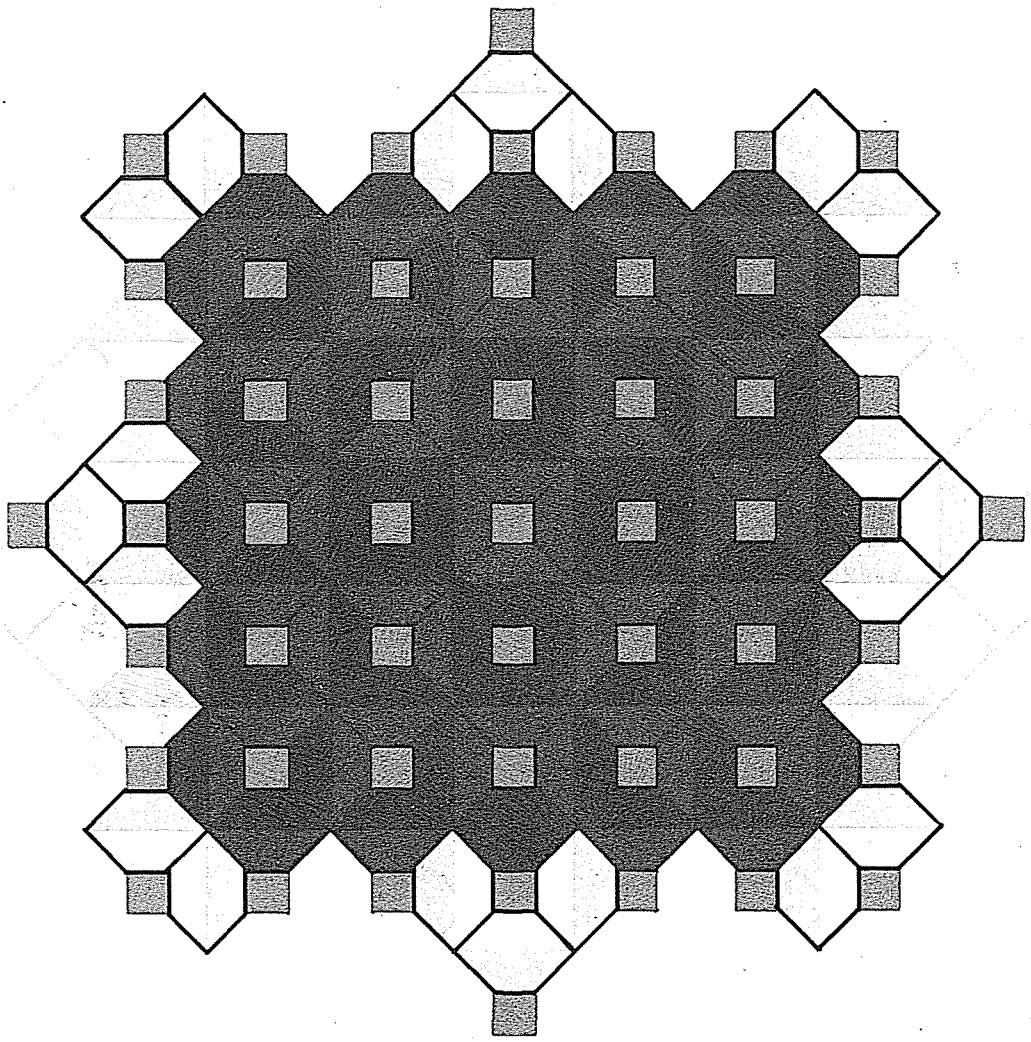
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THE ONTARIO ASSOCIATION FOR MATHEMATICS EDUCATION

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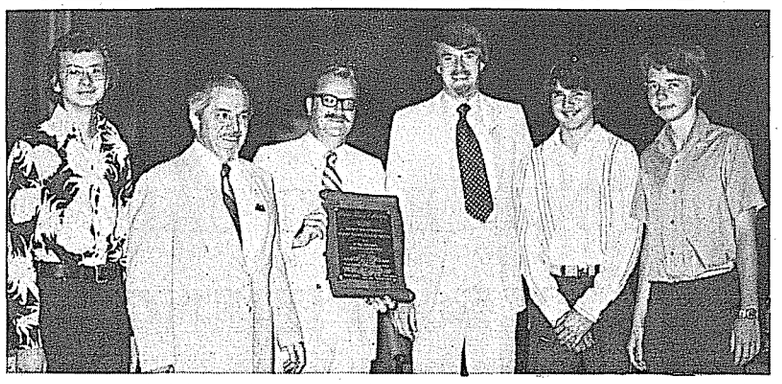
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THE FATHER FAUGHT AWARD



On Wednesday, June 16th, The Father Faught Award was presented to Scarlett Heights Collegiate Institute for achieving the highest Ontario standing in the Waterloo Junior Math Contest. Ed Silcox, an OAME councillor, presented the award to Mr. Bill Wilson, head of Math and 3 of the 5 team members. Neil Williamson, Vice-President of OAME, added congratulations from the executive, council and membership of OAME. Pictured from left to right are Rimas Ciplijauskas, Ed Silcox, Bill Wilson, Neil Williamson, Shawn James and Edgar Laengert.

The two team members who placed highest, Gordon Lee and Jeff Air were at the University of Waterloo for special seminars.

EDITORIAL

This magazine's editors aim to put out useful issues. Our readers (and non-readers) have urged us to do so. We begin a new school year eager to please you by increasing the practicality of the contents of the Gazette. However, recent developments tend to keep us in a defensive posture.

The BTB ("back to basics") controversy and the recent publication of the Science Council report, **The Mathematical Sciences in Canada,** require us to comment. I hope the members of O.A.M.E. will bear with us, recognizing the need for some discussion of fundamental issues of mathematics education by their official mouthpiece.

We welcome Frank Ebos of F.E.U.T. as Research Editor. Frank wishes it to be known that he is mainly concerned with researching for good articles in the magazines of our sister organizations and getting permission to have them re-printed in the Gazette. His bias is practical rather than theoretical. However, he did find time to be philosophical in an article for this issue.

The recent newspaper publicity regarding the Science Council report has led our Book Review Editor, Bill Higginson to write a comprehensive review. Every member should get a copy of this milestone document and read it. A Canadian Press article in the London Free Press was headlined: "Study Charges New Math Bust, Texts a Ripoff". Hard-working and well-meaning authors must feel discouraged to be singled out for such criticism. However, that is the price of recognition. It is to be hoped that such studies as those of Madgett and Weygang described in September 1975 and the U.W.O. Statistics Project, reported on in this issue, will lead the the development of teaching materials which show students how to "mathematize", a key requirement according to the Science Council report.

Look for Doug Crawford's two-part article on Modelling, beginning in this issue. Isn't that what we want?

Arn

OUR MISTAKE

We regret the error in numbering the March, 1976 issue. It should have read: Volume 14 Number 3 March, 1976. Librarians are asked to take note and make the necessary corrections.

FOURTH ANNUAL OAME CONFERENCE

SEE DETAILS ON PAGE 36

LETTERS TO THE EDITOR

Lake Superior High School
January 15, 1976

Prof. Arn Harris
Editor, Ontario Mathematics Gazette

Dear Prof. Harris:

In my letter to you dated January 14, I made some comments on C. Pinnegar's article in December's Gazette. After writing that letter, I have done some more thinking. Below you will find a more general definition of multiplicity of roots of a function, a simple description of some such functions, a statement of what I call a "Generalized Pinnegar's Conjecture", and a proof of the truth of that statement. Surprisingly perhaps, of the four points mentioned above, the second is the meat of the discussions, and hence is dealt with last.

Definition:

Let m be a real number, and n be a natural number.
Let $y=f(x)$ be a function such that
1) $f^{(k)}(m)=0$, $k=0,1,2,\ldots,n-1$
(where, as usual, $f^{(0)}=f, f^{(1)}=f^1, f^{(2)}=f'', \ldots$)
2) $f^{(n)}(m)\neq 0$
Then m is called a root of f of multiplicity n.
Also, if n=1, m is called a simple root of f.

Theorem 1 (Generalized Pinnegar's Conjecture)

Let m be a real number, and n be a natural number.
Let $y=f(x)$ be a function such that m is a root of f' of multiplicity n.
Then:
a) if n is even, then x=m is the location of an inflection point of $y=f(x)$.
b) if n is odd, then x=m is the location of a local extremum of $y=f(x)$.
(We assume throughout that $f^{(n+1)}$ is continuous around x=m).

Proof: We deduce immediately that $f^{(k)}(m)=0$, $k=1,2,\ldots,n$ and that $f^{(n+1)}(m)\neq 0$.

The result then follows immediately from standard calculus. (See, for example, "Advanced Calculus", second edition, by David V. Widder, Prentice-Hall, 1961, pages 119-120).

Remark:

The above is all well and good, but is somewhat unsatisfying in that it doesn't seem consistent with Pinnegar's spirit of the problem:

1) What does this have to do with multiplicity, as conceived in Pinnegar's article?
2) How can we use the above without checking so many derivatives to ascertain the multiplicity involved?

I believe these objections are handled below:

Theorem 2:

Let me be a real number, and n a natural number.
For each i=1,2,...,n, let $f_i(x)$ be a function for which m is a simple root.
Let g(x) be a function such that $g(m)\neq 0$.
Let $f(x)=g(x)f_1(x)f_2(x)\ldots f_n(x)$.
Then m is a root of f of multiplicity n.

Proof: Clearly f(m)=0.

By using the product rule, and mathematical induction on k, it is easily seen that

$$f^{(k)}(x)=\sum_i c_i g^{(t_{0,i})}(x)f_1^{(t_{1,i})}(x)f_2^{(t_{2,i})}(x)\ldots f_n^{(t_{n,i})}(x)$$

where each c_i is a constant other than 0

and $\{(t_{0,i},t_{1,i},t_{2,i},\ldots,t_{n,i})\}$ is the set of all solutions to $t_0+t_1+t_2+\ldots+t_n=k$, with each t a non-negative integer.

For $k\leq n-1$, at least two of t's, regardless of i, must be 0.
Hence, for each i, there is a natural number q with $t_{q,i}=0$,

for which $f_q^{(t_{q,i})}(m)=f_q^{(o)}(m)=f_q(m)=0$.

Thus, $f^{(k)}(m)=0$, k=0,1,2,...,n-1.
For k=n, at least one of the t's, regardless of i, must be 0.

If $t_{0,i}\neq 0$, then for that i, $f_q^{(t_{q,i})}(m)=0$, for some q and that term will then be 0.

In fact, the only way $t_{q,i}\neq 0$ for all q=1,...,n, is if $t_{q,i}=1$ for all q=1,...,n.

Hence $f^{(n)}(m)=c_i g(m)f_1'(m)f_2'(m)\ldots f_n'(m)$ for i; every factor is non-zero.

Thus, $f^{(n)}(m)\neq 0$.

The result follows from the definition.

Some Examples:

1) $y'=(x-\frac{\pi}{2})^6\cos x$

$\frac{d}{dx}(x-\frac{\pi}{2})=1$ and so $\frac{\pi}{2}$ is a simple root of $x-\frac{\pi}{2}$

$\frac{d}{dx}\cos x\Big|_{x=\frac{\pi}{2}}=-1\neq 0$, $\cos\frac{\pi}{2}=0$, and so $\frac{\pi}{2}$ is a simple root of $\cos x$.

$\therefore$ By Theorem 2, $\frac{\pi}{2}$ is a root of $(x-\frac{\pi}{2})^6\cos x$ of multiplicity 7.

$\therefore$ By Theorem 1, y has a local extremum at $x=\frac{\pi}{2}$.

2) $y'=\ln x^2\equiv f(x)$.

$f(1)=\ln 1=0$ $f'(x)=\frac{2}{x}\therefore f'(1)=2\neq 0$.

$\therefore$ 1 is a simple root of $\ln x^2$.

$\therefore$ By Theorem 1, x=1 is the location of a local extremum of y.

3) $y'=x^3\ln(1+x)$

A similar analysis gives x=0 as the location of an inflection point of y.

Conjecture:

I suspect Theorem 2 can be strengthened to include products of functions with known multiplicities, giving a function with a multiplicity equal to their sum. This would, if true, allow quick analysis of, say, x=0 being an inflection point of y, for $y'=x^2\ln(1+x^2)$.

Respectfully submitted,

Dale S. Cooper

Jenny Wrenway,
May 19, 1976

Dear Mr. Harris,

I was a student at F.E.U.T. during the 1975-76 session. While there, I thought up an alternate approach to the problem of finding the roots of a quadratic equation (alternate to the traditional "completing the squares" approach), which you might like to consider putting into one of the OAME publications. Here it is:

beginning with

$Ax^2 + Bx + C = 0$, where A, B, $C \in R$ and x is variable.

Let $X = x + h$, where $h \in R$.

Then, the original equation above becomes

$$A(X-h)^2+B(X-h)+C=0$$
$$\iff A(X^2-2hX+h^2)+B(X-h)+C=0$$

$\iff AX^2+(B-2hA)X+Ah^2-Bh+C=0 \ldots(1)$

$\therefore$ $h \in Re$, suppose $h = \frac{B}{2A}$, $A \neq 0$

then $B-2hA=B-2(\frac{B}{2A})A=B-B=0$

and equation$_2$(1) reduces to

$AX^2+A(\frac{B^2}{4A}2)-B(\frac{B}{2A})+C=0$

$\iff AX^2+\frac{B^2}{4A} - \frac{B^2}{2A}+C=0$

$\iff AX^2=\frac{B^2}{4A} -C$

$\iff X^2=\frac{B^2}{4A^2}-\frac{C}{A}$

$=\frac{B^2}{4A^2}-\frac{4AC}{4A^2}$

$=\frac{B^2-4AC}{4A^2}$

$\therefore$ $X=\pm \frac{\sqrt{B^2-4AC}}{2A}$

but $x=X-h$

$=\pm \frac{\sqrt{B^2-4AC}}{2A} - \frac{B}{2A}$

$=\frac{-B\pm \sqrt{B^2-4AC}}{2A}$

Yours truly,
Murray Major

Pickering College
Newmarket, Ontario
L3Y 4X2

Prof. A. Harris
Ontario Mathematics Gazette

Dear Prof. Harris,

In an article in the March issue of the Gazette (Testing Theory) W. Longworth proposed an outline for the grading of pupils. The article suggested that norm-referencing be used i.e. fitting the students actual (raw) scores to the normal curve. Once this had been done an arbitrary line was drawn through the curve and those falling below the line "failed" whilst those above it "passed". In the example given in the article the line was drawn in such a way that approximately 93% of the students passed.

While the article provides some useful guidelines for the norm-referencing of marks it fails to answer or even mention the fundamental question of when and where we are justified in using norm-referencing techniques. Surely the answer is; only in those areas and activities where we are concerened with a students performance relative to the other students in his group.

In most areas of school work, especially academic subjects, norm-referencing of marks is most inappropriate. In mathematics and other sequential subjects we are (or should be) primarily interested in whether a student has successfully mastered a given set of knowledge or developed a specified number of requisite skills. In these subjects it is usually a reasonably simple task to specify the criteria that must be met before a student can be considered to have satisfactorily completed the course. If the required criteria are not met, irrespective of the student's

performance relative to the others in his group, then that student is not ready to move on to the next course. This kind of measurement (called criterion-referenced measurement) is usually defined as the proportion the student has learned in relationship to what he should have learned. The usual system (in Ontario Secondary schools) of giving percentage grades is, technically, criterion-referenced measurement. A grade of 50% is taken to mean that the student has mastered one half of the material he was expected to learn. In actual fact the past ten or fifteen years has seen our grading system erode from one which was primarily criterion-referenced to one that is predominantly norm-referenced.

Pressures beyond the control of the teacher have forced him to adjust his marks (always upwards) so that a specified number of students were successful. Typically the teacher is expected to pass at least 80 or 90% of his students. This is exactly the same as fitting the marks to the normal curve and arbitrarily drawing a line so that the required number of students are successful. When this situation occurs the final marks bear no relationship to a student's mastery of the subject matter.

This trend to norm-referencing in areas which are completely inappropriate has contributed, in no small way, to the decline in standards which has occurred in the past few years.

Yours truly,
John F. Leightell

Editor's Note: Read Banesh Hoffmann's *The Tyranny of Testing* (Collier Books 1963)

Canadian Education Showplace,
Resources and Communications for Education and Industry,
April 15, 1976.

Editor, Ontario Mathematics Gazette.

Dear Sir:

The program Advisory Committee for CES'77 has asked me to drop you a line about the activities program under discussion for the next Canadian Education Showplace, Toronto, February 22-3-4, 1977, with the thought that your readers may appreciate an early announcement.

To review briefly, CES'76 meetings and features ranges from P1J1 to Project Aristotle. A copy of the official catalogue is enclosed for your reference. Despite the once-in-a-lifetime bad weather, the activities were well attended, lively and innovative.

In our programs and features we try to explore areas of current significance with a fresh and stimulating approach. For CES'77 we are working on a broad range of ideas for activities as outlined in the Program Proposals, attached.

Your comments and/or the feed-back from your audience would be greatly appreciated. You may wish to make use of the brief questionnaire attached.

One further point: we have received a number of letters from educators from across Canada who were unable to attend CES'76, asking for copies of the official catalogue of exhibitors and events. We would be glad to send a copy to any of your readers on request.

Cordially,
Lloyd Haines
Show Manager

THE MANNER OF COUNTING IS METRIC

FACE, PLACE, BASE, POWER, AND REAL VALUES

Nettie Sharkey, Charles Hulse School, Ottawa

Place value in the base ten system of numeration is one aspect of math which corresponds to the metric stystem. Undoubtedly, the principle of place value and power value inherent in it, is often vaguely understood. The system applies to using a set of numerals to keep track of number value in any base, but the metric system is a base ten system.

The counting procedure is a structured sequence of a repeat pattern of three counting stages: 1. Cubes, 2. Strips, 3. Flats. Let us take an in-depth look at these stages:

1 cm^3
1 ml
1 g



1 single-units cube

1. The Cubes (Single Units)

As counting objects, let us start with a small cube and call it one single unit, represented by numeral 1. When we have counted out nine of these single cubes we have the maximum number which can be represented by a numeral in a specific place, in this case the single-units, (power zero). Now if we count out one more object, we bind (package) the 9 + 1 objects into a strip and cell this a 'one' (1) ten-strip.



1 TEN-STRIP

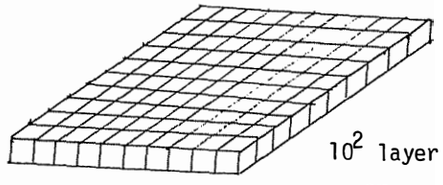
The numeral 1 has the same face - but it has a different value, a base power one: (b^1) and must occupy a new place. So we now think of the numerals 1 to 9 which represent the counting numbers in terms of their face value, (symbol shape), their place value, (position in numeration) and their real value (when counted singly).

2. The Strips

After completing 1 strip, really valued at ten, we may join *(group) single-units into 9 identical ten-strips; whereupon we count out, and group enough single-units for one more ten-strip (9 ten-strips + 1 more ten-strip), and bind *(package) this together calling it one (1), but again the 'one', while having the same face, has a different

value and must occupy a different numeration place (hundreds place). For the actual value of this 1, is one-hundred, counted singly. We will call this a 1 hundred-flat or a ten-square layer, and it is base power (b^2).

(10 X 10→10^2)



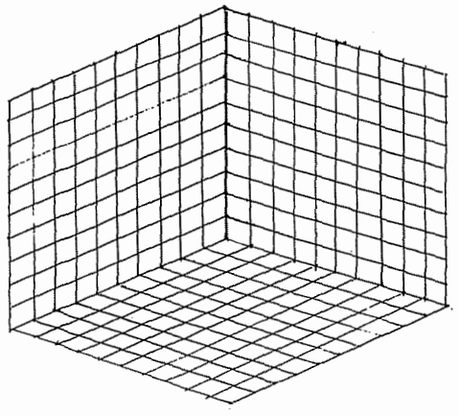
10^2 layer
or
1 HUNDRED-FLAT

*Group→count out and join→number
Package→bind together→power

3. The Flats or Layers

Continuing the same counting procedures we now count and join 9 identical hundred-flats. After making 1 more hundred-flat, put these flats in layers and bind the ten layers of flats together into a cube. We will call this a 1 thousand-cube. As before while having the same face, a different value of 1 and a different numeral place (thousands place). Here the base power is three - (b^3), (10 X 10 X 10→10^3). The actual (real numerical value is represented by the expression n X b^3, or nb^3.

Recall that we started counting using a single cube and we have a cube again at this point, but of course a cube of a different value with representative numerals in a different place. Note the thousand-cube diagram, the multiple cube.



1 THOUSAND-CUBE

Now using this new cube as a single counter repeat:

(1) the singles

(2) the strips, and

(3) the flats or layers

counting structures. This sequence takes us to the cube stage again, and we have completed single thousands (b^3) 10^3, ten thousands (b^4) 10^4, and hundred thousands (b^5) 10^5, values and places, and have reached base power six - (b^6), with this power six multiple (the million cube) cube, to start and perform the sequence of the next three repeat-count pattern stages, to construct the billion cube. Every three number value places a new value-cube is formed to start the counting pattern over again. The base powers in multiples of three (the ternary powers) represent each cube-singles stage in the counting sequence of procedures. This counting pattern continues and infinitum in both positive and negative directions.

What we have now explained can be shown in the following tables:

Structure	Place Value Procedure	Expression	Power (Exponential)
1) Cubes	single-units	n X 1	zero
2) Strips	tens	n X b^1	one
3) Flats	hundreds	n X b^2	two

For these three counting stages single cubes are used as counting objects.

Repeating these three stages we use as counting objects the multiple-cube which is power three, of whatever base in which the counting is being done.

Structure	Place Value Procedure	Expression	Power (Exponential)
1) Cubes	single-thousands	nb^3	three
2) Strips	ten-thousands	nb^4	four
3) Flats	hundred-thousands	nb^5	five

It can be seen how these three counting stages will repeat through millions, using as counters the power-six multi-cube; billions, using as counters the power-nine multi-cube; trillions, using as counters the power-twelve multi-cube, etc. These are base ten number names. Each strip raises the power by

l, each flat raises that power by one more, and each multi-cube raises the power by one more again, arriving at a ternary power for each new multi-cube. This is the decimal system of numeration and the aspect which corresponds to the metric system.

Now if I haven't lost you, and I do prevail upon you to stick with thinking this through, for once you get the idea it is quite simple and you can put it all into a place value chart thus:

hundred quintillions		0	b	
ten quintillions		0	b	
single quintillions		0	b^{18}	
hundred quadrillions		0	b	
ten quadrillions		0	b	
single quadrillions		0	b^{15}	
hundred trillions		0	b	
ten trillions		0	b	
single trillions		0	b^{12}	ter T
hundred billions		0	b	
ten billions		0	b	
single billions		0	b^{9}	giga G
hundred millions		0	b^{8}	
ten millions		0	b^{7}	
single millions		0	b^{6}	mega M
hundred thousands		0	b^{5}	
ten thousands	2	0	b^{4}	
single thousands	5	0	b^{3}	kilo
hundreds	3	0	b^{2}	hecto
tens	4	0	b^{1}	deca
single units	6	0	b^{0}	metre, litre, gram, dollar
decimal point				
tenths		0	b^{-1}	
hundredths		0	b^{-2}	
thousandths		0	b^{-3}	milli
ten thousandths		0	b^{-4}	
hundred thousandths		0	b^{-5}	
millionths		0	b^{-6}	micro

of principal use

Continuing the count past the quintillions, we have the sextillions (b^{21}), septillions (b^{24}), octillions (b^{27}), nonillions (b^{30}), decillions (b^{33}).

Continuing the negative ternary powers to the right of the decimal point, we have the:

billionths	(b^{-9})	nano	n	0.000 000 000
trillionths	(b^{-12})	pico	p	0.000 000 000 000
quadrillionths	(b^{-15})	femto	f	0.000 000 000 000 000
quintillionths	(b^{-18})	atto	a	0.000 000 000 000 000 000

	symbol	prefix	power	factors and subfactors: number name	numeral symbol
Greek	T	tera	10^{12}	one trillion times	1 000 000 000 000
	G	giga	10^{9}	one billion times	1 000 000 000
	M	mega	10^{6}	one million times	1 000 000
	k	kilo	10^{3}	one thousand times	1 000
	h	hecto	10^{2}	one hundred times	100
	da	deca	10^{1}	ten times	10
		(root)	10^{0}	one times (metre, m-litre, 1 gram, g)	1
Latin	d	deci	10^{-1}	one tenth of	0.1
	c	centi	10^{-2}	one one-hundredth of	0.01
	m	milli	10^{-3}	one one-thousandth of	0.001
	u	micro	10^{-6}	one one-millionths of	0.00 001
	n	nano	10^{-9}	one one-billionth of	0.00 000 001
	p	pico	10^{-12}	one one-trillionth of	0.00 000 000 001
	f	femto	10^{-15}	one one-quadrillionths of	0.000 000 000 000 001
	a	atto	10^{-18}	one one-quintillionths of	0.000 000 000 000 000 001

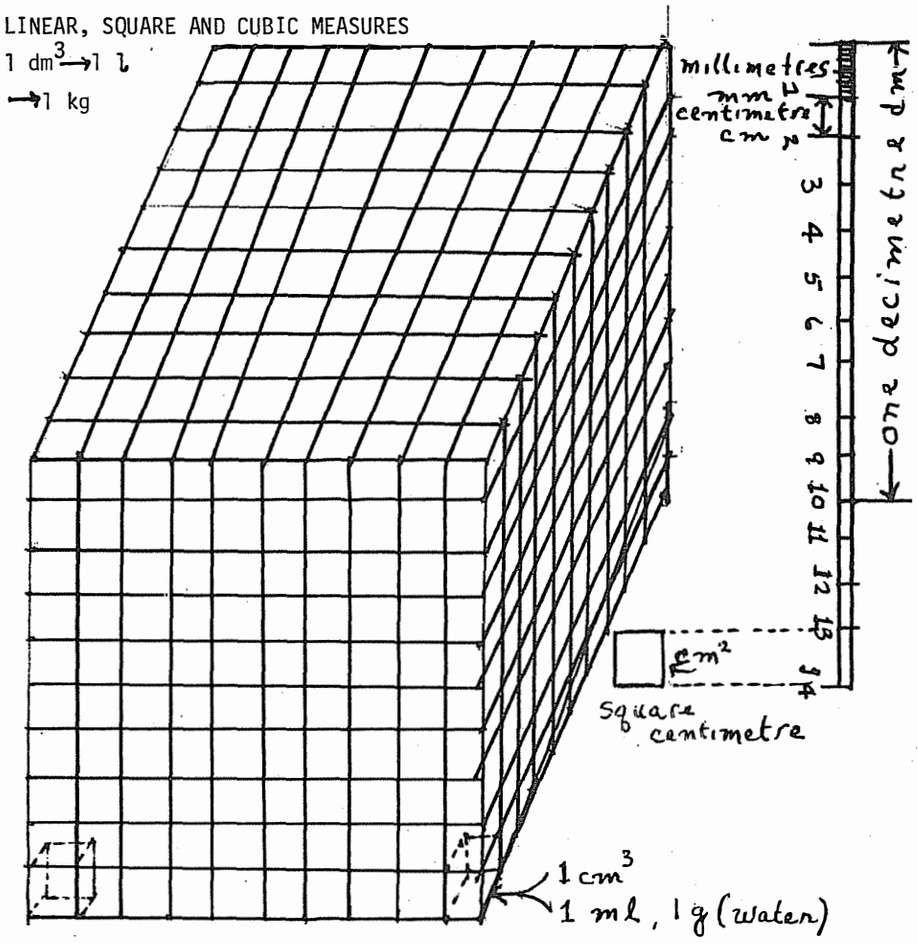
of principal use

The magnitude and the minuteness of number is incomprehensible perhaps even with all the devices there are as aids in thinking of number.

The prefixes for the multiples above the root words are of Greek origin. Those for the subdivisions below the root words are of Latin origin.

Minute quantities are applicable to molecules and atomes; immense quantities to outer space and interplanetary measurements.

LINEAR, SQUARE AND CUBIC MEASURES



This small figure represents one cubic centimetre, 1 cm^3 (one centimetre cubed). The large figure represents one decimetre cubed, 1 dm^3.

Cubic volume, liquid capacity, and mass are directly related as to numerical value in the metric system of measurements when using water. Thus a 1 cm^3 container holds 1 millilitre of water which weighs 1 gram, and 1 dm^3 container holds 1 litre of water and weighs 1 kg.

1 cm^3 → 1 ml → 1 g

1 dm^3 → 1 l → 1 kg = 1000 g

1 m^3 → 1 kl → 1 t = 1000 kg

Now that we understand these three structural steps in this 'base powers' counting procedure, we are capable of counting and calculating quantities, and of recording these quantities numerically in a number system of any base. In

base ten we use a set of ten numerals (1,2,3,4,5,6,7,8,9,0) and never count past ten in any value or place for the counting steps and structures.

Every time we count to ten,
Make a package; start again.

For the 'power' concept use the term 'package' with very young children. To count in base nine use a set of nine numerals, (1,2,3,4,5,6,7,8,0), and never count past nine.

Every time we count to nine,
Make a package; start again.

In base eight use a set of eight numerals (1,2,3,4,5,6,7,0) and never count past eight.

Every time we count to eight,
Make a package; start again.

Likewise in base seven, six, five ... etc., right down to a set of two numerals (1,0), for the binary system, base two, never counting past two.

Every time we count to two, make a package, start anew.

Let us look at the face, power and real values of the number 25 346 given in the place value chart.

Base	Face	Power	Real	
ten	the 2 means	$2 \times 10^4 =$	20 000	
	5	$5 \times 10^3 =$	5 000	
	3	$3 \times 10^2 =$	300	
	4	$4 \times 10^1 =$	40	
	6	$6 \times 01^0 =$	6	
			25 346	REAL
nine	the 2 means	$2 \times 9^4 =$	13 122	
	5	$5 \times 9^3 =$	3 745	
	3	$3 \times 9^2 =$	243	
	4	$4 \times 9 =$	36	
	6	$6 \times 1 =$	6	
			$17\ 152_{10}$	

Thus 25 346 in base nine is the equivalent of 17 152 in base ten.

eight	the 2 means	2×8^4	=	8 192
	5	5×8^3	=	2 560
	3	3×8^2	=	192
	4	4×8	=	32
	6	6×1	=	6
				$10\ 982_{10}$

Thus 25 346 in base eight is the equivalent of 10 982 in base ten.

seven	the 2 means	2×7^4	=	4 802
	5	5×7^3	=	1 715
	3	3×7^2	=	147
	4	4×7	=	28
	6	6×1	=	6
				$6\ 698_{10}$

Thus 25 346 in base seven is the equivalent of 6 698 in base ten.

In this same manner we see how this picture continues right along to counting in base two.

Of course counting systems in base eleven, twelve, etc., would follow the same procedures for place and power values but additional one-place numeral symbols would need to be invented.

From this it can be seen how easily numbers in base ten are computed, compared to using other units. This illustrates the simplicity of the metric system with its units of ten, over working in quantities of other ratios.

Very young children are capable of performing these counting procedures and of a rudimentary understanding of the concepts. Start them in base 5 or 6. Starting with base 10 or anything above 6 is too extended a counting activity to be able to grasp the total pattern of sequences quickly enough.

In a base below 5, the counting activity provides too few examples for developing a reinforced picture of the total pattern of sequences.

This is the crux of the whole counting system including decimal and metric values. It is a pity that children continue one by one counting far past ten. This establishes a block greatly impeding or even preventing the understanding of place and power value or of understanding the relationship of face, place and real number values.

Returning now to the counting procedures, let the strip represent distance, the flat represent area, and the cube represent volume and there we have the derivations of linear measure in metric; as it is related to area and to cubic measure; to volume and mass:

distance	the metre	m, L
area	the metre squared	m^2, L X W
cubage	the metre cubed	m^3, L X W X D

Think now of the first little cube with which we started counting, or of each subsequent cube, or any multiple or part of it, as taking up space, having volume, and as enclosing space, having capacity: and when the capacity is filled and we think of this fill as having mass: we have embodied the basis of weights and measures.

METRIC POLICY STATEMENT, MINISTRY OF EDUCATION

by R. J. Stevenson, Curriculum Officer, Western Ontario Region, Ministry of Education, London, with responsibilities for Senior Division Education. Mr. Stevenson is also Chairman of the Regional Metric Conversion Committee.

On pages 6 and 7 of the Policy Statement section 2:6 and 2:7, the direction for implementing Metric Conversion in the Secondary Schools of the Province of Ontario is spelled out. In part it states "Classroom implementation of the changeover ..... should commence, at the very latest, by the fall of 1976 and should be completed, in most applications, by June 1978".

"Conversion to the **predominant use** of common metric units is expected to be completed by June 1978".

Several questions come to mind

1. What does **"predominant use"** mean specifically?

2. Have secondary teachers prepared themselves? If not, how can they do so?

3. Is there a concerted effort being made to join up the secondary programs to the elementary ones in a planned, sequenced way?

Question 1 has a straight-forward answer - at least for me. "Predominant use" means "most of the time use by everyone". What are the exceptions? Note - it's not who are the exceptions, it's what! Why, because the use of SI is necessary by all persons including all teachers, all secretaries and all custodial staff, Everyone. There are a few situations where **total use** of SI would be vastly impractical if not fool hardy; for example, the machine Shop. It is not economically feasible to change all lathes, milling machines and shapers to fully SI. Industry is moving toward SI but not completely at this time. Certain piece of equipment will not be converted. Eventually they will be replaced, that may be ten years from now. This does not mean that we won't be predominantly metric; it does mean that duality will exist, that general processes are still the key learning taking place, not a particular measurement system. Alternate scales, electronic print-outs and partial conversions will be sufficient.

Question 2. With regard to the second question for those not prepared - the Inter State Consortium on Metric Education representing one half of the U.S.A. - after much debate came to concensus on 23 metric principles. One of these is that twelve hours of instruction are the minimum for any person who will be teaching and using SI. Would not a full day of learning using all the resources at hand (O.S.S.T.F. and other Federations, the Ministry of Education Regional Offices, O.I.S.E. Field Offices) during a P.A. day be in order? Maybe two half-days would be even better!! In any case some self help will be necessary. Here is a good opportunity to look at the other side of the teaching-learning coin.

Question 3 is also a golden opportunity to link the curricula in the Intermediate Division. It's also a very interesting opportunity for those with only secondary experience to find out how younger children learn. It could also provide you with the background to help someone whose real problem goes back a long way. Time and again it seems that going over the work twice more in the abstract (secondary?) way didn't help for long. There must be more about learning that could be of help to teachers. Often a primary, junior specialist could add a great deal to our understanding of how children learn.

OPERATION DO-IT

Frank Lillman, Northbrae Public School, London

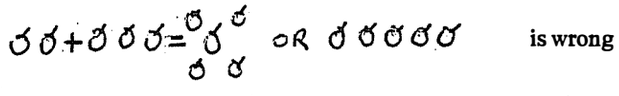
Have you had an operation lately? When a doctor operates he **does** something. In mathematics when you or the pupils operate or do an operation what do you do? You could watch a doctor operate. Could we "watch" your pupils add, subtract, multiply, and divide? What would we see? Would we see them write down several numerals on a piece of paper? Is this what is meant by a mathematical operation?

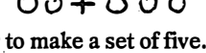
Primary teachers have manipulated everything from pop bottle caps to felt apples to get children to associate the numeral 5 with a set of five objects. Ask most children to **"show** you the meaning of the numeral 5 and they will show you a set of five objects. Ask them to "show" you the meaning of +, and few if any can do it. They will **tell** you add or plus but they will not **"do"** anything!

If we examine carefully living and non-living objects in the universe, we would find them constantly changing. They are either increasing or decreasing in some way. Mathematics is our way of trying to measure this change. The basic of all mathematics is addition and subtraction.

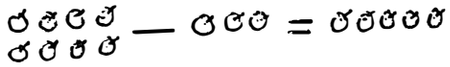
When you see a child take two sets of objects and form a new set by putting them together, you may say he has done the operation of addition. When he has a set of objects and makes them into two smaller sets he has done the operation of subtraction. These are the **concepts** of addition and subtraction.

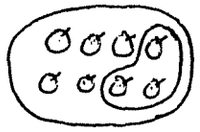
The first **skill** a child should be taught is to **write** the mathematical symbols for particular operations. i.e. A set of two objects put together with a set of three objects gives you a set of five objects. $2 + 3 = 5$. This **should NOT be displayed** on a flannel board using **ten** objects.

i.e.  is wrong

A better procedure would be to use only five objects 
remove the plus sign, and move the objects together to make a set of five. Once this is done, record the mathematical sentence $2 + 3 = 5$.

The skill for associating the mathematical sentence with the concept of subtraction would be handled in the same way. A set of eight objects, having a set of three of those objects taken away would leave five of the objects. Any concrete respresentation should not use more than eight objects.

i.e.  is wrong

A more desireable illustration would be 

The mathematical sentence is 8 - 3 = 5

When the teachers of the elementary school can make **every** mathematical symbol meaningful by having the children **show** the meaning, they will have succeeded in helping the children realize that the manipulation of mathematical numerals is simply a written record of what actually happens in the real world. Your challenge is: How do you "show" the operations of multiplication and division? or How do you teach the **"concepts"** of these two operations?

"ARE YOU TRYING TO SET ME UP?"

by David & Sharon McPhail, Lambton County Board of Education

Most teachers who have had the opportunity to teach Mathematics in the elementary school panel, especially the middle grades, have no doubt been faced by the topic of "Sets". Probably, those who have been in this situation, have thought--"How can I involve the kids?" Often the reaction is, "Well, I will get them to give me the name of a set, perhaps, luggage, dishes, or football cards. We will build a beautiful list on the board of all the sets that we can think of, perhaps 20 or 30 or even 40 and then while the students are keen on sets, because the lists have put them onto something about which they have some knowledge, we throw into the middle of the discussion that the Natural Numbers start at one and the Whole Numbers at zero therefore the students are to write the set of the first 5 numbers in each of these number groups!" What a rip off! What an absolute let down!

Sure, the children go ahead and do just that, however they have no idea of why they wrote all those numbers other than, "the teacher told me to do it!" If you ask the pupil how it is that we can call this a set, he has little or no understanding of the **concept** of set. I believe that this is a tragedy, since we are missing a golden opportunity to have the child conceptualize sets and have him fully understand the fact that a set can be written or created because of certain attributes that are common to each member or element within that set.

We have the pupils in the classroom, they have to be there by law, so let's use them to drive home the very important concept of "set". If we back up to the list of sets that we had created on the board earlier, we can extend our work from that point. Children even at grade three or four have at least heard of a set of dishes or a set of luggage, they may in fact have a set of football cards. Therefore we can now use the students in their rows or groups in the classroom--we can have the children create sets using the children in the room, they can determine the criteria, or attributes for their own sets. After all--"Why is a set--a set?"

Why is it a set of luggage, of dishes, of cutlery or why can you call that, the set of football cards? If we determine the reasons for placing items into a certain grouping we can begin to develop the thought, that every item in a set is there for a **specific** reason.

Each item has at least **one attribute** that each of the other items also possesses. The luggage for example, all has the same fabric, the dishes all have the same pattern and the football cards are a set because all of these are related to the game of football. The use of the pupils is good since we might have all those in group 1, or row 1, all the boys or all the girls or all left handers, right handers or those taller than Mary but shorter than Harold. Thus we start to develop the concept of "set" and the attribrutes for creating that set. "Too far out,"--"Children can't work with this concept, it is too deep" you might say--I couldn't disagree more!

Have you ever watched kids outside on the yard during a free sports activity time or at a recess? They know and understand the idea of attributes, even though they are unable to verbalize it in terms of "sets". For example, John is picked to be quarterback in a football game because he can run faster, throw more accurately and further and he is a better thinker than anyone else. Harold is big, but he is slow so he is not an early choice perhaps, Fred is tall and deceptive in his movements so that he is gone in the first round as a pass catcher and so it goes. The children have decided who they want and don't want on their teams because of the attributes that those players possess relative to the expectations of the captains and ultimately the team.

In the classroom we can draw upon this knowledge of our children to teach "sets". We can relate the thoughts of **"why"** we choose people for our team or **"why"** we put them into a particular group. After all a set is a group.

Furthering this concept of the "why" and relating this to attributes, we can now begin to do the mathematics very graphically. The use of beads, of children's blocks, of centicubes and the Invicta Attribute Blocks[1] all help bring the idea of sets closer to the mathematics lesson and still teach the concept. Note that as yet, we have not used numerals. The pupils can perform many "hands on" activities creating sets based on the attributes that they wish to use for their groups. Perhaps colour will be used, or size, or shape, will form the attribute for the set. Using candies is good. The pupils can put these into sets based on colour, size, mass, shape, wrappings, texture and so on using as many ways as they can ending up by eating the candies and even creating sets by taste or perhaps by the time it takes the candy to melt. The candy that is not enjoyed could be put into a water glass, thus creating a timing device for other classroom activities.

Further sets can be created by the introduction of the conjunction "and". We can now create sets that are red and round for example, so that two attributes have to be satisfied before an element can become part of a certain set. Once again this is another way to stress the idea of attributes in creating sets, however two points have to be satisfied now rather than only one. At this point the concept of "set" is developing to a far greater degree than it would have before. The feeling for "sub-set" or "intersecting sets" or "union sets" has far more meaning than many earlier approaches might offer to the student too since the concept of "set" has been so well learned.

To this point, much work has been covered. The understanding of "set" and "**why**" we can write sets as we do, is far better understood. The student becomes more critical of the attributes for sets and can relate to sets in more areas than merely mathematics. Yet, at no time has he written a numeral or a set of numerals. I believe that for elementary students, this is merely the icing on the cake after the understanding has been fully mastered.

The way I like to see the numerical aspects of sets develop is through the use of an ordinary deck of playing cards. A full 52 card deck has the 4 obvious suits and the 13 cards in each. Allowing the pupils to play with the cards, they can make sets without writing, and these are strictly done on the attributes that they wish to apply to their sets and manipulating the actual cards to show the answer. After some playing, the move into the writing of sets is now quite simple. Directions such as "give me all of the cards of one suit that will not beat the 5". "Write them down!"--"Separate them!"--"Now give that set a name with as many attributes as you can!" Such ideas like "the first 4 natural numbers," the numbers greater than zero but less than 5 and so on might be forth coming. He has however done two things. He has conceptualized that sets are created because of certain attributes, and he has now started to write sets using numerals. The mere addition of more attributes will only broaden the understanding. Give me the cards greater than 7 but less than the Jack and on it goes.

I believe that this recipe for sets is an excellent way to teach sets in a meaningful and valuable manner. The writing of the sets is quite simple if the full concept for "a set" is developed. The thought of attributes is automatic with any set, therefore we should teach them. If we do, our pupils will have a better working knowledge of "sets", "what they are" and "why they are."

If this fails, all is not lost, since the students have at least had the opportunity of eating some candy and playing cards all at the school's expense.

[1] Invicta Attribute Blocks, Invicta Educational Aids, England

MATH PROGRAMS CRITICIZED

Western News, July 29, 1976 (U.W.O.)

Too many university mathematics programs are geared only to the training of mathematics teachers, says a recently-released study sponsored by the Science Council of Canada.

Mathematical Sciences in Canada, written by Dr. A.J. Coleman, Dr. G. Edwards and Mr. K. P. Beltzner, describes the role and status of mathematical sciences in this country.

The Science Council study argues the need for the applications of mathematical sciences to be more clearly understood and appreciated.

The study also insists that "research is the life-blood of mathematics" and maintains that "a university department not committed to active research cannot remain alive for long".

Mathematics programs at Western are not directed primarily at the training of mathematics teachers, Faculty of Science Dean A. E. Scott told *Western News.*

"Western's mathematics training provides a broad base in mathematics applicable to many fields," said Dean Scott whose faculty includes the Departments of Mathematics, Applied mathematics and Computer Science.

Dean Scott is full agreement with the study's statement on research.

"We'revery conscious of the value of research at Western and we're pressing on in that field," he said.

Prof. S.C. Dennis, Acting Chairman of the Applied Mathematics Department, said that most of the faculty in the department are involved in some aspect of mathematical research, both practical and theoretical.

"We have quite a strong research program," Prof. Dennis said. "The main research being conducted is the application of mathematics to physical and engineering problems."

Assistant to the Dean of Science A.E.M. Bode heads an advisory committee currently looking into the entire question of mathematical sciences at Western with particular reference to the teaching of first year calculus and the position of statistics in the faculty.

Dean Scott said that a report will be forthcoming in the near future and indicated that the committee has broad terms of reference enabling it to look into the possible restructuring of mathematics programs at the university.

The *Mathematical Sciences in Canada* study calls for an increased emphasis by government on block grants for mathematical research, and urges that the development side of mathematics be encouraged.

Prof. Dennis expressed agreement with the study's recommendation for the creation of a Canadian Institute of Applied Mathematics to promote interaction between mathematical researchers and users.

He said there is already strong interaction on a small scale between university applied mathematics departments.

Prof. Dennis was instrumental in organizing a national meeting in fluid dynamics two years ago. The meeting at Western was the first of its kind in Canada, and a second meeting was held at the University of British Columbia this year. A third national meeting is planned at the University of Toronto in 1978.

The Science Council's study also urged immediate action on several other recommendations:

- Universities, business, government, and industry must make efforts to enhance the mutual understanding of the practical applications of new types of mathematics and of the attitudes and skills needed to overcome the communications gap.
- Provincial Ministers of Education should take vigorous action to improve the teaching of mathematics in elementary and secondary schools.
- A task force should be appointed by the sponsoring organizations to study and improve the structure and content of the university mathematical sciences programs from undergraduate to doctoral.
- The Ministry of State for Science and Technology should review federal granting policies to determine how financial support for mathematical research can be sustained at an adequate level and whether NRC grants to individuals could be modified to emphasize support of projects causing interaction among researchers in the mathematical sciences.
- Officers of Canadian universities should provide increased incentives for academics in mathematical sciences to communicate professionally with mathematics users.
- A Canadian Council for the Mathematical Sciences should be formed to promote the developments of the mathematical sciences in Canada.

PRECEPTIONS OF THE MATHEMATICS PROGRAM OF THE ONTARIO SECONDARY SCHOOL SYSTEM:

A Sample Survey of the Principals.*

by

L. K. Chan, Department of Mathematics, University of Western Ontario
A. W. Harris, Faculty of Education, University of Western Ontario
I. B. MacNeill, Department of Applied Mathematics, University of Western Ontario**
F. B. Rachar, Mathematics Coordinator, London Board of Education

1. INTRODUCTION

In the course of assessing the position of statistics as a topic for study in the Ontario high school mathematics program, the Secondary School Statistics Project of the University of Western Ontario deemed it important to survey the principals with a view to obtaining their perceptions of the present and future state of the mathematics program. The importance of the principal's view stems from the unique position of his office as the focal point for forces emanating from students, teacher, senior administrators, parents, employers and post-secondary educational institutions. Unhappiness among any of these groups with an aspect of the educational system is quickly translated into pressure upon the principal. The decade of the 1960's was a period of revolution for the mathematics curriculum. New topics and new jargon were introduced to such an extent that parents had difficulty understanding much of what their children were being taught. The last few years have provided some opportunity for assessing the effects of the revolution called the "new math". How is it being received? Are further changes necessary? Have we had enough time to see what is good and what is bad in the "new math"? Should we go back to the old syllabus that stood so firmly for so many years? Are the needs of certain groups of students being badly served by the present program? Because of his diverse sources of information we have sought answers to questions such as these from the principals of Ontario secondary schools.

*This research was funded by the Ministry of Education, Ontario through a grant-in-aid of research.

**Research supported in part by the Canada Council.

2. DESCRIPTION OF THE SURVEY

If, as we felt, it was useful to know what the principals think then we thought it best to obtain their thoughts by using a scientifically selected sample rather than by some hit-or-miss procedure that might contain large hidden biases. Accordingly, a random sample consisting of approximately one third of the Ontario secondary schools was selected. The sample was stratified with respect to region according to the 1973-74 regional division of the school districts. The ten regions were numbered roughly as follows: the numbers begin in the north-west of the province, cover the north, then move down to the south-west and continue across southern Ontario to the Quebec border. The separate schools form a region by themselves. The first three regions are in the north and the remaining seven are in the south with regions seven and eight including Metropolitan Toronto. The response rate was approximately 70%. Table 1 indicates the numbers of schools chosen and the numbers responding from the various regions.

Region	Total Mailed	Total Returned
Separate Schools	24	9
1	8	7
2	10	8
3	9	7
4	21	18
6	18	9
6	22	14
7	36	17
8	43	22
9	14	13
10	22	14
Unknown		4
Total	202	142

Table 1. Number of schools chosen in the survey and the number responding by region.

3. ADEQUACY OF THE MATHEMATICS PROGRAM AND RESISTANCE TO CHANGE

The first set of questions asked the principals to give their opinions on how well the present mathematics program meets the needs of several types of students. The student body was segmented first with respect to ultimate destination and secondly with respect to ability. The destinations were i) university, ii) CAAT and iii) no further post-secondary education. The "university" category was further subdivided into a) natural sciences and b) social sciences. Each principal was asked to rate the adequacy of the mathematics program using a scale running from 1 to 5 with 1 indicating "very badly" and 5 indicating "very well". Table 2 contains the averages of the responses to this set of questions broken down by region. There seems to be a tendency for region 8 to be higher than the others and for region 1 to be lower than the others and a multivariate analysis of variance does show differences among regions to be statistically significant. One striking feature of this table is the fact that principals from almost all regions tend to feel that almost all groups of students are well served by the mathematics program. The scores are quite high.

	REGION											
	Total	1	2	3	4	5	6	7	8	9	10	Separate
By Destination												
University bound students												
Natural Sc.	4.74	4.57	4.50	4.57	4.83	4.67	4.86	4.47	5.00	4.62	4.93	4.88
Social Sc.	3.98	4.00	3.67	3.57	4.24	3.25	3.93	4.24	4.32	4.25	3.50	3.86
CAAT's	4.19	2.83	4.43	3.43	4.29	3.56	4.43	4.47	4.52	4.46	4.08	4.22
School leavers	3.63	2.71	3.63	2.86	3.59	3.44	4.15	3.88	4.00	3.67	3.62	3.22
By Ability												
"Above average" students	4.54	4.86	3.88	4.57	4.50	4.56	4.50	4.76	4.64	4.31	4.57	4.67
"average"	4.18	3.71	3.75	3.71	4.18	4.00	4.29	4.41	4.45	4.23	4.07	4.44
"Below average"	3.09	2.43	3.38	2.00	3.00	2.78	3.14	3.47	3.32	3.46	3.00	3.00

Table 2
Average response of principals to adequacy of mathematics program.
Scale ranges from 1 (very bad) to 5 (very good).

The principals tend to perceive the program as best serving the university bound students and least well serving those not planning further education. Those going on to the CAAT's fall in between. Among those going on to university, the mathematics program is thought to do a very good job for the natural sciences and a good job for the social sciences. This latter perception is at odds with the assessments conveyed by the social scientists of Ontario universities in a survey conducted by the Secondary School Statistics Project and reported by MacNeill (1975). It would be interesting to have comparable assessments from the natural scientists and the CAAT's.

Turning now to the segmentation of the student body according to ability, the data in table 2 indicate that the mathematics program is perceived as being more than adequate for average and above-average students but less adequate for below-average students.

Region	School Size					
	500>	500-999	1000-1499	1500-1999	≥2000	not reported
Total	21	43	57	18	1	2
1	3	2	2			
2		4	2	1		1
3	2	2	2	1		
4	2	4	9	2		1
5	4	4	1			
6	1	7	5	1		
7		5	9	3		
8		5	9	7	1	
9	2	4	7			
10	1	5	6	2		
Separate Schools	6		2	1		
Unknown		1	3			

Table 3. School Sizes

The principals were asked to provide some information on the size of their schools. Table 3 summarizes this data. The principals' perceptions of the adequacy of the mathematics program have been categorized according to the size of the school. This data appears in Table 4. Since there was only one school in the sample whose student body exceeded 2,000 this category has been ignored. There is a marked tendency for the principals of the large schools to perceive the program as being more adequate than do the principals of smaller schools. This tendency applies to students of all destinations and all abilities. A multivariate analysis of variance detects a size effect at levels between 5% and 10%. One could speculate as to reasons for this tendency. Perhaps larger schools are able to offer a wider range of courses thus catering to more varied interests. Perhaps larger schools are able to stream students and hence better accommodate different ability groups. Perhaps principals of smaller schools are closer to their students and are giving more realistic assessments.

	School Size				
	500>	500-999	1000-1499	1500-1999	≥2000
By Destination					
University bound students					
Natural sciences	4.55	4.61	4.84	4.94	5.00
Social sciences	3.42	3.95	4.13	4.13	4.00
CAAT's	3.81	4.05	4.33	4.44	4.00
School leavers	3.33	2.62	3.62	4.06	3.00

By ability					
"Above average" students	4.29	4.35	4.68	4.83	5.00
"Average"	4.00	4.09	4.27	4.39	4.00
"Below average"	2.81	3.07	3.07	3.65	1.00

Table 4. Average response by school size of principals to adequacy of mathematics program. Scale ranges from 1 (very bad) to 5 (very good).

A second set of question asked principals to speculate on the amount of resistance that would arise in the event that efforts were made to introduce statistics and mathematics-for-the-social-sciences into the school system. The rating scale used ran from 1 to 5 with 1 indicating heavy resistance and 5 indicating enthusiastic acceptance. The various possible times for the changes in the program were i) next year, ii) next two years and iii) next 5 years. The average response to these questions categorized by region are given in Table 5. A score of 3 indicates no resistance (also no enthusiasm, perhaps) and this is the most common score. The regions do not differ significantly one from the other. There are, however, significant differences in the acceptance - resistance averages for the three possible times for changes. Early change will meet with more resistance than later change according to the high school principals.

		Region										
	Total	1	2	3	4	5	6	7	8	9	10	Separate
Next year	2.13	2.80	2.00	2.14	2.06	2.11	1.92	2.00	2.26	2.15	2.36	2.29
Next 2 years	3.06	3.50	3.14	3.43	3.11	3.11	3.00	2.69	2.89	3.15	3.08	3.29
Next 5 years	3.57	3.25	4.14	3.83	3.33	3.44	3.50	3.64	3.25	3.62	3.73	4.00

Table 5. Resistance to change in mathematics program catagorized by region. Scale ranged from 1 (heavy resistance) to 5 (enthusiastic acceptance).

The same data can be categorized according to school size. Table 6 contains the relevant averages. A multivariate statistical analysis shows no size effect.

	School Size				
	<500	500-999	1000-1499	1500-1999	≥ 2000
Next year	2.35	2.24	1.86	2.50	2.00
Next 2 years	3.65	3.05	2.84	3.19	2.00
Next 5 years	4.16	3.60	3.36	3.53	4.00

Table 6. Resistance to Change in the mathematics Program categorized according to school size. Scale ranges from 1 (heavy resistance) to 5 (enthusiastic acceptance).

In summarizing the results of the questionnaire so far, it would appear that there is general satisfaction on the part of the principals with the present program and little enthusiasm for change. A possible exception to this might be the program as it applies to the weaker students who do not continue their education at the tertiary level.

4. COMMENTS

The principals were asked to contribute information gained from their Guidance Departments or from other sources pertaining to the practicability or applicability of the current mathematics program. The comments give the impression that the mathematics program is less adequate than the averages of the survey indicate. The comments also seem to indicate a bit more enthusiasm for change than do the averages. This, of course, is quite understandable since those who take time to comment will likely be more interested in change and more likely to have some dissatisfaction with the present program. The comments, for the most part, are thoughtful and represent a wide range of attitudes.

Editor's Note: In the original article nearly 75 comments were included. In the interest of brevity, and so as not to further bias a possibly biased sample, I used random procedures to make a further selection. The same method was used with the comments regarding statistics and social science. Satisfied?

The failure of the current program to develop fundamental skills in operations is the primary concern of both parents and students.

Mathematics is the largest single problem area in the school. Parents and students feel that Mathematics is geared primarily to the above average student.

Mathematics is the stumbling block for many students. Although some attempt is made for the 4 level student this subject lacks the fundamentals and still has the theoretical frills. Today the attitude is, "You are good in mathematics and science or forget taking Grade 13 in any of those subjects". Look at the requirements of any calendar and this is clear.

No parent reaction is available. Students, from their reaction to our mathematics programme, seem to be well prepared for post secondary courses that require Mathematics as a prerequisite. Our programme seems to attract a good number of students into Math, programmes and engineering at university and into technological programme at CAAT's.

There is general acceptance of our mathematics program as a high calibre and efficient program for preparing students for university.

We have programs to cover below average, average and above average students, as well as remedial mathematics. The general feeling is that ours is a good mathematics program.

There is never a reaction to a non-compulsory additive; only when it is withdrawn. We introduced statistics because we thought it would be good not as an answer to student requests.

It is becoming obvious that to satisfy the needs of a greater number of student ambitions, a common approach to mathematics is not tenable.

My general impression is that the present Math. courses beyond grade 9, phase 5, and beyond grade 10, phase 4, are largely theoretical in nature and of little use to any student except to study math "next year" and eventually at the university level. (very little practical application and hence it is difficult to motivate the below average student).

Your attempts to introduce more statistics into the secondary school programs are well intended. keep in mind: (1) Our programs are intended to prepare students for many future programs. Thus, they must be broadly based. (2) The very general admission requirements at university for the Social Science areas results in many students not taking mathematics in Grade 13, thus, the introduction of additional content into our mathematics courses serves only those who take the courses.

In addition to my administrative responsibilities, I also teach Grade 9, 11 and 13 mathematics courses, none of which include a statistics unit. I have always regretted not including statistics because of its practical application and would be enthusiastic about including it - but - what goes to make room for it? Of all the Ministry guidelines, those for mathematics make the greatest demands of teachers in my estimation for the quantity of skills to be mastered. Everything

presently being done in our very busy programmes seem to be a vital part of the background required by some students.

Additional work in Statistics in our present courses of Math. 250 and/or Math. 551 would be possible if topics now taught could be identified for omission. Time is the problem.

"New Math" is just beginning to stabilize - further change would not be welcome at this time - and what should be deleted for another inclusion?

Our general level grades 11, 12 mathematics courses involve some statistics. In general, I prefer our present situation because the student going to Grade 13 has some idea whether or not he is heading into special sciences. If he is and he is a strong mathematics student - then he will be taking a very strong mathematics in Grade 13, without statistics - this strong background in mathematics, in my experience, allows one to handle statistics with ease - partly because it contains related topics such as probability. If a student is not strong in mathematics and is interested in Social Sciences then our grade 13 course in mathematics for the Social Sciences gives him a suitable background. At the same time, our students who head into CAAT usually study general mathematics and thus gain a knowledge of statistics. Although I'm a mathematician myself, I am concerned about the present proliferation of courses which tends to have students specializing at a young age.

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MATHEMATICAL APPLICATIONS AND MODELS FOR AGES 11 - 14 IN BRITISH SCHOOLS

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This is Part I of a 2-Part article. It introduces the idea and significance of mathematical modelling, and its relation to elementary applications of mathematics, and gives some examples of useful mathematical models from the Fife Mathematics Project in Scotland.

In the second part, which will appear in the next issue, the Mathematics for the Majority Project and its Continuation Project, centred at Exeter University, are used to illustrate common elementary applications of mathematics. The article concludes with an account of how mathematical modelling was used in an English secondary school to solve a common every day problem.

Introduction

In recent years, school mathematics has gone through a series of stages of action and reaction. Crudely speaking, the revolution in school mathematics led to a heavy emphasis on structure, with a parallel reduction in routine computation, manipulation and problem solving. This was followed by a wave of enthusiasm for discovery and activity learning, based to a great extent on the views of Jean Piaget.

In North America, it is probably safe to say that these movements have been less controlled than in the U.K., and that the imbalances in learning and achievement caused by them have been greater. Thus the very good pupil has probably benefitted greatly whereas the poorer pupil, although possibly enjoying mathematics more, has not been required to reach the levels of rote learning and operation with rational numbers of say 15 years ago. The problem of finding the judicious mixture of structure, and ability to handle adequately everyday numerical and mathematical problems, for pupils of greatly varying aptitudes and capabilities is still with us, and demands our continuing attention.

In this article, two concepts much in vogue today, particularly in the U.K., will be used to discuss and illustrate this central problem in school mathematics. They are the ideas of (1) mathematical modelling and (2) applications of elementary mathematics; and I shall consider them in the context of the middle school in the U.K. which corresponds roughly to Grades 6 through 9 in Ontario or ages 11 to 14.

What is mathematical modelling?

Suppose you are planning a new house, and in particular, the lay-out of the kitchen, with its sinks, appliances, counter tops, and cupboards. It would be very useful to make a scale plan of the room itself, and to draw each of the pieces of furniture involved, and make cut-outs of them. You can then try them out in various positions to see how they fit, both spatially, and for convenience. When you have found a particular arrangement which fits, you know that the corresponding original objects - stove, refrigerator, etc. will fit into the corresponding space in the kitchen. **This is an example of mathematical modelling.**

Let us look briefly at the processes involved to see what the essential features are:

1. We had a problem.
2. To help us to solve it, we made a (physical scale) model.
3. The model used only the relevant features of the original problem, i.e. **we abstracted only the relevant qualities** - size, shape, and function - for representation and use in the model.
4. We moved about (manipulated) the components (variables) in the model, to find an acceptable arrangement (solution).
5. We transferred (translated) the arrangement (solution) obtained with the model back to the real problem, and obtained a real solution.

The model used in solving the kitchen problem was **physical.** Two other major types are (2) algebraic, and (3) visual, which can be further subdivided into geometrical, diagrammatic and graphical.

You can see then that this idea of mathematical modelling is not really new! We have been using mathematical models all the time in school mathematics! What is new perhaps is that by examining how we use them, what features particular models need to have for particular problems, and similar questions, we can help students do and understand mathematics more effectively, for mathematical modelling is the essence of what mathematicians actually do.

In concluding this brief look at mathematical modelling, it may be worthwhile to point out that modelling occurs in mathematics in two special ways. These are

(1) when we have a concrete realisation (B) of something abstract (A). For example, the axioms of Euclidean geometry are modelled by Cartesian geometry.

(2) when we have a mathematical description (B) of a physical situation (A).

The "concrete" form of modelling is clearly more relevant to pure mathematics, and the "abstract" form to applied mathematics, and hence much more appropriate at lower levels of mathematics learning. We shall confine our attention to "abstract" models.

Applying Mathematics

There have always been pleas for school mathematics to be more relevant to life. The originators of the School Mathematics Project in England were very conscious of this in 1962 when they had the rare opportunity to develop a new secondary course in school mathematics. The Deputy Director of the Project, Douglas Quadling, described their point of view as follows:

> ... the mathematics must justify itself at its own level, not in terms of its importance later on. Thus we saw in the teaching of mathematics a social function, which should in some measure displace the technical, skill-acquiring emphasis of traditional syllabuses ...

> ... the mathematics teacher should aim at appreciation as well as knowledge and accuracy. This implies that the fundamental patterns of mathematics - sets, functions, probability, symmetry, rate of change, groups, to take just a few examples - and their manifestation in mathematical models (represented through graphs and diagrams as well as algebraic formulae) should be seen in a variety of forms applied to many different situations; and that mathematics should be taught in relation to its potential as well as its actuatlity. Hence the introduction into the syllabus of items such as linear programming, networks, circular and exponential functions, statistics and a numerical approach to calculus, as typifying ways in which mathematics can help us to interpret and control our environment.
>
> Note especially the sentence
>
> This implies that the fundamental patterns of mathematics - sets, functions, probability, symmetry, rate of change, groups, to take just a few examples - and their manifestation in mathematical models (represented through graphs and diagrams as well as algebraic formulae) should be seen in a variety of forms applied to many different situations.

This then is the link between models and applications. We need many situations and contexts which pupils can see as interesting and relevant to them, in which to use the mathematical models we create, otherwise as Griffiths and Howson say elsewhere (2 , p. 319) the pupil may tend to say that "stupid computations are mathematics, but serious reflection is science".

Two mathematical Models Used in the Fife mathematics Project

1. 3 Dimensional Sketching

The Fife Mathematics Project began in 1970 as an attempt to help solve the problems of teaching mathematics to mixed-ability classes ushered in by the arrival of comprehensive secondary school education in Scotland. Geoffrey Giles, of the University of Stirling, began to experiment with sets of mathematics workcards with various educational and mathematical aims, including active personal student involvement and "exploration of carefully-chosen situations that will help him to appreciate the meaning and significance of the more formal mathematics that will come later".

Among the 8 series of workcards produced so far is one entitled 3D- sketching, the main aim of which is to develop the ability both to interpret and to draw isometric sketches. The work involved uses unit cubes, and what are called S and L cubes (see Figure 1), and isometric dotty paper (see figure 2), and develops spatial perceptions by moving from the reality of 3-D configuration to visual 2-D models of them, and back again. The idea is a simple yet powerful one, which enables a very useful technical ability to be developed in an interesting yet controlled way.

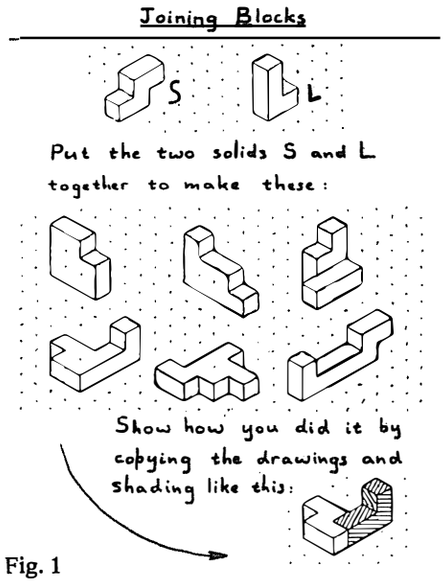


Fig. 1

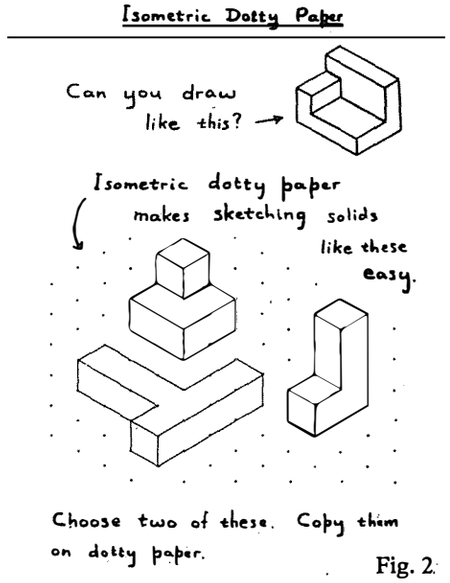


Fig. 2

2. In Understanding Large Numbers, Basic Arithmetic and Elementary Algebra

More recently, Geoffrey Giles has been developing "The Dot Array Grid", which allows modelling of several numerical and algebraic situations. He has described it in the British Journal **Mathematics Teaching** #71, of June 1975.

The teaching material consists of sheets of dots arranged in rectangular array fashion, together with a transparent grid which groups the dots, and initially directs attention to 100's, 10's and 1's, thus emphasizing both the meaning and importance of place value (see Fig. 3). The grid can be used very flexibly. Here only two examples of its use can be given.

Example 1. Multiplication as Repeated Addition

By placing the grid over the dot array pictured in Figure 4, we see that there are 15 rows of dots each containing 21 dots, and that the total number of dots is

$$2 \times 100 + 5 \times 10 + 5 \times 10 + 10 + 5 = 315$$

i.e. $15 \times 21 = 315$

Example 2. The Generalization $(x + 1^2) = x^2 + 2x + 1$

By using the variable grid, pictured in Figure 5, we readily see that

$$9^2 = 8^2 + 2 \times 8 + 1$$

$$10^2 = 9^2 + 2 \times 9 + 1$$

and in general that $(x + 1)^2 = x^2 + 2x + 1$

This approach to the development of numberical and algebraic concepts uses a very simple visual representation or model yet its flexibility enbables it to model many basic situations very effectively.

Dot Array and Standard Grid

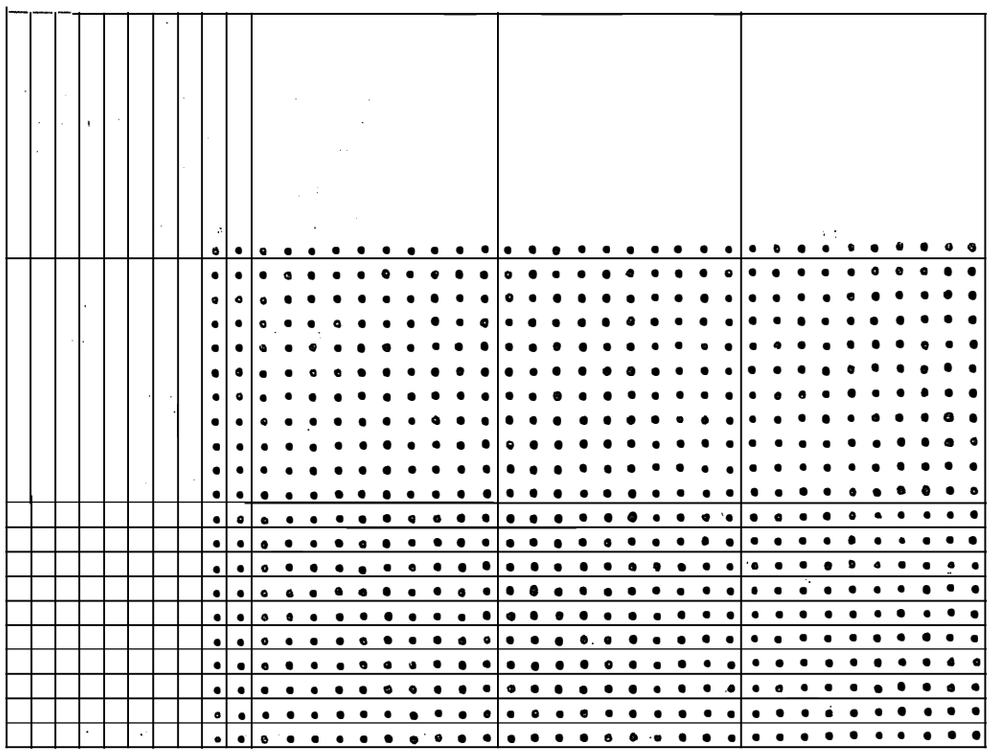
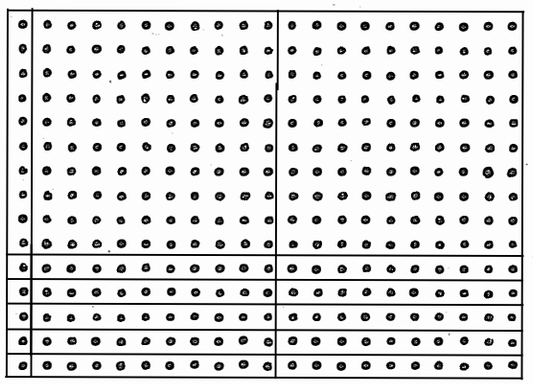


Fig. 3

How many dots? Use your grid



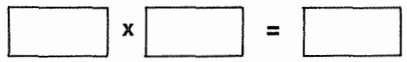
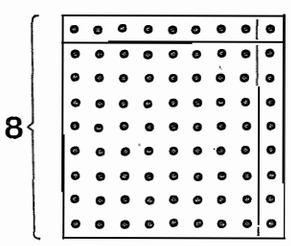
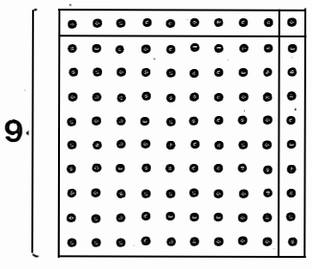


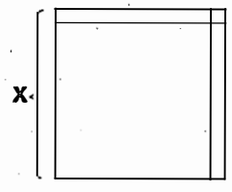
Fig. 4

a. 

$81 = 64 + 2 \times 8 + 1$

$9^2 = 8^2 + 2 \times 8 + 1$

b. 

c. 

$100 = 81 + 2 \times 9 + 1$

$10^2 = 9^2 + 2 \times 9 + 1$

Fig. 5

THE ORDER OF SUBJECT MATTER

by V. Byers
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In 1958 Morris Kline found it necessary to remind the Thirty-sixth Annual Meeting of the National Council of the Teachers of Mathematics that:

"Mathematics is ... a cumulative development. Hence the older portions do not become antiquated or useless ... The geometry of Euclid and the trigonometry of Hipparcus and Ptolemy, although two thousand years old, are still not replaced by a new geometry and trigonometry that show the old subjects to be incorrect or useless." (1)

In 1973 in his book **"Why Johnny Can't Add: The Failure of the New Math"** Kline wrote:

"The argument is essentially that each person must go through about the same experiences that his forefathers did if he is to attain the level of thought that many generations have achieved...

If it took mathematicians a thousand years from the time first-class mathematics appeared to arrive at the concept of negative number -- and it did -- and if it took another thousand years for mathematicians to accept negative numbers -- and it did -- we may be sure that students will have difficulties with negative numbers." (2)

These statements are strikingly true. Yet, with respect to the sequential organization of mathematics instruction, they curiously miss the point. Kline has not proposed that we teach the trigonometry of Hipparcus and Ptolemy. Nor has he ever suggested that we cannot teach a child how to write the number ten in the Hindu-Arabic notation because the great mathematicians of Ancient Greece failed to develop the concept of zero. Nor would Kline, an advocate of introducing an oscilloscope into a mathematics laboratory, want us on the basis of history and students' difficulties with negative numbers to smash all Celsius thermometers or at least to keep the dash-board ammeter chastely covered until our offspring has mastered Euclid and Apollonius.

It is examples such as these that have caused Edwin Moise to reject "the so-called **genetic principle,** under which mathematics should be presented in the order in which it was discovered." He writes:

"Indeed, to rescue the genetic principle from absurdity we must construe it so loosely that it can hardly be used at all."

Moise, who is no defender of "traditional" teaching, is concerned with "the problem of arranging a curriculum in an order which is logically sensible and psychologically well-motivated..." He objects to situations in which "first-rate mathematics is presented apart from the context which makes it significant; ... the solution ... presented before the problem." (3)

But surely the presentation of the problem before the solution, the consideration of concrete cases before their abstract generalization, is teaching mathematics in the order in which it developed, which is equivalent to the adoption of the historical order, if not for the whole of the mathematics curriculum, then for the topics under study!

Other principles for ordering subject matter have been proposed. One of them is the principle of increasing difficulty, another might be called the principle of decreasing utility. Since mathematicians seem to be agreed that calculus is easier than algebra, according to the first principle the study of calculus should precede that of algebra! According to the second principle, in this day of adding machines and pocket computers, the study of the fundamental operations of arithmetic should be postponed till graduate school! We shall try to show that what we are dealing with here is not a matter of one or another "principle" but is rather a matter of deliberate decision.

One thing is undeniable: whether it be in curriculum planning or in classroom teaching, subject matter must be selected and arranged in a temporal order. Actually, selection may be considered as a limiting case of ordering, the omitted subject matter being placed at the tail-end of an instructional sequence. Instructional sequences are by no means unique. We shall assume, with Bruner, that among them there exist optimal sequences.

"Instruction consists of leading the learner through a sequence of statements and restatements of a problem or body of knowledge that increase the learner's ability to grasp, transform, and transfer what he is learning ... There is no unique sequence for all learners, and the optimum in any particular case will depend on a variety of factors, including past learning, stage of development, nature of the material, and individual differences." (4)

In curriculum design the terms of such a sequence are the branches of a discipline or the major topics within an individual branch. But every teacher when developing a new topic has to order to best advantage the facts and ideas within the topic. The order he adopts constitutes a teaching strategy, which should reflect in addition to himself not only the class he is teaching but also what he hopes to accomplish with it. In general, the criteria for optimality of a sequence depend not only on the material and the learner but also on the goals of instruction. For example, measures taken to increase the speed of learning tend to increase cognitive strain, i.e., the difficulty of learning. On the other hand, such measures tend to decrease the transferability of what had been learned to new situations, probably because of an adverse effect on understanding. In any case, experience has shown that one cannot use the same instructional sequence for mathematics teachers that one would use for future research mathematicians.

We shall call the order of an optimal instructional sequence the **pedagogical order**. It should be emphasized that the pedagogical order is not the actual order of instruction. The latter may be far from optimal. It may be set for administrative convenience or determined by tradition and so be ill-adapted to the needs of the students or to the goals of instruction.

The factors which affect the pedagogical order require further discussion. The specificity in the teaching of mathematics as compared with the teaching of other subjects stems from the sequential nature of this discipline (see examples 1, 2 and 3 below). This means, in the first place, that the material to be taught comes already ordered by the discipline (we shall see that this order is not immutable). Secondly, it means that the results of "past learning" must be used during instruction and that the material which is being taught will be required for "future learning", either immediately or in the long-term. This means in turn that the classroom teacher must be aware of the kind of mathematics his pupils know when they come to him and what mathematical knowledge they will have to have when they leave him. Now, it is easy to determine what a student has been or will be **taught,** but teaching is not synonymous with learning. The distinction is even more important in curriculum design. The "genetic principle" is an attempt to infer the course of the individual student's learning of mathematics from the course of the acquisition of corresponding knowledge by the mathematicians of the past. By and large this principle arranges material in the order of decreasing familiarity and increasing abstraction. Finally, the "stage of development" refers to the level reached by a child in the growth of his general logico-mathematical know-how. The existence of stages of cognitive development was discovered by developmental or genetic psychology but mathematicians have long recognized similar phenomena in themselves and their students. These developmental stages and their substages are also ordered. Thus, goals and individual differences aside, the other factors which affect the pedagogical order have orders of their own - at least in mathematics.

We shall distinguish between: (a) the logical or mathematical order, (b) the historical order, and (c) the psychological order.*

The logical or the mathematical order is the order directly imposed on the teaching of mathematics by the sequential nature of the discipline. The historical order is the order in which mathematics actually developed. Each of these is quite definite and, in the absence of other factors, each would determine an order for mathematics instruction. In fact, following the mathematical order might be described, with Whitehead, as adhering to the "principle of necessary antecedence" in subject matter, according to which "the study of integers must precede the study of fractions." (5) Of course, in this case, the same result is obtained if one accepts the "genetic principle" and follows the historical order.

*Our use of this term differs from Moise's use. See pg. 32.

The psychological order is more vague and controversial. Mathematicians speak of the difficulty of a topic, concept, or proof, of the "naturalness" of a definition or approach, of the need for "mathematical maturity". For children and adolescents these ideas merge with the concept of "readiness" for certain types of mathematical learning described by the stages of cognitive development. These ideas really refer to the ordering of the student's logico-mathematical background in terms of the availability of concepts and the development of cognitive structures needed for new learning.

In the preceding discussion we have not dealt with motivational factors which affect the pedagogical order such as the need to arouse and maintain interest and to decide whether prerequisite material will be presented well in advance or as close to the point of application as possible. It seems clear, however, that the determination of the pedagogocal order may involve a choice, a modification, or a compromise between the other three orders.

Let us look at some examples to see how these ideas might work out in practice. The first three illustrate the ordering terminology.

Example 1. Since the theorems of Euclidean geometry are deduced from each other, they must be taught in logical order. Apparently there is no choice. Actually, though Euclid's system has been called a mathematical discourse, it forms a tree rather than a chain - the order of the theorems can often be changed (with or without a change of proof) and yet still satisfy the rules of logic. Thus a high school geometry text might give two proofs of Pythagoras' Theorem, first the one which appears in Book I of the **Elements** to be followed much later by a proof based on ratio and proportion. We see therefore, that even in this case, the logical order is not unique and thus the pedagogical order is not completely determined by it.

Example 2. It is impossible to solve quadratic equations without knowing something about factoring - unless one simply memorizes the quadratic formula only to forget it right after an examination (it usually gets forgotten in any case). But the mathematical order demands only the factoring of trinomials, in fact, only of square trinomials if the methods of solution are restricted to completion of the square, a method never used in practice. If the teaching of other types of factoring before going on to quadratic equations is deemed optimal because of the need to consolidate the concept of factoring, then the pedagogical order will be determined by the psychological rather than the mathematical order.

Example 3. It is impossible to learn algebra without knowing at least some arithmetic and algebra is needed for calculus. Hence, it appears that the pedagogical order must coincide with the mathematical order which also happens to be the historical order of development of these branches of mathematics. Let us examine the matter more closely. Is it possible to learn algebraic fractions without understanding arithmetical ones? Formal mathematics must say "yes", psychology - "probably not". However historically, **more** arithmetic was developed than is strictly necessary for algebra and **more** algebra than is essential for calculus. If we assume that the traditional sequence of teaching these subjects is optimal, then we must say the pedagogical order here is determined by the historical rather than by the mathematical order.

Our next example illustrates the fact that the pedagogical order may not be in doubt even when it contradicts the historical order and guidelines from the other two orders are absent.

Example 4. Whether complex numbers are introduced by the adjunction of $i = \sqrt{-1}$ to the reals or as a set of ordered pairs, their introduction is usually motivated by the need to solve all cases of the quadratic equation. Historically, mathematicians avoided the recognition of imaginaries during the solution of quadratics by maintaining that an equation like $x^2 + 1 = 0$ was not solvable. Imaginary numbers made their first appearance in mathematics during the solution of cubics by the Cardano-Tartaglia method, in cases where the cubic equation was known to have a real root. There is no mathematical reason why complex numbers should not be introduced this way. We must ask, therefore, why the historical order is not followed in this instance. One reason is simply that Cardano's method is not taught in our schools. The second would be that the procedure actually followed is more natural. What this really means is that the adopted order is advantageous to future learning; the complex field is a quadratic extension of the reals so that whichever procedure is used for the extension (there are two) the extension must be based on an irreducible polynomial of degree two.

The pedagogical order in our remaining examples is much more controversial. The first example deals with an old problem in the teaching of calculus. The other three are the result of the introduction of "new mathematics" into the schools and colleges.

Editor's Note: Examples 7 and 8 have been ommitted.

Example 5. Since differentiation and integration are inverse operations, there is no mathematical reason why the study of one should precede that of the other. Historically, the origins of integration can be traced further back than those of differentiation. However, experience has shown that most students find the study of calculus easier if differentiation is done first. Hence, the pedagogical order should reverse the historical one, except perhaps in the case of outstanding students (where these can be segregated) for the sake of future learning. Needless to say, this does not mean that the introduction of integration must wait till the completion of a course in differential calculus.

On the other hand, both derivatives and integrals are limits. Thus, logically the study of limits should precede that of differentiation and integration. However, for 250 years great mathematicians have developed calculus and differential equations far beyond anything required in an introductory course and have applied these successfully to difficult problems in physics and astronomy, all without the benefit of the limit concept. We certainly cannot go back to original innocence and forget about limits. But since students find the concept of a limit difficult to grasp, the proponents of an intuitive introduction to calculus, of which I am one, hold that the limit concept is best learned in use, so that in this case the pedagogical order for most students should essentially follow the historical order and that a detailed discussion of limits should follow, rather than precede, the study of differentiation and integration.

Example 6. Although the algebraic theory of sets (the logic of classes) was not discovered till the middle of the last century, it may be viewed as independent of the rest of mathematics. Hence, there is no logical or mathematical reason why it should not be the first branch of mathematics to be studied. Developmental psychology has shown that children begin to classify according to similarity, i.e., to use implicitly the common property definition of a set, at the age of 7 or 8. Since children start school at 6, the psychological order lags behind the mathematical one. We have a suggestion, however, from a psychologist that the pedagogical order should follow the mathematical rather than the psychological order.

"One wonders ... whether it might not be interesting to devote the first two years of school to a series of exercises in manipulating, classifying, and ordering objects in ways that highlight the basic operations of logical addition (symmetric difference, disjoint union) multiplication (intersection), inclusion, serial ordering and the like. For surely these logical operations are the basis of more specific operations and concepts of all mathematics and science." (6)

The difficulty here is with the last sentence, the truth of which is by no means selfevident. Personally, I have never found that an acquaintance with the elementary operations of symbolic logic helped a student with logical reasoning. In my opinion, sets and set notation should be taught in school because they provide a convenient language for other parts of mathematics, i.e., for use in specific future learning rather than for cognitive development. Their study should not be undertaken until the student has had some experience with algebraic symbolism and is ready to study mathematical topics to which the language of sets may be applied. Set notation should not be used if it confuses rather than aids the student (e.g., in elementary calculus).

I trust we have shown that the establishment of an optimal sequence of instruction in mathematics, whether in the classroom or in curriculum design, is not a matter of deciding between "old" and "new" mathematics; nor is it a matter of choosing in advance the logical, the historical, or the psychological order to the exclusion of all others. The determination of such a sequence requires the careful weighing of a variety of factors. In particular, the sequential nature of mathematics demands close attention to the students' previous background and the use they are likely to make of the mathematics they are learning.

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7. J. Piaget, **Genetic Epistemology**, Norton, N.Y., 1970.

FOURTH ANNUAL O.A.M.E. CONFERENCE

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Ottawa Board of Education

Enclosed with this Gazette you will find your copy of a flyer announcing this 4th Annual Conference. The conference committee expects to have one of these in all Ontario schools by the end of October. The committee would appreciate it, if you would post your personal copy as well.

ARE WE SHORTCHANGING OUR STUDENTS?
OR
BACK TO BASICS? NEW BASICS?
OR OLD BASICS?

Frank Ebos, Faculty of Education, University of Toronto

Probably each of us in our role as a teacher has been confronted by an anxious student inquiring about some of the math we teach. How many times have each of us heard:

Why are we taking this stuff?
What's it good for .... sir?

I would be the first to admit that we can't justify every thing we do **all the time.** To explain how a specific topic in mathematics "fits" into the scheme of things is often difficult. The student must trust that what we are doing day by day in the math classroom is useful, is relevant, and is needed for today's activities as well as for tomorrow's. Unfortunately, for many students tomorrow never comes. The students then become parents, and the cycle of asking "Why are we taking this stuff" is continued. here are certainly many indications that there is concern about the curricula we teach. Many studies have been, and are being completed in both the United states and Canada to ascertain "What should be the math curricula?" Unfortunately, to predict the content, the skills, or even the methods needed at some future date is difficult. If you listen to the experts and read the jounals, you soon would develop a complex about what we are not doing in the math classroom, but thank goodness for the so-called new math. We have in it at least a scape-goat. We have seen the headlines "New Math has failed! Back to basics!", and how we have a replacement for the new math, the BTB (Back To Basics). When the "New Math" was in vogue, each person you talked to had a different "understanding" of what the "New Math" was: There was the **Set-New-Math** followers; the **Base-new-math** followers; the **Structure-new-math** followers, etc. (I apologize to those new-math groups not identified at this time). The parents identified new math from another point of view. They only saw what their children brought home; sets, and words, commutative, associative, distributive, inverse, base 2, base 8, third base - to name a few - soon the scene is set for the BTB to be formed, and the parents are willing to join.

These days, I am hearing more and more of the BTB, but I think that before we change we need to decide **"what are the basics".** Parents whose experience with math was almost completely computation, evaluate a new program or curriculum according to their backgrounds. We must ask: are the basics solely related to computation? I think not. No one would disagree that "the old basics" are essential, but there are "new basics" that parents as adults use in their every day living, and which need to be dealt with. The computation content, so familiar to many and part of the "Happy Days Syndrome" to return to basics is not, by itself, suitable even for today's world, let alone tomorrow's. My main concern is not that the BTB wants change (improvement, or whatever) but that they appear to want the pendulum to swing back to a "shut-up: do-this" curriculum, based on computation.[1] We can't afford to have the pendulum swing back too far. Everyone will basically agree that computational skills are important and basic to the students mathematical development, but to stop there would short-change our students. Before the BTB make the same errors as the New Math groups, they (there are probably more than one) should decide what the basics are!

I want to help the BTB by offering the following suggestions. (These suggestions could be added to the computational platform already advocated by the BTB. (A little review is useful here:
BTB = a group of persons who want to return Back To Basics)

1. Students want us to be accountable. We should have some reasonable explanation or justification for "Why are we taking this stuff?" If we were to provide examples of applications for the present curriculum, some students and parents would be partially satisfied. To tell a

1 A description of the "shut-up: do this" curriculum was in a speech delivered by Eric MacPherson of the Faculty of Education, University of Manitoba at the Annual Meeting of the National Council of Teachers of Mathematics in Denver, Colorado, April, 1975.

student he is taking a topic because he needs it for next years math is a weak argument. Many topics have applications not only to satisfy the present questions asked by students but also to provide a foundation for their future study of the topic in next year's math class. There are at present at least five studies that are looking at the curriculum from this point of view. Hopefully, the results of these studies will be expressed in practical terms and eventually made available to practising teachers. **Applications should be basic to any curriculum.**

2. In our everyday living, we are never given "neat problems" solved in a "neat way" that result in a "neat solution". A real-life problem requires us to sort out the useful (needed) information from the extraneous information. Do we every give students problems that contain more information than is really required to solve the problem? Will our students ever be given a problem to solve when they enter the "world of work" for which the "boss" provides only the information that is necessary? I doubt it! You doubt it! Yet how much of this practice in solving do we provide our students in our math classrooms (if you do provide these types of problems, then consider this section a brief review). **To decide on what information is necessary or needed to solve a problem, to me, is a basic skill.** How many times have you given a problem which has a missing piece of information that the student is to provide in order to solve the problem? (Try assigning 5 problems, each having an essential piece of information missing. Provide a second sheet that contains the 5 missing "pieces", as well as 25 other pieces of extraneous but closely related information). This skill will probably be used more often than the skills advocated by the BTB. Without this skill the BTB skills are often confusingly applied by students to solve problems. (I think a mathematician might say the BTB skills are sufficient, but not necessary or is that .... necessary but not sufficient ..... ?)

   However, "let me make it perfectly clear" that we need the BTB basic skills once the information is properly interpreted and the essential computational operations decided upon. Unfortunately, "good old" Euclidean geometry provided an opportunity for a student to "sort out" the needed information, but an EGID movement (Euclidean Geometry Is Dying) seems to have sprung up. (E GAD!)

3. Every day, as adults, we read about the Gallup Poll, the latest statistics on "why we are paid more and more but are eating less and less" and this poll and that poll. However, do we provide any basics for students to tackle the world of statistics? We give a brief look at the topic in grade 8 (if at all in some classrooms), completely ignore it in grades 9 and 10, but give some hope to those who stay on and finish hish school and study the topic again (the key question here is - how many will finish high school?) Could these students not find the skill of working with statistics useful just for everyday living and thus be given the opportunity to study the topic in grades 9 and 10? **Working with, interpreting and reading statistics is a basic skill.** Statistics permeate too much of our everyday living to be ignored as it has been in our mathematics curriculum. We are shortchanging our students by not teaching this skill. Continuously through Grade 9 and 10 we are especially shortchanging those students who do not reach grade 11.

4. How many students have asked you "Is this right?". How many are completely lost if they cannot find the answer at the back of the book? Students should be taught (and taught and taught ....) to know how to check that their answers are reasonable. **To know the answer is reasonable is a basic skill.** A student must by grade 12 "feel" whether the answer is reasonable. The "boss" does not assign problems with the answer at the back of the book. Teaching this skill partially can be accomplished by providing students with skills of approximating, estimating, as well as decision making. Too often students are given problems that have "neat" answers. The following problem about a corn roast seems trivial at first but when it was assigned to grade 8 students it introduced the need to make decisions.

   "How much would it cost our class to go on a corn roast?"

   The students list the assumptions, and make decisions to arrive at the cost of going on the corn roast. I have assigned this problem to teacher groups and have had to referee agruments, as well as to impose a maximum time limit of fifteen minutes because the problem, although simple in appearance, can be complex in finding a "reasonable" solution. The solution will eventually involve answers to these questions:

   Where are we going? How do we get there? Do we take drinks? Do we need salt, butter, napkins, pepper, etc. etc.? These are but a few of the problems hidden in arriving at a "reasonable" price for a "reasonable" corn roast. What's your answer?

Problems suitable for different grade levels should be assigned to develop and strengthen this basic skill "Is my answer reasonable?"

5. I am sure many teachers already provide students with a variety of strategies for "starting" to solve a problem. I will never forget when I posed a problem to a class and got the immediate reply, "We haven't taken that yet .... sir!" We all remember too well the complete blanks left for some problems tackled by students writing exams. Some students freeze as soon as we say "word problem", and will sit and look but really do nothing to "start" the problem.
Teaching students to "sketch" the problem or make a diagram to help them solve the problem is a basic skill that needs to be emphasized continually. Many problems have been solved by "doodlers". Often a problem is solving by "doing" something rather than "waiting" for an inspiration. A blank page provides very little inspiration (but there are exceptions of course).

My list of recommendations to the BTB is not exhaustive or complete. Each of us have probably many other recommendations to add, but **we do** have to make recommendations.

There is much going on in curriculum development as well as studying how students learn. The process is painstakingly slow. Studies are being conducted on developing the problem-solving abilities of our students. All you need to do is name a problem in education today and there is probably a study going on somewhere investigating the problem and formulating a solution.

Curriculum development is progressing, but to a new teacher the educational scene must appear confusing. I once heard a description of curriculum development that seems to describe the present scene. The scene opens in the cockpit of a 747 (big bird). One pilot remarks to the other "We seem to be in a fog". The other pilot remarked, "Yes, but **we are** making headway". The evidence, as valid or invalid as we wish to make it, seems to indicate that there are problems in our curriculum, and different pressure groups are making it known that our present curriculum is not making headway. Students do not want to be short-changed, nor do we want to short-change them. They want to be able to understand "why we are taking this stuff"; and perhaps their reoccurring question might indicate a weakness in our curriculum. We are so over-preparing for the future that we are neglecting the present. We do have to have informed consumers. We do want them to use Math in their everyday living (accurately too!) We do want our students to be mathematically - literate when they read the newspaper, pay that charge account or calculate the percentage increase of their raise (a very conroversial issue). At the same time, we do not want to short-change the students by providing them with a curriculum that accommodates the present but ignores the future. They must have some "computer sense", some appreciation and working knowledge of calculators, as well as an appreciation of mathematics, "as a science". At the same time, we need to prepare them to be mathematically sound, and for this we would need to provide a foundation involving relations and functions, transformations, vectors - the list could go on and one.
To keep everybody happy is impossible, but we all have to participate in lifting the fog.

RECTILINEAR MOTION PROBLEMS: A GRAPHICAL APPROACH

J. A. Irvine, Mayfield Secondary School

In the March issue of the Gazette Dr. William Bruce presented a tabular method for handling rectilinear motion problems. Here is a graphical approach to the same problem which requires very little computation and requires only an ability to sketch basic polynomial graphs.

A particle moves horizontally on a straight line such that its displacement at any time t is given by

$$s(t) = t^3 - 3t^2 - 24t$$
$$= t(t^2 - 3t - 24)$$

from which s=0 for t=0, $\frac{3 + \sqrt{105}}{2}$, $\frac{3 - \sqrt{105}}{2}$

The velocity of the particle is given by

$$v(t) = s'(t)$$
$$= 3t^2 - 6t - 24$$
$$= 3(t + 2)(t - 4)$$

So v=0 for t=-2,4.

Similarly the acceleration is

$$a(t) = v'(t)$$
$$= 6t - 6$$

and a=0 for t=1.

Note that in this particular problem, s(t) is a cubic equation, v(t) is a quadratic, and a(t) is linear. Below are sketches of the three curves, using the same horizontal scale:

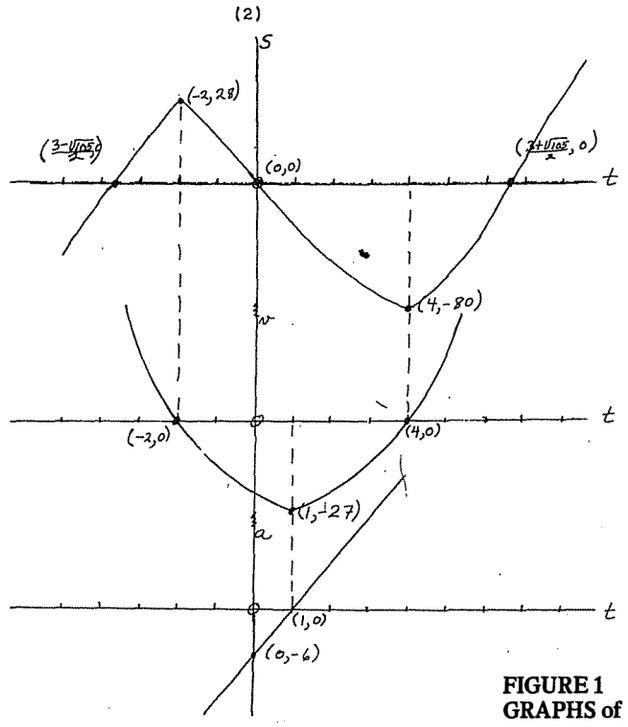


FIGURE 1
GRAPHS of
s vs t
v vs t
a vs t

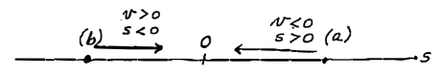


FIGURE 2
CONDITIONS FOR PARTICLE APPROACHING THE ORIGIN

Desired information concerning the motion of the particle can now be read directly from the graphs. Once the required conditions on s, v, and a for a particular state of the particle are identified, the corresponding time intervals are readily found from the graphs:

1. Direction of motion: The particle moves right for $v > 0$ $(t < -2, t > 4)$, or left for $v < 0$ $(-2 < t < 4)$.

2. The particle is approaching the origin if
either (a) $v < 0$ and $s > 0$ $(-2 < t < 0)$
or (b) $v > 0$ and $s < 0$ $\left(t < \frac{3 - \sqrt{105}}{2}, \ 4 < t < \frac{3 + \sqrt{105}}{2}\right)$

(see Figure 2)

3. Acceleration is directed towards the origin
if (a) $a < 0$ and $s > 0$ $\left(\frac{3 - \sqrt{105}}{2} < t < 0\right)$
or (b) $a > 0$ and $s < 0$ $\left(1 < t < \frac{3 + \sqrt{105}}{2}\right)$

4. Speed (magnitude of velocity) is
Increasing if (a) $v > 0$ and $a > 0$ $(t > 4)$
or (b) $v < 0$ and $a < 0$ $(-2 < t < 1)$
Decreasing if (a) $v > 0$ and $a < 0$ $(t < -2)$
or (b) $v < 0$ and $a > 0$ $(1 < t < 4)$

The motion of the particle is thus concisely described using only the three sketches.

This grachical approach, if done early in the term, also provides an excellent leadin to determination of maxima and minima, or, if done later, reinforces the same concepts.

The Great Debate is Over - At Least for the Time Being. Taken from **Western Metric News,** June 1976, published by Western Ontario Region, Ministry of Education, London. A - 4 -- NO MORE

The Canadian Government Specifications Board on recommendation of its Committee on Paper Sizes has just released the standards for paper sizes for this country. The A series has not been accepted.

In a recent decision by Standards Council of Canada, the standards for paper sizes in this country have been set. Although contrary to the policy of the Government of Ontario, the most common size will be P4, 21.5 cm by 28 cm., very close to 8½" by 11". It appears that the Committee of persons representing the manufacturers and users finally came to concensus or a majority vote situation. With so much to be said for the logic of the A series it appears that no plan for conversion would satisfy the manufacturers and consumers.

In our opinion it's too bad that the logical time to change will be missed. Is this another example of a small selfish nationalistic group who are ignoring the global implications, making decisions that will be a nuisance for many years to come? Or is this an example of poor leadership from the Metric Commission in that they put forward vague suggestions of a proposed change and let people get trapped into assuming the changes were right around the corner and inevitable?

Your reactions are solicited. (Send to R. Stevenson, Regional Metric Co-ordinator, Ministry of Education, London, Ontario.)

"THOUGHTS ON MATHEMATICS"

Steven Hatfield, Marshall University

This article originally appeared in the March 1976 issue of "Mathematics in Michigan", published by M.C.T.M., and is reprinted with the kind permission of the author and the editor, Dr. Horall Mourer.

Professor Kline in his book **Why Johnny Can't Add: The Failure of the New Math** has presented a good argument for his case. In his book he states that mathematics is not an isolated self-sufficient body of knowledge. According to him mathematics should be presented as an integrated part of a liberal education which is based on the nature of knowledge. Had mathematics not been this 'part of a whole', I feel it would have disappeared as a required course from our school curriculum much the way Latin has. To me mathematics is the queen of the sciences because it helps us to understand the physical world.

Kline believes every mathematical topic should be motivational in order to make the study of mathematics interesting to all levels of students. He does not believe math should be studied for the sake of mental discipline, but rather as a source of knowledge that can be used in the real world. After teaching for several years, I have often heard the student question the value of learning a particular topic. The student is more willing to learn the material when I am able to point to someone in his field of study who uses it. This type of motivation can also come from everyday situations that the student faces, such as figuring the true interest on a car payment. A study by Jack Price, "Discovery: Its Effect on Critical Thinking and Achievement in Mathematics," showed the discovery and transfer classes showed slight but not significant gains over the control classes in achievement in mathematics. They showed a positive attitude change toward mathematics, while the control group showed a negative change.

Professor Kline states that too much terminology and symbolism scare students. The definitions in the modern mathematics textbooks are based on precise language instead of first being developed by intuitive ideas. Even though I agree with this, I feel by the end of the seventh grade the skill in computation in basic arithmetic should be well established and the student should be introduced to more mathematical terminology and to some symbolism. This would result in a larger number of students having more success in algebra one and plane geometry.

He feels that the mathematics laboratory is a fine pedagogical aid that has been neglected. The mathematics laboratory could help the student discover a mathematics property by using concrete material in addition to helping the student discover the benefit of mathematics in the everyday world. I agree that the mathematics laboratory has not been used to its potential. Many educators see the value of the math lab as is evidenced by the supporting research. Robert Reys, in his book **The Mathematics Laboratory,** states few mathematics laboratories were in existence before 1960. However, by 1968, 2,000 laboratories were operating in elementary, junior and senior high school programs. Since then we have seen a rapid increase in the number of mathematics laboratories. In Marguerite Klaltz's article "The Mathematics Laboratory - A Meaningful Approach to Mathematics Instruction" she states that students, if given the time available for independent investigation and experimentation individually and in groups, discover mathematical facts and concepts through the manipulation of objects, through studious inquiry and testing of hypotheses, through the application of theory, and through reading and discussion. Blaine Worthen in "A Comparison of Discovery and Expository Sequencing in Elementary Mathematics Instruction" holds the following implications for educational practice.

1. To the extent that pupil ability to retain mathematical concepts and pupil ability to transfer hueristics of problem solving are valued outcomes of education, discovery techniques should be an integral part of the methodology used in presenting mathematics in the elementary classroom.
2. To the extent that immediate recall is a valued outcome of education, expository instruction should be continued as the typical instruction practice used in the elementary classroom.
3. The present study also suggests that pupil's ability to transfer concepts will likely be increased in proportion to the degree to which discovery techniques are used in the classroom.

Kline would like to see the genetic method used as much as possible. This method shows the student the development of a mathematical concept from the inductive stage to the deductive stage. By seeing how a particular concept evolves, the student is better able to see how a mathematician thinks. As a student during my high school and undergraduate days. I can remember only a few instances where the history of a mathematical concept was discussed. After taking a class in the history of mathematics, I had a better understanding of mathematics development. If would have been beneficial to have had some of this history introduced at an earlier stage. Hopefully every student preparing to teach mathematics should have a course in the history of mathematics or at least be required to study some of this material in a methods course.

Kline finds the training of many Ph.D students inadequate, especially those who end up teaching. Usually the Ph.D program is set-up to prepare the student for research in a narrow area of mathematics. Those who intend to teach in two and four year colleges should spend more time gaining general mathematics knowledge. He suggests the desirability of a new degree to better prepare the teacher, a degree that is not so overwhelmingly geared to research. Being a teacher in a smaller university which has hired approximately ten new PH.D.'s, I have heard many complaints about teaching general mathematics courses and about the lack of research time when teaching ten or twelve hours.

Kline is of the opinion that the training of the prospective high school teacher is also inadequate. He feels that science courses in which mathematics is used should be required. From my own experience, I have found that students enrolled in teachers college majoring in mathematics have more difficulty with applications in calculus than the other students who are required to take chemistry and physics. I believe they have this problem because they are not required to take science courses in which mathematics is used. These students on becoming teachers of mathematics will be handicapped by not being able to show where mathematics is used in science.

According to Kline topics in new mathematics such as Boolean algebra, abstract algebra, and certain topics in logic, should not be included in the high school mathematics curriculum. he feels these areas of mathematics are too far removed from any practical application. Even though I agree with him on the above areas, I feel that the study of real numbers by using axioms should begin in the eighth grade for the better students in order to help the student develop plane geometry by using the properties of the real numbers.

Kline's basic premise that mathematics should be applicable includes the content of high school mathematics programs. Many schools need to up-date their curriculums to include the use of computers and the study of statistics, areas that have become more of a way of life for today. For the same reason the study of the metric system should be increased in the elementary and secondary schools. There are many articles appearing on applied mathematics. As an example, "Teaching Applicable Mathematics" by E. A. Bender deals with restructuring calculus in college. While modern mathematics was the rage of the last decade, I feel applied mathematics will be the main thrust in this decade and hopefully for a long time.

NEWS RELEASE FROM SSMA

Career education and its implications for science and mathematics teachers are the theme of the special issue just published by the School Science and Mathematics Association. The recent emphasis in the schools on career education has stressed that teachers from elementary school through the university level are to have extensive input into the career education of their students. What input have mathematics and science teachers had in the implementation of career education in their schools? Will career education be a vehicle through which more students become interested in mathematics and science? The articles in this special issue provide some of the answers to these questions.

Single copies of this publication are available for $1.50 from the School Science and Mathematics Association, P. O. Box 1614, Indiana University of Pennsylvania, Indiana, PA 15701.

A LOOK AT SOME MATHEMATICS FILMS

by Richard Baron, Ontario Science Centre

High school mathematics teachers seldom if ever use film to supplement classroom activities. Unlike other subjects which lend themselves so well to the use of film to increase students awareness, mathematics on film is often boring and unimaginative. Even when we do browse through the film catalogues there is no way of knowing if the film we finally order will not be another poorly conceived effort.

The Ontario Science Centre makes extensive use of film as a method of communicating scientific information to the public. Feature length documentaries on topics of current scientific interest are shown in the main auditorium on a regular basis. Throughout the exhibit areas twenty mini-theatres, with seating capacities ranging from two to fifty, feature shorter science films.

As part of a long range program to enhance the Centre's math exhibits, I have been looking for films that would be suitable for public viewing - films that would help the average visitor appreciate mathematics as a living discipline. While previewing films for use in the Centre, it occurred to me that many teachers were probably unaware of the existence of some of the films I was watching. Furthermore, I could not recall ever having seen math films reviewed in the Ontario Mathematics Gazette. So I would like to take advantage of this unique opportunity to offer my opinion on some of the films I have seen and their suitability to the current Ontario curriculum. All films mentioned in this article are available from: Educational Film Distributors, 285 Lesmill Road, Don Mills, Ontario M3B 2V1.

Certainly the most exciting new mathematics film available today is a 25 minute colour production entitled SPACE FILLING CURVES. This film brilliantly illustrates the concept of a limit curve as specified by a sequence of approximation curves, and applies it to the construction of a space filling curve. The film poses some fascinating questions. How can a curve of infinite length be drawn to enclose a finite area (say the size of a postage stamp)? How can a curve, which by definition has zero width, be drawn to completely fill a finite area? Computer animation is used to answer these questions and to explore fascinating ideas about limits and infinity. The intriguing visual patterns with their motion and magnification serve to simplify and clarify the concepts involved, and indeed illustrate how math problems lead to works of art.

This film would serve as an excellent introduction to the work on limits in grade 13 Calculus. Because of its strong aesthetic appeal as well as its suitability to curriculum, you would probably be better off to purchase this film ($295) for board wide use than to count on renting it ($15) at the time you need it. This motion picture was produced as the first in a series called the Topology Films Project. Other titles in the series will be available for preview later in 1976.

Another interesting series is the COLLEGE GEOMETRY films that were developed by the College Geometry Project at the University of Minnesota. Detailed descriptions of the content of these twelve films are given in the Supplementary Catalog of the International Film Bureau, available from the above address. The films vary in length from 10 to 20 minutes, with rental fees of $10.00 and $12.50. Rather than review all twelve titles individually, I have grouped the films according to suitability for classroom use.

GEOMETRIC VECTORS. The subject matter of this film could be understood by grade 10 or 11. However, there is nothing in the content that a good teacher couldn't duplicate on a blackboard in a regular class period.

INVERSION, CENTRAL PERSPECTIVES, PROJECTIVE GENERATION OF CONICS. These three films do not directly relate to any of the topics on the senior math curriculum. They are probably too advanced even as enrichment for grade 13.

ISOMETRIES, CENTRAL SIMILARITIES, ORTHOGONAL PROJECTIONS, CAROMS. These films could serve as a source of enrichment material for grade 13 classes. Students who have a basic understanding of rotations, reflections, translations, similarities and conics will be led to some interesting and unexpected results regarding the inter-relation of these topics.

EQUIDECOMPOSABLE POLYGONS, CURVES OF CONTANT BREADTH, DIHEDRAL KALEIODOSCOPES, SYMMETRIES OF THE CUBE.

While the last four films do not relate directly to curriculum content at any grade level, they would be suitable as sources of motivation and enrichment for superior students in any grade from 9 to 13. The language gets a bit heavy at times - as when the narrator refers to the "cyclic subgroups of the second dihedral group". However, this shortcoming is offset by the effective use of live action and animation to produce some attractive and unusual visual effects. Good math students will be sufficiently motivated to explore the topics in more depth. These films will find their best audience in enrichment classes, math club meetings, and groups of students looking for math project topics.

My final reference is to a film which has been running continuously at the Centre's math mini-theatre for the past six years. "Mathematical Peep Show" was created by Charles Eames, a renowned architect and designer, specifically for science museums and world's fairs. The film is divided into five segments, each illustrating a concept of sophisticated mathematics in a clear simplified way. Clever animation and photography are used to convey basic principles of topology, symmetry, functions, exponents and Eratosthenes' measurement of the earth's circumference. This 12 minute colour film, suitable for general viewing at all grade levels, is available from the Visual Education Centre, 115 Berkely St., Toronto.

BOOK REVIEWS

GEOMETRY: AN INVESTIGATIVE APPROACH
PHARES, G. O'DAFFER AND STANLEY R. CLEMENS
Reading, Mass.: Addision-Wesley, 1976.
455 pp. Bibliography. Price not set. Probably about $10.00. 0-201-05420-5

Teachers in the junior and intermediate grades who are charged with the task of revising their geometry curricula in response to new guidelines from the Ministry of Education in this area will find this attractive handbook of great assistance. Its eleven chapters cover topics such as 'Motions in the Physical World' (good material on Slides, Turns and Flips), 'Magnification and Similarity' and 'Measuring with Metric Units'.

The book isprofusely illustrated with both diagrams and photographs. Well-constructed but fairly standard sets of exercises are supplemented by more open-ended investigation problems and suggestions of pedagogical activities for teachers. Throughout the book an inductive approach to geometry is stressed. Manipulative aids such as miras and geoboards are used extensively. This well planned and successfully executed publication should be available to teachers in every elementary and junior high school.

W.H.

STATLAB: AN EMPIRICAL INTRODUCTION TO STATISTICS,
J. L. HODGES, DAVID KRECH, RICHARD S. CRUTCHFIELD.
New York: McGraw-Hill, 1975.
378 pp. Paper. $13.00. 0-07-029134-9

This book was used in my school as the basis for a grade 13 course in Statistics. It is divided into four sections, each one expanding on the topics of its predecessors. By the end of the book, the student has been exposed to a variety of statistical techniques. The basic ideas communicated are:

i) How to group and graph samples of data
ii) How to derive population parameters from sample statistics
iii) How to attach accuracy to measurements and predictions
iv) Methods of testing the relationship of variables.

The complete package consists of a textbook which contains a census giving 32 sets of information on a population of 1296 families; a pair of dice for drawing a random sample from this population; and a set of worksheets and inquiries.

Most students found the book useful and interesting--they all enjoyed generating their own samples using the dice and derived an intuitive feeling for the subject. Some of their comments follow:

"Statlab is the most interesting and absorbing 'teach-yourself' math text I have read. The language is earthy, yet not condescending. Its practical tone parallels and accentuates the practical nature of statistics."

"Statlab is a well organised book ... at the back is a glossary of symbols and formulae."

One of the few negative comments which occurred had to do with the fact that the textbook could not be resold, once the worksheets and inquiries had been completed. Also at a cost of $13. it seems expensive for a paperback.

D. M. Handley
Belleville Collegiate

MEASUREMENTS IN SCHOOL MATHEMATICS
1976 Yearbook of the National Council of Teachers of Mathematics.
DOYAL NELSON, Ed.
Reston, Va.: NCTM, 1976. 244 pp. $7.50. ($5.50 to NCTM members)

This is the first of the "new breed" of NCTM Yearbooks. These "new" yearbooks are to be published annually on a central theme of current interest to NCTM members. Rather than commissioning articles, as has been the case for past yearbooks, the editors now select articles from papers submitted in open competition. Each yearbook is also to include "a nonthematic essay whose length, quality, and timely ideas warrant broad dissemination".

To produce a single publication which will interest both the kindergarten teacher and the college instructor is, as OAME editors know only too well, a very challenging task. It would seem that Dr. Nelson has succeeded fairly well in meeting this challenge. In the four parts of this yearbook there are 16 chapters grouped under the four general headings 'Learning and Teaching Measurement', 'Selected Measurement Activities', 'Selected Measurement Resources for Teaching' and 'Nonthematic Essay'.

The chapters are short, classroom-oriented and generally well-illustrated and quite readable. The difficulty range in the activities section goes from "Using the Carpenter's Square" to "Measuring the Velocity of a Projectile". The two chapters in the Resources section on "Manipulative Devices" and "A Metric Bibliography" would seem to be quite adequate. Canadian publications and distributors are, however, given very little coverage. Only one Canadian distributor is listed and such excellent books as **The Metric Book of Amusing Things To Do** by Elisabeth Hallamore (Toronto: Greey de Pencier, 1974) and **Exploring Metric Measure** by John Bates, Terry Clifford and Paul Park (Toronto: McGraw-Hill Ryerson, 1973) are not listed although some rather obscure American publications are recommended. The yearbook could not therefore, without being supplemented, stand as a comprehensive sourcebook on measurement in the Canadian context. To say that the nonthematic essay on "Low-Stress Algorithms" is a paper "whose length, quality and timely ideas warrant broad dissemination" is, in this review's opinion, to make somewhat grandiose claims for a rather modest effort.

The format and size of the new series seem much less intimidating than in the old yearbooks. The lower price and empahsis on teacher useability also seem to be moves in the right direction. With this publication NCTM seems to have a solid, if not spectacular, start to its new series.

W.H.

BASIC SKILLS, HIGH SCHOOL FINAL EXAMINATIONS AND UNIVERSITY PLACEMENT TESTS: RESULTS OF A SURVEY COMMISSIONED BY THE COMMITTEE OF CHAIRMEN OF ONTARIO UNIVERSITY MATHEMATICS DEPARTMENTS.
ALAN J. GOLD
Windsor: Department of Mathematics, University of Windsor, 1975
45 pp. Mimeo. $1.00

In March of 1975 Dr. A. J. Gold of the University of Windsor was invited by the Committee of Chairmen of Ontario University Mathematics Departments to "survey the present state of high school-university interaction on the question of basic mathematical skills and the extent to which regional high school mathematics examinations are either in effect or in the planning stage across the province". (p. 2.) Dr. Gold has done a very competent job of summarizing the findings of his survey in this report. The results, if not surprising (they are in many ways quite similar to the findings of the OAME Senior Division Curriculum Review Committee as reported in this journal in September 1975, 14-18) are of considerble interest. This is due, at least in part, to the scale of the survey which solicited responses from 645 secondary school mathematics department heads, 23 university departments, 83 boards of education and 9 ministry consultants. Responses were received from 409 of these 760 parties.

As frequently happens in this type of survey some general trends were observed with quite strong minority opinions being voiced in opposition to these trends. The general consensus among those responding to the survey was:

"(i) More interaction on a continuing basis is needed between secondary and post-secondary teachers of mathematics on a variety of topics, including that of basic skills."

"(ii) Most teachers would like to see some kind of common yardstick by means of which schools and their students could be compared. There does not seem to be any strong consensus on the form this should take, nor how it would affect the student's grade."

"(iii) Most teachers would welcome some testing procedure to place students entering into university in courses suitable to their mathematical background. Many would not have this used as the sole criterion for admission to programmes and/or courses. Most High School teachers wish to be kept informed, or be consulted, about what universities do in this area." (pp. 36-37)

Copies of this report are available from Dr. Gold (for five or more copies sent to one address the price for each report is $0.90; prepaid copies will be sent post free) at the University of windsor. Educators concerned about the High School-University interface will find it of considerable interest.

W.H.

MATHEMATICAL SCIENCES IN CANADA
KLAUS P. BELTZNER, A. JOHN COLEMAN, GORDON D. EDWARDS
Ottawa: Science Council of Canada, 1976.
333 pp. Paper. $6.50 0-660-00388-0
Available at Canadian Government Bookstores or by mail from Printing and Publishing, Supply and Services Canada, Ottawa K1A OS9.

"**MATHEMATICS CANADA** is not another bureaucracy. It is a study which will attempt to bring into sharp focus the present state and role of the Mathematical Sciences in Canada, and recommend policies for the future". (p 37). Such was the hope of the Science Council of Canada in 1973 when the Mathematics Study was first being organized. With the publication, almost three years later, of the results of the study it seems fair to say that these aims have been successfully met. Dr. John Coleman and his colleagues have produced a comprehensive document which raises a number of crucial questions about the state of the discipline in this nation, and insofar as the Canadian case is typical, to other Western nations as well. The path recommended is a difficult one and early reactions to the study do not make one optimistic about the chances of the mathematical community making a united effort to respond to the problems outlined. Although they appear to be heretical to some, the arguments Coleman makes in support of his views are compelling and if it achieves nothing else the Math Study has already done something quite

significant in arousing mathematicians in this country to discuss and debate the implications and future direction of their subject.

The Math Study was sponsored by six mathematical organizations on behalf of the Science Council of Canada. The six societies were the Canadian mathematical Congress (CMC), the Canadian Operational Research Society (CORS), the Canadian Institute of Actuaries (CIA), the Canadian Information Processing Society (CIPS), the Statistical Science Association of Canada (SSAC) and the American Statistical Association, District 7 (ASA). Dr. Coleman, Head of the Department of Mathematics at Queen's University, and at that time President of the CMC was chosen as Study Director. He was assisted by Mr. Klaus Beltzner and Dr. Gordon Edwards. The objectives of the study were:

"1. To establish what kinds of mathematics are currently used in Canada, and to what extent. Also to suggest areas of mathematics which are not being studied or used to the extent which would be desirable.
2. To describe and evaluate various possible types of research in the mathematical sciences.
3. To estimate present and future manpower supplies and needs at various levels of mathematical competence.
4. To examine current objectives and methods of training in mathematical activities and to suggest possible improvements at all levels.
5. To recommend methods by which significant real problem areas amenable to mathematical treatment can be identified and publicized on a systematic and continuing basis." (pp. 45-46).

In an attempt to meet these ambitious aims the staff of the study constructed a detailed questionnaire which was sent to some 3000 mathematics graduates. Visits were made to 24 university campuses and eight one-day seminars involving users of mathematics were held. The proceedings of three of these seminars were published, unfortunately in quite a limited edition, under the title **Mathematics in Today's World.** A preliminary edition of the results of these and other activities was circulated in September 1975. The final edition, which does not differ significantly from the draft, appeared in July, 1976.

The report consists of eleven chapters and a large number of appendices. The first three chapters deal with the procedural aspects of the study, the "Mathematical Ecosystem" and "Mathematics in Business, Industry and Government". Chapters four through six deal with the teaching of mathematics in schools, community colleges and universities respectively. In chapters seven and eight "Research" and "The Reward System" are examined. Proposals for "A Canadian Institute of Applied Mathematics" and a "Canadian Council for the Mathematical Sciences" constitute chapters nine and ten. chapter eleven summarizes the preceeding material.

Among the highlights of the study are the following:

Industry and government do know how to make effective use of the Mathematical Sciences. Consequently, there is much unnecessary error, inefficiency and waste (Chapter III).

The ministers of education should take energetic steps to improve the teaching of mathematics in elementary and secondary schools; liaison between university professors of mathematics and school teachers should be enhanced; the current trend in some of the provincial departments of education to turn everyone into generalized administrators should be quickly reversed so that subject specialists within the departments may again play a decisive role as consultants in the schools and in shaping curriculum guidelines (Chapter IV).

Canada needs more, not less, PhD's in mathematics but of a radically new type. The curriculum and method of training specialists in the mathematical Sciences need to be rethought (Chapter VI). (p. 33).

As is the case in his consideration of the applications of mathematics in business and industry and of the teaching of mathematics at university, Coleman finds little to be complacent about in the teaching of mathematics in school. Unlike so many of his less enlightened colleagues, however, he does not merely castigate the classroom teacher but goes on to show that responsibility for the deficiencies of mathematics education in our school system must be shared by all members of the mathematical community including teacher educators, ministry of education officials and

university professors of mathematics. Coleman's thinking about the development of the discipline leads him to "propose as the chief and overriding aim for the teaching of mathematics in the schools **the widest possible dissemination of an understanding of what mathematics is and what it is not.** As far as we know, this has never been adopted as a conscious aim of any school system. When it is achieved, the average citizen will understand that mathematics is a) **a way of thinking** which provides a powerful tool for analyzing subtle and unobvious aspects of our experience, and b) a **cultural resource** which can ad interest and enjoyment to life. Further, c) it will be apparent that the symbolism of mathematics - algebraic and graphical - constitute an important **language** which is essential for communication of ideas and for the formulation of societal goals". (p. 117)

An axiom of Coleman's about the nature of mathematics which underlies much of his writing in the study is made explicit in the fourth chapter.

"However, one fact of overriding importance must be in the forefront of everyones' efforts. **To mathematize is to joy!** It is to exult in one of the powers distinctive of our humanity. It is to employ explicitly structures which are implicit in the higher levels of our brain. To convey a tone of confidence and satisfaction in doing mathematics should predominate over obsession about "covering" a predetermined curriculum. This should be true in teaching students. But at this juncture, it should be especially true in teaching teachers and prospective teachers. There is no other way of dissipating the widespread anxiety with which mathematics is regarded." (p. 126)

The reaction of the press in Canada to the study has, in general, been distressing and supportive of the thesis that intelligent and responsible journalism is a rare phenomenon in our country at this time. Given a well written report (despite a few slips such as "less PhD's") filled with recommendations of substance most reporters seized the peripheral, battle-scarred but attention-grabbing issue of "New Maths" to serve as the main focus of their article. This misplaced focus would have been bad enough by itself, but frequently the interpretation in articles was barely consistent with the position in the report. That this is perhaps not an unexpected response can be concluded from the study itself. "A common method of evading the real issues is to indulge in vituperation of the 'New Maths' and the monstrous abstract mathematicians who foisted these dreadful irrelevancies upon us! (p. 115)

It is to be hoped that the reaction of members of the mathematical community will be more reasoned and responsible than that of the media. To his preface Coleman states, "My ideas may be wrong. But I ask that they be considered calmly." (p. 30)

"The **main** concern of this study is not the creation of new mathematics. Rather, its main thrust is towards the promotion of better liason between mathematicians and everybody else. Few of its recommendations will be easy to implement. They will require good will on the part of business, government, pre-university teachers as well as mathematicians themselves. Only the future will reveal whether this will be forthcoming." (p. 30)

In his foreword to the study Dr. McTaggart-Cowan, the distinguished scientist and former Executive Director of the Science Council, calls it "a charming essay of great importance" (p. 27). to disagree with the first part of Dr. McTaggart-Cowan's contention seems, to this reviewer, to be a question of taste; to disagree with the second a question of intelligence. If we accept Francis Bacon's dictum that, "some books are to be tasted, others to be swallowed, and some few to be chewed and digested", then this publication should surely fall into the last category. May the process of chewing and digesting be a profitable one.

W.H.

DIVERSIONS

by T. G. Friday

The provision of diversions is much more easily accomplished if one has a diversity of material to provide. To put this in simpler terms, Help!!!

This section of the Gazette needs your contributions. Please send in your favourite mathematical party pieces, and your solutions to our problems. Even better send in your students offerings, solutions, essays on mathematics and mathematics projects. Even if they do not achieve solutions to the problems, their methods of approach would be most interesting.

DERIVATIVES

The enemy stared at her; black, grim-faced, and bold. It came alive, lifted itself from the page and marched through her head into the depths of her brain where it was poked at, squeezed and squished. It did not emerge; rather, it formed long equations of y's and x's and h's limiting to zero. The eyes turned inward in an exasperating attempt to understand, but the enemy remained vague and indefinite. The eyes looked harder until they almost pierced the blackened reaches of the brain's invaders, but finally, they, too, surrendered and slowly twisted back into their positions. The enemy had accumulated by now and she felt the floating mass reach into the inner crevasses of the greyish jellified organ fixed within her skull. Panic seized her; she realized her resistance would soon collapse. More and more of them were entering her head; they began to laugh inside of her -- laughing at her! There were different kinds now; she could recognize a few $\sqrt{2}$'s and Δx -- she even thought she saw--it couldn't be--yes, it was--a $\frac{dy}{dx}$! The cavern walls pushed harder but the enemy would not succumb. Grey slime slid through the indecipherable congestion of figures but was unable to smother the vipers. Pain etched into her brain as the enemy drove into the walls of the pulsating matter. In the next second, she experienced total blankness as her resistance tumbled down. She had expected that one day this would happen; it came as no surprise; she had even found it comforting to know the end was near. As the eyelids slid up, a fuzzy, disturbing sound gradually became clearer, then faded away. "sure you understand what you're doing? This work is important and I want to be sure you have it clear in mind. You do, eh? Well, study it some more just to be sure..."

Only after all the bells had rung and all the streets had been crossed, had she breathed a sigh of relief. For, there it was, The Book, sitting there waiting for her. And as she pursued each word, the enemy sickened and withered and died. It was left behind while she explored her imaginary paradise which became more real with the turning of each page.

Connie Armitage
Student North Lambton Secondary School

The following offering is an excerpt from the Eureka of May, 1976. The Eureka is an excellent bilingual publication produced by C.O.M.A. a chapter of O.A.M.E., and is devoted to the presentation and discussion of interesting mathematical problems.

Dear editor:

Your superb handling of the Steiner-Lehmus Theorem in the February 1976 issue of EUREKA (1976: 19-24) provides the interested reader with such an abundance of references that you can just about claim to have exhausted everything that can be said on the subject! That is, everything except one little item! This was discovered quite by accident.

In 1968 while browsing in Gibert's on Boulevard St-Michel in Paris, I came across a fascinating old geometry book. It was not dated, but judging by its white-on-black diagrams I estimated it to be about a hundred years old. As I flipped through the pages, my eye was attracted by a familiar diagram-- one associated with such an unbelievably simple proof of the Steiner-Lehmus Theorem that my pulse raced with excitement. What a rare gem! At last--a proof to annihilate all rival proofs! The world-famous problem that had been responsible for universal frustration was here neatly and incredibly disposed of in two brief sentences!

I hurriedly paid for my newly-found treasure and could hardly wait to get back to my hotel to dash off letters to friends across the Atlantic. Back at my room, I ran to the desk, opened my precious acquisition and re-read the proof. Slowly my jaw dropped with consternation; then suddenly, to the utter amazement of my wife, I burst out in hysterical laughter.

Enclosed is a photocopy of this astounding proof that I had thought was too good to be true but which turned out not true enough to be good. Later, in a private communication, Professor Coxeter aptly characterized the proof as "charmingly stupid."

Can your readers tell why--and, it is hoped, more quickly than I could?

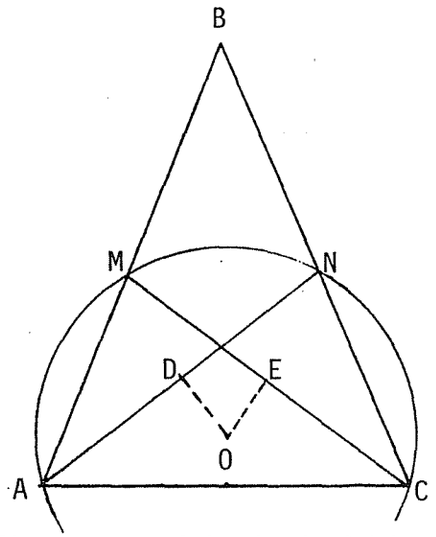
LEON BANKOFF,
Los Angeles, California.

Editor's comment.
Bankoff found his serendipitous "proof" on p. 314 in **Exercices de Géométrie,** by M. Ph. André, 14th edition, Librairie classique de F.-E. André-Guedon, E. André fils, Successeur (no date).

To find such a "proof" in a presumably reputable treatise on geometry is remarkable enough, but to find it in a *fourteenth* edition (and how many later ones?) is simply astounding. The similarity in names between author and publisher leads one to speculate whether the publication of this work might not have been more an act of filial piety rather than a desire to aid the cause of mathematical education. (Well, there's nothing wrong with nepotism, as long as it's kept in the family.)

Here, in its entirety, is the "proof" found by Bankoff.

What is wrong with the "proof"?



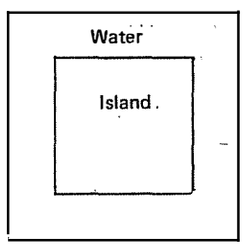
If in a triangle two angle bisectors are equal, then the triangle is isoscles.
At the midpoints of the angle bisectors, I erect two perpendiculars which meet in O; with O as center and AO as radius, I describe a circle which will evidently pass through the points A, M, N, C.

Now the angles MAN, MCN are equal since the measure of each is arc $\frac{MN}{2}$; hence BAC = ACB, and triangle ABC is isosceles.

PROBLEMS

15-1-1 Up to grade 9

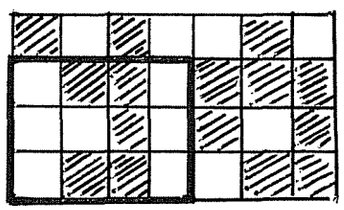
A homeworkless student can find sanctuary from his irate teacher if he crosses to a square island surrounded by a 4 metre wide moat. He has only two 3 metre long planks to cross with. Can he escape, and if so, how?



15-1-2 Up to grade 12

An integral lattice is a Cartesian co-ordinate graph in which the two elements of the ordered pairs are integers. What is the greatest number of non-collinear points you can select such that the mid-point of any line joining any pair of selected points is not a lattice point.

15-1-3 From the 1976 USA Olympiad paper



(a) Suppose that each square of a 4 x 7 chessboard, as shown above is coloured either black or white. Prove that with **any** such colouring, the board must contain a rectangle (formed by the horizontal and vertical lines of the board), such as the one outlined in the figure, whose four distinct corner squares are all the same colour.

(b) Exhibit a black-white colouring of a 4 x 6 board in which the four corner squares of every such rectangle as described above, are not the same colour.

PUBLICATIONS AVAILABLE

TO O.A.M.E. MEMBERS

	Regular Price	O.A.M.E. Member Price
Handbook of Trigonometry (1975)	$2.00	N/C
A Preparedness Test for Grade Nine Students (1973)	2.00	N/C
Sample Grade 13 Examinations	1.00	N/C
Grade 11 (Level 5) Sample Examinations	1.00	N/C
Reprints of 1974 & 1975 Gauss Contest Papers (Elementary)	.10	N/C

To receive these please contact:

Mr. R. G. Scoins,
Faculty of Mathematics,
University of Waterloo,
WATERLOO, Ontario.

Boxes, Squares and Other Things (Elementary)		
A leather guide for a unit in informal geometry	1.80	N/C

Available from:

Mr. M. MacGregor,
Secretary-Treasurer O.A.M.E.

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FOR

MATHEMATICS EDUCATION

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CHAPTER ITEMS

SWOAME Annual Conference, Saturday, November 6, 1976 at University of Windsor. Contact Dr. Alan Gold, Department of Mathematics.
NOMA Annual Conference, North Bay, October 2, 1976.
COMA continues to publish EUREKA. Its good! Write Fred Maskell

ONTARIO ASSOCIATION for MATHEMATICS EDUCATION

REPORT OF THE PRESIDENT

Report of the President of OAME, Joan Routledge
to the Annual Meeting, St. Catharines, May 15, 1976

This year has been an active and productive one for OAME in spite of pressures on the educational community and a lengthy postal strike at prime promotional time. The council met three times -June 14, 1975, November 8, 1975 and April 10, 1976. Some of the major actions this past year were as follows:

The Past President, the President-Elect, and the President met with Mr. Crossley and Garth Kaye of the Ministry of Education to discuss the Brief on the Qualifications and Preparation of Teachers of Mathematics Grades K-8. With the approval of Council, The Deans of the Faculties of Education at Ontario Universities were contacted, and their reactions to this brief were sought. To date, a few replies have been received and several more are forthcoming. Council will consider these reactions next year.

Under the joint chairmanship of Morley MacGregor and Bruce Jay, another successful leadership conference for 96 elementary teachers of mathematics (K-8) was held in Niagara Falls October 22-25, 1975. Plans are underway for a similar conference, October 20-23, 1976 at the Ascot Inn in Toronto.

OAME will present, annually, a Father Faught memorial award in the form of a set of books and a wall plaque to the Ontario School placing highest in the Junior Mathematics Contest.

An OAME-NCTM Joint Membership Drive was conducted. promotional materials were mailed to all elementary schools in Ontario. NCTM gave a large grant of $700 for this drive.

Grants - $400 to COMA to allow them to continue to publish "Eureka" while they looked for alternative ways and means.
- $305 to NOMA to assist in their October 1976 conference.

An offer to host the NCTM 1982 Annual Meeting in Toronto was sent to the Meetings Committee of NCTM. This proposal will be considered at the committee's winter meeting in 1977.

COMMITTEES

The effectiveness of OAME depends largely upon the work done by its committees.

EXECUTIVE COMMITTEE

The Executive Committee met six times during the year to act for the Council and members of OAME. All such actions were reported to the Council at the regular Council meetings.

PROJECT COMMITTEE

The primary task of this committee is to consider projects presented to them. In the light of priorities and moneys available, the committee decides upon suitable projects and establishes a sub-committee to initiate them.

The Trading Post project was very successful at the OAME Annual Conference in London last year. It is being repeated this year in St. Catharines.

The OAME Resource Project "Scholarship and Research Funding" sub-committee is meeting, and will be reporting next year.

The Senior Cyclical Review Committee has presented an interim report to Council for discussion. The final report is expected next year.

MEMBERSHIP AND PUBLICITY COMMITTEE

Through a Joint Membership Drive with NCTM, all elementary schools in Ontario were contacted by mail in an attempt to make elementary teachers aware of the advantages of membership in OAME and NCTM. A postage free return membership application envelope was enclosed. This committee has prepared an OAME membership Certificate suitable for framing and hanging. This certificate will be sent to every 1976-1977 OAME member. Seals to affix to this certificate will be sent yearly to each member at the time of renewal of membership.

PUBLICATIONS COMMITTEE

This committee has worked on terms of reference for the editors of the Gazette and the Abacus. Their report was adopted by the Council on April 10. The two editors - Arn Harris of the Gazette, and Andy Czempinski of the Abacus - have completed the first year of a two-year term.

MEETINGS AND MANPOWER

This committee's primary task is to obtain locations for the annual Meetings. The 1977 meeting will be in Ottawa, May 13-15. Duff Butterill of the Ottawa Board of Education and Bruce topping of

Carleton County Board of Education are co-chairmen. Negotiations are underway for the 1978 meeting. The committee also offers assistance in obtaining speakers for chapter meetings.

CHAPTERS COMMITTEE

SWOAME held a meeting on November 1, 1975 in Sarnia. They are planning one for November 6, 1976, in Windsor.

NOMA held a meeting in Sudbury October 25, 1975, and are planning one in North Bay for October 2, 1976.

GVMA had several meetings during the year. More publications are being considered.

The chapter in eastern Ontario has adopted the name Eastern Ontario Mathematics Association (EOMA) Meetings are planned for next year.

Quinte-St. Lawrence held two meetings - one in December and one in April.

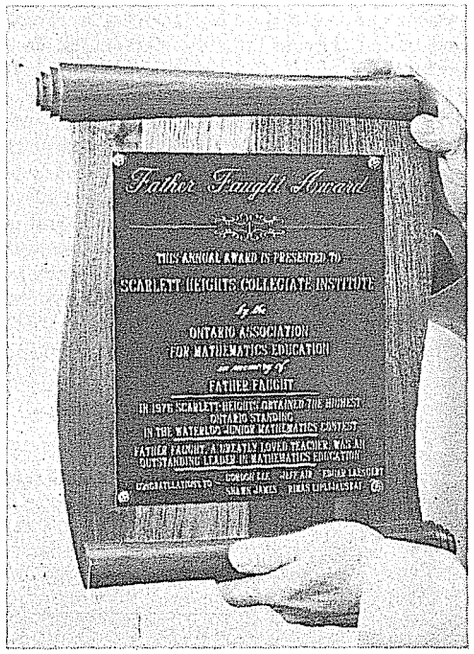
COMA had three meetings this year. The members are deeply involved in the planning for the 1977 Ottawa Annual Meeting of OAME. They are continuing to publish "Eureka".

NOMINATIONS COMMITTEE

The Nominations Committee is responsible for the slate of offices that will be presented and voted on at this meeting.

May I take this opportunity to thanks the members of the Executive, The Council, and all the other OAME members for their support during my years as President. A special note of thanks is owing to Morley and Mona MacGregor, our very able Secretary-Treasurer and his wife. Their hard work and devotion to OAME is immeasurable.

You have made an excellent choice for your new president. I am sure that OAME **will have** another excellent year under the leadership of Tom Griffiths, our capable president for 1976-1977.



Picture of The Father Faught Award, which is to be presented annually by the OAME

INTERNATIONAL MATHEMATICS EDUCATION

KARLSRUHE CONFERENCE (ICME): August 16-21, 1976

Perhaps one of our readers would like to write a description of the conference.

N.C.T.M. NAME-OF-SITE MEETING - WINNIPEG

Plan to attend this outstanding event, October 21-23, 1976. Among others, one Ontario speaker will be Dave Campbell, Faculty of Education, U.W.O.

N.C.T.M. NIAGARA FALLS, NEW YORK MEETING

The Association of Mathematics Teachers of New York State invites you to Niagara Falls October 7-9, 1976. The program includes E. J. Barbeau, U. of T., Charles Brainerd, U. of Alberta (cardinal vs. ordinal, Scientific American, March 1973) Anna Maria Pataky, North York, and many other outstanding American and Canadian teachers.

NOTICE TO MEMBERS

The teachers of Mathematics had in the early years of Education in Ontario an opportunity to be members of the Ontario Association of Teachers of Mathematics and Physics and later the Ontario Association of Teachers of Mathematics and more recently the Ontario Association for Mathematics Education.

Anyone purchasing a membership in OAME after January 1, 1976 received a membership certificate. It is now the wish of the OAME that all charter members of the above named organizations should receive a seal to be placed on this certificate. These seals will be available after January 1, 1977 and will be mailed out with the 1977-78 receipts. In order that the office may have an estimate of the number of seals required we ask that members cut off the lower portion of this page and send to the Sarnia office at their earliest convenience. Since records are not complete for the association we are asking members to request seals on the honour system. A member is eligible for an OATMP seal, if he has been a continuous member of the three organizations beginning at some date prior to 1965. We recognize this is not within the context of a "charter member" but we wish to give recognition to those teachers who were members of OATMP. A member is eligible for the OATM seal if he has been a continuous member of OATM and OAME since 1965. For these two seals the member will have joined OEA and indicated his or her interest section as the Mathematics Section or her or she will have joined OEA and attended the Mathematics Meeting of OEA or an OATMP meeting or an OATM meeting. These were usually held during the Easter Holidays. A memer is eligible for an OAME seal if he has been a continuous member since May 1973. These members have joined directly and not through OEA.

I desire an

OATMP seal _____ OATM seal _____ OAME seal _____

check on one line - one seal per member

_____

Signature

Last date of return Dec. 1/76

SPECIAL NOTICE TO MEMBERS OF OAME

In past years, the policy of the Association had been to extend privileges of membership and the right to receive publications to a member whose membership expired on August 31 of any calendar year to December 31st of that year without a renewal payment. This entitled a member to the September and December issues of the Gazette and the September and November issues of the Abacus.

BEGINNING September 1, 1976 privileges of membership and publications will cease on October 31st if a renewal payment is not made. A self addressed remittance envelope was sent to members in late August for your convenience or may mail directly to OAME office, 247 Bright St., Sarnia, Ont. N7T 4E9

Coming Events	Date	Inquiries
Elementary Leadership Seminar	Oct. 20-23/76	Sarnia Office
NOAMA Conference-North Bay	Oct. 2/76	R. Tuck
SWOAME Conference - Windsor	Nov. 6/76	D. Skoyles

For information about these conferences write R. Tuck, or D. Skoyles

THE ABACUS
247 Bright St.,
Sarnia, Ont.
N7T 4E9

SUBSCRIPTION RATES

$5.00 FOR ONE YEAR'S SUBSCRIPTION
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O.A.M.E. members receive the Gazette as an adjunct to their membership.

Invoices will be sent 30 days prior to expiry of a subscription, and 60 days is allowed for payment.

Orders for subscription and single copies should be sent to the Secretary-Treasurer.

M.B.MacGregor

REVIEW OF SENIOR DIVISION CURRICULUM

The Council of OAME invites responses to the report of the Committee published in the September, 1975 Gazette. They should be directed to Neil Williamson