 4, 5" as an example of length of sides of a right triangle. Someone will add 6, 8, 10 and discussion will bring out and prove that there is an infinite family of triples of the form 3x, 4x, 5x. When another student brings up 5, 12, 13 and its related infinite family, the stage is set for a sequence of questions alternating with motivations and advances: Are there other families? Can we find an algorithm or formula for generating them? Does our formula generate all possible families? I sketch one sequence which has worked.

Note the sequence of diagrams for the square numbers:



This leads to the two discoveries, $n^2+(2n+1) = (n+1)^2$ and the fact that the sum of the first n consecutive odd numbers is n^2. (Pythagorean figurate numbers have many properties which can be "discovered" geometrically or numerically and proved by induction).

Observing that if (2n+1) were a square the formula above would give a Pythagorean triple suggests letting $m^2 = 2n+1$. Then $n = \frac{m^2-1}{2}$ and $n+1 = \frac{m^2+1}{2}$. Now it is easy to verify this numerically, to prove it algebraically and to compute a table showing that if $a = m$, $b = \frac{m^2-1}{2}$, $c = \frac{m^2+1}{2}$, then $a^2+b^2 = c^2$.

m	a	b	c
2	2	$\frac{3}{2}$	$\frac{5}{2}$
3	3	4	5
4	4	$\frac{15}{2}$	$\frac{17}{2}$
5	5	12	13
⋮	⋮	⋮	⋮

From this we see that if m is even, two of the triple, a, b, c will not be integers. However, we can get a formula which will always produce a triple of integers by multiplying each expression by 2. Thus if we take

$$a = 2m,\ b = m^2-1,\ c = m^2+1$$

we get

m	a	b	c
2	4	3	5
3	6	8	10
4	8	15	17
⋮	⋮	⋮	⋮

This will give an infinity of infinite families of triples. Will it give all?--No, because it omits, 5, 12, 13 (but it includes 10, 24, 26), and 20, 21, 29 (if anyone has spotted the latter as a triple). It is interesting to note, if not easy to show, that $a = u^2-v^2$, $b = 2uv$, $c = u^2+v^2$ will give all possible triples!

Here we can raise such questions as, would all the triples be primitive (having no common factor) and, if not, how could we restrict u and v to make them all primitive? We also could pursue the connection of the sequence 1+3+5+ --- a terms = a^2 with finite differences, curve fitting and the summation of series.

However, let's follow up the theme of famous problems, some solved, some "settled" by being shown unsolvable, and some unsolved. Note that a mathematician having settled $a^2+b^2=c^2$ would naturally turn to $a^3+b^3 = c^3$ or $a^2+b^2+c^2 = d^2$. The former of course leads to the famous unsolved "Last theorem" of Fermat, that $a^n+b^n = c^n$ has no solution in integers. The latter can lead to the theorem that every

integer can be represented as the sum of not more than four squares. Both of these theorems have interesting histories and both can lead to interesting activities. The second theorem is a good candidate for a discovery lesson as is also the Goldbach conjecture that every even number greater than 2 is the sum of two primes. The Goldbach conjecture is another example of a simple unsolved problem.

The number theoretic problems just cited have few if any practical connections, however, there are many historical sequences in which intellectual curiousity and pure mathematics lead to developments which later found major applications. Two examples are Leibniz' development of the binary notation which has been so important in computers, and Cardan's first hesitating steps toward complex numbers which are now major tools in electrical and aeronautical engineering. There is not time to do more than sketch Cardan's treatment in 1545 in his Ars Magna of the problem: Find two numbers whose sum is ten and whose product is 40. After stating this problem he went on to state "Obviously this is impossible. Nevertheless, let us operate." A good question for student discussion is why it is "obviously impossible." Cardan went ahead to solve the related quadratic equation by following the sequence of operations (completing the square) which he had just taught. He obtained $5 + \sqrt{-15}$ and $5 - \sqrt{-15}$ as the two numbers and checked that their sum was ten and product 40 (if $(\sqrt{15})^2$ is -15). He then called this result "truly sophisticated" and stated that if one were to go farther he would have to develop a new arithmetic which would be "as subtle as it would be useless?!

This last example gives an opportunity again to (1) "**humanize**" via discussions of Cardan, his friends, enemies, students, and fraud, (2) to **enrich teaching** via discussion and discovery of how Cardan could know in advance that $x(10 - x) = 40$ is "impossible" (in the real domain), (3) to **help students understand mathematics** as a man-made logical structure where we can construct new numbers and operations whenever we wish, but also (4) to understand that physical and geometric **analogies or models may both motivate the construction of new mathematics and provide the key to using old mathematics to solve physical problems.**

If the history can be used to accomplish these four goals, I believe that it can play a very important role in the teaching of mathematics.

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free materials

The following materials are available free to OAME members:

Elementary - "Boxes, Squares, and Other Things". A teacher guide for a unit in informal Geometry. (Cost to non-members $1.80) from Mr. M. MacGregor Sec. Treas. OAME.

Secondary - Grade 9 Preparedness Test. Grade 13 Sample Examinations. A Handbook of Trigonometry. (Cost to non-members $1.00 per item) from Grand Valley Mathematics Association.

DIVERSIONS

by

T.G. Friday

The purpose of this column is to provide easy to present, pleasant enrichment topics and puzzles for such times as the afternoon of the last day of the week.

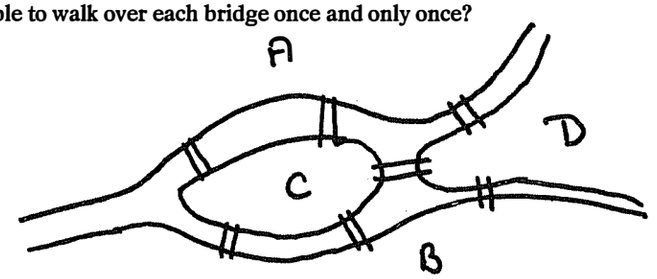
We should welcome your ideas, successful puzzles and other goodies. These need not be original. Remember, the good old chestnuts are good.

Goodie

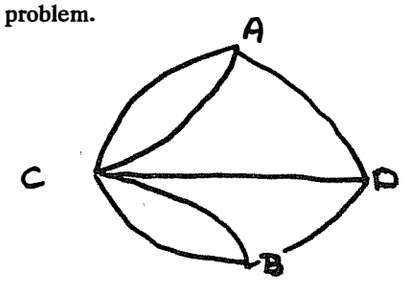
Level: Grade 7 up. The Konigsberg Bridge Problem Solved by Leonard Euler 1701 - 1783 Considered to be historcally important as the originating problem in topology

The city of Konigsberg in Prussia is on both sides of the river Pregel and includes the island of Kneiphof. The parts of the city are connected by seven bridges.

Problem: Is it possible to walk over each bridge once and only once?

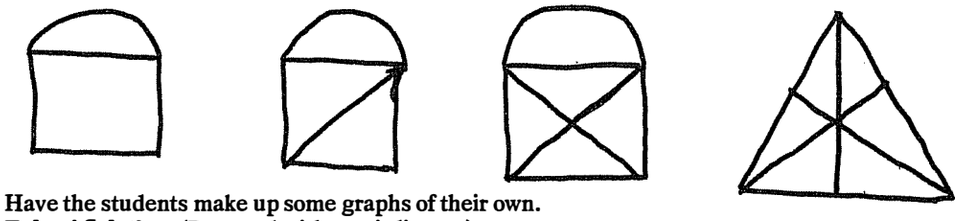


Euler simplified the problem by reducing the map to its barest essentials. He reduced the land to points and the bridges to lines. This is a mathematical model of the situation, and is a very important step in solving the problem.



Restatement of problem: Is it possible to draw this diagram without lifting your pencil from the paper?

Class exercise: Which of the following diagrams is it possible to draw without lifting your pencil from the paper? These diagrams are called graphs.



Have the students make up some graphs of their own.

Eulers' Solution: (Restated with poetic licence)

In a diagram which can be drawn without lifting the pencil from the paper there are three kinds of points:

S - A starting point where the pencil is placed on the paper

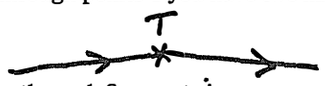
T - Through points which the pencil travels through on its journey

F - Finish point where you lift the pencil from the paper.

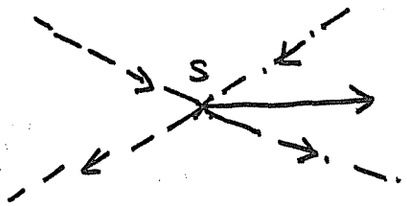
When the pencil is put on a starting point S and a line is drawn away from it you have one line leaving S.



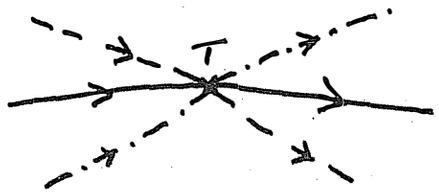
As lines are drawn to and from through points T you have two lines to T



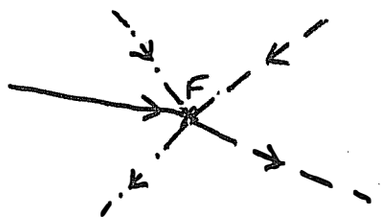
During the journey you may pass through S once, twice ~~or more~~ times, which will give 3, 5 or more odd numbers of lines from S. Call this an odd point.



You may also pass through the through points several times, giving an even number of lines to through points. Call these even points.

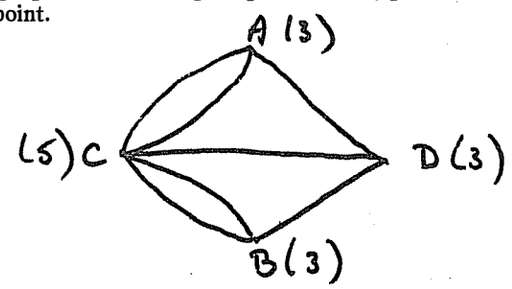


At the finish point F you will draw a line in and lift the pencil. This will give one line to F, unless this point has been passed through before, in which case you will have 3, 5, or an odd number of lines to the point. Hence this will be an odd point.



Therefore the only odd points are the start and finish points. If you start and finish at the same point, this point will also be even, since odd plus odd is even.

Conclusion: It is possible to draw a graph without lifting the pencil from the paper if the graph has exactly two odd points, in which case one is the starting point and the other the finishing point. It is also possible to draw a graph without lifting the pencil if every point is even, in which case you start and finish at the same point.



The Konigsberg problem has 4 odd points and therefore there is no solution.

Exercise: Try more graphs.

PROBLEMS

Problems — Reconstruct the Division

14-1-1
Up to Grade 9

```
                    8 x x x x
  x x x ) x x x x x x x x x
          x x x
          x x x x
          x x x
            x x x x
            x x x x
                  0
```

14-1-2
Up to Grade 12

What is the rule for this sequence
0, 1, 10, 2, 100, 11, 1000, 3, 20, 101, 10000,
12, 100000, 1001, 110, 4, 1000000, 21, ...

(Eureka)

14-1-3

Prove that $[5x] + [5y] > [3x + y] + [3y + x]$
for $x; y > 0$, where $[\ ]$ is the greatest integer function i. e. $[\sqrt{3}] = 1$ (1975 U.S.A. Olympiad)

Really? Proof that all rational numbers are equal.

Let $a = b$

(xa) $\quad a^2 = ab$

$(-b^2)$ $\quad a^2 - b^2 = ab - b^2$

$\therefore\ (a - b)(a + b) = b(a - b)$

$[\div(a - b)]$ $\quad a + b = b$

but $a = b$

$\therefore\ 2b = b$

$(\div b)$ $\quad \therefore\ 2 = 1$

$a = b$

(xa^2) $\quad a^3 = a^2b$

$(-b^3)$ $\quad a^3 - b^3 = a^2b - b^3$

$\therefore\ (a - b)(a^2 + ab + b^2) = b(a - b)(a + b)$

$\div(a - b)$ $\quad \therefore\ a^2 + ab + b^2 = b(a + b)$

but $a = b$ $\quad \therefore\ b^2 + b^2 + b^2 = b(2b)$

$\therefore\ 3b^2 = 2b^2$

$\div b^2$ $\quad \therefore\ 3 = 2$

but $2 = 1$

$\therefore\ 3 = 2 = 1$ etc.

$\therefore$ All natural numbers are equal.

INTERNATIONAL MATHEMATICS EDUCATION

THIRD INTERNATIONAL CONGRESS ON MATHEMATICAL EDUCATION

Karlsruhe, Federal Republic of Germany, will be the site of the Third International Congress on Mathematical Education (ICME), 16-21 August 1976. The Congress is being organized by the West German Subcommittee of ICME in collaboration with the Local Organizing Committee. The program will include plenary sessions, survey reports, panel discussions, special projects, and commercial exhibitions. Further information and application forms may be obtained from Professor H. Kunle, 3d International Congress on Mathematical Education, University, D 75 Karlsruhe, Federal Republic of Germany.

FOURTH INTER-AMERICAN CONFERENCE ON MATHEMATICAL EDUCATION

This conference will take place in Caracas, Venezuala December 1-6, 1975. Focus will be on mathematics teaching at the elementary and secondary levels. Information may be obtained from Claude Gaulin, Faculty of Education, Universite Laval, Quebec City, who is Canadian representative on Comite Interamericano de Educaion Matematica.

N.C.T.M. DETROIT

Please keep in mind the Detroit Name-of-Site meeting of N.C.T.M. next spring. This convention will be held at Cobo Hall, March 4-6, 1976. At this time the Program Committee is lining up speakers. It appears there will be about two hundred section meetings.

Plan ahead! Come to Detroit next year!

O.A.M.E.
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ONTARIO ASSOCIATION for MATHEMATICS EDUCATION

REPORT OF THE PRESIDENT

Report of President of OAME, John C. Egsgard
to Annual Meeting, London, Ontario, Sunday, May 25, 1975.

In its second year of existence OAME has been very active. The Executive and Council of OAME has met several times; the Council three times and the Executive five times.

Some of the major actions taken by OAME during the year were the following:

1. Submitted a brief to the Minister of Education on the Qualifications and Preparation of Teachers of Mathematics K-8. We received a positive response from the Minister to this brief and intend to continue the dialogue with the Minister regarding the matters dealt with in brief. Copies of the brief are available from the executive secretary.

2. Conducted a leadership conference for elementary teachers of mathematics. This conference was held October 23-26, 1974 in Niagara Falls. The conference was attended by over 100 elementary teachers. A similar conference will be held in 1975 again in Niagara Falls.

3. Began a system of grants to Chapters to help them with special projects. This year grants were given to GVMA ($500), NOMA ($70.40) and COMA ($145.).

4. Established a Senior Cyclic Curriculum Committee to make recommendations that the OAME will submit to the Minister of Education at the time of the review of the Senior Curriculum in Mathematics.

5. Set up an ad hoc committee to obtain information regarding scholarships for mathematics teachers.

6. Nominated Dr. Magee of the University of Western Ontario as a life member of OEA. Our nomination was accepted by OEA.

7. At the suggestion of last years annual meeting the idea of proxy voting for the executive and council of OAME was considered. Proxy voting was rejected by the Council.

Much of the work of OAME is done by its committees of which there are seven. The following is a summary of the actions taken by these committees.

EXECUTIVE COMMITTEE

The Executive Committee met five times during the year to consider the recommendations of the various committees and to act upon them.

PROJECTS COMMITTEE

The brief submitted to the Minister of education was primarily the work of a sub-committee of the Projects Committee. The Projects Committee established a Trading Post for the annual meeting in London. It is the hope of the Committee that the Trading Post will be a means for the sharing of ideas among the mathematics teachers. The Committee was also responsible for obtaining information about courses that are available to teachers of mathematics at various universities.

MEMBERSHIP AND PUBLICITY

This Committee made a concentrated effort to obtain new members for the Council and had members assigned to junior high, secondary, university and CAAT, industrial membership and faculties of education. There will be a special push for elementary teacher membership this coming year. The Committee is in process of designing a membership certificate.

CHAPTERS COMMITTEE

The Chapters Committee has the special task of setting up guidelines for grants that will be given by OAME to its Chapters. The Committee is also making an effort to encourage all of the regions of the Province to set up Chapters to affiliate with OAME.

PUBLICATIONS COMMITTEE

The Publications Committee is responsible for the Ontario Mathematics Gazette and the **ABACUS.** With the completion of the terms of the previous editors, Bill Eames and John Griffith for the Gazette and Brock Rachar for the Abacus, new editors were obtained. Arnold Harris was appointed as editor of the Gazette and Andy Czempinski for the Abacus. These appointments will be from May 1, 1975 to April 30, 1977. The Publications Committee has also made available for each member of OAME two publications from the Grand Valley Mathematics Association Chapter for secondary school teachers and the NCTM Publication "Boxes, Squares and Other Things" for elementary school teachers. The committee is also in the process of recommending terms of reference for the editors of the two journals.

MEETINGS AND MANPOWER

This Committee has as its primary job that of obtaining locations for the annual meetings. The next meeting of OAME in 1976 will be held May 14, 15, 16 at Brock University. This meeting will be co-sponsored by the Lincoln County Board of Education and Brock University.

NOMINATIONS COMMITTEE

The Nominations Committee is responsible for the slate of nominations that are presented at each annual meeting.

I believe that this has been an excellent second year of OAME. I wish to thank the members of the Executive and Councillors of OAME for their assistance in making this year successful. A special thanks is due to our executive secretary, Morley MacGregor and his wife Mona. I wish our new president, Joan Routledge, the best of luck for the coming year. OAME is in good hands!

SIX CHAPTERS OF OAME

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Sec. Treas. Bill Scott

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We are planning a very selective programme with exceptional key speakers and a close look at current issues.

Consider participating in the Trading Post.

Details concerning the Trading Post and all other Activities will be available in future publications.

THEMES - OAME CONFERENCE - 1976

Where Have We Been and Where are We Going

- a look at what we have done well in Ontario, what we need to improve and how these factors relate to social need and world developments in mathematics

Standards, Achievement & Evaluation In Mathematics

- a basic look at basic skills at all levels, the need for basic skills, what basic skills will be in the last part of this decade and in early 80's, the question of examinations, accountability to the public, evaluation techniques

- liaison between levels and schools

Geometry: Approaches and Their Influence

- what constitutes a geometry program for K-6, transformations, a vector approach to geometry, a deductive approach using traditional Euclidean methods, how transformations/vectors affect other areas of the mathematics program

Meaningful Arithmetic/Algebra

- making arithmetic (algebra) useful to the student, what arithmetic (algebra) is appropriate in todays mathematics programs

Remedial Mathematics

- describing who, what, where, when, why and how in remediating skills for reluctant and/or disadvantaged and/or handicapped learners

Mathematical Models Representing Reality

- using mathematics to gather and process data, interpreting mathematics in every day situations, problem solving, mathematical models for solving concrete problems

Calculating Devices - A Help or a Hindrance

- how hand held calculating devices are/not useful in our mathematics programs, changes in mathematics program to accomodate hand held calculators, using hand held calculators to best effect

What About Statistics

- the use of statistics in today's world, making statistics a profitable learning experience for students of all ages, skill building through statistics and probability, media to facilitate learning in a statistics strand

Computers

- the appropriateness of desk top units, advantages and/or disadvantages of desk top or central installation computers, making computers a tool for the use of all (some) departments in the secondary school, computers and elementary school students

Metric Measure and Mathematics

- what about decimals, are fractions necessary, changes in the PJ area, the influence of metric measure on Intermediate Dvision programs

Beautiful Mathematics

- mathematics workshops and presentations for the sake of mathematics, the beauty of pure mathematics

Motivating Mathematics

- games, making mathematics fun, ideas for motivation at all levels, mathematics in art, mathematics in music, mathematics in pure and applied science, mathematics and the social sciences

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M.B.MacGregor

BRIEF TO MINISTER OF EDUCATION

Copies of a brief on the qualifications and preparation of Teachers of Mathematics (K-8) are available from M. MacGregor, 247 Bright Street, Sarnia, N7T 4E9. This brief was prepared from comments from OAME members and Faculties of Education who were sent a preliminary draft. The brief has now been approved by the OAME Council and sent to the Minister.